CIRCLES

1. Equation of the circle passing through the points (0, 0), (a, 0), (0, b) is $x^2 + y^2 - ax - by = 0$
2. Equation of the circle passing through origin and making intercepts a and b on X an Y axes respectively is $x^2 + y^2 - ax - by = 0$
3. If x_1, x_2 are the roots of $x^2 + ax + b = 0$ and y_1, y_2 are the roots of $y^2 + cy + d = 0$ then the equation of the circle having $(x_1, y_1), (x_2, y_2)$ as ends of a diameter is $(x^2 + ax + b) + (y^2 + cy + d) = 0$.
4. If (x_1, y_1) is one end of a diameter of the circle $x^2 + y^2 + 2gx + 2fy + c = 0$, the other end is $(-2g - x_1, -2f - y_1)$.
5. The vertices of the square which is inscribed in the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ of radius r and whose sides are parallel to the coordinate axes are $\left(-g \pm \frac{r}{\sqrt{2}}, -f \pm \frac{r}{\sqrt{2}}\right)$.
6. Shortest distance from $P(x_1, y_1)$ to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is $|CP - r|$.
7. Longest distance from $P(x_1, y_1)$ to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is $CP + r$.
8. The area of the triangle formed by the tangent at (x_1, y_1) on the circle $x^2 + y^2 = a^2$ with the coordiante axes is $\frac{a^4}{2|x_1 y_1|}$
9. The area of the triangle formed by the pair of tangents drawn from $P(x_1, y_1)$ to the circle S = 0 and its chord of contact is $\frac{rS_{11}^{3/2}}{S_{11}+r^2}$.
10. Locus of the point of intersection of perpendicular tangents to
    (i) $x^2 + y^2 = a^2$ is $x^2 + y^2 = 2a^2$
    (ii) $x^2 + y^2 + 2gx + 2fy + c + 0$ is $x^2 + y^2 + 2gx + 2fy + c = g^2 + f^2 - c$.(i.e. $S = r^2$)
11. Locus of the point of intersection of perpendicular tangents one each to the circle
    (i) $x^2 + y^2 = a^2$, $x^2 + y^2 = b^2$ is $x^2 + y^2 = a^2 + b^2$.
    (ii) $x^2 + y^2 + 2gx + 2fy + c = 0$, $x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$ is $x^2 + y^2 + 2gx + 2fy + c = g_1^2 + f_1^2 - c_1$.
12. Length of the tangent from a poin on the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ to the circle $x^2 + y^2 + 2g'x + 2f'y + c' = 0$ is $\sqrt{c' - c}$.
13. If two lines $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$ cut the coordinate axes at four concyclic points, then $a_1a_2 = b_1b_2$, Equation of the circle passing through these concyclic points is $(a_1x+b_1y+c_1)(a_2x + b_2y + c_2) - (a_1b_2 + a_2b_1)xy = 0$
14. The equation of the circumcircle of the triangle formed by the line $ax + by + c = 0$ with the coordinate axes is $ab(x^2+y^2) + c(bx + ay) = 0$.
15. The area of the quadrilateral formed by the tangents from (x_1, y_1) to the circle S = 0 of radius r, with a pair of radii joining the points of contact of these tangents is $r\sqrt{S_{11}}$.
16. Locus of midpoints of chords of the circle $x^2 + y^2 = a^2$ subtending a right angle at the centre of the circle is $x^2 + y^2 = a^2/2$.
17. If A, B are conjugate points with respect to $x^2 + y^2 = a^2$ then $OA^2 + OB^2 = AB^2 + 2a^2$.
18. If A, B are conjugate points with respect to $S = x^2 + y^2 + 2gx + 2fy + c = 0$ and t_1, t_2 are the lengths of tangents from A, B to S = 0, then $AB^2 = t_1^2 + t_2^2$.
19. The condition that the pair of tangents drawn from the origin to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ may be at right angles is $g^2 + f^2 = 2c$.

20. The condition that the pair of tangents drawn from (g, f) to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ may be right angle is $g^2 + f^2 + c = 0$.

21. The length of the chord joining the points θ_1, θ_2 on the circle $x^2 + y^2 = a^2$ is $2a\left|\sin\left(\frac{\theta_1 - \theta_2}{2}\right)\right|$.

22. If two circles touch externally and $r_1 : r_2 = 1 : 3$ or $3 : 1$, then the triangle formed by the common tangents to the circles is an equilateral triangle.

23. The condition that the two circles touch each other

    (i) $x^2 + y^2 + 2ax + c = 0$, $x^2 + y^2 + 2by + c = 0$ is $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c}$.

    (ii) $x^2 + y^2 + 2g_1 x + 2f_1 y = 0$, $x^2 + y^2 + 2g_2 x + 2f_2 y = 0$ is $g_1 f_2 = g_2 f_1$.

    (iii) $x^2 + y^2 + 2ax + 2by + c = 0$, $x^2 + y^2 + 2bx + 2ay + c = 0$ is $(a + b)^2 = 2c$.

24. Equation of the circle with centre (a, b) and touching

    (i) X-axis is $x^2 + y^2 - 2ax - 2by + a^2 = 0$

    (ii) Y-axis is $x^2 + y^2 - 2ax - 2by + b^2 = 0$

25. Equation of the circle with radius 'r' touching both the axes is $x^2 + y^2 \pm 2rx \pm 2ry + r^2 = 0$.

26. The inverse point of P w.r.t. the circle is the foot of the perpendicular of C or P on the polar of P w.r.t. the circle.

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SPHERE

1. Equation of the sphere concentric with $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$ is in the form $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + k = 0$.

2. For the sphere $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$,

   (i) Length of X-intercept = $2\sqrt{u^2 - d}$.

   (ii) Length of Y-intercept = $2\sqrt{v^2 - d}$.

   (iii) Length of Z-intercept = $2\sqrt{w^2 - d}$.

3. The sphere $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$, $(d > 0)$ touches

   i) X-axis if $u^2 = d$

   ii) Y-axis if $v^2 = d$

   iii) Z-axis if $w^2 = d$

   iv) all the three axes if $u^2 = v^2 = w^2 = d$.

4. The smallest radius of the sphere passing through the points (1, 0, 0), (0, 1, 0), (0, 0, 1) is $\sqrt{\frac{2}{3}}$

5. The number of spheres of radius r touching the coorinate planes is 8.

6. A sphere of constant radius k passes through the origin and meets the coordinate axes in A, B, C repectively. The centroid of the triangle ABC lies on $9(x^2 + y^2 + z^2) = 4k^2$.

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SYSTEM OF CIRCLES

1. Angle between circles $\frac{S}{r}+\frac{S'}{r'}=0$ and $\frac{S}{r}-\frac{S'}{r'}=0$ is 90^0.
2. The centres of the circles which cuts two given circles orthogonally lies on their radical axis.
3. If n circles are given, then
   (i) the maximum number of radical axes taken two circles at a time is ${}^nC_2$.
   (ii) the maximum number of radical centres taken three circles at a time is ${}^nC_3$.
4. Length of the common chord of the circles.

   i) $x^2 + y^2 + 2hx = 0$, $x^2 + y^2 + 2ky = 0$ is $\frac{2hk}{\sqrt{h^2+k^2}}$.

   ii) $(x - a)^2 + (y - b)^2 = c^2$, $(x - b)^2 + (y - a)^2 = c^2$ is $\sqrt{4c^2-2(a-b)^2}$.

   iii) $x^2 + y^2 + 2gx + c = 0$, $x^2 + y^2 + 2fy - c = 0$ is $\sqrt{\frac{(g^2-c)(f^2+c)}{g^2+f^2}}$.

   iv) $x^2 + y^2 + ax + by + c = 0$, $x^2 + y^2 + bx + ay + c = 0$ is $\sqrt{\frac{(a+b)^2-8c}{2}}$.

5. Two circles of radii r_1 and r_2 intersect at right angles, then the length of the common chord is $\frac{2r_1r_2}{\sqrt{r_1^2+r_2^2}}$.
6. If r_1, r_2 are radii of two circles and θ is the acute angle of intersection, then the length of common chord is $\frac{2r_1r_2\sin\theta}{\sqrt{r_1^2+r_2^2-2r_1r_2\cos\theta}}$.
7. The radical centre of three circles described on three sides of a triangle as diameter is the orthocentre.
8. If A, B, C are the centres of three circles touching mutually, then the radical centre of the circles is the incentre of ΔABC.
9. The radical centre of three circles which cut each other orthogonally is the orthocentre.
10. If A, B, C are centres of three circles of equal radii which do not touch or intersect each other, then their radical centre is circumcentre of ΔABC.
11. If r_1, r_2 are radii of two circles and d distance between their centres, then

    i) Length of the direct common tangent is $\sqrt{d^2-(r_1-r_2)^2}$.

    ii) Length of the transvers common tangent is $\sqrt{d^2-(r_1+r_2)^2}$

12. Radical axis is the perpendicular bisector of the line segment joining limiting points of the coaxial system.
13. The two limiting points are inverse points with respect to every circle of the coaxial system.

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PARABOLA

1. Equation of the parabola whose axis is parallel to
   i) X-axis is $x = \ell y^2 + my + n$ or $(y - \beta)^2 = \pm 4a(x - \alpha)$.
   ii) Y-axis is $y = \ell x^2 + mx + n$ or $(x - \alpha)^2 = \pm 4a(y - \beta)$.
2. Equation of axis of the parabola S = 0 is
   i) $\frac{\partial S}{\partial x} = 0$ if x^2 is present ii) $\frac{\partial S}{\partial y} = 0$ if y^2 is present.
3. Equation of tangent to the parabola $y^2 = 4a(x + a)$ is $y = m(x + a) + \frac{a}{m}$.
4. Locus of the point, of intersection of two tangents to the parabola which make complementary angles with the axis of the parabola is its tangent at the vertex.
5. Locus of the point of intersection of pair of tangents to the parabola $y^2 = 4ax$ which make an angle α is $y^2 - 4ax = (x + a)^2 \tan^2\alpha$.
6. Length of latursrectum of (i) $x = ay^2 + by + c$ is $1/|a|$.
   (ii) $y = ax^2 + bx + c$ is $1/|a|$.
7. Two straight lines are perpendicular to each other. One of them touches the parabola $y^2 = 4a(x + a)$ and the other touches $y^2 = 4b(x + b)$. Locus of the point of intersection of two lines is $x + a + b = 0$.
8. Locus of poles of chords of $y^2 = 4ax$, which subtend a right at its vertex is $x + 4a = 0$.
9. If θ is the angle between the tangents from $P(x_1, y_1)$ to
   i) $y^2 = 4ax$, then $\theta = \tan^{-1}\left(\frac{\sqrt{S_{11}}}{x_1 + a}\right)$
   ii) $x^2 = 4ay$, then $\theta = \tan^{-1}\left(\frac{\sqrt{S_{11}}}{y_1 + a}\right)$
10. Equation of normal having slope 'm' to $y^2 = 4ax$ is $y = mx - 2am - am^3$ and the foot of the normal is $(am^2, -2am)$.
11. Equation of common tangent to $y^2 = 4ax$ and $x^2 = 4by$ is $a^{1/3}x + b^{1/3}y + (ab)^{2/3} = 0$.
12. The area of the triangle formed by three points on the parabola $y^2 = 4ax$ is $\frac{1}{8a}|(y_1 - y_2)(y_2 - y_3)(y_3 - y_1)|$ is twice the area of the triangle formed by their tangents $\frac{1}{16a}|(y_1 - y_2)(y_2 - y_3)(y_3 - y_1)|$.
13. A focal chord of $y^2 = 4ax$ meets it at P and Q. If S is focus then $\frac{1}{SP} + \frac{1}{SQ} = \frac{1}{a}$
14. If the point 't' is one extremity of a focal chord of the parabola $y^2 = 4ax$, then the length of the chord is $a\left(t + \frac{1}{t}\right)^2$.
15. The length of focal chord of the parabola $y^2 = 4ax$ which makes an angles θ with its axis is $4a \operatorname{cosec}^2\theta$.
16. The tangents at the points $(at_1^2, 2at_1)$, $(at_2^2, 2at_2)$ on the parabola $y^2 = 4ax$ are at right angles if $t_1t_2 = -1$.
17. The normals at the points $(at_1^2, 2at_1)$, $(at_2^2, 2at_2)$ on the parabola $y^2 = 4ax$ are at right angles if $t_1t_2 = -1$.
18. The orthocentre of the triangle formed by three tangents to $y^2 = 4ax$ lies on its directrix.

19. If the normal at 't' on $y^2 = 4ax$ substends a right angle
i) at its focus then $t = \pm 2$
ii) at its vertex then $t = \pm\sqrt{2}$

20. If t_1, t_2, t_3 are the feet of the three normals drawn from the point $P(x_1, y_1)$ to the parabola $y^2 = 4ax$, then
i) $t_1 + t_2 + t_3 = 0$ ii) $t_1t_2 + t_2t_3 + t_3t_1 = \frac{2a-x_1}{a}$ 3) $t_1t_2t_3 = \frac{y_1}{a}$.

21. The area of the triangle formed by the tangents and the chord of contact from (x_1, y_1) to the parabola $y^2 = 4ax$ is $S_{11}^{3/2}/2a$.

22. For any conic if ℓ is the semilatusrectum and d is the perpendicular distance from focus on the directrix. Then $\frac{\ell}{d}$ = eceentricity.

23. The condition that the line $\ell x + my + n = 0$ to be a normal to the parabola $y^2 = 4ax$ is $a\ell^3 + 2a\ell m^3 + m^2n = 0$.

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ELLIPSE

1. If the length of the major axis is n times its minor axis for the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$, then $e = \frac{\sqrt{n^2-1}}{n}$

2. The condition that the line $\ell x + my + n = 0$ may be tangent to the ellipse $\frac{x_2}{a_2}+\frac{y_2}{b_2} = 1$ is $a^2\ell^2 + b^2m^2 = n^2$.

3. The condition that the line $\ell x + my + n = 0$ to be a normal to the hyperbola $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$ is $\frac{a^2}{\ell^2}+\frac{b^2}{m^2}=\frac{(a^2-b^2)^2}{n^2}$.

4. The product of the perpendiculars from the foci on any tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$ is b^2 (smaller square).

5. The tangent at any point on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$ meets the tangents at A and A' in L and M respectively, then $AL.A'M = b^2$.

6. C is the centre of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$ and L is an end of a latursrectum. If the normal at L meets the major axis in G, then $CG=ae^3$.

7. If the normal at an end of latusrectum of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$ of ecentricity e passes through one end of the minor axis, then $e^4 + e^2 = 1$.

8. If $\ell x + my + n = 0$ is a chord of an ellipse S = 0, then the midpoint of the chord is $\left(\frac{-a^2\ell n}{a^2\ell^2+b^2m^2}, \frac{-b^2mn}{a^2\ell^2+b^2m^2}\right)$

9. If tangent at α on an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}$ = 1 meets the auxiliary circle at two points which subtend a right angle at the centre. Then e = $\frac{1}{\sqrt{1+\sin^2\alpha}}$.
10. The sum of the eccentric angles of the feet of normals from a point to an ellipse is an odd multiple of π.
11. The sum of the eccentric angles of the four concylic points on an ellipse is an even multiple or π.
12. If θ is the angle between the tangents from (x_1, y_1) to the ellipse, then tan θ = $\frac{2ab\sqrt{S_{11}}}{|x_1^2+y_1^2-a^2-b^2|}$.
13. Area of the parallelogram formed by the tangents at the points whose eccentric angles are θ, $\theta+\frac{\pi}{2}$, $\theta+\pi$, $\theta+\frac{3\pi}{2}$ on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}$ = 1 is 4ab.
14. S, S' are foci and P is any point on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}$ = 1, then maximum area of $\Delta PSS'$ = abe.
15. Area of the quadrilateral $SBS'B'$ is 2ab.
16. Distance between the polars of foci of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}$ = 1 is the distance between the dirctrices $\frac{2a}{e}$.
17. Length of the double ordinate of $\frac{x^2}{a^2}+\frac{y^2}{b^2}$ = 1 which is conjugate to the directrix is length of the latus rectum.

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HYPERBOLA

1. A point A (x_1, y_1) is said to be an
   (i) external point to the hyperbola if $S_{11} < 0$.
   (ii) internal point to the hyperbola if $S_{11} > 0$.
   (iii) lies on the hyperbola if $S_{11} = 0$.
2. If e_1 and e_2 are eccentricities of a hyperbola and its conjugate hyperbola, then
   (i) $\frac{1}{e_1^2}+\frac{1}{e_2^2}=1$ or $e_1^2 + e_2^2 - e_1^2e_2^2 = 0$
   (ii) $ae_1 = be_2$
3. The equations of the hyperbola and its pair of asymptotes differ by a constant.
4. Asymptotes of a hyperbola passes through the centre of hyperbola.
5. The equation to the pair of asymptotes of the hyperbola $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ is $ax^2 + 2hxy + by^2 + 2gx + 2fy + k = 0$ where k is given by $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$
6. The condition that the line $\ell x + my + n = 0$ to be tangent to $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $a^2\ell^2 - b^2m^2 = n^2$.

7. The condition that the line $\ell x + my + n = 0$ to be normal to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $\frac{a^2}{\ell^2}-\frac{b^2}{m^2}=\frac{\left(a^2+b^2\right)^2}{n^2}$.

8. Centre of the hyperbola $S \equiv ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ is given by $\frac{\partial S}{\partial x}=0, \frac{\partial S}{\partial y}=0$.

9. The product of the perpendiculars from any point on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ to its asymptotes is $\frac{a^2b^2}{a^2+b^2}$.

10. The parametric equations of the rectangular hyperbola $xy = c^2$ are $x = ct$, $y = c/t$.

11. Equation of the tangent at (x_1, y_1) to $xy = c^2$ is $\frac{x}{x_1}+\frac{y}{y_1}=2$.

12. Equation of the normal at (x_1, y_1) to $xy = c^2$ is $\frac{x}{y_1}-\frac{y}{x_1}=\frac{x_1}{y_1}-\frac{y_1}{x_1}$ or $xx_1 - yy_1 = x_1^2 - y_1^2$.

13. Equation of the chord joining the points (x_1, y_1), (x_2, y_2) to $xy = c^2$ is $\frac{x}{x_1+x_2}+\frac{y}{y_1+y_2}=1$.

14. If $\ell x + my + n = 0$ is a chord of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, then the midpoint of the chord is $\left(\frac{-a^2\ell n}{a^2\ell^2-b^2m^2}, \frac{b^2mn}{a^2\ell^2-b^2m^2}\right)$.

15. If θ is the angle between the tangents from (x_1, y_1) to the hyperbola $S = 0$ then $\tan\theta = \frac{2ab\sqrt{-S_{11}}}{x_1^2-y_1^2-a^2+b^2}$.

16. The product of the perpendiculars from foci to any tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is b^2.

17. The orthocentre of the triangle formed by any three points on a rectangular hyperbola lies on the hyperbola itself.

18. The area of the triangle formed by any tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ with its asymptotes is ab.

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POLAR COORDINATES

1. The area of the triangle whose vertices are $(a, 0)$, $(2a, \theta + \frac{\pi}{3})$ and $(3a, \theta + 2\frac{\pi}{3})$ is $\frac{5\sqrt{3}}{4}a^2$.

2. Polar equation of the straight line with intercepts 'a' and 'b' on the rays $\theta = 0$, $\theta = \frac{\pi}{2}$ respectively is $r(b\cos\theta + a\sin\theta) = ab$.

3. If the vectorial angle of a point P on the line joining (r_1, θ_1), (r_2, θ_2) is $\frac{\theta_1+\theta_2}{2}$, then the radial distance of P is $\frac{2r_1r_2}{r_1+r_2}\cos\frac{\theta_1+\theta_2}{2}$.

4. The point of intersection of the lines $\frac{k}{r} = \cos\theta + \cos(\theta-\alpha)$ and and $\frac{k}{r} = \cos\theta + \cos(\theta-\beta)$ is $\left(\frac{k}{2\cos\frac{\alpha}{2}\cos\frac{\beta}{2}}, \frac{\alpha+\beta}{2}\right)$.

5. The lines $a_1\cos\theta + b_1\sin\theta = \frac{c_1}{r}$, $a_2\cos\theta + b_2\sin\theta = \frac{c_2}{r}$ are
   (i) parallel then $a_1b_2 = a_2b_1$ (ii) perpendicular then $a_1a_2 + b_1b_2 = 0$.
6. The lines $r\cos(\theta-\alpha) = p$, $r\sin(\theta-\alpha) = q$ are perpendicular to each other.
7. The equation of the line passing through the pole and at right angles to the line $r\cos(\theta-\alpha) = p$ is $\theta = \alpha$.
8. The equation of the line passing through the pole and parallel to the line $r\cos(\theta-\alpha) = p$ is $\theta = \frac{\pi}{2} + \alpha$.
9. The condition for the lines $r\cos(\theta-\alpha) = a$, $r\cos(\theta-\beta) = b$, $r\cos(\theta-\gamma) = c$ to be concurrent is $\sum a\sin(\beta-\gamma) = 0$.
10. The angle between the circles $r = a\cos(\theta-\alpha)$ and $r = b\cos(\theta-\beta)$ is $|\alpha-\beta|$.
11. The angle between the circles $r = a\cos(\theta-\alpha)$ and $r = b\sin(\theta-\alpha)$ is $\frac{\pi}{2}$.
12. If PQ is a chord of the conic with focus S and semilatusrectum ℓ, then $\frac{1}{SP}+\frac{1}{SQ}=\frac{2}{\ell}$.
13. If PSP' and QSQ' are two perpendicular focal chords of a conic, then
    (i) $\frac{1}{SP.SP'}+\frac{1}{SQ.SQ'}=\frac{2-e^2}{\ell^2}$ (ii) $\frac{1}{SP+SP'}+\frac{1}{SQ+SQ'}=\frac{1}{PP'}+\frac{1}{QQ'}=\frac{2-e^2}{2\ell}$
14. Vertex of the parabola $\frac{\ell}{r}=1+\cos(\theta-\alpha)$ is $(\frac{\ell}{2}, \alpha)$.
15. A chord PQ of a conic $\ell/r = 1+e\cos\theta$ subtends a right angle at the focus S, then $\left(\frac{1}{SP}-\frac{1}{\ell}\right)^2+\left(\frac{1}{SQ}-\frac{1}{\ell}\right)^2=\frac{e^2}{\ell^2}$.

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QUADRATIC EXPRESSIONS

1. If α, β are the root of $ax^2 + bx + c = 0$, then
   (i) $\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$ (ii) $\alpha^3+\beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$
   (iii) $\alpha^4+\beta^4 = (\alpha^2+\beta^2)^2 - 2\alpha^2\beta^2$ (iv) $\alpha^5+\beta^5 = (\alpha^3+\beta^3)(\alpha^2+\beta^2) - (\alpha\beta)^2(\alpha+\beta)$
   (v) $\alpha^6+\beta^6 = (\alpha^3+\beta^3)^2 - 2(\alpha\beta)^3$
2. If the roots of $ax^2 + bx + c = 0$ are in the ratio m: n, then $mnb^2 = (m+n)^2ac$.

3. If a > 0, then $\sqrt{a+\sqrt{a+\sqrt{a+........\infty}}} = \frac{1+\sqrt{4a+1}}{2}$

4. $(a+\sqrt{b})^{x^2-k} + \left(a-\sqrt{b}\right)^{x^2-k} = 2a$ and $a^2 - b = 1$, then $x^2 - k = \pm 1$.

5. If x > 0, then the least value of $x + \frac{1}{x}$ is 2.

6. If a + b + c = 0, then the roots of $ax^2 + bx + c = 0$ are 1 and $\frac{c}{a}$.

7. If a + c = b, then the roots of ax^2 bx + c = 0 are -1 and $\frac{-c}{a}$.

8. If a = c, then the roots of $ax^2 + bx + c = 0$ are reciprocal to one another.

9. If the ratio of the roots of $ax^2 + bx + c = 0$ and $px^2 + qx + r = 0$ are the same $\frac{b^2}{ac} = \frac{q^2}{pr}$.

10. If the difference of the roots of $ax^2 + bx + c = 0$ is
(i) k, then $k^2a^2 = b^2 - 4ac$. (ii) unity, then $a^2 = b^2 - 4ac$.

11. If one root of $ax^2 + bx + c = 0$ is n^{th} power of the other, $(a^n c)^{1/n+1} + (a c^n)^{1/n+1} = -b$.

12. The condition that the expression $ax^2 + 2hxy + by^2 + 2gx + 2fy + 2fy + c$ can be resolved into product of two linear factors is $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$.

* * * * * * * *

THEORY OF EQUATIONS

1. The equation of lowest degree with rational coefficients having a root
(i) $\sqrt{a}+\sqrt{b}$ is $x^4 - 2(a+b)x^2 + (a-b)^2 = 0.$
(ii) $\sqrt{a}+\sqrt{b}$ is $x^4 - 2(a-b)x^2 + (a+b)^2 = 0.$

2. The condition that the roots of $ax^3 + bx^2 + cx + d = 0$ may be in
(i) A.P is $2b^3 + 27a^2d = 9abc$ (ii) G.P is $ac^3 = b^3d$ (iii) H.P is $2c^3 + 27ad^2 = 9bcd$.

3. The condition that one root of $ax^3 + bx^2 + cx + d = 0$ may be the sum of the other two roots is $b^3 + 8a^2d = 4abc$.

4. The condition that the product of two of the roots of $ax^3 + bx^2 + cx + d = 0$ may be - 1 is a (a + c) + d(b + d) = 0.

5. If the product of two roots of $x^4 + px^3 + qx^2 + rx + s = 0$ is equal to the product of the other two roots, then $p^2 s = r^2$.

6. If the roots of $ax^3 + bx^3 + cx + d = 0$ are in (i) A.P. then mean root is $\frac{-b}{3a}$ (ii) H.P, then mean root is $\frac{-3d}{c}$.

7. The second term of $f(x) = p_0 x^n + p_1 x^{n-1} + p_2 x^{n-2} + ... + p_n = 0$ can be removed by diminishing the roots by 'h' where $h = \frac{-p_1}{np_0}$ and the required transformed equation is f (x + h) = 0.
In the above the concept, to remove 3rd term, h should satisfy the relation $\frac{n(n-1)}{2} p_0 h^2 + (n - 1) p_1 h + p_2 = 0$.

8. **Descarte's rule of signs:** An equation f(x) = 0
(i) cannot have more positive roots than the number of changes of sign in f(x).
(ii) can not have more negative roots than the number of changes of sign in f(-x).

9. If f(x) is polymonial such that f(a) and f(b) have oppostive signs, then one root of of f(x) = 0 must lie between a and b.

10. **Netwton's method:** Consider the equation $x^4 + p_1x^3 + p_2x^2 + p_3x + p_4 = 0$.
$S_1=\alpha+\beta+\gamma+\delta$ $S_2=\alpha^2+\beta^2+\gamma^2+\delta^2$ $S_3=\alpha^3+\beta^3+\gamma^3+\delta^3$
$S_{-1}=\alpha^{-1}+\beta^{-1}+\gamma^{-1}+\delta^{-1}$ $S_{-2}=\alpha^{-2}+\beta^{-2}+\gamma^{-2}+\delta^{-2}$ $S_1+p_1S_1+2p_2=0$
$S_2+p_1S_1+2p_2=0$
$S_3+p_1S_2+p_2S_1+3p_3= 0$
$S_4+p_1S_3+p_2S_2+p_3S_1+4p_4=0$ and so on.

MATRICES

1. **Idempotent matrix:** A square matrix A is said to be an idempotent matrix if $A^2 = A$. Eg $A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$
2. If A an idempotent matrix, then $|A| = 0$ or 1
3. If A and B are square matrices of same order such that AB = A and BA = B, then A and B are idempotent matrices.
4. **Involutory matrix:** A square matrix A is said to be an involutory matrix if $A^2 = I$.
Eg: Identity matrix is an involutory matrix
5. If A is an involutory matrix, then $|A| = \pm 1$.
6. Nilpotent matrix: A square matrix A is said to be a 'Nilpotent matrix' if there exists a positive integer n such that $A^n = O$. If n is the least postive integer such that $A^n = O$, then n is said to be the 'index' of the nilpotent matrix A.

Ex: $A = \begin{bmatrix} 1 & -3 & -4 \\ -1 & 3 & 4 \\ 1 & -3 & -4 \end{bmatrix}$ is an nilpotent matrix of index 2.
7. Every nilpotent matrix is a singular matrix.i.e $|A| = 0$.
8. **Orthogonal matrix:** A square matrix A is said to be 'orthogonal' if $AA^T=I$. Eg: $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$
9. If A is an orthogonal matrix, then $|A| = \pm 1$.
10. If AB = O, then A and B need not be null matrices. Eg: $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}=\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.
11. If AB = AC, then B need not be equal to C.
12. $AB = AC \Rightarrow B = C$ if A is a non - singular matrix.
13. If A is a matrix of order m x n, the $A I_n = I_m A = A$.
14. Every square matrix can be uniquely expressed as a sum of a symmetric matrix and a skew symmetric matrix. i.e. $A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$.
15. If A is a square matrix, then
(i) $A + A^T$. AA^T, A^TA are symmetric matrices (ii) $A - A^T$ is a skew symmetric matrix
16. If A, B are square symmetric matrices of the same order, then
(i) AB is a symmetric matrix if AB = BA (ii) AB - BA is a skew symmetric matrix.
17. If A is a symmetric matrix and $n \in N$, then A^n is symmetric.

18. If A is a skew symmetric matrix and
(i) n is a odd + ve integer, then A^n is skew symmetric.
(ii) n is an even + ve integer, then A^n is symmetric.
19. If A is a symmetric matrix or a skew symmetric matix, then A^2 is a symmetric matrix.
20. **Conjugate of a matrix:** The matrix obtained from a matrix A on replacing its elements by the corresponding conjugate complex numbers is called the conjugate of A and it is denoted by $\overline{A}$.

Eg: If A = $\begin{bmatrix} 1+i & 2-i \\ 4i & 7 \end{bmatrix}$, then $\overline{A}$ $\begin{bmatrix} 1-i & 2+i \\ -4i & 7 \end{bmatrix}$.

21. **Transposed conjugate of a matrix:** The transpose of the conjugate of a matrix A is called 'transposed conjugate of A and is denoted by A^θ of A*.

If A = $\begin{bmatrix} 1+i & 2-i \\ 4i & 7 \end{bmatrix}$ then A^θ = $\begin{bmatrix} 1-i & -4i \\ 2+i & 7 \end{bmatrix}$

22. **Hermitian matrix:** A square matrix A is said to be a 'Hermitian matrix' if A^θ =A. Eg: A = $\begin{bmatrix} 1 & 2+i \\ 2-i & 5 \end{bmatrix}$

23. **Skew Hermitian matrix:** A square matrix A is said to a 'Skew - Hermitian matrix' it A^θ = -A. Eg: A = $\begin{bmatrix} 0 & 2+i \\ -2+i & 0 \end{bmatrix}$

DETERMINANTS:

1. Determinant of a triangular matrix is the product of the elements of the principal diagonal of the matrix

$$\begin{vmatrix} a & p & q \\ o & b & r \\ o & o & c \end{vmatrix} = abc.$$

2. $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ = 3abc - a^3 - b^3 - c^3.

3. $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$ = abc + 2fgh - af^2 - bg^2 - ch^2.

4. $\begin{vmatrix} 1+a & b & c \\ a & 1+b & c \\ a & b & 1+c \end{vmatrix}$ = 1 + a + b + c.

5. $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix}$ = abc $\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$ = abc + bc + ca + ab.

6. $\begin{vmatrix} 1+a^2 & ab & ac \\ ab & 1+b^2 & bc \\ ac & bc & 1+c^2 \end{vmatrix} = 1 + a^2 + b^2 + c^2$.

7. $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = \begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix} = (a - b)(b - c)(c - a)$.

8. $\begin{vmatrix} 1 & a & a^3 \\ 1 & b & b^3 \\ 1 & c & c^3 \end{vmatrix} = \begin{vmatrix} 1 & a^2 & bc \\ 1 & b^2 & ca \\ 1 & c^2 & ab \end{vmatrix} = (a - b)(b - c)(c - a)(a + b + c)$.

9. $\begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix} = \begin{vmatrix} a & a^2 & bc \\ b & b^2 & ca \\ c & c^2 & ab \end{vmatrix} = (a - b)(b - c)(c - a)(ab + bc + ca)$.

10. $\begin{vmatrix} 1^2 & 2^2 & 3^2 \\ 2^2 & 3^2 & 4^2 \\ 3^2 & 4^2 & 5^2 \end{vmatrix} = -8$

11. If ω is complex root of unity, then $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix} = 0$, $\begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^n & \omega^{2n} & 1 \\ \omega^{2n} & 1 & \omega^n \end{vmatrix} = 0$

12. $\begin{vmatrix} 0 & a-b & a-c \\ b-a & 0 & b-c \\ c-a & c-b & 0 \end{vmatrix} = 0$

13. $\begin{vmatrix} b+c & a & 1 \\ c+a & b & 1 \\ a+b & c & 1 \end{vmatrix} = 0$.

14. $\begin{vmatrix} a+b & b+c & c+a \\ b+c & c+a & a+b \\ c+a & a+b & b+c \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 2(3abc - a^3 - b^3 - c^3)$.

15. $\begin{vmatrix} a-b-c & 2b & 2c \\ 2a & b-c-a & 2c \\ 2a & 2b & c-a-b \end{vmatrix} = (a+b+c)$.

16. $\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$.

17. $\begin{vmatrix} y+z & x & x \\ y & z+x & y \\ z & z & x+y \end{vmatrix} = 4xyz.$

18. $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix}$ = 0, then abc = -1.

19. $\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix}$ = $(a - 1)^3$.

20. If α, β, γ are the roots of $x^3 + px + q = 0$, then $\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix}$ = 0.

21. If a, b, c are in A.P. then $\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix} = 0$.

22. If x, y, z $\in$ N, $\begin{vmatrix} x & x+y & x+y+z \\ 2x & 3x+2y & 4x+3y+2z \\ 3x & 6x+3y & 10x+6y+3z \end{vmatrix} = 125$

$\Rightarrow x^3 = 125 \Rightarrow x = 5$

Inverse of a matrix:

1. If A is a square non-singular matrix, then A (adjA) = (adj A) A = (det A) I.
2. If A is a square singular matrix, then A (adj A) = (adj A) A = O.
3. A square matrix is said to be an 'invertible matrix' if there exists a square matrix B such that AB = BA = I. The matrix B is called inverse of A.
4. If A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then Adj A = $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ and $A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

5. If $A = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$ and abc $\neq 0$, then $A^{-1} = \begin{bmatrix} 1/a & 0 & 0 \\ 0 & 1/b & 0 \\ 0 & 0 & 1/c \end{bmatrix}$.

6. If $A_\theta = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & -\cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ then $A_\theta^{-1} = A_{-\theta}$.

7. For any Scalar k, adj (kA) = k^{n-1} adjA.
8. adj (adjA) = $|A|^{n-2}$. A.
9. $|adj\,(adj\,A)| = |A|^{(n-1)^2}$.
10. $|adj\,\{adj\,(adj\,A)\}| = |A|^{(n-1)^3}$.
11. $adj\,(A^T) = (adj\,A)^T$.
12. $(adj\,A)^{-1} = \dfrac{A}{|A|}$.
13. Inverse of a symmetric matrix is symmetric.
14. $(kA)^{-1} = \dfrac{1}{k}A^{-1}$.
15. $AA^T = I \Rightarrow A^{-1} = A^T$.
16. If $AA^T = 9I \Rightarrow A(\frac{1}{9}A^T) = I \Rightarrow A^{-1} = \frac{1}{9}A^T$.
17. Elementary transformations do not charge the rank of a matrix.
18. The rank of a matrix in the 'ECHELON form' is equal to the number of non-zero rows of the matrix.
19. Rank of a non- singular matrix of order n is n.
20. Rank of a unit matrix of order n is n.

PERMUTATIONS AND COMBINATIONS

1. ${}^nP_r = r.\ {}^{n-1}P_{r-1} + {}^{n-1}P_r$ Number of permutations of n dissimilar things taken r things at a time
   (i) which contain a particular thing is $r.\ {}^{n-1}P_{r-1}$
   ii) which do not contain a particular thing is ${}^{n-1}p_r$.
2. The number of ways in which n candidates $A_1, A_2, \ldots A_n$ can be ranked if
   (i) 'A_1 is always above A_2' is $\dfrac{n!}{2!}$
   (ii) 'A_1 is always above A_2' and 'A_2 is always above A_3' is $\dfrac{n!}{2!}$
   (iii) 'A_2 should follow immediately after A_1' is (n - 1)!.
3. The sum of n digit numbers formed by permuting all the given n digits.
   (i) when '0' is absent in n digits is (n - 1)! (sum of n digits) $(111 \ldots\ldots 1)_{n\text{ times}}$.
   (ii) when '0' is present in n digits is (sum of n digits) $[(n-1)!\,(111 \ldots 1)_{n\text{ times}} - (n-2)!\,(111 \ldots 1)_{n-1\text{ times}}]$.
4. The sum of all 'r' digited numbers formed with the given 'n' digits
   (i) When '0' is absent in n digits is (sum of n digits) $({}^{n-1}P_{r-1})\,(111 \ldots 1)_{r\text{ times}}$

(ii) When ‘0’ is present in n digits is (sum of n digits) $[({}^{n-1}P_{r-1})\,(11.....1)_{r\ times} - ({}^{n-2}P_{r-2})\,(11....1)_{r-1\ times}]$.

5. The number of combinations of n dissimilar things taken r things at a time when
   (i) ‘k’ particular things always occur is ${}^{n-k}C_{r-k}$.
   (ii) ‘ℓ’ particular things never occur is ${}^{n-\ell}C_r$.
   (iii) ‘k’ particular things always occur and ‘ℓ’ particular things never occur is ${}^{n-k-\ell}C_{r-k}$.
6. A father wants to take his n children to a park. He can take exactly r children at a time and will not take the same group more than once.
   (i) Number of visits by father is ${}^nC_r$.
   (ii) Number of visits by each child is ${}^{n-1}C_{r-1}$.
7. The number of ways in which n identical objects that can be distributed among r persons
   (i) each one of whom can receive 0 or 1or 2 or more objects is ${}^{n+r-1}C_{r-1}$.
   (ii) each one of whom can receive atleast one object is ${}^{n-1}C_{r-1}$.
8. If a set of m parallel lines are intersected by another set of n parallel lines, then the number of parallelograms that can be formed is ${}^mC_2 . {}^nC_2$.
9. If n points lie on a circle (or no three of them are collinear), then
   (i) Number of straight lines formed is ${}^nC_2$.
   (ii) Number of triangles formed is ${}^nC_3$.
   (iii) Number of quadrilaterals formed is ${}^nC_4$.
10. Number of ways of answering one or more out of n questions is $2^n - 1$.
11. Number of ways of answering one or more out of n questions when each question has an alternative is $3^n - 1$.
12. Number of rectangles (including squares) in a chess board is ${}^9C_2 . {}^9C_2 = 1296$.
13. Number of squares in a chess board is $\Sigma 8^2 = 204$.
14. Number of rectangles in a chess board which are not squares is 1092.
15. If N, n_1, n_2, ... n_k are postive integers and N = $2^{n_1}\ 3^{n_2}\ 5^{n_3}$ .... then
    (i) the number of divisors of N is $(n_1 + 1)(n_2 + 1) ...... (n_k + 1)$.
    (ii) the number of divisors of N except unity is $(n_1 + 1)(n_2 + 1) .... (n_k + 1) - 1$.
    (iii) the number of divisors of N except unity and itself is $(n_1 + 1)(n_2 + 1) ...... (n_k + 1) - 2$.
    (iv) the number of odd divisiors of N is $(n_2 + 1)(n_3 + 1) .... (n_k + 1)$
    (v) the number of even divisiors of N is $(n_1 + 1)(n_2 + 1)(n_3 + 1) .... (n_k + 1) - (n_2 + 1)(n_3 + 1) ...... (n_k + 1)$.
    (vi) sum of the positive divisiors of N is $\left(\frac{2^{n_1+1}-1}{2-1}\right)\left(\frac{3^{n_2+1}-1}{3-1}\right)\left(\frac{5^{n_3+1}-1}{5-1}\right)$.......

BINOMIAL THEOREM

1. Coefficient of x^m in the expansion of $\left(ax^p + \frac{b}{x^q}\right)^n$ is the coefficient of T_{r+1} where r = $\frac{np - m}{p + q}$.
2. To get the term independent of x in $\left(ax^p + \frac{b}{x^q}\right)^n$, find T_{r+1} where r = $\frac{np}{p+q}$.
3. Number of terms in $(x + y + z)^n$ is $\frac{(n+1)(n+2)}{2!}$.
4. Number of terms in $(x + y + z + t)^n$ is $\frac{(n+1)(n+2)(n+3)}{3!}$.

5. If the coefficient of x^r, x^{r+1} in the expansion of $\left(a+\frac{x}{b}\right)^n$ are equal, then n = (r + 1) (ab + 1) -1.
6. If the coefficients of r^{th}, $(r + 1)^{th}$, $(r + 2)^{th}$ terms of $(1 + x)^n$ are in A.P. (or ${}^nC_{r-1}$, ${}^nC_r$, ${}^nC_{r+1}$ are in A.P), then $n^2 - (4r + 1) n + 4r^2 - 2 = 0$.
7. When (i) n is even, then ${}^nC_{n/2}$ is the greatest binomial coefficient.
   (ii) n is odd, then ${}^nC_{(n-1)/2}$, ${}^nC_{(n+1)/2}$ are the greatest binominal coefficients. They are equal in value.
8. The general term in the expansion of $(x_1 + x_2 + ... + x_p)^n$ is $\frac{n!}{n_1!n_2!....n_p!}\, x_1^{n_1} x_2^{n_2} .... x_p^{n_p}$ where $n = n_1 + n_2 + ... + n_p$.
   Eg: coefficient of x^2y^2z in the expansion of $(2x + 3y -z)^5 = \frac{5!}{2!2!1!} 3^2 . 3^2 . (-1)^1 = 1080$.
9. $(1 + x)^n - nx - 1$ is divisible by x^2.
   Eg. $16^n - 15n -1$ is divisible by 15^2 (225)
10. If $(a + \sqrt{b})^n = I + f$ where I, n are positive integers $0 < f < 1$, $a^2 - b = 1$, then
    (i) I is an odd positive integer (ii) (I + f) (1 - f) = 1
11. The integral part of $(\sqrt{2} + 1)^6 + (\sqrt{2} - 1)^6 - 1 = 197$.
12. Number of non - zero terms in the expansion of $(x + a)^n + (x - a)^n$ is
    (i) $\frac{n}{2} + 1$ if n is even (ii) $\frac{n+1}{2}$ if n is odd.
13. Number of non - zero terms in the expansion of $(x + a)^n - (x - a)^n$ is
    (i) $\frac{n}{2}$ if n is even (ii) $\frac{n+1}{2}$ if n is odd.
14. $\frac{C_1}{C_0} + 2.\frac{C_2}{C_1} + 3.\frac{C_3}{C_2} + ... + n.\frac{C_n}{C_{n-1}} = \frac{n(n+1)}{2}$.
15. $aC_0^{\,2} + (a + d) C_1^{\,2} + (a + 2d) C_2^{\,2} + ... + (a + nd) C_n^{\,2} = \frac{1}{2}(2a + nd)\, {}^{2n}C_n$.
16. $C_0^{\,2} - C_1^{\,2} + C_2^{\,2} - ..... + (-1)^n C_n^{\,2} = {}^nC_{n/2} (-1)^{n/2}$ if n is even
    = 0 if n is odd.
17. $C_0 - C_2 + C_4 - .... = 2^{n/2} \cos\left(\frac{n\pi}{4}\right)$
    $C_1 - C_3 + C_5 - ..... = 2^{n/2} \sin\left(\frac{n\pi}{4}\right)$.
18. In a polynomial expansion
    (i) sum of all the polynomial coefficients = f (1).
    (ii) sum of the coefficients of even powers of x = $\frac{f(1)+f(-1)}{2}$.
    (iii) sum of the coefficients of odd poweers of x = $\frac{f(1)-f(-1)}{2}$.
    (iv) $a_1 + 2a_2 + 3a_3 + ... + 2na_{2n} = f'(1)$.
    (v) $a_1 - 2a_2 + 3a_3 - .... -2na_{2n} = f'(-1)$
    (vi) $a_0 + a_3 + a_6 + ... = 3^{n-1}$.

PARTIAL FRACTIONS

1. $\dfrac{1}{x^3(x+a)} = \dfrac{1}{a^3x} - \dfrac{1}{a^2x^2} + \dfrac{1}{ax^3} - \dfrac{1}{a^3(x+a)}$.
2. The remainder of f(x) when divided by the polynomial of degree n, is a polynominal of degree atmost n-1.

EXPONENTIAL AND LOGARITHMIC SERIES

1. $\dfrac{e+e^{-1}}{2} = 1 + \dfrac{1}{2!} + \dfrac{1}{4!} + \dfrac{1}{6!} + ...\infty = \cosh 1$.
2. $\dfrac{e-e^{-1}}{2} = \dfrac{1}{1!} + \dfrac{1}{3!} + \dfrac{1}{5!} + ...\infty = \sinh 1$.
3. $\sum_{n=1}^{\infty}\left(\dfrac{1 + a + a^2 + ... + a^{n-1}}{n!}\right) = \dfrac{e^a - e}{a-1}$.
4. $-\log_e(1-x) = x + \dfrac{x^2}{2} + \dfrac{x^3}{3} + ... \infty$.
5. $\frac{1}{2}\log_e\left(\dfrac{1+x}{1-x}\right) = x + \dfrac{x^3}{3} + \dfrac{x^5}{5} + ... \infty = \tan h^{-1} x$.

PROBABILITY

1. Odds in favour of the event A is P (A): P $(\overline{A})$.
2. Odds against the event A is P$(\overline{A})$: P(A).
3. If P(A) : P(A) = a : b, then P (A) = $\dfrac{a}{a+b}$.
4. If P $(\overline{A})$: P(A) = a : b, then P(A) = $\dfrac{b}{a+b}$.
5. When two dice are thrown:

Sum of the numbers on the top faces	2	3	4	5	6	7	8	9	10	11	12
No. of ordered pairs	1	2	3	4	5	6	5	4	3	2	1

6. When three dice are thrown:

$P(3 \le K \le 8) = \frac{(K-1)(K-2)}{432}$

$P(9 \le K \le 14) = \frac{21K - K^2 - 83}{216}$

$P(15) = \frac{10}{216}$, $P(16) = \frac{6}{216}$, $P(17) = \frac{3}{216}$, $P(18) = \frac{1}{216}$

7. Number of prime numbers from 1 to 100 is 25. They are
2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.
No. of prime numbers between 1 to 50 = 15.

8. Out of 2n + 1 tickets consecutively numbered, three are drawn at random. The probability that numbers on the are in A.P. is $\frac{3n}{4n^2 - 1}$.

9. Out of 2n tickets consecutively numbered, three are drawn at random. The probability that the numbers on them are in A.P. is $\frac{3n}{2(2n-1)}$.

10. If A and B play a game, p and q are probabilities of success and failure respectively, then

(i) $P(A) = \frac{p}{1-q^2}$ (ii) $P(B) = \frac{qp}{1-q^2}$ (iii) $P(A) : P(B) = 1 : q$

11. If A, B and C play a game, p and q are the probabilities of success and failure respectively, then

(i) $P(A) = \frac{p}{1-q^3}$ (ii) $P(B) = \frac{qp}{1-q^3}$ (iii) $P(C) = \frac{q^2p}{1-q^3}$ (iv) $P(A) : P(B) : P(C) = 1 : q : q^2$.

12. If n letters are put at random in n addressed envelopes, the probability that

(i) all the letters are in right envelopes = $\frac{1}{n!}$

(ii) atleast one letter may be in wrongly addressed envelope = $1 - \frac{1}{n!}$

(iii) all the letters may be in wrong envelopes = $\frac{1}{2!} - \frac{1}{3!} + \ldots\ldots + \frac{(-1)^n}{n!}$

RANDOM VARIABLE & DISTRIBUTIONS

1. If $|r| < 1$, sum of the infinite arthmetico geometric series a.1 + (a + d) r + (a + 2d) r^2 + ... $\infty = \frac{a}{1-r} + \frac{dr}{(1-r)^2}$
2. If X is a Binomial variate which denotes the number of times heads occur in n tosses of a fair coin, then $p = \frac{1}{2} = q \Rightarrow P(X = r) = {}^nC_r . \frac{1}{2^n}$.
3. If λ is the parameter of a Poission variate, then
   (i) $\lambda = np$. (ii) $P(X = 0) = P(X = 1) \Rightarrow \lambda = 1$ (ii) $P(X = 1) = P(X = 2) \Rightarrow \lambda = 2$

SUCCESSIVE DIFFERENTIATON

1. If $y = \frac{1}{(ax+b)^r}$, then $y_n = \frac{(-1)^n (n+r-1)! a^n}{(r-1)!(ax+b)^{n+r}}$.

   eg: If $y = \frac{1}{(ax+b)^2}$ then $y_n = \frac{(-1)^n (n+1)! a^n}{(ax+b)^{n+2}}$

   If $y = \frac{1}{(4x-7)^4}$ then $y_n = \frac{(-1)^n (n+3)! 4^n}{3!(4x-7)^{n+4}}$
2. If $y = ax^{n+1} + bx^{-n}$, then $x^2 y_2 = n(n + 1) y$.
3. If $y = \frac{ax+b}{cx+d}$, then $2y_1 y_3 = 3y_2^2$.
4. If $I_n = \frac{d^n}{dx^n}(x^n \log x)$, then $I_n = n! [\log x + 1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{n}]$.
5. If $y = \frac{\log x}{x}$, then $y_n = \frac{(-1)^n n!}{x^{n+1}} [\log x - 1 - \frac{1}{2} - \frac{1}{3} - .... - \frac{1}{n}]$.
6. If $ax^2 + 2hxy + by^2 = 1$ then $y_2 = \frac{h^2 - ab}{(hx+by)^3}$.
7. If $y = \frac{1}{x^2+a^2}$ then $y_n = \frac{(-1)^n n!}{ar^{n+1}} \sin (n + 1) \theta$ where $\theta = \tan^{-1}\left(\frac{a}{x}\right)$, $r = \sqrt{x^2+a^2}$.
8. If $y = \tan^{-1} x$, then $y_n = (-1)^{n-1} (n - 1)! \sin^n \theta \sin n\theta$, where $\theta = \tan^{-1}\left(\frac{1}{x}\right)$.

   Eg: $y = \tan^{-1}\left(\frac{1+x}{1-x}\right)$, $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$, $\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$.
9. If $f(x) = e^{mx}$, $0 < m < 1$, then $\sum_{r=0}^{\infty} f^{(r)}(0) = \frac{1}{1-m}$.
10. If $f(x) = x^n$, then $\sum_{r=0}^{n} \frac{f^{(r)}(1)}{r!} = 2^n$.

INDEFINITE INTEGRALS

1. If $\int \frac{dx}{a^2\sin^2 x + b^2\cos^2 x} = \frac{1}{ab}\tan^{-1}\left(\frac{a}{b}\tan x\right) + c$

2. $\int \frac{dx}{a\sin x + b\cos x} = \frac{1}{\sqrt{a^2+b^2}} \log\tan\left[\frac{x}{2} + \frac{1}{2}\tan^{-1}\frac{b}{a}\right] + c$

3. $\int \frac{dx}{(a\sin x + b\cos x)^2} = \frac{1}{a(a\tan x + b)} + c$

4. $\int \frac{a\cos x + b\sin x}{c\cos x + d\sin x}\,dx = \left(\frac{ac+bd}{c^2+d^2}\right)x + \left(\frac{ad-bc}{c^2+d^2}\right)\log(c\cos x + d\sin x) + c.$

5. $\int \frac{1}{x\left(x^n+1\right)}\,dx = \frac{1}{n}\log\left|\frac{x^n}{x^n+1}\right| + c.$

6. $\int \frac{1}{x\left(x^n-1\right)}\,dx = \frac{1}{n}\log\left|\frac{x^n-1}{x^n}\right| + c.$

7. $\int \frac{1}{x\left(1-x^n\right)}\,dx = \frac{1}{n}\log\left|\frac{x^n}{1-x^n}\right| + c.$

8. $\int \frac{\sec x}{(\sec x + \tan x)^n}\,dx = \int \frac{1}{n(\sec x + \tan x)^n} + c.$

9. $\int \frac{\operatorname{cosec} x}{(\operatorname{cosec} x + \cot x)^n}\,dx = \frac{1}{n(\operatorname{cosec} x + \cot x)^n} + c$

10. If u and v are functions of $x; u', u'', u'''$.............. denote the successive derivatives of u, and v_1, v_2, v_3 ...... denote the successive integrals of v, then the extension of integration by parts is $\int uv\,dx = uv_1 - u'v_2 + u''v_3 - u'''v_4 + \ldots\ldots\ldots\ldots$

11. $\int e^{ax}\left[f(x) + \frac{f'(x)}{a}\right]dx = \frac{e^{ax} f(x)}{a} + c$

12. If the integrand is a function of rational powers of x, i.e. $\int (x^{p/q}, x^{a/b}, x^{r/s})\,dx$, then put $x = t^{\text{LCM of } q,\, b,\, s}$

Eg: $\int \frac{x^{1/2}}{1+x^{1/3}}\,dx$ or $\int \frac{1}{x^{1/2}+x^{1/3}}\,dx$ put $x = t^6$.

DEFINITE INTEGRALS

1. If $\int_0^a x(a-x^n)dx = \frac{a^{n+2}}{(n+1)(n+2)}.$ Eg: $\int_0^2 x(2-x)^4 dx = \frac{2^6}{5.6} = \frac{32}{15}.$

2. $\int_a^b \frac{f(x)}{f(x)+f(a+b-x)}dx = \frac{b-a}{2}$ Eg: $\int_2^3 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{5-x}}dx = \frac{3-2}{2} = \frac{1}{2}$

3. $\int_0^{\pi/2} \frac{f(\sin x)}{f(\sin x)+f(\cos x)}dx = \int_0^{\pi/2} \frac{f(\tan x)}{f(\tan x)+f(\cot x)}dx = \int_0^{\pi/2} \frac{f(\sec x)}{f(\sec x)+f(\operatorname{cosec} x)}dx = \frac{\pi}{4}.$

   Eg: $\int_0^{\pi/2} \frac{\sin^{5/2} x}{\sin^{5/2} x+\cos^{5/2} x}dx = \frac{\pi}{4}; \int_0^{\pi/2} \frac{\sqrt{\tan x}}{\sqrt{\tan x}+\sqrt{\cot x}}dx = \frac{\pi}{4}$

4. $\int_0^{\pi/2} \frac{a\sin x+b\cos x}{\sin x+\cos x}dx = \frac{\pi}{4}(a+b) = \int_0^{\pi/2} \frac{a\tan x+b\cot x}{\tan x+\cot x}dx = \int_0^{\pi/2} \frac{a\sec x+b\operatorname{cosec} x}{\sec x+\operatorname{cose} x}dx$

   Eg: $\int_0^{\pi/2} \frac{3\sin x+4\cos x}{\sin x+\cos x}dx = \frac{7\pi}{4}$

5. $\int_0^{\pi} \frac{dx}{a^2\cos^2 x+b^2\sin^2 x} = \frac{\pi}{ab}$ Eg. $\int_0^{\pi} \frac{dx}{25\cos^2 x+16\sin^2 x} = \frac{\pi}{5(4)} = \frac{\pi}{20}$

6. $\int_0^{\pi} \frac{xdx}{a^2\cos^2 x+b^2\sin x} = \frac{\pi}{2ab}$ Ex. $\int_0^{\pi} \frac{xdx}{25\cos^2 x+16\sin^2 x} = \frac{\pi}{2(5)(4)} = \frac{\pi}{40}$

7. $\int_a^b \sqrt{\frac{b-x}{x-a}}\,dx = \int_a^b \sqrt{\frac{x-a}{b-x}}\,dx = (b-a)\frac{\pi}{2}$ Eg. $\int_3^7 \sqrt{\frac{7-x}{x-3}}\,dx = (7-3)\frac{7}{2} = 2\pi$

8. $\int_a^b \sqrt{(x-a)(b-x)}\,dx = (b-a)^2\frac{\pi}{8}$ Ex. $\int_3^7 \sqrt{(x-3)(7-x)}\,dx = (7-3)^2\frac{\pi}{8} = 2\pi$

9. $\int_a^b \frac{1}{\sqrt{(x-a)(b-x)}}dx = \pi$ Ex. $\int_2^9 \frac{1}{\sqrt{(x-2)(9-x)}}dx = \pi$

10. $\int_a^b \frac{1}{x\sqrt{(x-a)(b-x)}}dx = \frac{\pi}{\sqrt{ab}}$ Ex. $\int_4^9 \frac{1}{x\sqrt{(x-4)(9-x)}}dx = \frac{\pi}{\sqrt{4(9)}} = \frac{\pi}{6}$

11. $\int_a^b \frac{x}{\sqrt{(x-a)(x-b)}}dx = \frac{\pi}{2}(a+b)$ Ex. $\int_3^7 \frac{x}{\sqrt{(x-3)(7-x)}}dx = \frac{\pi}{2}(3+7) = 5\pi$

12. $\int_0^a \sqrt{a^2-x^2}\,dx=\frac{\pi a^2}{4}$ EX. $\int_0^4 \sqrt{16-x^2}\,dx=\frac{\pi}{4}4^2=4\pi$

13. $\int_0^a \frac{1}{x^2+a^2}dx=\frac{\pi}{2a}$ Ex. $\int_0^2 \frac{1}{x^2+4}dx=\frac{\pi}{2.2}=\frac{\pi}{4}$

14. $\int_0^a \frac{1}{\sqrt{a^2-x^2}}dx=\frac{\pi}{2}$ Eg. $\int_0^3 \frac{1}{\sqrt{9-x^2}}dx=\frac{\pi}{2}$

15. $\int_0^a \frac{x}{\sqrt{a^2-x^2}}dx=a$ Ex. $\int_0^3 \frac{x}{\sqrt{9-x^2}}=3$

16. $\int_0^a \frac{x^2\,dx}{\sqrt{a^2-x^2}}=\frac{\pi a^2}{4}$ Ex. $\int_0^3 \frac{x^2\,dx}{\sqrt{9-x^2}}=\frac{\pi(9)}{4}$

17. $\int_0^{\pi/2} \log\sin x\,dx=\int_0^{\pi/2}\log\cos x\,dx=\frac{-\pi}{2}\log 2$ Ex. $\int_0^{\pi/2}\log\sec x\,dx=\int_0^{\pi/2}\log\operatorname{cosec} x\,dx=\frac{\pi}{2}\log 2$

18. $\int_0^{\pi/2}\log\tan x\,dx=\int_0^{\pi/2}\log\cot x\,dx=0$

19. $\int_0^{\pi}\log(1+\cos x)\,dx=-\pi\log 2$

20. $\int_0^{\pi/4}\log(1+\tan x)\,dx=\frac{\pi}{8}\log 2$

21. $\int_0^{\infty} e^{-ax}\sin bx\,dx=\frac{b}{a^2+b^2}$ Ex. $\int_0^{\infty} e^{-3x}\sin 4x\,dx=\frac{4}{9+16}=\frac{4}{25}$

22. $\int_0^{\infty} e^{-ax}\cos bx\,dx=\frac{a}{a^2+b^2}$ Ex. $\int_0^{\infty} e^{-4x}\cos 5x\,dx=\frac{4}{16+25}=\frac{4}{41}$

23. $\int_0^{\infty}\frac{x^2}{(x^2+a^2)(x^2+b^2)(x^2+c^2)}dx=\frac{\pi}{2(a+b)(b+c)(c+a)}$

Ex. $\int_0^{x}\frac{1}{(x^2+4)(x^2+9)}dx=\int_0^{\infty}\frac{x^2}{(x^2+0)(x^2+4)(x^2+9)}dx=\frac{\pi}{2(0+2)(2+3)(3+0)}=\frac{\pi}{60}$

24. If f(x) is a periodic function with period a, then $\int_0^{na} f(x)\,dx=n\int_0^a f(x)\,dx$

25. $\int_0^{\pi} x f(\sin x)\,dx = \frac{\pi}{2}\int_0^{\pi} f(\sin x)\,dx$.

26. $\frac{d}{dx}\int_{f(x)}^{g(x)} F(t)\,dt = F[g(x)]g'(x) - F[f(x)]f'(x)$.

27. $\int_0^{\pi/4} [\mathrm{Tan}^n x + \mathrm{Tan}^{n-2} x]\,dx = \frac{1}{n-1}$.

28. $\int_{\pi/4}^{\pi/2} [\cot^n x + \cot^{n-2} x]\,dx = \frac{1}{n-1}$.

29. $\int_0^{n} [x]\,dx = \frac{n(n-1)}{2}$.

30. $\int_{n-1}^{n} [x]\,dx = n-1$.

31. $\int_0^{\infty} (a^{-x} + b^{-x})\,dx = \frac{1}{\log a} - \frac{1}{\log b}$ if $a, b > 1$

$= \infty$ if $a, b < 1$.

32. $\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x}\,dx = \frac{\pi^2}{4}$.

33. $\int_0^{\pi} \frac{x \tan x}{\sec x + \tan x} = \frac{\pi(\pi-2)}{2}$.

34. $\int_0^{\pi/2} \frac{1}{\cos x + \sin x}\,dx = \sqrt{2}\log(\sqrt{2}+1)$.

35. $\int_0^{\pi} \frac{x}{1+\sin x}\,dx = \pi$.

36. $\int_0^{\pi} \frac{\sin x}{1+\cos^2 x}\,dx = \int_0^{\pi} \frac{\tan x}{\sec x + \cos x}\,dx = \frac{\pi^2}{4}$.

37. $\int_0^{\pi} \frac{\tan x}{\tan x + \sec x}\,dx = \pi - 2$.

38. $\int_0^{\pi/2} \frac{x}{\sin x + \cos x}\,dx = \frac{\pi}{2\sqrt{2}}(\sqrt{2}+1)$.

39. $\int_0^{\pi} \frac{\sin^2 x}{\sin x + \cos x} dx = \frac{1}{2}\log(\sqrt{2}+1)$.

40. $\int_0^{1} \frac{\log(1+x)}{1+x^2} dx = \frac{\pi}{8}\log 2.$

41. $\int_0^{\pi/2} \log(\tan x + \cot x)\, dx = \pi \log 2.$

42. $\int_0^{\infty} \frac{\log(1+x^2)}{1+x^2} dx = \pi \log 2$

NUMERICAL INTEGRATION

1. The area of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is πab.

2. AOB is the positive quadrant of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where OA = a, OB = b. The area between the arc AB and the chord AB of the ellipse is $\frac{(\pi-2)ab}{4}$.

3. The area of the region bounded by

   (i) $y^2 = 4ax$ and $x^2 = 4by$ is $\frac{16ab}{3}$. (ii) $y^2 = 4ax$ and $x^2 = 4ay$ is $\frac{16a^2}{3}$.

4. The area of the region bounded by $y^2 = 4ax$ and $y = mx$ is $\frac{8a^2}{3m^3}$.

5. The area of the region bounded by $y^2 = 4ax$ and its latusrectum is $\frac{8a^2}{3}$.

6. The area of the region bounded by one arc of cos ax or sin ax and x-axis is $\frac{2}{a}$.

7. The area of the region bounded by the curve $\sqrt{x}+\sqrt{y}=\sqrt{a}$ where $x > 0$ and the coordinate axes is $\frac{a^2}{6}$.

8. The area of the region bounded by the parabola $y = ax^2 + bx + c$ and X-axis is $\frac{(b^2-4ac)^{3/2}}{6a^2}$.

9. The area of the region bounded by the parabola $x = ay^2 + by + c$ and Y=axis is $\frac{(b^2-4ac)^{3/2}}{6a^2}$.

10. The area enclosed by the curve $a^2y^2 = x^2(a^2 - x^2)$ is $\frac{4a^2}{3}$.

11. The area of the region bounded by the curve [x] + |y| = a is $2a^2$.

12. The area of the region bounded by $y^2 = 4ax$ and the line x = b is $\frac{8a^{1/2}b^{3/2}}{3}$.

13. The area of the region bounded by $x^2 = 4by$ and the line y=a is $\frac{8b^{1/2}a^{3/2}}{3}$.

DIFFERENTIAL EQUATIONS

1. $xdy + ydx = d(xy)$.

2. $\frac{xdy - ydx}{x^2} = d\left(\frac{y}{x}\right)$

3. $\frac{ydx - xdy}{y^2} = d\left(\frac{x}{y}\right)$

4.

Given solution	**Correspnding D.E.**	**Arbitary constants**
(i) $y = A_1e^{ax} + A_2e^{bx}$	$y_2 - (a + b)\, y_1 + aby = 0$	(A_1, A_2)
(ii) $y = e^{ax}(A_1x + A_2)$	$y_2 - 2ay_1 + a^2y = 0$	(A_1, A_2)
(iii) $y = A_1e^{ax} + A_2e^{bx} + A_3e^{cx}$	$y_3 - (a + b + c)y_2 + (ab + bc + ca)y_1 - abcy = 0$	(A_1, A_2, A_3)
(iv) $y = e^{ax}(A_1x^2 + A_2x + A_3)$	$y_3 - 3ay_2 + 3a^2y_1 - a^3y = 0$	(A_1, A_2, A_3)
(v) $y = A_1x^m + A_2x^n$	$x^2y_2 - (m + n - 1)\, xy_1 + mny = 0$	(A_1, A_2)
(vi) $y = e^{ax}(A_1 \cos bx + A_2 \sin bx)$	$y_2 - 2ay_1 + (a^2 + b^2)\, y = 0$	(A_1, A_2)
(vii) $y = A_1 \sin ax + A_2 \cos ax$	$y_2 + a^2y = 0$	(A_1, A_2)
(viii) $y = A \sin(mx + B)$	$y_2 + m^2y = 0$	(A, B)
(ix) $y = A \cos(mx + B)$	$y_2 + m^2y = 0$	(A, B)

5. Bernoulli's equation : An equation of the form $\frac{dy}{dx} + Py = Qy^n$, where P and Q are functions of x only, is called a Bernoulli's equation.

$$\Rightarrow \frac{1}{y^n}\frac{dy}{dx} + \frac{1}{y^{n-1}}P = Q \text{ put } \frac{1}{y^{n-1}} = z.$$

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