## PREREQUISITES

## ( 2-D GEOMETRY )

## DISTANCE BETWEEN TWO POINTS

(i) The distance between two points $\mathrm{A}\left(\mathrm{x}_{1}, \mathrm{y}_{1}\right)$ and $\mathrm{B}\left(\mathrm{x}_{2}, \mathrm{y}_{2}\right)$
$A B($ or $B A)=\sqrt{\left(x_{1}-x_{2}\right)^{2}+\left(y_{1}-y_{2}\right)^{2}}$
(ii)The distance from origin O to the point $\mathrm{A}\left(\mathrm{x}_{1}, \mathrm{y}_{1}\right)$ is $\mathrm{OA}=\sqrt{x_{1}^{2}+y_{1}^{2}}$
(iii) The distance between two points $\mathrm{A}\left(\mathrm{x}_{1}, 0\right)$ and $\mathrm{B}\left(\mathrm{x}_{2}, 0\right)$ lying on the X - axis is $A B=\sqrt{\left(x_{1}-x_{2}\right)^{2}+(0-0)^{2}}=\sqrt{\left(x_{1}-x_{2}\right)^{2}}=\left|x_{1}-x_{2}\right|$
(iv) The distance between two points $\mathrm{C}\left(0, \mathrm{y}_{1}\right)$ and $\mathrm{D}\left(0, \mathrm{y}_{2}\right)$ lying on the Y -axis is $C D=\left|y_{1}-y_{2}\right|$

## SECTION FORMULA

(i)The point P which divides the line segment joining the points $\mathrm{A}\left(\mathrm{x}_{1}, \mathrm{y}_{1}\right)$, $B\left(x_{2}, y_{2}\right)$ in the ratio $m: n$ internally is given by
$\mathrm{P}=\frac{\mathrm{c}}{\mathrm{c}} \frac{m x_{2}+n x_{1}}{m+n}, \frac{m y_{2}+n y_{1}}{m+n} \underset{\dot{\bar{\circ}}}{\stackrel{\ddot{\dot{\sigma}}}{ }}\left(\begin{array}{ll}m+n^{1} & 0\end{array}\right)$
(ii)If P divides in the ratio m:n externally then $\mathrm{P}=\frac{\oiiint_{8}-n x_{2}-n x_{1}}{m-n}, \frac{m y_{2}-n y_{1}}{m-n} \frac{\ddot{\partial}}{\dot{\dot{\varphi}}} \quad\left(m^{1} n\right)$

Note: If the ratio $\mathrm{m}: \mathrm{n}$ is positive then P divides internally and if the ratio is negative P divides externally.

## MID POINT

The mid point of the line segment joining $\mathrm{A}\left(\mathrm{x}_{1}, \mathrm{y}_{1}\right)$ and $\mathrm{B}\left(\mathrm{x}_{2}, \mathrm{y}_{2}\right)$ is $\left(\frac{x_{1}+x_{2}}{2}, \frac{y_{1}+y_{2}}{2}\right)$

## POINTS OF TRISECTION

The points which divide the line segment $\overline{A B}$ in the ratio $1: 2$ and $2: 1$ (internally) are called the points of trisection of $\overline{A B}$.

## AREA OF A TRIANGLE

The area of the triangle formed by the points $\mathrm{A}\left(\mathrm{x}_{1}, \mathrm{y}_{1}\right), \mathrm{B}\left(\mathrm{x}_{2}, \mathrm{y}_{2}\right), \mathrm{C}\left(\mathrm{x}_{3}, \mathrm{y}_{3}\right)$ is

$$
\begin{aligned}
\text { Area }= & \frac{1}{2}\left|\left(x_{1} y_{2}-x_{2} y_{1}\right)+\left(x_{2} y_{3}-x_{3} y_{2}\right)+\left(x_{3} y_{1}-x_{1} y_{3}\right)\right| \\
& \text { i.e., Area of ABC }=\frac{1}{2}\left|\sum\left(x_{1} y_{2}-x_{2} y_{1}\right)\right|
\end{aligned}
$$

Note:1. The area of the triangle formed by the points
$\left(\mathrm{x}_{1}, \mathrm{y}_{1}\right)\left(\mathrm{x}_{2}, \mathrm{y}_{2}\right) .\left(\mathrm{x}_{3}, \mathrm{y}_{3}\right)$ is the positive value of the determinant $\frac{1}{2}\left|\begin{array}{ll}x_{1}-x_{2} & y_{1}-y_{2} \\ x_{2}-x_{3} & y_{2}-y_{3}\end{array}\right|$.
2. The area of the triangle formed by the points $\left(\mathrm{x}_{1}, \mathrm{y}_{1}\right)\left(\mathrm{x}_{2}, \mathrm{y}_{2}\right)$ and the origin is $\frac{1}{2}\left|x_{1} y_{2}-x_{2} y_{1}\right|$

## AREA OF A QUADRILATERAL

The area of the quadrilateral formed by the points
$\left(\mathrm{x}_{1}, \mathrm{y}_{1}\right),\left(\mathrm{x}_{2}, \mathrm{y}_{2}\right),\left(\mathrm{x}_{3}, \mathrm{y}_{3}\right),\left(\mathrm{x}_{4}, \mathrm{y}_{4}\right)$ taken in that order is
$\frac{1}{2}\left|x_{1} y_{2}-x_{2} y_{1}+x_{2} y_{3}-x_{3} y_{2}+x_{3} y_{4}-x_{4} y_{3}+x_{4} y_{1}-x_{1} y_{4}\right|$
Note 1: The area of the quadrilateral formed by the points $\left(x_{1}, y_{1}\right),\left(x_{2}, y_{2}\right),\left(x_{3}, y_{3}\right),\left(x_{4}, y_{4}\right)$ taken in order is

$$
\text { Area }=\frac{1}{2}\left|\begin{array}{ll}
x_{1}-x_{3} & y_{1}-y_{3} \\
x_{2}-x_{4} & y_{2}-y_{4}
\end{array}\right|
$$

## CENTRES OF A TRIANGLE

Median : In a triangle, the line segment joining a vertex and the mid point of its opposite side is called a median of the triangle. The medians of a triangle are concurrent.

The point of concurrence of the medians of a triangle is called the centroid (or) centre of gravity of the triangle. It is denoted by G.

## In centre of a Triangle

Internal bisector : The line which bisects the internal angle of a triangle is called an internal angle bisector of the triangle.

The point of concurrence of internal bisectors of the angles of a triangle is called the incentre of the triangle. It is denoted by I.
$\mathrm{I}=\left(\frac{a x_{1}+b x_{2}+c x_{3}}{a+b+c}, \frac{a y_{1}+b y_{2}+c y_{3}}{a+b+c}\right)$

## Excentres of a Triangle:

The point of concurrence of internal bisector of angle A and external bisectors of angles $\mathrm{B}, \mathrm{C}$ of $A B C$ is called the ex-centre opposite to vertex $A$. It is denoted by $I_{1}$. The excentres of ABC opposite to the vertices $\mathrm{B}, \mathrm{C}$ are respectively denoted by $\mathrm{I}_{2}, \mathrm{I}_{3}$.
$\mathrm{I}_{1}=$ Excentre opposite to $\mathrm{A}=\left(\frac{-a x_{1}+b x_{2}+c x_{3}}{-a+b+c}, \frac{-a y_{1}+b y_{2}+c y_{3}}{-a+b+c}\right)$
$\mathrm{I}_{2}=$ Excentre oppposite to $\mathrm{B}=\left(\frac{a x_{1}-b x_{2}+c x_{3}}{a-b+c}, \frac{a y_{1}-b y_{2}+c y_{3}}{a-b+c}\right)$
$\mathrm{I}_{3}=$ Excentre opposite to $\mathrm{C}=\left(\frac{a x_{1}+b x_{2}-c x_{3}}{a+b-c}, \frac{a y_{1}+b y_{2}-c y_{3}}{a+b-c}\right)$

## Ortho centre of a Triangle

Altitude: The line passing through vertex and perpendicular to opposite side of a triangle is called an altitude of the triangle. Altitudes of a triangle are concurrent. The point of concurrence is called the ortho centre of the triangle. It is denoted by " O " or ' H '.

Circum centre of a Triangle:
Perpendicular bisector: The line passing through mid point of a side and perpendicular to the side is called the perpendicular bisector of the side.

The perpendicular bisectors of the sides of a triangle are concurrent. The point of concurrence is called the circum centre or the triangle. It is denoted by S .

## EXERCISE

I. Find the distance between the following pairs of points. i) $(4,5),(5,4) \mathbf{i i})(-3,1),(3,2)$
iii) $(a \cos a, a \sin a),(a \cos b, a \sin b)$

Sol.
i) Let $\mathrm{A}=(4,5), \mathrm{B}=(5,4) \Rightarrow A B=\sqrt{\left(x_{1}-x_{2}\right)^{2}+\left(y_{1}-y_{2}\right)^{2}}$

$$
\Rightarrow \mathrm{AB}=\sqrt{(4-5)^{2}+(5-4)^{2}}=\sqrt{1+1}=\sqrt{2}
$$

ii) Let $\mathrm{A}=(-3,1), \mathrm{B}=(3,2) \Rightarrow \mathrm{AB}=\sqrt{(3+3)^{2}+(2-1)^{2}}=\sqrt{36+1}=\sqrt{37}$
iii) Let $\mathrm{A}=(a \cos a, a \sin a), \mathrm{B}=(a \cos b, a \sin b)$

$$
\begin{aligned}
\mathrm{AB} & =\sqrt{(a \cos \alpha-a \cos \beta)^{2}+(a \sin \alpha-a \sin \beta)^{2}}=\sqrt{a^{2}[2-2 \cos (\alpha-\beta)]}=\sqrt{2 a^{2}[1-\cos (\alpha-\beta)]} \\
& =\sqrt{4 a^{2} \sin ^{2}\left(\frac{\alpha-\beta}{2}\right)}=2\left|a \sin \left(\frac{\alpha-\beta}{2}\right)\right|
\end{aligned}
$$

2. Find the value of ' $\mathbf{a}$ ' if the distance between the points $(\mathbf{a}, \mathbf{2}),(\mathbf{3}, \mathbf{4})$ is $2 \sqrt{2}$.

Sol. P (a, 2), Q (3, 4) are the given points.

$$
\begin{aligned}
& \mathrm{PQ}=2 \sqrt{2} \Rightarrow \mathrm{PQ}^{2}=8 \\
& \Rightarrow(\mathrm{a}-3)^{2}+(2-4)^{2}=8 \\
& \Rightarrow(\mathrm{a}-3)^{2}=8-4=4 \\
& \Rightarrow \mathrm{a}-3= \pm 2 \Rightarrow \mathrm{a}=3 \pm 2=5 \text { or } 1 .
\end{aligned}
$$

3. Find the point on the $x$-axis, which is equidistant from $(7,6)$ and $(-3,4)$.

Sol. Let A(7, 6), B(-3, 4)
Let $\mathrm{P}(\mathrm{x}, 0)$ be the point on x -axis which is equidistant from A and B .
Then $\mathrm{PA}=\mathrm{PB} \Rightarrow \mathrm{PA}^{2}=\mathrm{PB}^{2}$
$\Rightarrow(x-7)^{2}+(0-6)^{2}=(x+3)^{2}+(0-4)^{2}$
$\Rightarrow \mathrm{x}^{2}-14 \mathrm{x}+49+36=\mathrm{x}^{2}+6 \mathrm{x}+9+16$
$\Rightarrow-14 \mathrm{x}+85=6 \mathrm{x}+25 \Rightarrow 20 \mathrm{x}=60 \Rightarrow \mathrm{x}=3$
The required point is $\mathrm{P}(3,0)$.
4. Find the relation between $x$ and $y$, if the point $(x, y)$ is to be equidistant from $(6,-1)$ and $(2,3)$.

Sol. P(x, y), A $(6,-1), \mathrm{B}(2,3)$ are the given points.
$\mathrm{PA}=\mathrm{PB} \Rightarrow \mathrm{PA}^{2}=\mathrm{PB}^{2}$
$\Rightarrow(x-6)^{2}+(y+1)^{2}=(x-2)^{2}+(y-3)^{2}$
$\Rightarrow \mathrm{x}^{2}-12 \mathrm{x}+36+\mathrm{y}^{2}+2 \mathrm{y}+1=\mathrm{x}^{2}-4 \mathrm{x}+4+\mathrm{y}^{2}-6 \mathrm{y}+9$
$\Rightarrow 8 \mathrm{y}=8 \mathrm{x}-24 \Rightarrow \mathrm{y}=\mathrm{x}-3$
5. Find the points which divide the line segment joining $A(1,-3)$ and $B(-3,9)$ in the ratio 1:3 (i) internally and (ii) externally.


Sol. (i) Given points are $\mathrm{A}(1,-3)$ and $\mathrm{B}(-3,9)$
Let P be the point dividing AB internally in the ratio $1: 3$.

Then coordinates of P are

$$
\left(\frac{1(-3)+3 \cdot 1}{1+3}, \frac{1 \cdot 9+3(-3)}{1+3}\right)=\left(\frac{3-3}{4}, \frac{9-9}{4}\right)=(0,0)
$$

Let Q be the point dividing AB externally in the ratio $1: 3$. Then coordinates of Q are

$$
\left(\frac{(1)(-3)-3 \cdot 1}{1-3}, \frac{1.9-3(-3)}{1-3}\right) \quad=\left(\frac{-3-3}{-2}, \frac{9+9}{-2}\right)=(3,-9)
$$

## 6. Find the points of trisection of the line segment joining $(5,-6)$ and $(-3,4)$.

Sol. Gvein points are A $(5,-6), \mathrm{B}(-3,4)$


Let P and Q be the points of trisection of AB .
Let $P$ divides $A B$ in the ratio 1: 2 then Coordinates of $P$ are $\left(\frac{1(-3)+2(5)}{1+2}, \frac{1(4)+2(-6)}{1+2}\right)$

$$
=\left(\frac{-3+10}{3}, \frac{4-12}{3}\right)=\left(\frac{7}{3}, \frac{-8}{3}\right)
$$

Let Q divides AB in the ratio $2: 1$, then Coordinates of Q are $\left(\frac{2(-3)+1(5)}{2+1}, \frac{2(4)+1(-6)}{2+1}\right)$

$$
=\left(\frac{-6+5}{3}, \frac{8-6}{3}\right)=\left(\frac{-1}{3}, \frac{2}{3}\right)
$$

7. Find the points which divide the line segment joining $(8,12)$ and $(12,8)$ into four equal parts.

Sol. Given points $\mathrm{A}(8,12), \mathrm{B}(12,8)$.


Let $\mathrm{P}, \mathrm{Q}, \mathrm{R}$ be the points dividing AB into four equal parts.
Let P divides AB in the ratio $1: 3$, then
Coordinates of P are $\quad\left(\frac{1(12)+3(8)}{1+3}, \frac{1(8)+3(12)}{1+3}\right)=(9,11)$
Q divides AB in the ratio 1: 1
Coordinates of Q are $\left(\frac{8+12}{1+1}, \frac{12+8}{2}\right)=\left(\frac{20}{2}, \frac{20}{2}\right)=(10,10)$
$R$ divides $A B$ in the ratio 3: 1 , then Coordinates of $R$ are:

$$
\left(\frac{3(12)+1(8)}{3+1}, \frac{3(8)+1(12)}{3+1}\right)=(11,9)
$$

$\mathrm{P}(9,11), \mathrm{Q}(10,10), \mathrm{R}(11,9)$ are the required points.
8. Find the ratio in which the point $(1 / 2,6)$ divides the line segment joining $(3,5)$ and $(-7,9)$.

Sol. Let A $(3,5), \mathrm{B}(-7,9), \mathrm{P}(1 / 2,6)$. P divides AB in the ratio m: n .

$$
\begin{gathered}
\mathrm{A}(3,5) \mathrm{m} \frac{\mathrm{n}}{\mathrm{P}(1 / 2,6)} \mathrm{B(-7,9)} \\
\frac{\mathrm{AP}}{\mathrm{~PB}}=\frac{\mathrm{m}}{\mathrm{n}}=\frac{\mathrm{x}-\mathrm{x}_{1}}{\mathrm{x}_{2}-\mathrm{x}}=\frac{\frac{1}{2}-3}{-7-\frac{1}{2}}=\frac{-\frac{5}{2}}{-\frac{15}{2}}=\frac{1}{3}
\end{gathered}
$$

$\therefore \mathrm{P}$ divides AB in the ratio $1: 3$.
9. In what ratio do the coordinate axes divide the line segment joining ( $-2,5$ ) and ( $3,-4$ ).

Sol. $\mathrm{A}(-2,5), \mathrm{B}(3,-4)$ are the given points. AB meets the X -axis in P and Y -axis in Q .


Ratio in which X -axis divides $\mathrm{AB}=-\mathrm{y}_{1}: \mathrm{y}_{2}$ i.e., $\quad=-5:-4=5: 4$
Ratio in which $Y$-axis divides $\mathrm{AB}=-\mathrm{x}_{1}: \mathrm{x}_{2}$ i.e., $\quad=2: 3$
10. If $(-2,-1),(1,0)$ and $(4,3)$ are three successive vertices of a parallelogram, find the fourth vertex.

Sol. Vertices of a parallelogram are A $(-2,-1), \mathrm{B}(1,0), \mathrm{C}(4,3)$ Let $4^{\text {th }}$ vertex be $\mathrm{D}(\mathrm{x}, \mathrm{y})$

Diagonals AC and BD bisect each other.
Midpoint of $\mathrm{AC}=$ Midpoint of BD

$\Rightarrow\left(\frac{-2+4}{2}, \frac{-1+3}{2}\right)=\left(\frac{1+\mathrm{x}}{2}, \frac{0+\mathrm{y}}{2}\right) \Rightarrow \frac{\mathrm{x}+1}{2}=\frac{2}{2} \Rightarrow \mathrm{x}+1=2 \Rightarrow \mathrm{x}=1$
$\Rightarrow \frac{\mathrm{y}}{2}=\frac{2}{2} \Rightarrow \mathrm{y}=2$. Therefore, Coordinates of D are (1,2).
11. A $(2,6)$ is one of the extremities of a diameter of a circle with centre (3,5), find the other point.
Sol. A (2, 6), B(x, y) are the ends of the diameter.

And $C(3,5)$ is the centre.
Now C is the mid point of AB. $\therefore\left(\frac{2+\mathrm{x}}{2}, \frac{6+\mathrm{y}}{2}\right)=(3,5)$
$\frac{2+x}{2}=3 \Rightarrow 2+x=6 \Rightarrow x=4$ and $\frac{6+y}{2}=5 \Rightarrow 6+y=10 \Rightarrow y=4$
Coordinates of the other point $B$ are $(4,4)$.
12. Find the area of the triangle formed by the following points.
i) $(0,0),(1,0),(0,1)$
ii)
$(5,2),(-9,-3),(-3,-5)$

Sol. i) $\mathrm{A}(0,0), \mathrm{B}(1,0), \mathrm{C}(0,1)$ are the vertices of the triangle.
Area of $\Delta \mathrm{OAB}$ is $=\frac{1}{2}\left|\mathrm{x}_{1} \mathrm{y}_{2}-\mathrm{x}_{2} \mathrm{y}_{1}\right|$
$=\frac{1}{2}|1.1-0.0|=\frac{1}{2}$ sq.units

ii) $\mathrm{A}(5,2), \mathrm{B}(-9,-3), \mathrm{C}(-3,-5)$ are the vertices of the triangle.
$=\frac{1}{2}\left|\begin{array}{cc}x_{2}-x_{1} & y_{2}-y_{1} \\ x_{3}-x_{1} & y_{3}-y_{1}\end{array}\right|=\frac{1}{2}\left|\begin{array}{ll}-9-5 & -3-2 \\ -3-5 & -5-2\end{array}\right|$
$=\frac{1}{2}\left|\begin{array}{ll}-14 & -5 \\ -8 & -7\end{array}\right|$
$=\frac{1}{2}|98-40|=\frac{58}{2}=29$ sq.units

13. Show that the following points are collinear.
i) $(0,-2),(-1,1),(-2,4)$ ii) $(-1,7),(3,-5),(4,-8)$

Sol. i) $\mathrm{A}(0,-2), \mathrm{B}(-1,1), \mathrm{C}(-2,4)$ are the given points.
Area of $\Delta \mathrm{ABC}=\frac{1}{2}|0(1-4)-1(4+2)-2(-2-1)|=\frac{1}{2}|0-6+6|=0$
$\therefore \mathrm{A}, \mathrm{B}, \mathrm{C}$ are collinear.
ii) $\mathrm{A}(-1,7), \mathrm{B}(3,-5), \mathrm{C}(4,-8)$

Area of $\Delta \mathrm{ABC}=\frac{1}{2}|-1(-5+8)+3(-8-7)+4(7+5)|=\frac{1}{2}|-3-45+48|=0$
$\therefore \mathrm{A}, \mathrm{B}, \mathrm{C}$ are collinear.
14. Find the value of $k$, if the points $(k,-1),(2,1)$ and $(4,5)$ are collinear.

Sol. $\mathrm{A}(\mathrm{k},-1), \mathrm{B}(2,1), \mathrm{C}(4,5)$ are the given points.
$\mathrm{A}, \mathrm{B}, \mathrm{C}$ are collinear $\Rightarrow \triangle \mathrm{ABC}=0$
$\frac{1}{2}|k(1-5)+2(5+1)+4(-1-1)|=0$
$|-4 \mathrm{k}+12-8|=0 \Rightarrow|-4 \mathrm{k}+4|=0$
$\Rightarrow 4 \mathrm{k}=4 \Rightarrow \mathrm{k}=1$
15. Find the centroid of the triangle whose vertices are $(2,7),(3,-1),(-5,6)$.

Sol. A $(2,7), \mathrm{B}(3,-1), \mathrm{C}(-5,6)$ are the vertices of the triangle.
Let $G$ be the centroid of $\triangle A B C$.
$\mathrm{G}=\left(\frac{\mathrm{x}_{1}+\mathrm{x}_{2}+\mathrm{x}_{3}}{3}, \frac{\mathrm{y}_{1}+\mathrm{y}_{2}+\mathrm{y}_{3}}{3}\right)$

$=\left(\frac{2+3-5}{3}, \frac{7-1+6}{3}\right)=\left(0, \frac{12}{3}\right)=(0,4)$
16. $A(4,8), B(-2,6)$ are two vertices of a triangle $A B C$. Find the coordinates of $C$ if the centroid of the triangle is $(2,7)$.
Sol. Let $\mathrm{C}(\mathrm{x}, \mathrm{y})$ be the third vertex. Given vertices are $\mathrm{A}(4,8), \mathrm{B}(-2,6)$ and centroid of the triangle ABC is $\mathrm{G}(2,7)$

$$
\begin{aligned}
& \Rightarrow G=\left(\frac{4-2+x}{3}, \frac{8+6+y}{3}\right) \\
& \Rightarrow\left(\frac{x+2}{3}, \frac{y+14}{3}\right)=(2,7) \\
& \Rightarrow \frac{x+2}{3}=2 \Rightarrow x+2=6 \Rightarrow x=4 \text { and } \\
& \frac{y+14}{3}=7 \Rightarrow y+14=21 \Rightarrow y=7
\end{aligned}
$$



Third vertex of C is $(4,7)$.

## II.

1. Show that the following points taken in order from the figure mentioned against the points.
i) $(-2,5),(3,-4),(7,10):$ right angled isosceles triangle.

Sol. $\mathrm{A}(-2,5), \mathrm{B}(3,-4), \mathrm{C}(7,10)$ are the given points.
$\mathrm{AB}^{2}=(-2-3)^{2}+(5+4)^{2}=25+81=106$
$\mathrm{BC}^{2}=(3-7)^{2}+(-4-10)^{2}=16+196=212$

$\mathrm{CA}^{2}=(7+2)^{2}+(10-5)^{2}=81+25=106$
$\mathrm{AB}^{2}=\mathrm{CA}^{2} \Rightarrow \mathrm{AB}=\mathrm{CA}$
$\mathrm{AB}^{2}+\mathrm{CA}^{2}=106+106=212=\mathrm{BC}^{2}$
$\mathrm{AB}^{2}+\mathrm{AC}^{2}=\mathrm{BC}^{2}$
$\therefore \mathrm{By}(1)$ and (2), A, B, C are the vertices of a right angled isosceles triangle.
ii) $(1,3),(3,-1),(-5,-5)$ : right angled triangle.

Sol. $\mathrm{A}(1,3), \mathrm{B}(3,-1), \mathrm{C}(-5,-5)$ are the given points

$$
\begin{aligned}
& \mathrm{AB}^{2}=(1-3)^{2}+(3+1)^{2}=4+16=20 \\
& \mathrm{BC}^{2}=(3+5)^{2}+(-1+5)^{2}=64+16=80 \\
& \mathrm{AC}^{2}=(1+5)^{2}+(3+5)^{2}=36+64=100
\end{aligned}
$$

From the above values
$\mathrm{AB}^{2}+\mathrm{BC}^{2}=20+80=100=\mathrm{AC}^{2}$
$\therefore \mathrm{ABC}$ is a right angled triangle.
iii) $(2,4),(2,6),(2+\sqrt{3}, 5)$ : equilateral triangle.

Sol. $\mathrm{A}(2,4), \mathrm{B}(2,6), \mathrm{C}(2+\sqrt{3}, 5)$ are the given points.
$\mathrm{AB}^{2}=(2-2)^{2}+(4-6)^{2}=0+4=4$
$\mathrm{BC}^{2}=(2-2-\sqrt{3})^{2}+(6-5)^{2}=3+1=4$
$\mathrm{CA}^{2}=(2+\sqrt{3}-2)^{2}+(5-4)^{2}=3+1=4$

$\therefore \mathrm{AB}^{2}=\mathrm{BC}^{2}=\mathrm{CA}^{2} \Rightarrow \mathrm{AB}=\mathrm{BC}=\mathrm{CA}$
$\therefore \mathrm{ABC}$ is an equilateral triangle.
iv) $(-3,1),(-6,-7),(3,-9),(6,-1)$ : parallelogram.

Sol. Let $\mathrm{A}(-3,1), \mathrm{B}(-6,-7), \mathrm{C}(3,-9), \mathrm{D}(6,-1)$

$$
\begin{aligned}
\Rightarrow & \mathrm{AB}^{2} \\
\mathrm{BC}^{2} & =(-3+6)^{2}+(1+7)^{2}=9+64=73 \\
\mathrm{CD}^{2} & =(3-6)^{2}+(-9+1)^{2}=9+64=73 \\
\mathrm{DA}^{2} & =(6+3)^{2}+(-1-1)^{2}=81+4=85
\end{aligned}
$$

$\mathrm{AB}^{2}=\mathrm{CD}^{2}$ and $\mathrm{BC}^{2}=\mathrm{DA}^{2}$
$\therefore \mathrm{AB}=\mathrm{CD}$ and $\mathrm{BC}=\mathrm{DA}$
$\Rightarrow \mathrm{ABCD}$ is a parallelogram.
v) $(8,4),(5,7),(-1,1),(2,-2)$ : rectangle.

Sol. Let $\mathrm{A}(8,4), \mathrm{B}(5,7), \mathrm{C}(-1,1), \mathrm{D}(2,-2)$
$\mathrm{AB}^{2}=(8-5)^{2}+(4-7)^{2}=9+9=18$
$\mathrm{BC}^{2}=(5+1)^{2}+(7-1)^{2}=36+36=72$
$\mathrm{CD}^{2}=(-1-2)^{2}+(1+2)^{2}=9+9=18$
$\mathrm{DA}^{2}=(2-8)^{2}+(-2-4)^{2}=36+36=72$
$\mathrm{AB}^{2}=\mathrm{CD}^{2} \Rightarrow \mathrm{AB}=\mathrm{CD}$
$\mathrm{BC}^{2}=\mathrm{DA}^{2} \Rightarrow \mathrm{BC}=\mathrm{AD}$
$\mathrm{AC}^{2}=(8+1)^{2}+(4-1)^{2}=81+9=90$
$\mathrm{BD}^{2}=(5-2)^{2}+(7+2)^{2}=9+81=90$
$\mathrm{AC}^{2}=\mathrm{BD}^{2} \Rightarrow \mathrm{AC}=\mathrm{BD}$
$\therefore \mathrm{ABCD}$ is a rectangle.
vi) $(3,-2),(7,6),(-1,2),(-5,-6)$ : rhombus.

Sol. Let A(3, -2), B(7, 6), C(-1, 2), D(-5, -6)

$$
\mathrm{AB}^{2}=(3-7)^{2}+(-2-6)^{2}=16+64=80
$$

$\mathrm{BC}^{2}=(7+1)^{2}+(6-2)^{2}=64+16=80$
$\mathrm{CD}^{2}=(-1+5)^{2}+(2+6)^{2}=16+64=80$
$\mathrm{DA}^{2}=(-5-3)^{2}+(-6+2)^{2}=64+16=80$
$\mathrm{AB}=\mathrm{BC}=\mathrm{CD}=\mathrm{DA}$
$A C^{2}=(3+1)^{2}+(-2-2)^{2}=16+16=32$
$B D^{2}=(7+5)^{2}+(6+6)^{2}=144+144=288$
$\mathrm{AC} \neq \mathrm{BD}$
$\therefore \mathrm{ABCD}$ is a rhombus.
2. Find the value of $k$, if the area of the triangle formed by $(k, 0),(3,4)$ and $(5,-2)$ is $\mathbf{1 0}$.

Sol. Let A(k, 0), B(3, 4) and C(5, -2)
Area of $\Delta \mathrm{ABC}=\frac{1}{2}|\mathrm{~K}(4+2)+3(-2-0)+5(0-4)|=10$

$$
|6 \mathrm{k}-6-20|=20 \Rightarrow 6 \mathrm{k}-26= \pm 20
$$


$\Rightarrow 6 \mathrm{k}=46$ or $6 \Rightarrow \mathrm{k}=1$ or $\frac{46}{6}=\frac{23}{3}$
3. Find the value of $k$ if $(k, 2-2 k),(-k+1,2 k),(-4-k, 6-2 k)$ are collinear.

Sol. $A(k, 2-2 k), B(-k+1,2 k), C(-4-k, 6-2 k)$ are the given points.
$\mathrm{A}, \mathrm{B}, \mathrm{C}$ are collinear $\triangle \mathrm{ABC}=0$
$\frac{1}{2}|\mathrm{k}(2 \mathrm{k}-6+2 \mathrm{k})+(-\mathrm{k}+1)(6-2 \mathrm{k}-2+2 \mathrm{k})+(-4-\mathrm{k})(2-2 \mathrm{k}-2 \mathrm{k})|=0$
$\Rightarrow \mathrm{k}(4 \mathrm{k}-6)+4(-\mathrm{k}+1)+(-4-\mathrm{k})(2-4 \mathrm{k})=0$
$\Rightarrow 4 \mathrm{k}^{2}-6 \mathrm{k}-4 \mathrm{k}+4-8+16 \mathrm{k}-2 \mathrm{k}+4 \mathrm{k}^{2}=0$
$\Rightarrow 8 \mathrm{k}^{2}+4 \mathrm{k}-4=0 \Rightarrow 2 \mathrm{k}^{2}+\mathrm{k}-1=0$
$\Rightarrow 2 \mathrm{k}^{2}+2 \mathrm{k}-\mathrm{k}-1=0 \Rightarrow 2 \mathrm{k}(\mathrm{k}+1)-1(\mathrm{k}+1)=0 \Rightarrow(\mathrm{k}+1)(2 \mathrm{k}-1)=0 \Rightarrow \mathrm{k}=-1$ or $1 / 2$
III.

1. Find the in-centre of the triangle whose vertices are $\mathbf{A}(3,2), B(7,2), C(7,5)$.

Sol. Vertices are $\mathrm{A}(3,2), \mathrm{B}(7,2), \mathrm{C}(7,5)$
Sides of the triangle are
$\mathrm{a}=\mathrm{BC}=\sqrt{(7-7)^{2}+(2-5)^{2}}=\sqrt{0+9}=3$
$\mathrm{b}=\mathrm{CA}=\sqrt{(7-3)^{2}+(5-2)^{2}}=\sqrt{16+9}=5$

$\mathrm{c}=\mathrm{AB}=\sqrt{(3-7)^{2}+(2-2)^{2}}=\sqrt{16+0}=4$
In centre $I=\left(\frac{a x_{1}+b x_{2}+c x_{3}}{a+b+c}, \frac{a y_{1}+b y_{2}+c y_{3}}{a+b+c}\right)$

$$
=\left(\frac{3 \cdot 3+5 \cdot 7+4 \cdot 7}{3+5+4}, \frac{3 \cdot 2+5 \cdot 2+4 \cdot 5}{3+5+4}\right)
$$

$$
=\left(\frac{9+35+28}{12}, \frac{6+10+20}{12}\right)
$$

$$
=\left(\frac{72}{12}, \frac{36}{12}\right)=(6,3)
$$

## PROBLEMS FOR PRACTICE

1. Find the area of the triangle formed by the points $(1,2),(3,-4)$ and $(-2,0)$.

Ans. 11
2. Find the area of the triangle formed by the points $(9,-7),(2,4)$ and $(0,0)$.

Ans. 25
3. Find the value of $x$, if the area of the triangle formed by $(10,2),(-3,-4)$ and $(x, 1)$ is 5 . Ans. 19/2 or 37/6
4. Show that the triangle formed by the points $(4,4),(3,5)$ and $(-1,-1)$ is a right angled triangle.
5. The centroid of the triangle $A B C$ is $(2,7)$. The points $B$ and $C$ lie on $X, Y$ axes respectively and $A=(4,8)$. Find $B$ and $C$.
Ans. B (2, 0), C (0, 13)
6. Find the in-centre of the triangle formed by the points $A(7,9), B(3,-7)$ and $C(-3,3)$.

ANS. $(13-8 \sqrt{2}, 2 \sqrt{2}-1)$

