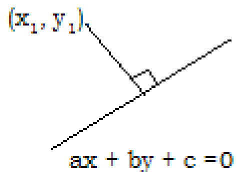
LENGTH OF THE PERPENDICULAR FROM A POINT TO A STRAIGHT LINE AND DISTANCE BETWEEN TWO PAPALLEL LINES

THEOREM

The perpendicular distance from a point P(x_1, y_1) to the line ax + by + c = 0 is

$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

Proof:



Let the axes be translated to the point P(x_1, y_1).

Let (X,Y) be the new coordinates of (x, y). Then $x = X + x_1$, $y = Y + y_1$ The transformed equation of the given line is

$$a(X + x_1) + b(Y + y_1) + c = 0$$

$$\Rightarrow aX + bY + (ax_1 + by_1 + c) = 0$$

The perpendicular distance from the new origin P to the line is $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$.

(from normal form) The perpendicular distance from a point P(x_1, y_1) to the line ax+ by + c = 0 is $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$.

DISTANCE BETWEEN PARALLEL LINES THEOREM

The distance between the two parallel lines ax + by + c_1 = 0 and ax + by + c_2 = 0 is

$$\frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}} .$$

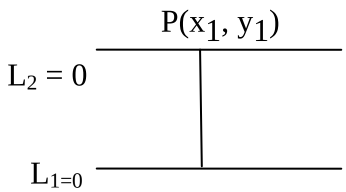
Proof:

Given lines are $ax + by + c_1 = 0$ --- (1)

$ax + by + c_2 = 0$ --- (2)

Let P(x_1, y_1) be a point on the line (2).

Then



$ax_1 + by_1 + c_2 = 0$

$ax_1 + by_1 = -c_2$.

Distance between the parallel lines = Perpendicular distance from P to line (1)

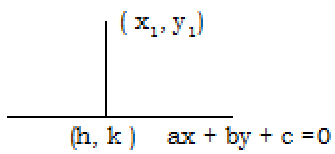
$$= \frac{|ax_1 + by_1 + c_1|}{\sqrt{a^2 + b^2}} = \frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}$$

FOOT OF THE PERPENDICULAR

THEOREM

If (h, k) is the foot of the perpendicular from (x_1, y_1) to the line ax + by + c = 0

(a0, b0) then $\frac{h - x_1}{a} = \frac{k - y_1}{b} = \frac{-(ax_1 + by_1 + c)}{a^2 + b^2}$.



Proof :

Let A = (x_1, y_1) P = (h, k)

P lies on ax + by + c = 0

ah + bk + c = 0

ah + bk = – c

Slope of $\overline{AP}$ is $\frac{k - y_1}{h - x_1}$

Slope of given line is $-\frac{a}{b}$

$\overline{AP}$ is perpendicular to the given line

$$\Rightarrow \left(\frac{k - y_1}{h - x_1}\right)\left(-\frac{a}{b}\right) = -1$$

$$\Rightarrow \frac{k - y_1}{b} = \frac{h - x_1}{a}$$

By the law of multipliers in ratio and proportion

$$\frac{h - x_1}{a} = \frac{k - y_1}{b} = \frac{a(h - x_1) + b(k - y_1)}{a^2 + b^2}$$

$$= \frac{ah + bk - ax_1 - by_1}{a^2 + b^2} = \frac{-ax_1 - by_1 - c}{a^2 + b^2}$$

$$= \frac{-(ax_1 + by_1 + c)}{a^2 + b^2}$$

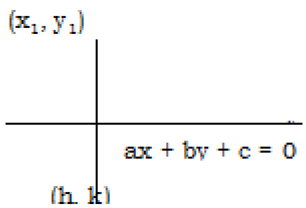
Hence $\frac{h - x_1}{a} = \frac{k - y_1}{b} = \frac{-(ax_1 + by_1 + c)}{a^2 + b^2}$

IMAGE OF A POINT

THEOREM

If (h, k) is the image of (x_1, y_1) w.r.t the line ax + by + c = 0 (a $\neq$ 0, b $\neq$ 0),

then $\frac{h - x_1}{a} = \frac{k - y_1}{b} = \frac{-2(x_1 + by_1 + c)}{a^2 + b^2}$ **.**



Proof:

Let A(x_1, y_1), B(h, k)

Mid point of is P = $\left(\frac{x_1 + h}{2}, \frac{y_1 + k}{2}\right)$

Since B is the image of A,therefore mid point P lies on ax + by + c = 0.

$\Rightarrow a\left(\frac{x_1 + h}{2}\right) + b\left(\frac{y_1 + k}{2}\right)$ +c =0

$\Rightarrow$ ax_1 + by_1 + ah + bk + 2c = 0

$\Rightarrow$ ah + bk = –ax_1 + by_1 – 2c.

Slope of $\overline{AB}$ is $\frac{k - y_1}{h - x_1}$

And Slope of given line is $-\frac{a}{b}$

$\overline{AB}$ is perpendicular to the given line

$\Rightarrow \left(\frac{k - y_1}{h - x_1}\right)\left(-\frac{a}{b}\right) = -1$

$\Rightarrow \frac{k - y_1}{b} = \frac{h - x_1}{a}$

By the law of multipliers in ratio and proportion

$$\frac{h-x_1}{a}=\frac{k-y_1}{b}=\frac{a\left(h-x_1\right)+b\left(k-y_1\right)}{a^2+b^2}$$

$$=\frac{ah+bk-ax_1-by_1}{a^2+b^2}$$

$$=\frac{-ax_1-by_1-2c-ax_1-by_1}{a^2+b^2}$$

$$=\frac{-2(ax_1+by_1+c)}{a^2+b^2}$$

Hence $\dfrac{h-x_1}{a}=\dfrac{k-y_1}{b}=\dfrac{-2(x_1+by_1+c)}{a^2+b^2}$

Note 1: The image of (x_1, y_1) w.r.t the line x = y is (y_1, x_1)

Note 2: The image of (x_1, y_1) w.r.t the line x + y = 0 is $(-y_1, -x_1)$

THEOREM

If the four straight lines ax + by + p = 0, ax + by + q = 0, cx + dy + r = 0 and cx + dy + s = 0 form a parallelogram. Then the area of the parallelogram so formed is

$$\left|\frac{(p-q)(r-s)}{bc-ad}\right|$$

Proof:

Let L_1 = ax + by + p = 0

L_2 = ax + by + q = 0

L_3 = cx + dy + r = 0

L_4 = cx + dy + s = 0

Clearly

$L_1 \| L_2$ and $L_3 \| L_4$. So L_1 and L_3 are nonparallel. Let be the angle between L_1 and L_3.

Let d_1 = distance between L_1 and L_2= $\dfrac{|p-q|}{\sqrt{a^2+b^2}}$

Let d_2 = distance between L_3 and L_4 = $\dfrac{|r-s|}{\sqrt{c^2+d^2}}$

Now $\cos\theta=\dfrac{|ac+bd|}{\sqrt{(a^2+b^2)(c^2+d^2)}}$ and $\sin\theta=\sqrt{\dfrac{(a^2+b^2)(c^2+d^2)-(ac+bd)^2}{(a^2+b^2)(c^2+d^2)}}$

$$=\frac{|bc-ad|}{\sqrt{(a^2+b^2)(c^2+d^2)}}$$

Now area of the parallelogram is $\dfrac{d_1d_2}{\sin\theta}=\left|\dfrac{(p-q))(r-s)}{bc-ad}\right|$

ANGLE BETWEEN TWO LINES THEOREM

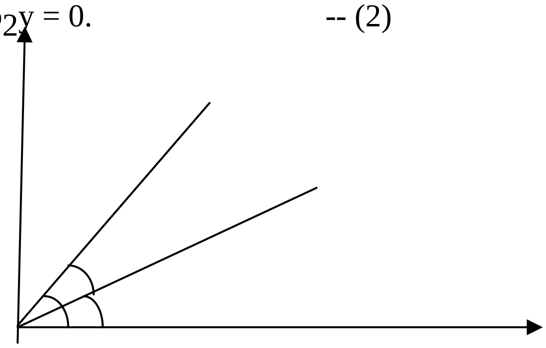
If θ is an angle between the lines $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$ then

$$\cos\theta = \pm\frac{a_1a_2 + b_1b_2}{\sqrt{a_1^2 + b_2^3}\sqrt{a_2^2 + b_2^2}}$$

Proof: The lines passing through the origin and parallel to the given lines are

$a_1x + b_1y = 0,$ -- (1)

$a_2x + b_2y = 0.$ -- (2)



Let θ_1, θ_2 be the inclinations of (1) and (2) respectively $(\theta_1 > \theta_2)$

Now θ is an angle between (1) and (2)

$$\theta = \theta_1 - \theta_2$$

$P(-b_1, a_1)$ satisfies eq(1), the point lies on (1)

Similarly, $Q(-b_2, a_2)$ lies on (2)

Let L, M be the projection of P,Q respectively on the x - axis.

$$\therefore \cos\theta_1 = \frac{OL}{OP} = \frac{-b_1}{\sqrt{a_1^2 + b_1^2}}, \sin\theta_1 = \frac{PL}{OP} = \frac{a_1}{\sqrt{a_1^2 + b_1^2}}$$

$$\therefore \cos\theta_2 = \frac{OM}{OQ} = \frac{-b_2}{\sqrt{a_2^2 + b_2^2}}, \sin\theta_2 = \frac{OM}{OQ} = \frac{a_2}{\sqrt{a_2^2 + b_2^2}}$$

$$\theta = \theta_1 - \theta_2$$

cos = cos $(\theta_1 - \theta_2)$

$= \cos\theta_1 \cos\theta_2 + \sin\theta_1 \sin\theta_2$

$$= \frac{(-b_1)}{\sqrt{a_1^2 + b_1^2}} \frac{(-b_2)}{\sqrt{a_1^2 + b_1^2}} + \frac{a_1}{\sqrt{a_1^2 + b_1^2}} \frac{a_2}{\sqrt{a_1^2 + b_1^2}}$$

$$= \frac{a_1a_1 + b_1b_2}{\sqrt{a_1^2 + b_1^2}\sqrt{a_2^2 + b_2^2}}$$

Note 1: If is the acute angle between the lines then

$$\cos\ \theta = \frac{|a_1 a_1 + b_1 b_2|}{\sqrt{a_1^2 + b_1^2}\sqrt{a_2^2 + b_2^2}}$$

Note 2: If is an angle between two lines, then is another angle between the lines.

Note 3: If is an angle between two lines are not a right angle then the angle between the lines means the acute angle between the lines.

Note 4: If θ is an angle between the lines $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$

then $\tan\theta = \frac{a_1 b_2 - a_2 b_1}{a_1 a_2 + b_1 b_2}$

Note 5: If is the acute angle between the lines

$a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$ then

$$\tan\theta = \left|\frac{a_1 b_2 - a_2 b_1}{a_1 a_2 + b_1 b_2}\right| = \left|\frac{a_1/b_1 - a_2/b_2}{(a_1 a_2)/\ \ (b_1 b_2) + 1}\right|$$

$= \left|\frac{(-a_1/b_1) - (-a_2/b_2)}{1 + (-a_1/b_1)(-a_2/b_2)}\right|$ where m_1, m_2 are the slopes of the lines.

THEOREM

The equation of the line parallel to ax + by + c = 0 and passing through (x_1, y_1) is $a(x - x_1) + b(y - y_1) = 0$.

Proof:

Slope of the given line is –a/b.

$\Rightarrow$ Slope of the required line is –a/b.(lines are parallel)

Equation of the required line is

$$y - y_1 = -\frac{a}{b}(x - x_1)$$

$b(y - y_1) = -a(x - x_1)$

$a(x - x_1) + b(y - y_1) = 0$.

Note 1: The equation of a line parallel to ax + by + c = 0 may be taken as ax + by + k = 0.

Note 2: The equation of a line parallel to ax + by + c = 0 and passing through the origin is ax + by = 0.

THEOREM

The equation of the line perpendicular to ax + by + c = 0 and passing through (x_1, y_1) is $b(x - x_1) - a(y - y_1) = 0$.

Proof:

Slope of the given line is –a/b. ⇒ Slope of the required line is b/a.

(since product of slopes = -1)

Equation of the required line is $y - y_1 = \frac{b}{a}(x - x_1)$

$a(y - y_1) = b(x - x_1)$

$b(x - x_1) - a(y - y_1) = 0.$

Note 1: The equation of a line perpendicular to ax + by + c = 0 may be taken as

bx – ay + k = 0

Note 2: The equation of a line perpendicular to ax + by + c = 0 and passing through the origin is bx – ay = 0.

EXERCISE -3 (d)

I. **Find the angle between the following straight lines.**

1. $y = 4 - 2x,\ y = 3x + 7$

Sol: given lines are

y = 4 – 2x ⇒ 2 x + y – 4 = 0and 3x – y + 7 = 0

Let θ be the angle between the lines, then

$$\cos\theta = \frac{|a_1a_2 + b_1b_2|}{\sqrt{a_1^2 + b_1^2}\sqrt{a_2^2 + b_2^2}}$$

$$= \frac{|2.3 + 1(-1)|}{\sqrt{4+1}\sqrt{9+1}} = \frac{5}{\sqrt{5}\sqrt{10}} = \frac{1}{\sqrt{2}}$$

$$\therefore \theta = \frac{\pi}{4}$$

2. **3x + 5y = 7, 2x – y + 4 = 0**

Sol. $\text{ans}: \theta = \cos^{-1}\left(\frac{1}{\sqrt{170}}\right)$

3. **y =** $-\sqrt{3}x+5, y=\frac{1}{\sqrt{3}}x-\frac{2}{\sqrt{3}}$

Sol. slope of 1st line is $m_1=-\sqrt{3}$

Slope of 2nd line is $m_2=\frac{1}{\sqrt{3}}$.

$m_1m_2=\left(-\sqrt{3}\right)\frac{1}{\sqrt{3}}=-1.$

The lines are perpendicular,hence angle between the lines is $\theta=\frac{\pi}{2}$

4. **ax + by = a + b, a(x – y) + b(x + y) = 2b**

Sol. given lines ax + by = a + b, (a + b) x + (- a + b) y =2b

let θ be the angle between the lines, then

$$\cos\theta=\frac{|a(a+b)+b(-a+b)|}{\sqrt{a^2+b^2}+\sqrt{(a+b)^2+(-a+b)^2}}$$

$$=\frac{|a^2+ab-ab+b^2|}{\sqrt{a^2+b^2}\sqrt{2(a^2+b^2)}}=\frac{a^2+b^2}{\sqrt{2}(a^2+b^2)}=\frac{1}{\sqrt{2}}$$

$$\therefore\theta=\frac{\pi}{4}$$

Find the length of the perpendicular drawn form the point given against the following straight lines.

5. **5x – 2y + 4 = 0, (-2, -3)**

Sol. Length of the perpendicular $=\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}=\frac{|5(-2)-2(-3)+4|}{\sqrt{25+4}}=\frac{|-10+10|}{\sqrt{29}}=0$

6. **3x – 4y + 10 = 0, (3, 4)**

Sol. Length of the perpendicular $=\frac{|3.3-4.4+10|}{\sqrt{9+16}}=\frac{3}{5}$

7. **x – 3y – 4 = 0, (0, 0)**

Sol. Ans; $\frac{4}{\sqrt{10}}$

Find the distance between the following parallel lines.

8. **3x – 4y = 12, 3x – 4y = 7**

Sol. Given lines are 3x – 4y = 12, 3x – 4y = 7

Distance between parallel lines $=\frac{|c_1-c_2|}{\sqrt{a^2+b^2}}=\frac{|-12+7|}{\sqrt{9+16}}=\frac{5}{5}=1$

9. 5x - 3y – 4 = 0, 10x – 6y – 9 = 0

Sol. Equations of the lines can be taken as 10x – 6y – 8 = 0 ,10x – 6y – 9 = 0

Distance between parallel lines $=\frac{|-8+9|}{\sqrt{100+36}}=\frac{1}{2\sqrt{34}}$

10. Find the equation of the straight line parallel to the line 2x + 3y + 7 = 0 and passing through the point (5, 4).

Sol. Given line is 2x + 3y + 7 =0

Equation of the parallel to 2x + 3y + 7 =0 is 2x + 3y = k.

This line is passing through P (5, 4)

$\Rightarrow$ 10 + 12 = k $\Rightarrow$ k = 22

Equation of the required line is 2x + 3y – 22 = 0

11. Find the equation of the straight line perpendicular to the line 5x – 3y + 1 = 0 and passing through the point (4, -3).

Sol. Equation of the given line is 5x –3 y + 1 = 0

Equation of the perpendicular to 5x –3 y + 1 = 0 is 3x + 5y + k = 0

This line is passing through P (4, -3)

$\Rightarrow$ 12 – 15 + k = 0 $\Rightarrow$ k = 3

Equation of the required line is 3x + 5y + 3 = 0

12. Find the value of k, if the straight lines 6x – 10y + 3 = 0 and k x – 5y + 8 = 0 are parallel.

Sol. Given lines are 6x – 10y + 3 = 0 and k x – 5y + 8 = 0.

Since the lines are parallel $\Rightarrow \frac{a_1}{a_2}=\frac{b_1}{b_2}$ $\Rightarrow$ -30 = -10 k $\Rightarrow$ k = 3

13. Fund the value of P, if the straight lines 3x + 7y – 1 = 0 and 7x – p y + 3 = 0 are mutually perpendicular.

Sol. Given lines are 3x + 7y – 1 = 0, 7x – p y + 3 = 0

Since the lines are perpendicular

$$\Rightarrow a_1a_2+b_1b_2=0 \Rightarrow 3.7+(-p)=0 \Rightarrow 7p=21 \Rightarrow p=3$$

14. Find the value of k, the straight lines y – 3kx + 4 = 0 and (2k – 1) x – (8k – 1) y = 6 are perpendicular.

Sol. Given lines are -3kx + y + 4 = 0

$(2k – 1)x – (8k – 1)y – 6 = 0$

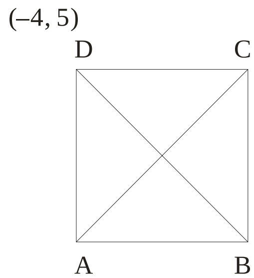
These lines are perpendicular $\Rightarrow a_1a_2 + b_1b_2 = 0$

$\Rightarrow -3k(2k-1)-1(8k-1)=0 \Rightarrow -6k^2+3k-8k+1=0$

$6k^2+5k-1=0 \Rightarrow (k+1)(6k-1)=0$

$k=-1$ or $1/6$

15. (-4, 5) is a vertex of a square and one of its diagonal is 7x – y + 8 = 0. Find the equation of the other diagonal.



Sol. let ABCD be the square.

Let the equation of the diagonal AC be 7x – y + 8 = 0

The point (-4, 5) is not satisfying the equation.

Let D=(-4, 5)

The other diagonal BD is perpendicular to AC.

Equation of BD is can be taken as x + 7y + k = 0

D (-4, 5) is a point on this line $\Rightarrow$ -4 + 35 + k = 0 $\Rightarrow$ k = 4 – 35 = - 31

Equation of BD is x + 7y – 31 = 0

II.

1. Find the equation of the straight lines passing through (1, 3) and

i)parallel to ii) perpendicular to the line passing through the points (3, -5) and (-6, 1).

Sol. Given points A (3, -5), B (-6, 1)

Slope of AB = $\frac{-5-1}{3+6}=\frac{-6}{9}=\frac{-2}{3}$

i) slope of the line parallel to AB is $\frac{-2}{3}$ equation of the line parallel to AB and passing through (1, 3) is

$y-3=\frac{-2}{3}(x-1) \Rightarrow 3y-9=-2x+2 \Rightarrow 2x+3y-11=0$

ii) slope of the line perpendicular to AB is 3/2 .

Equation of the line passing through (1, 3) and having slope 3/2 is $y-3=\frac{-2}{3}(x-1)$

$\Rightarrow$ 2x+3y-5=0.

2. The line $\frac{x}{a}-\frac{y}{b}=1$ meets the X – axis at P. Find the equation of the line perpendicular to this line at P.

Sol. given line is $\frac{x}{a}-\frac{y}{b}=1$------(1)

On x- axis y= 0 $\Rightarrow \frac{x}{a}-\frac{0}{b}=1 \Rightarrow x = a$

Point P=(a ,0)

Equation of the line perpendicular to (1) is $\frac{x}{b}+\frac{y}{a}=k$

This line is passing through P (a,0) $\Rightarrow \frac{a}{b}+0=k \Rightarrow k=a/b$

Equation of the line is $\frac{x}{b}+\frac{y}{a}=\frac{a}{b}$.

3. Find the equation of the line perpendicular to the line 3x + 4y + 6 = 0 and making intercept -4 on X – axis .

Sol. Given line is 3x + 4y + 6 =0.

Equation of the perpendicular to 3x + 4y + 6 =0 is 4x – 3y =k

$$\Rightarrow \frac{4x}{k}-\frac{3y}{k}=1 \Rightarrow \frac{x}{\left[\frac{k}{4}\right]}+\frac{y}{\left[\frac{-k}{3}\right]}=1$$

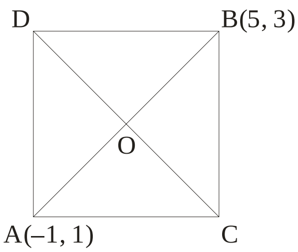
$$x-\text{intercept}=\frac{k}{4}=-4 \Rightarrow k=-16$$

Equation of the required line is 4x – 3y= –16 $\Rightarrow$ 4x – 3y + 16 = 0.

4. A (-1, 1), B (5,3) are opposite vertices of a square in the XY plane. Find the equation of the other diagonal (not passing through A, B) of a square.

Sol. A(-1,1) , B (5, 3) are opposite vertices of the square.

Slope of AB = $\frac{1-3}{-1-5}=\frac{-2}{-6}=\frac{1}{3}$



The other diagonal is perpendicular to AB

Slope of CD = $-\frac{1}{m}$ = -3

Let 'O' is the point of intersection of the diagonals then O = $\left(\frac{-1+5}{2},\frac{1+3}{2}\right)=(2,2)$

Diagonal CD is passing through O (2,2), Equation of CD is y – 2 = –3 (x – 2)
= –3x + 6 $\Rightarrow$ 3x + y – 8 = 0.

5. Find the foot of the perpendicular drawn from (4,1) upon the straight line 3x – 4y + 12=0.

Sol. Equation of the line is 3x - 4y + 12 = 0

If (h,k) is the foot of the perpendicular from (x_1,y_1) on the line ax + by + c = 0, then

$$\frac{h-x_1}{a}=\frac{k-y_1}{b}=-\frac{(ax_1+by_1+c)}{a^2+b^2}$$

$$\frac{h-4}{3}=\frac{k-1}{-4}=-\frac{(12-4+12)}{9+16}$$

$$\frac{h-4}{3}=\frac{k-1}{-4}=-\frac{20}{25}=-\frac{4}{5}$$

$$h-4=-\frac{12}{5}\Rightarrow h=4-\frac{12}{5}=\frac{20-12}{5}=\frac{8}{5}$$

$$k-1=\frac{16}{5}\Rightarrow k=1+\frac{16}{5}=\frac{5+16}{5}=\frac{21}{5}$$

$\therefore$ Foot of the perpendicular = $\left(\frac{8}{5},\frac{21}{5}\right)$

6. Find the foot of the perpendicular drawn from (3,0) upon the straight line 5x + 12y – 41 =0.

Sol. Ans: $\left(\frac{49}{13},\frac{24}{13}\right)$

7. x - 2y - 5 = 0 is the perpendicular bisector of the line segment joining the points A,B. If A = (-1,-3), find the co-ordinates of B.

Sol. If PQ is the perpendicular bisector of AB, then B is the image of A in the line PQ.

Equation of the line is x – 3y -5 = 0

Given point A = (-1,-3) . Here B is the image of A w.r.t x – 3y -5 = 0.

Let $B(h,k)$ be the image of A (-1,-3), then

$$\frac{h-x_1}{a}=\frac{k-y_1}{b}=\frac{-2(ax_1+by_1+c)}{a^2+b^2}$$

$$\frac{h+1}{1}=\frac{k+3}{-3}=\frac{-2(-1+9-5)}{1+9}$$

$$\frac{h+1}{1}=\frac{y_2+3}{-3}=-\frac{3}{5}$$

$$\frac{h+1}{1}=\frac{-3}{5}\Rightarrow h+1=-\frac{3}{5}$$

$$h=-1-\frac{3}{5}=-\frac{8}{5}\Rightarrow\frac{y_2+3}{-3}=\frac{-3}{5}$$

$$k+3=\frac{9}{5}\Rightarrow k=\frac{9}{5}-3=-6/5$$

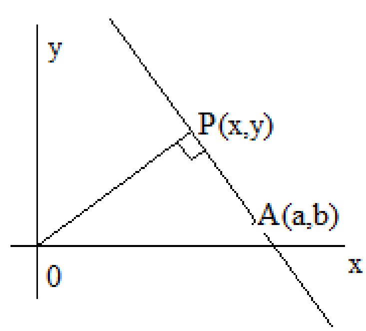
Co-ordinates of B are $\left(\frac{-8}{5},\frac{-6}{5}\right)$.

8. Find the image of the point straight line 3x + 4y – 1 = 0.

Sol. Ans: $\left(-\frac{7}{5},-\frac{6}{5}\right)$

9. Show that the distance of the point (6,-2) from the line 4x + 3y = 12 is half of the distance of the point (3,4) from the line 4x – 3y = 12.

10. Find the locus of foot of the perpendicular from the origin to a variable straight line which always passes through a fixed point (a,b).



Sol.

Let P(x,y) be the foot of the perpendicular from O(0,0) to the line.

Slope of OP = $\frac{y}{x}$

Line is passing through A(a,b).

Slope of AP = $\frac{y-a}{x-b}$

Line AP is perpendicular to OP, $\Rightarrow$ product of slopes = -1

$\Rightarrow \frac{y}{x}.\frac{y-b}{x-a} = -1 \Rightarrow y^2 - by = -\left(x^2 - ax\right)$

$\Rightarrow x^2 + y^2 - ax - by = 0$

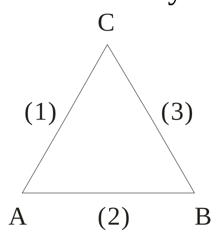
III.

1. Show that the lines x – 7y – 22 = 0, 3x + 4y + 9 = 0 and 7x + y – 54 = 0 form a right angled isosceles triangle.

Sol. Given lines x – 7y – 22= 0 -------(1)

3x + 4y + 9 = 0 -------(2)

7x + y – 54 = 0 -------(3)



Let 'A' be the angle between (1) ,(2)

cos A = $\frac{|a_1a_2 + b_1b_2|}{\sqrt{a_1^2 + b_1^2}\sqrt{a_2^2 + b_2^2}} = \frac{|3-28|}{\sqrt{1+49}\sqrt{9+16}} = \frac{25}{5\sqrt{2}.5} = \frac{25}{25\sqrt{2}} = \frac{1}{\sqrt{2}}$

$\Rightarrow A = 45°$

Let B be the angle between (2) ,(3) then cos B = $\frac{21+4}{\sqrt{9+16}\sqrt{49+1}} = \frac{25}{25\sqrt{2}} = \frac{1}{\sqrt{2}}$

$\Rightarrow B = 45°$

Let 'C' be the angle between (3), (1)

cosC = $\frac{7-7}{\sqrt{1+49}\sqrt{49+1}} = 0 \Rightarrow \quad C = 90°$

Since $\underline{|A} = \underline{|B} = 45° \text{and} \underline{|C} = 90°$

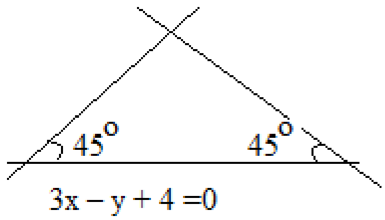
$\therefore$ Given lines form a right angled isosceles triangle.

2. Find the equation of the straight lines passing through the point (-3,2) and making an angle of $45°$ with the straight line 3x – y + 4 =0.

Sol. Given point P (-3,2)

Given line 3x – y + 4 = 0 ------(1)

Slope $m_1 = -\frac{a}{b} = 3$



Let m be the slope of the required line.

Then $\tan 45° = \frac{m-3}{1+3m}$

$\Rightarrow \left|\frac{m-3}{1+3m}\right| = 1 \Rightarrow \frac{m-3}{1+3m} = 1$

$\Rightarrow$ m-3 = 1+ 3m $\Rightarrow$ 2m = -4 or m = -2

$\Rightarrow \frac{m-3}{1+3m} = -$ $\Rightarrow$ m – 3 = - 1 – 3m

$\Rightarrow$ 4m = 2 $\Rightarrow$ m= ½

case (1) m = - 2 and point (-3,2)

Equation of the line is

y -2 = -2(x + 3) = -2x – 6 $\Rightarrow$ 2x + y + 4 = 0

case (2) $m = \frac{1}{2}$,point (-3,2)

Equation of the line is

$y - 2 = \frac{1}{2}(x+3) \Rightarrow 2y - 4 = x + 3$ $\Rightarrow$ x -2y + 7 = 0

3. Find the angle of the triangle whose sides are x + y – 4 = 0, 2x + y – 6 = 0, 5x + 3y – 15 = 0.

Sol. Ans: $\cos^{-1} = \left(\frac{4}{\sqrt{17}}\right)$, $\cos^{-1}\left(\frac{13}{\sqrt{170}}\right)$, $\cos^{-1}\left(\frac{3}{\sqrt{10}}\right)$

4. Prove that the foot of the perpendiculars from the origin on the lines x + y = 4, x + 5y = 26 and 15x – 27y = 424 are collinear.

5. Find the equation of the lines passing through the point of intersection of the lines 3x + 2y + 4 = 0, 2x + 5y = 1 and whose distance form (2, -1) is 2.

Sol. Equation of the lines passing through the point of intersection of the line

$L_1 \equiv 3x + 2y + 4 = 0, L_2 \equiv 2x + 5y - 1 = 0$ is

$L_1 + \lambda L_1 = 0$

$(3x + 2y + 4) + \lambda\ (2x + 5y - 1) = 0$

$\Rightarrow (3+2\lambda)x + (2+5\lambda)y + (4-\lambda) = 0$ -----(1)

Given distance from (2, -1) to (1) = 2

$$\frac{|3+2\lambda)2 + (2+5\lambda)(-1) + (4-\lambda)|}{\sqrt{(3+2\lambda)^2 + (2+5\lambda)^2}} = 2$$

$$\Rightarrow \frac{|-2\lambda + 8|}{\sqrt{(3+2\lambda)^2 + (2+5\lambda)^2}} = 2$$

$$\Rightarrow (-\lambda+4)^2 = 9 + 4\lambda^2 + 12\lambda + 4 + 25\lambda^2 + 20\lambda$$

$$\Rightarrow 28\lambda^2 + 40\lambda - 3 = 0$$

$$\Rightarrow 28\lambda^2 - 2\lambda + 42\lambda - 3 = 0$$

$$\Rightarrow (2\lambda+3)(14\lambda-1) = 0 \Rightarrow \lambda = \frac{1}{14}, \lambda = -\frac{3}{2}$$

From (1)

If $\lambda = \frac{1}{14}$, then equation of the line is $4x + 3y + 5 = 0$

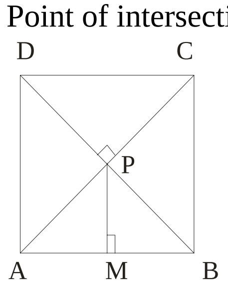
If $\lambda = -\frac{3}{2}$, then equation of the line is

y – 1 = 0.

6. Each side of a square is of length 4 units. The center of the square is (3, 7) and one of its diagonals is parallel to y = x. Find the Co-ordinates of its vertices.

Sol. Let ABCD be the square. Side AB =4

Point of intersection of the diagonals is the center P(3, 7)



From P drawn PM $\perp$ AB. Then M is midpoint of AB

$\therefore$ AM = MB = PM = 2

Since a diagonal is parallel to y = x, its sides are parallel to the co-ordinate axes.

M(3, 5)

$\Rightarrow$ A (3-2, 5), B(3+2, 5), C(3+5, 7+2),D(3-2,7+2)