# PREREQUISITES

### (2-D GEOMETRY)

## **DISTANCE BETWEEN TWO POINTS**

(i) The distance between two points  $A(x_1, y_1)$  and  $B(x_2, y_2)$ 

$$AB(orBA) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

(ii) The distance from origin O to the point  $A(x_1, y_1)$  is  $OA = \sqrt{x_1^2 + y_1^2}$ 

(iii) The distance between two points  $A(x_1, 0)$  and  $B(x_2, 0)$  lying on the X - axis

is 
$$AB = \sqrt{(x_1 - x_2)^2 + (0 - 0)^2} = \sqrt{(x_1 - x_2)^2} = |x_1 - x_2|$$

(iv) The distance between two points C(0, y<sub>1</sub>) and D(0, y<sub>2</sub>) lying on the Y-axis is  $CD = |y_1 - y_2|$ 

### SECTION FORMULA

(i) The point P which divides the line segment joining the points  $A(x_1, y_1)$ ,

 $B(x_2, y_2)$  in the ratio m : n internally is given by

$$P = \underbrace{\overset{\mathfrak{W}}{\mathfrak{g}}}_{m+n}^{mx_2 + nx_1}, \frac{my_2 + ny_1 \overset{\mathsf{O}}{\overset{\mathsf{T}}{\mathfrak{g}}}}{m+n} \underbrace{(m+n^1 \ \mathsf{O})}_{\overset{\mathsf{T}}{\mathfrak{g}}} \quad (m+n^1 \ \mathsf{O})$$
(ii) If P divides in the ratio m:n externally then 
$$P = \underbrace{\overset{\mathfrak{W}}{\mathfrak{g}}}_{m-n}^{mx_2 - nx_1}, \frac{my_2 - ny_1 \overset{\mathsf{O}}{\overset{\mathsf{T}}{\mathfrak{g}}}}{m-n} \underbrace{(m^1 \ n)}_{\overset{\mathsf{T}}{\mathfrak{g}}} \quad (m^1 \ n)$$

**Note:** If the ratio m : n is positive then P divides internally and if the ratio is negative P divides externally.

### **MID POINT**

The mid point of the line segment joining A(x<sub>1</sub>, y<sub>1</sub>) and B(x<sub>2</sub>, y<sub>2</sub>) is  $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ 

### POINTS OF TRISECTION

The points which divide the line segment  $\overline{AB}$  in the ratio 1 : 2 and 2 : 1 (internally) are called the points of trisection of  $\overline{AB}$ .

### **AREA OF A TRIANGLE**

The area of the triangle formed by the points  $A(x_1, y_1)$ ,  $B(x_2, y_2)$ ,  $C(x_3, y_3)$  is

Area=
$$\frac{1}{2} |(x_1y_2 - x_2y_1) + (x_2y_3 - x_3y_2) + (x_3y_1 - x_1y_3)|$$
  
i.e., Area of ABC =  $\frac{1}{2} |\sum (x_1y_2 - x_2y_1)|$ 

Note:1. The area of the triangle formed by the points

 $(x_1, y_1) (x_2, y_2) (x_3, y_3)$  is the positive value of the determinant  $\frac{1}{2} \begin{vmatrix} x_1 - x_2 & y_1 - y_2 \\ x_2 - x_3 & y_2 - y_3 \end{vmatrix}$ .

2. The area of the triangle formed by the points  $(x_1, y_1) (x_2, y_2)$  and the origin is  $\frac{1}{2}|x_1y_2 - x_2y_1|$ 

### AREA OF A QUADRILATERAL

The area of the quadrilateral formed by the points  $(x_1, y_1), (x_2, y_2), (x_3, y_3), (x_4, y_4)$  taken in that order is

$$\frac{1}{2} |x_1y_2 - x_2y_1 + x_2y_3 - x_3y_2 + x_3y_4 - x_4y_3 + x_4y_1 - x_1y_4|$$

**Note 1:** The area of the quadrilateral formed by the points (x<sub>1</sub>, y<sub>1</sub>), (x<sub>2</sub>, y<sub>2</sub>), (x<sub>3</sub>, y<sub>3</sub>),(x<sub>4</sub>, y<sub>4</sub>) taken in order is

Area = 
$$\frac{1}{2} \begin{vmatrix} x_1 - x_3 & y_1 - y_3 \\ x_2 - x_4 & y_2 - y_4 \end{vmatrix}$$

### **CENTRES OF A TRIANGLE**

**Median** : In a triangle, the line segment joining a vertex and the mid point of its opposite side is called a median of the triangle. The medians of a triangle are concurrent.

The point of concurrence of the medians of a triangle is called the centroid (or) centre of gravity of the triangle. It is denoted by G.

### IN CENTRE OF A TRIANGLE

**Internal bisector** : The line which bisects the internal angle of a triangle is called an internal angle bisector of the triangle.

The point of concurrence of internal bisectors of the angles of a triangle is called the incentre of the triangle. It is denoted by I.

 $\mathbf{I} = \left(\frac{ax_1 + bx_2 + cx_3}{a + b + c}, \frac{ay_1 + by_2 + cy_3}{a + b + c}\right)$ 

### **EXCENTRES OF A TRIANGLE:**

The point of concurrence of internal bisector of angle A and external bisectors of angles B, C of ABC is called the ex-centre opposite to vertex A. It is denoted by  $I_1$ . The excentres of ABC opposite to the vertices B, C are respectively denoted by  $I_2$ ,  $I_3$ .

$$I_1 = \text{Excentre opposite to A} = \left(\frac{-ax_1 + bx_2 + cx_3}{-a + b + c}, \frac{-ay_1 + by_2 + cy_3}{-a + b + c}\right)$$
$$I_2 = \text{Excentre opposite to B} = \left(\frac{ax_1 - bx_2 + cx_3}{a - b + c}, \frac{ay_1 - by_2 + cy_3}{a - b + c}\right)$$
$$I_2 = \text{Excentre opposite to B} = \left(\frac{ax_1 + bx_2 - cx_2}{a - b + c}, \frac{ay_1 + by_2 - cy_3}{a - b + c}\right)$$

I<sub>3</sub> = Excentre opposite to C = 
$$\left(\frac{ax_1 + bx_2 - cx_3}{a + b - c}, \frac{ay_1 + by_2 - cy_3}{a + b - c}\right)$$

### **ORTHO CENTRE OF A TRIANGLE**

Altitude: The line passing through vertex and perpendicular to opposite side of a triangle is called an altitude of the triangle. Altitudes of a triangle are concurrent. The point of concurrence is called the ortho centre of the triangle. It is denoted by "O" or 'H'.

Circum centre of a Triangle:

Perpendicular bisector: The line passing through mid point of a side and perpendicular to the side is called the perpendicular bisector of the side.

The perpendicular bisectors of the sides of a triangle are concurrent. The point of concurrence is called **the** circum centre or the triangle. It is denoted by S.

#### EXERCISE

I. Find the distance between the following pairs of points. i) (4, 5), (5, 4) ii) (-3, 1), (3, 2) iii)  $(a \cos a, a \sin a)$ ,  $(a \cos b, a \sin b)$ 

Sol.

i) Let A = (4, 5), B = (5, 4) 
$$\Rightarrow AB = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$
  
 $\Rightarrow AB = \sqrt{(4 - 5)^2 + (5 - 4)^2} = \sqrt{1 + 1} = \sqrt{2}$ 

ii) Let A = (-3, 1), B = (3, 2)  $\Rightarrow$  AB =  $\sqrt{(3+3)^2 + (2-1)^2} = \sqrt{36+1} = \sqrt{37}$ 

iii) Let  $A=(a\cos a, a\sin a), B=(a\cos b, a\sin b)$ 

$$AB = \sqrt{\left(a\cos\alpha - a\cos\beta\right)^2 + \left(a\sin\alpha - a\sin\beta\right)^2} = \sqrt{a^2 \left[2 - 2\cos\left(\alpha - \beta\right)\right]} = \sqrt{2a^2 \left[1 - \cos\left(\alpha - \beta\right)\right]}$$
$$= \sqrt{4a^2 \sin^2\left(\frac{\alpha - \beta}{2}\right)} = 2\left|a\sin\left(\frac{\alpha - \beta}{2}\right)\right|$$

# 2. Find the value of 'a' if the distance between the points (a, 2), (3, 4) is $2\sqrt{2}$ .

**Sol.** P(a, 2), Q(3, 4) are the given points.

$$PQ = 2\sqrt{2} \implies PQ^{2} = 8$$
  
$$\implies (a - 3)^{2} + (2 - 4)^{2} = 8$$
  
$$\implies (a - 3)^{2} = 8 - 4 = 4$$
  
$$\implies a - 3 = \pm 2 \implies a = 3 \pm 2 = 5 \text{ or } 1.$$

## 3. Find the point on the x-axis, which is equidistant from (7, 6) and (-3, 4).

**Sol.** Let A(7, 6), B(-3, 4)

Let P(x, 0) be the point on x-axis which is equidistant from A and B.

Then 
$$PA = PB \Rightarrow PA^2 = PB^2$$
  
 $\Rightarrow (x - 7)^2 + (0 - 6)^2 = (x + 3)^2 + (0 - 4)^2$   
 $\Rightarrow x^2 - 14x + 49 + 36 = x^2 + 6x + 9 + 16$   
 $\Rightarrow -14x + 85 = 6x + 25 \Rightarrow 20x = 60 \Rightarrow x = 3$ 

The required point is P(3, 0).

4. Find the relation between x and y, if the point (x,y) is to be equidistant from (6,-1) and (2,3).

Sol. P(x, y), A (6, -1), B (2, 3) are the given points.

 $PA = PB \Rightarrow PA^{2} = PB^{2}$  $\Rightarrow (x - 6)^{2} + (y + 1)^{2} = (x - 2)^{2} + (y - 3)^{2}$  $\Rightarrow x^{2} - 12x + 36 + y^{2} + 2y + 1 = x^{2} - 4x + 4 + y^{2} - 6y + 9$  $\Rightarrow 8y = 8x - 24 \Rightarrow y = x - 3$ 

5. Find the points which divide the line segment joining A (1, -3) and B (-3, 9) in the ratio 1: 3 (i) internally and (ii) externally.

**Sol.** (i) Given points are A(1, -3) and B(-3, 9)

Let P be the point dividing AB internally in the ratio 1: 3.

Then coordinates of P are

$$\left(\frac{1(-3)+3\cdot 1}{1+3}, \frac{1\cdot 9+3(-3)}{1+3}\right) = \left(\frac{3-3}{4}, \frac{9-9}{4}\right) = (0,0)$$

Let Q be the point dividing AB externally in the ratio 1 : 3. Then coordinates of Q are

$$\left(\frac{(1)(-3)-3\cdot 1}{1-3},\frac{1.9-3(-3)}{1-3}\right) = \left(\frac{-3-3}{-2},\frac{9+9}{-2}\right) = (3,-9)$$

### 6. Find the points of trisection of the line segment joining (5, -6) and (-3, 4).

**Sol**. Gvein points are A (5, -6), B (-3, 4)

Let P and Q be the points of trisection of AB.

Let P divides AB in the ratio 1: 2 then Coordinates of P are

$$=\left(\frac{-3+10}{3},\frac{4-12}{3}\right)=\left(\frac{7}{3},\frac{-8}{3}\right)$$

Let Q divides AB in the ratio 2 : 1, then Coordinates of Q are  $\left(\frac{2(-3)+1(5)}{2+1}, \frac{2(4)+1(-6)}{2+1}\right)$ 

$$=\left(\frac{-6+5}{3},\frac{8-6}{3}\right)=\left(\frac{-1}{3},\frac{2}{3}\right)$$

# 7. Find the points which divide the line segment joining (8, 12) and (12, 8) into four equal parts.

**Sol.** Given points A(8, 12), B(12, 8).

A(8,12) P Q R B(12,8)

A (5,-6) P Q B (-3, 4)

 $\left(\frac{1(-3)+2(5)}{1+2},\frac{1(4)+2(-6)}{1+2}\right)$ 

Let P, Q, R be the points dividing AB into four equal parts.

Let P divides AB in the ratio 1: 3, then

Coordinates of P are 
$$\left(\frac{1(12)+3(8)}{1+3}, \frac{1(8)+3(12)}{1+3}\right) = (9, 11)$$

Q divides AB in the ratio 1: 1

Coordinates of Q are 
$$\left(\frac{8+12}{1+1}, \frac{12+8}{2}\right) = \left(\frac{20}{2}, \frac{20}{2}\right) = (10, 10)$$

R divides AB in the ratio 3: 1, then Coordinates of R are:

$$\left(\frac{3(12)+1(8)}{3+1}, \frac{3(8)+1(12)}{3+1}\right) = (11,9)$$

P (9, 11), Q (10, 10), R (11, 9) are the required points.

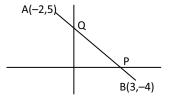
# 8. Find the ratio in which the point (1/2, 6) divides the line segment joining (3, 5) and (-7, 9).

**Sol.** Let A (3, 5), B (-7, 9), P (1/2, 6). P divides AB in the ratio m: n.

$$\frac{AP}{PB} = \frac{m}{n} = \frac{x - x_1}{x_2 - x} = \frac{\frac{1}{2} - 3}{-7 - \frac{1}{2}} = \frac{-\frac{5}{2}}{-\frac{15}{2}} = \frac{1}{3}$$

 $\therefore$  P divides AB in the ratio 1 : 3.

**9.** In what ratio do the coordinate axes divide the line segment joining (-2, 5) and (3, -4). Sol. A(-2, 5), B(3, -4) are the given points. AB meets the X-axis in P and Y-axis in Q.



Ratio in which X-axis divides  $AB = -y_1 : y_2$  i.e., = -5 : -4 = 5 : 4Ratio in which Y-axis divides  $AB = -x_1 : x_2$  i.e., = 2 : 3

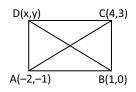
# 10. If (-2, -1), (1, 0) and (4, 3) are three successive vertices of a parallelogram, find the fourth vertex.

**Sol.** Vertices of a parallelogram are A (-2, -1), B (1, 0), C (4, 3)Let 4<sup>th</sup> vertex be D(x, y)

Diagonals AC and BD bisect each other.

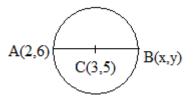
Midpoint of AC = Midpoint of BD

$$\Rightarrow \left(\frac{-2+4}{2}, \frac{-1+3}{2}\right) = \left(\frac{1+x}{2}, \frac{0+y}{2}\right) \Rightarrow \frac{x+1}{2} = \frac{2}{2} \Rightarrow x+1 = 2 \Rightarrow x = 1$$
$$\Rightarrow \frac{y}{2} = \frac{2}{2} \Rightarrow y = 2. \text{ Therefore, Coordinates of D are (1, 2).}$$



## 11. A (2, 6) is one of the extremities of a diameter of a circle with centre (3, 5), find the other point.

**Sol.** A (2, 6), B(x, y) are the ends of the diameter.



And C(3, 5) is the centre.

Now C is the mid point of AB.  $\therefore \left(\frac{2+x}{2}, \frac{6+y}{2}\right) = (3,5)$ 

$$\frac{2+x}{2} = 3 \Longrightarrow 2+x = 6 \Longrightarrow x = 4 \text{ and } \frac{6+y}{2} = 5 \Longrightarrow 6+y = 10 \Longrightarrow y = 4$$

Coordinates of the other point B are (4, 4).

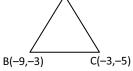
### 12. Find the area of the triangle formed by the following points.

i) (0, 0), (1, 0), (0, 1) ii) (5, 2), (-9, -3), (-3, -5)

Sol. i) A(0, 0), B(1, 0), C(0, 1) are the vertices of the triangle.

Area of 
$$\triangle OAB$$
 is  $=\frac{1}{2}|x_1y_2 - x_2y_1|$   
 $=\frac{1}{2}|1.1 - 0.0| = \frac{1}{2}$  sq. units  
ii) A(5, 2), B(-9, -3), C(-3, -5) are the vertices of the triangle.  
 $=\frac{1}{2}\begin{vmatrix}x_2 - x_1 & y_2 - y_1\\x_3 - x_1 & y_3 - y_1\end{vmatrix} = \frac{1}{2}\begin{vmatrix}-9 - 5 & -3 - 2\\-3 - 5 & -5 - 2\end{vmatrix}$   
 $=\frac{1}{2}\begin{vmatrix}-14 & -5\\-8 & -7\end{vmatrix}$ 

$$=\frac{1}{2}|98-40| = \frac{58}{2} = 29$$
 sq.units



A(0,0)

#### 13. Show that the following points are collinear.

i) (0, -2), (-1, 1), (-2, 4) ii) (-1, 7), (3, -5), (4, -8)

**Sol.** i) A(0, -2), B(-1, 1), C(-2, 4) are the given points.

Area of 
$$\triangle ABC = \frac{1}{2} |0(1-4) - 1(4+2) - 2(-2-1)| = \frac{1}{2} |0-6+6| = 0$$
  
: A B C are collinear

 $\therefore$  A, B, C are commear.

ii) A(-1, 7), B(3, -5), C(4, -8) Area of  $\triangle ABC = \frac{1}{2} \left| -1(-5+8) + 3(-8-7) + 4(7+5) \right| = \frac{1}{2} \left| -3 - 45 + 48 \right| = 0$  $\therefore$  A, B, C are collinear.

### 14. Find the value of k, if the points (k, -1), (2, 1) and (4, 5) are collinear.

Sol. A(k, -1), B(2, 1), C(4, 5) are the given points.

A, B, C are collinear 
$$\Rightarrow \Delta ABC = 0$$
  
 $\frac{1}{2} |k(1-5)+2(5+1)+4(-1-1)| = 0$   
 $|-4k+12-8| = 0 \Rightarrow |-4k+4| = 0$   
 $\Rightarrow 4k = 4 \Rightarrow k = 1$ 

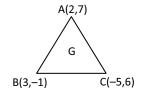
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### 15. Find the centroid of the triangle whose vertices are (2, 7), (3, -1), (-5, 6).

Sol. A(2, 7), B(3, -1), C(-5, 6) are the vertices of the triangle.

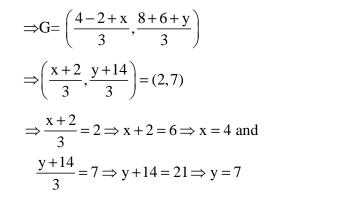
Let G be the centroid of  $\triangle ABC$ .

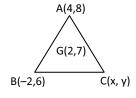
$$G = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$
$$= \left(\frac{2 + 3 - 5}{3}, \frac{7 - 1 + 6}{3}\right) = \left(0, \frac{12}{3}\right) = (0, 4)$$



# 16. A(4, 8), B(-2, 6) are two vertices of a triangle ABC. Find the coordinates of C if the centroid of the triangle is (2, 7).

Sol. Let C(x, y) be the third vertex. Given vertices are A(4, 8), B(-2, 6) and centroid of the triangle ABC is G(2,7)





Third vertex of C is (4, 7).

II.

- 1. Show that the following points taken in order from the figure mentioned against the points.
- i) (-2, 5), (3, -4), (7, 10) : right angled isosceles triangle.

**Sol.** A(-2, 5), B(3, -4), C(7, 10) are the given points.

$$AB^{2} = (-2-3)^{2} + (5+4)^{2} = 25 + 81 = 106$$
  

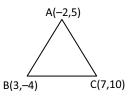
$$BC^{2} = (3-7)^{2} + (-4-10)^{2} = 16 + 196 = 212$$
  

$$CA^{2} = (7+2)^{2} + (10-5)^{2} = 81 + 25 = 106$$
  

$$AB^{2} = CA^{2} \Rightarrow AB = CA \qquad \dots(1)$$
  

$$AB^{2} + CA^{2} = 106 + 106 = 212 = BC^{2}$$
  

$$AB^{2} + AC^{2} = BC^{2} \qquad \dots(2)$$



: By (1) and (2), A, B, C are the vertices of a right angled isosceles triangle.

# ii) (1,3), (3,-1), (-5,-5): right angled triangle.

**Sol.** A(1,3), B(3,-1), C(-5,-5) are the given points

$$AB^{2} = (1-3)^{2} + (3+1)^{2} = 4+16 = 20$$
$$BC^{2} = (3+5)^{2} + (-1+5)^{2} = 64+16 = 80$$
$$AC^{2} = (1+5)^{2} + (3+5)^{2} = 36+64 = 100$$

From the above values

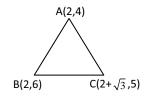
$$AB^2 + BC^2 = 20 + 80 = 100 = AC^2$$

: ABC is a right angled triangle.

iii) (2, 4), (2, 6),  $(2+\sqrt{3},5)$ , equilateral triangle.

**Sol**. A(2, 4), B(2, 6), C( $2 + \sqrt{3}$ , 5) are the given points.

AB<sup>2</sup> = 
$$(2-2)^2 + (4-6)^2 = 0 + 4 = 4$$
  
BC<sup>2</sup> =  $(2-2-\sqrt{3})^2 + (6-5)^2 = 3 + 1 = 4$   
CA<sup>2</sup> =  $(2+\sqrt{3}-2)^2 + (5-4)^2 = 3 + 1 = 4$   
∴ AB<sup>2</sup> = BC<sup>2</sup> = CA<sup>2</sup> ⇒ AB = BC = CA  
∴ ABC is an equilateral triangle.



iv) (-3, 1), (-6, -7), (3, -9), (6, -1): parallelogram.  
Sol. Let A(-3, 1), B(-6, -7), C(3, -9), D(6, -1)  

$$\Rightarrow AB^{2} = (-3+6)^{2} + (1+7)^{2} = 9+64 = 73$$

$$BC^{2} = (-6-3)^{2} + (-7+9)^{2} = 81+4 = 85$$

$$CD^{2} = (3-6)^{2} + (-9+1)^{2} = 9+64 = 73$$

$$DA^{2} = (6+3)^{2} + (-1-1)^{2} = 81+4 = 85$$

$$AB^{2} = CD^{2} \text{ and } BC^{2} = DA^{2}$$

$$\therefore AB = CD \text{ and } BC = DA$$

$$\Rightarrow ABCD \text{ is a parallelogram.}$$
v) (8, 4), (5, 7), (-1, 1), (2, -2): rectangle.  
Sol. Let A(8, 4), B(5, 7), C(-1, 1), D(2, -2)  

$$AB^{2} = (8-5)^{2} + (4-7)^{2} = 9+9 = 18$$

$$BC^{2} = (5+1)^{2} + (7-1)^{2} = 36+36 = 72$$

$$CD^{2} = (-1-2)^{2} + (1+2)^{2} = 9+9 = 18$$

$$DA^{2} = (2-8)^{2} + (-2-4)^{2} = 36+36 = 72$$

$$AB^{2} = CD^{2} \Rightarrow AB = CD$$

$$BC^{2} = DA^{2} \Rightarrow BC = AD$$

$$AC^{2} = (8+1)^{2} + (4-1)^{2} = 81+9 = 90$$

$$BD^{2} = (5-2)^{2} + (7+2)^{2} = 9+81 = 90$$

$$AC^{2} = BD^{2} \Rightarrow AC = BD$$

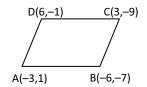
$$\therefore ABCD \text{ is a rectangle.}$$
vi) (3, -2), (7, 6), (-1, 2), (-5, -6): rhombus.  
Sol. Let A(3, -2), B(7, 6), C(-1, 2), D(-5, -6)
$$AC^{2} = (-1+5)^{2} + (2+6)^{2} = 16+64 = 80$$

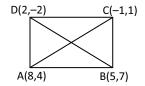
$$CD^{2} = (-1+5)^{2} + (-6+2)^{2} = 64+16 = 80$$

$$AC^{2} = (3+1)^{2} + (-2-2)^{2} = 16+16 = 32$$

$$BD^{2} = (7+5)^{2} + (6+6)^{2} = 144+144 = 288$$

$$AC \neq BD$$



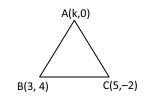


$$AB^{2} = (3-7)^{2} + (-2-6)^{2} = 16 + 64 = 80$$

### $\therefore$ ABCD is a rhombus.

## 2. Find the value of k, if the area of the triangle formed by (k, 0), (3, 4) and (5, -2) is 10.

Sol. Let A(k, 0), B(3, 4) and C(5, -2)  
Area of 
$$\triangle ABC = \frac{1}{2} |K(4+2) + 3(-2-0) + 5(0-4)| = 10$$
  
 $|6k - 6 - 20| = 20 \Rightarrow 6k - 26 = \pm 20$   
 $\Rightarrow 6k = 46 \text{ or } 6 \Rightarrow k = 1 \text{ or } \frac{46}{6} = \frac{23}{3}$ 



# 3. Find the value of k if (k, 2 - 2k), (-k + 1, 2k), (-4 - k, 6 - 2k) are collinear.

**Sol.** 
$$A(k, 2-2k)$$
,  $B(-k + 1, 2k)$ ,  $C(-4 - k, 6 - 2k)$  are the given points.

A, B, C are collinear 
$$\triangle ABC = 0$$
  

$$\frac{1}{2} |k(2k-6+2k)+(-k+1)(6-2k-2+2k)+(-4-k)(2-2k-2k)|=0$$

$$\Rightarrow k(4k-6)+4(-k+1)+(-4-k)(2-4k)=0$$

$$\Rightarrow 4k^2-6k-4k+4-8+16k-2k+4k^2=0$$

$$\Rightarrow 8k^2+4k-4=0 \Rightarrow 2k^2+k-1=0$$

$$\Rightarrow 2k^2+2k-k-1=0 \Rightarrow 2k(k+1)-1(k+1)=0 \Rightarrow (k+1)(2k-1)=0 \Rightarrow k=-1 \text{ or } 1/2$$

### III.

## 1. Find the in-centre of the triangle whose vertices are A(3, 2), B(7, 2), C(7, 5).

**Sol.** Vertices are A(3, 2), B(7, 2), C(7, 5)

Sides of the triangle are

$$a = BC = \sqrt{(7-7)^2 + (2-5)^2} = \sqrt{0+9} = 3$$
  

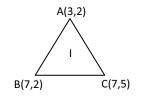
$$b = CA = \sqrt{(7-3)^2 + (5-2)^2} = \sqrt{16+9} = 5$$
  

$$c = AB = \sqrt{(3-7)^2 + (2-2)^2} = \sqrt{16+0} = 4$$
  
In centre I =  $\left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c}\right)$   

$$= \left(\frac{3 \cdot 3 + 5 \cdot 7 + 4 \cdot 7}{3+5+4}, \frac{3 \cdot 2 + 5 \cdot 2 + 4 \cdot 5}{3+5+4}\right)$$
  

$$= \left(\frac{9 + 35 + 28}{12}, \frac{6 + 10 + 20}{12}\right)$$
  

$$= \left(\frac{72}{12}, \frac{36}{12}\right) = (6,3)$$



## **PROBLEMS FOR PRACTICE**

1. Find the area of the triangle formed by the points (1, 2), (3, -4) and (-2, 0).

Ans. 11

2. Find the area of the triangle formed by the points (9, -7), (2, 4) and (0, 0).

**Ans.** 25

3. Find the value of x, if the area of the triangle formed by (10, 2), (-3, -4) and (x, 1) is 5.

**Ans**. 19/2 or 37/6

- 4. Show that the triangle formed by the points (4, 4), (3, 5) and (-1, -1) is a right angled triangle.
- 5. The centroid of the triangle ABC is (2, 7). The points B and C lie on X, Y axes respectively and A= (4, 8). Find B and C.

**Ans**. B (2, 0), C (0, 13)

6. Find the in-centre of the triangle formed by the points A(7, 9), B(3, -7) and C(-3, 3).

**ANS.**  $(13-8\sqrt{2}, 2\sqrt{2}-1)$