CHAPTER 3

STRAIGHT LINES – 1

TOPICS

1. Inclination, slope of a line when inclination is given, slope of a line through two points.
2. Equation of a line in point slope form, two point form , Slope intercept form, intercepts form.
3. Equation of a line in normal form
4. Equation of a line in symmetric form and parametric form.
5. Reduction of a line into various forms
6. Point of intersection of two lines
7. Family of straight lines .
8. Condition for concurrency of three straight lines.
9. Angle between two lines
10. Length of perpendicular from a point to line
11. Distance between two parallel lines
12. Foot of the perpendicular from a point to a straight Line
13. Image of a line w.r.t a line
14. Concurrent lines- properties related to a triangle.

STRAIGHT LINES

INCLINATION OF A LINE

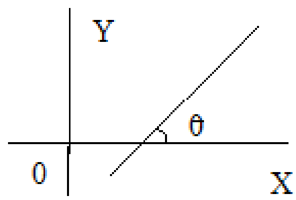
If a line L makes an angle θ with the positive direction of the x - axis, then θ is called the inclination of the line L.

The range of θ is $0 \leq \theta \leq \pi$.

If $\theta = 0^0$, then the line L is parallel to the x- axis.

If $\theta = 90^0$, then the line L is perpendicular to the x - axis.

Two lines have same inclination if and only if they are parallel.



Slope of a line:

If θ is the inclination of a line L then $\tan\theta$ is called the slope (or gradient) of the line L.

The slope of a line L is denoted by m. $m = \tan\theta$ $(\theta \neq 90^0)$

e.g., i) If the inclination of a line is 45^0, then its slope is 1.

ii) If the slope of a line is $\sqrt{3}$, than its inclination is 60^0.

Note 1: The slope of x - axis is 0 . $(\because \theta = 0)$

Note 2: The slope of a horizontal line is 0 . $(\because \theta = 0)$

Note 3 : The slope of y - axis is not defined .

Note 4 : The slope of a vertical line is not defined.

Note 5 : if 1. $\theta = 0^0$ m = 0 .

2. $0^0 < \theta < 90^0$ m > 0.

3. $\theta = 90^0$ m is not defined.

4. $90^0 < \theta < 180^0$ m < 0.

THEOREM

The slope of the line passing through the points $A(x_1, y_1)$ and $B(x_2, y_2)$ is $\frac{y_2 - y_1}{x_2 - x_1}, x_1 \neq x_2$.

(If $x_1 = x_2$, then the line is vertical and hence its slope is not defined.)

Note: Two non vertical lines are parallel iff their slopes are equal.
Note: Two non vertical lines are perpendicular iff the product of their slopes is –1.

EQUATIONS OF A LINE

Equation of horizontal line

The equation of the horizontal line passing through the point (x_1, y_1) is $y = y_1$

Note 1: The equation of x - axis is y = 0.
Note 2: The equation of any line parallel to x -axis is y = k, k is an unknown constant.

Equation of vertical line

The equation of the vertical line passing through the point (x_1, y_1) is $x = x_1$.

Note 1: The equation of y - axis is x = 0.
Note 2: The equation of any line parallel to y -axis is x = k, k is an unknown constant.

POINT -SLOPE FORM

The equation of the line passing through $A(x_1, y_1)$ and having slope m is $y - y_1 = m(x - x_1)$.

Note: The equation of the line passing through the origin and having slope m is y = mx.

TWO POINT FORM

The equation of the line passing through $A(x_1, y_1)$, $B(x_2, y_2)$ is

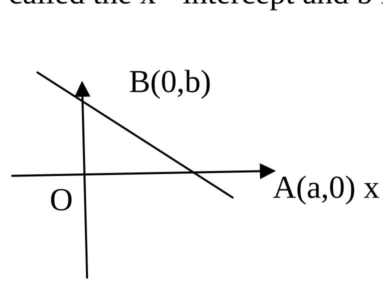
$$(y - y_1)(x_2 - x_1) = (x - x_1)(y_2 - y_1).$$

Note: The equation of the line passing through (x_1, y_1) and (x_2, y_2) $(x_1 \neq x_2)$ is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) \text{ (or) } \frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1}$$

INTERCEPTS:

If a line cuts x - axis at A(a, 0) and the y - axis at B(0, b) then a is called the x - intercept and b is called the y - intercept of the line.



SLOPE INTERCEPT FORM

The equation of the line having slope m and y - intercept c is $y = mx + c$.

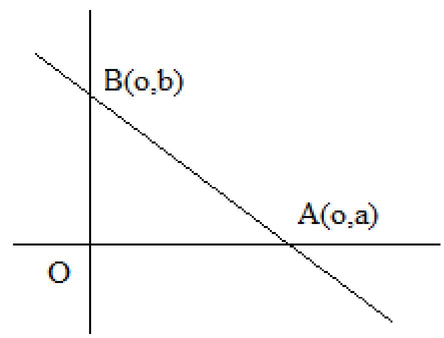
Note: The equation of the line having slope m and x - intercept a is $y = m(x - a)$.

INTERCEPT FORM

The equation of the line having x - intercept $a(\neq 0)$ and y - intercept $b(\neq 0)$ is $\frac{x}{a} + \frac{y}{b} = 1$.

Proof:

Since the x - intercept of the line is a, the line cuts the x - axis at (a, 0) and the y - intercept of the line is b, the line cuts the y - axis at (0, b).



Equation of the line is $y - 0 = \frac{b-0}{0-a}(x - a)$

$ay = -bx + ab \Rightarrow bx + ay = ab$

$\Rightarrow \frac{bx}{ab} + \frac{ay}{ab} = 1 \Rightarrow \frac{x}{a} + \frac{y}{b} = 1$

EXERCISE – 3(A)

1. Find the slope of the line x + y = 0 and x – y = 0.

Sol. Given line is x + y = 0 =>y = -x which is of the form y =mx.

Therefore slope m = -1

Equation of the line is x – y = 0 => y = x.

=> slope of the line is m =1.

2. Find the equation of the line containing the points (2, -3) and (0,-3).

Sol. Equation of the line is $(y-y_1)=\dfrac{(y_1-y_2)}{(x_1-x_2)}(x-x_1)$

$(y + 3) = \dfrac{(-3 + 3)}{(2 - 0)}(x-2)$

$2(y + 3) = 0 \Rightarrow y + 3 = 0$

3. Find the equation of the line containing the points (1, 2) and (1, -2).

Ans; x – 1 = 0

4. Find the angle which the straight line y = $\sqrt{3}x-4$ makes with the Y-axis.

Sol. Equation of the line is $y=\sqrt{3}x-4$

Slope m = $\sqrt{3}=\tan\dfrac{\pi}{6}$

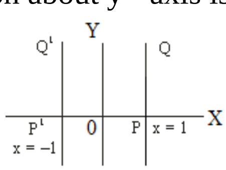
Angle made with x - axis = $\dfrac{\pi}{6}$

$\therefore$Angle made with y - axis = $\dfrac{\pi}{2}-\dfrac{\pi}{6}=\dfrac{\pi}{3}$

5. Write the equation of the reflection of the line x = 1 in the Y- axis.

Sol. Equation of PQ is x = 1

Reflection about y - axis is x = - 1 i.e., x + 1 = 0



6. Find the condition for the points (a, 0), (h, k) and (0,b)where $ab\neq 0$ to be collinear.

Sol. A(a, 0), B(h, k), C(0, b) are collinear.

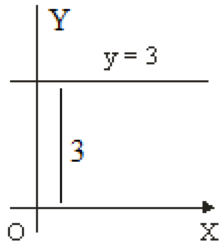
$\Rightarrow$ Slope of AB = Slope of AC

$\Rightarrow \dfrac{k-0}{h-a}=\dfrac{-b}{a}\Rightarrow ak=-bh+ab$

$\Rightarrow bh+ak=ab\Rightarrow \dfrac{h}{a}+\dfrac{k}{b}=1$

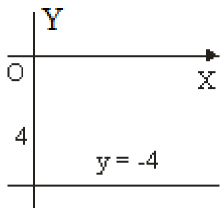
7. Write the equation of the straight lines parallel to X- axis is
i) at a distance of 3 units above the x-axis and
ii) at a distance of 4 units below the x- axis.

Sol. i)



Equation of the line which is at distance 3 units above the x- axis is y = 3

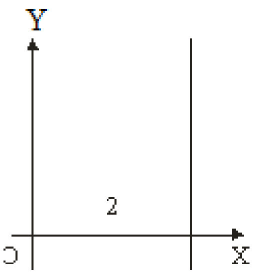
ii)



Equation of the line which is at distance 3 units below the x- axis is y = -3

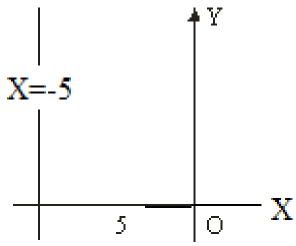
8. Write the equation of the straight line parallel to Y-axis and i) at a distance of 2 units from the Y - axis to the right of it.ii) at a distance of 5 units from the Y - axis to the left of it.

Sol. i)



Equation of the required line is x = 2 or x – 2 = 0

ii)



Equation of the required line is $x = -5 \Rightarrow x + 5 = 0$

II.

1. Find the slope of the straight line passing through the following points.

i) (-3, 8) (10, 5) ii) (3, 4) (7. -6) iii) (8, 1), (-1, 7) iv) (-p, q) (q. –p) (pq ≠ 0)

Sol. i) Slope = $\frac{y_1 - y_2}{x_1 - x_2} = \frac{8-5}{-3-10} = \frac{-3}{13}$

ii) Slope = $\frac{4+3}{3-7} = \frac{10}{-4} = \frac{-5}{2}$

iii) Slope = $\frac{1-7}{8+1} = \frac{-6}{9} = -\frac{-2}{3}$

iv) Slope = $\frac{q+p}{-p-q} = \frac{(p+q)}{-(p+q)} = -1$

2. Find the value of x, if the slope of the line passing through (2, 5) and (x, 3) is 2.

Sol. Slope of the line is 2.

But slope = $\frac{y_1 - y_2}{x_1 - x_2} \Rightarrow \frac{5-3}{2-x} = 2 \Rightarrow$ 2 = 2(2 – x) $\Rightarrow$ x = 2 – 1 = 1

3. Find the value of y if the line joining the points (3, y) and (2, 7) is parallel to the line joining the points (-1, 4) and (0, 6).

Sol. Let A (3, y), B (2, 7), P (-1, 4) and Q (0, 6) be the given points.

Slope of AB = $\frac{y-7}{3-2} = y - 7$ AND slope of PQ = $\frac{4-6}{-1-0} = \frac{-2}{-1} = 2$

lines AB and PQ are parallel $\Rightarrow$ their slopes are equal $\Rightarrow$ y – 7 = 2 $\Rightarrow$ y = 2 + 7 = 9

4. Find the slopes of the lines i) parallel to and ii) is perpendicular to the line passing through (6, 3) and (-4, 5).

Sol. Let A(6, 3) and B(-4, 5) $\Rightarrow$ Slope of AB = $\frac{3-5}{6+4} = \frac{-2}{10} = -\frac{1}{5}$

i)Slope of the line parallel to line AB = slope of AB = - $\frac{1}{5}$

ii) slope of a line perpendicular to AB = $\frac{-1}{\text{slope of AB}}$ = 5

5. Find the equation of the straight line which makes the following angles with positive X- axis and which passes through the points given below

i) $\frac{\pi}{4}$ and (0, 0) ii) $\frac{\pi}{3}$ and (1, 2) iii) 135º and (3, -2) iv) 150º and (-2, -1)

Sol. i) inclination $\theta = \frac{\pi}{4}$ and point is (0,0).

Slope of the line m= tan 45º = 1

Equation of the line is $y - y_1 = m(x - x_1)$

$y - 0 = 1(x-0)$ i.e., $y = x$ or $x - y = 0$

ii) **ANS:** $\sqrt{3}x - y + (2 - \sqrt{3}) = 0$

iii) inclination $\theta = 135^\circ$ and point (3,-2)

Slope $m = \tan 135^\circ = -\tan 45^\circ = -1$

Equation of the line is $y - y_1 = m(x - x_1)$

$\Rightarrow y + 2 = -1(x - 3) = -x + 3$ i.e., $x + y - 1 = 0$

iv) **ANS**: $x + \sqrt{3}y + (2 + \sqrt{3}) = 0$

6. Find the equation of the straight line passing through the origin and making equal angles with co- ordinates axes.

Sol. since require lines are passing through origin and making equal angles with the axes, therefore angles are 45^0 and 135^0.

i) $\theta = 45^0$ and point (0, 0)

slope $m = \tan 45^0 = 1$

$\Rightarrow$ equation of the line is $y = mx$ => $y = x$

ii) $\theta = 135^0$ and point (0, 0) $\Rightarrow$ slope $m = \tan 135^0 = -1$

$\Rightarrow$ equation of the line is $y = mx$ => $y = -x$

7. The angle made by a straight line with the positive X-axis in the positive direction and the Y-intercept cut of by it are given below. Find the equation of the straight line.

i) 60º, 3 **ii) 150º, 2** **iii) 45º, -2** **iv)** $\tan^{-1}\left(\frac{2}{3}\right), 3$

Sol. i) Slope $m = \tan 60^\circ = \sqrt{3}$ and y intercept is $c = 3$

Equation of the line is $y = mx + c$ $\Rightarrow y = \sqrt{3}x + 3$

$\sqrt{3}x - y + 3 = 0$

ii) **ANS** : $x + \sqrt{3}y - 2\sqrt{3} = 0$

iii) **ANS** : $x - y - 2$

iv) inclination $\theta = \tan^{-1}\left(\frac{2}{3}\right) \Rightarrow \tan\theta = \frac{2}{3}$

Slope $m = \tan\theta = \frac{2}{3}$ and y intercept $c=3$ Equation of the line is $y = mx + c$

$\Rightarrow y = \frac{2}{3}x + 3 \Rightarrow$ $3y = 2x + 9 \Rightarrow 2x - 3y + 9 = 0$

8. Find the equation of the straight line passing through (-4, 5) and cutting off equal non- zero intercepts on the co- 0rdinate axes?

Sol. Line is making equal intercepts, let the inter cepts be a,a

Equation of the line in the intercept from is

$\frac{x}{a}+\frac{y}{b}=1 \quad \Rightarrow \frac{x}{a}+\frac{y}{a}=1 \quad \Rightarrow x+y=a$

This line is passing through p (-4, 5) $\Rightarrow$ -4 + 5 = a $\Rightarrow$ a =1

Equation of the line is x + y = 1

9. Find the equation of the straight line passing through (-2, 4) and making non-zero intercepts whose sum is zero. ?

Sol. Let the intercepts be a,b.

Given a+b = 0 $\Rightarrow$ b = -a

Equation of the line in the intercept form is

$\frac{x}{a}+\frac{y}{b}=1 \Rightarrow \frac{x}{a}-\frac{y}{a}=1 \quad \Rightarrow x-y=a$

This line is passing through P(-2, 4)

$\therefore -2-4=a \Rightarrow a=-6$

Equation of the required line is x – y = -6$\Rightarrow$ x – y + 6 = 0

III.

1. Find the equation of the straight line passing through the point (3, -4) and making X and Y- intercepts which are in the ratio 2 : 3. ?

Sol. Let the intercepts be a,b.

Given $\frac{a}{b}=\frac{2}{3} \Rightarrow b=\frac{3a}{2}$

Equation of the line in the intercept form is

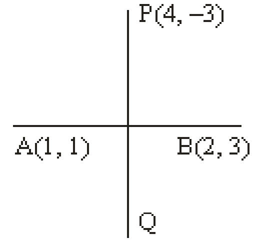
$\frac{x}{a}+\frac{y}{b}=1 \Rightarrow \frac{x}{a}+\frac{2y}{3a}=1 \Rightarrow$ 3x + 2y = 3a

This line passes through p(3, -4) $\Rightarrow$ 9 – 8 = 3a$\Rightarrow$3a = 1

Equation of the required line is 3x + 2y = 1 3x + 2y – 1 = 0

2. Find the equation of the straight line passing through the point (4, -3) and perpendicular to the line passing through the points (1, 1) and (2, 3).

Sol. Let A(1, 1), B(2, 3)



Slope of AB is m = $\frac{1-3}{1-2}=\frac{-2}{-1}=2$

Since require line is perpendicular to AB,

$\Rightarrow$ Slope of required line $= -\frac{1}{m}=-\frac{1}{2}$

Require line is passing through p(4, -3)

$\Rightarrow$Equation of the line is $y-y_1 = m(x-x_1)$

$\Rightarrow y+3=-\frac{1}{2}(x-4) \Rightarrow 2y+6=-x+4 \Rightarrow x+2y+2=0$

3. Show that the following sets of points are collinear and find the equation of the line L containing them.

i) (-5 1), (5, 5), (10, 7) ii) (1, -3), (-2, -6), (2, 6) iii) (a, b + c), (b, c + a), (c, a + b)

Sol. i) Given points A(-5, 1), B(5, 5), C(10, 7)

Equation of AB is

$(y-y_1)(x_1-x_2)=(x-x_1)(y_1-y_2)$

$(y-1)(-5-5)=(x+5)(1-5)$

$\Rightarrow$ -10 + 10 = -4x – 20

$\Rightarrow$4x – 10y + 30 = 0

or 2x – 5y + 15 = 0

substituting c (10, 7),

2x – 5y + 15 = 2. 10 - 5.7 + 15

= 20 – 35 + 15 = 0

Therefore C is a point on the line AB.

A, B, C are collinear and Equation of the line containing them is 2x – 5y + 15 = 0

ii) **ANS:** 3x – y = 0

iii) Given points A(a, b + c), B(b, c + a), C(c, a + b)

Equation of AB is (y –(b + c))(a – b)=(x – a)(b +c – c –a)

$\Rightarrow$ (y – b – c) (a – b) = - (a – b) (x – a)

$\Rightarrow$ x + y – (a + b+ c) = 0

Substituting C(c, a + b) in above equation

$c + a + b - a - b - c = 0 \Rightarrow$ C lies on AB

Therefore, A, B, C are collinear.

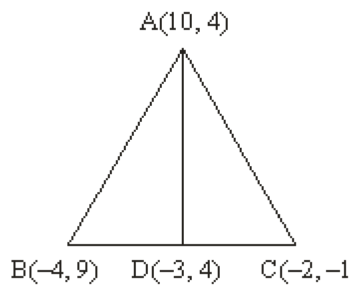
Equation of the line containing them is $x + y = a + b + c$

4. A(10, 4), B(-4, 9) and C(-2, -1) are the vertices of a triangle ABC. Find the equations of i) $\overleftrightarrow{AB}$ ii) the median through A iii) the altitude through B IV) the perpendicular bisector of AB

Sol. i) vertices of the triangle are A(10, 4), B(-4, 9) ,C(-2, -1)

$\Rightarrow$ Equation of AB is $(y - 4)(10 + 4) = (x - 10)(4 - 9)$

$\Rightarrow 14y - 56 = -5x + 50 \Rightarrow 5x + 14y - 106 = 0$



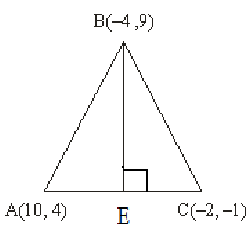
ii) let D be the mid-point of BC

$$\Rightarrow D = \left(\frac{-4-2}{2}, \frac{9-1}{2}\right) = (-3, 4)$$

Equation of AD is $(y - 4)(10 + 3) = (x + 3)(4 - 4)$

$\Rightarrow 13(y - 4) = 0 \Rightarrow y - 4 = 0 \Rightarrow y = 4$

iii)



Slope of AC is $m = \frac{4+1}{10+2} = \frac{5}{12}$

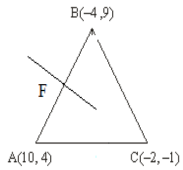
since BE is perpendicular to AC, Slope of BE $= \frac{-1}{m} = \frac{-12}{5}$

$\Rightarrow$ Equation of the altitude BE is $y - 9 = \frac{-12}{5}(x + 4)$

$\Rightarrow 5y - 45 = -12x - 48 \Rightarrow 12x + 5y + 3 = 0$

iv) let F be the mid-point of AB

$$\Rightarrow F = \left(\frac{10-4}{2}, \frac{4+9}{2}\right) = \left(3, \frac{13}{2}\right)$$



Slope of AB m = $\dfrac{4-9}{10+4} = \dfrac{-5}{14}$

$\Rightarrow$ Slope of the line perpendicular to AB $= \dfrac{-1}{m} = \dfrac{14}{5}$

Point is $= \left(3, \dfrac{13}{2}\right)$ and slope $\dfrac{14}{5}$

Equation of the perpendicular bisector AB is $y - \dfrac{13}{2} = \dfrac{14}{5}(x-3)$

$$\Rightarrow 5y - \frac{65}{2} = 14x - 42$$

$$\Rightarrow 14x - 5y - \left(\frac{65}{2} - 42\right) = 0$$

$$\Rightarrow 14x - 5y - \frac{19}{2} = 0$$

$$\Rightarrow 28x - 10y - 19 = 0$$