1. FUNCTIONS

1. Number of functions that can be defined from A to B is $n(B)^{n(A)}$.
2. If $n(A) \leq n(B)$, then the number of one - one functions from A to B is ${}^{n(B)}P_{n(A)}$.
3. If $n(A) > n(B)$, then the number of one - one functions from A to B is 0.
4. If $n(A) \leq n(B)$, then the number of many - one functions from A to B is $n(B)^{n(A)} - {}^{n(B)}P_{n(A)}$.
5. If $n(A) \geq n(B)$ and $n(B) = 2$, then the number of onto functions from A to B is $2^{n(A)} - 2$.
6. If $n(A) < n(B)$, then the number of onto functions from A to B is 0.
7. The number of constant functions that can be defined from A to B is n(B).
8. The number of identity functions that can be defined from A to A is 1.
9. If $n(A) = n(B) = n$, then the number of bijections from A to B is n!.
10. Rule to find the domain of $\frac{1}{\log f(x)}$ is $f(x) > 0$ and $f(x) \neq 1$.
11. If $f(x) = \frac{ax + b}{cx - a}$ then
    i) $(fof)(x) = x$ ii) $(fofof)(x) = f(x)$ iii) $f = f^{-1}$.
12. If $a^2 + b^2 + c^2 = p$, then the range of $ab + bc + ca$ is $\left[\frac{-p}{2}, p\right]$.
13. If $x > 0$, then the range of $x + \frac{1}{x}$ is $[2, \infty)$.
14. If f(x) is a polynomial function such that $f(x)\, f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right)$, then $f(x) = x^n + 1$ or $1 - x^n$.
15. If $f(x) = (a - x^n)^{1/n}$ then : (i) $(fof)(x) = x$ ii) $f = f^{-1}$.
16. Domain and Range table

Function	Domain	Range
1) $\lvert x \rvert$	R	$[0, \infty)$
2) a^x $(a > 0)$	R	$(0, \infty)$
3) $\log_a x$	$(0, \infty)$	R
4) $[x]$	R	Z
5) $\sqrt{x}$	$[0, \infty)$	$[0, \infty)$
6) $\sqrt{a^2 - x^2}$	$[-a, a]$	$[0, a)$
7) $\frac{1}{\sqrt{a^2 - x^2}}$	$(-a, a)$	$\left[\frac{1}{a}, \infty\right)$
8) $\sqrt{x^2 - a^2}$	$(-\infty, -a] \cup [a, \infty)$	$[0, \infty)$
9) $\frac{1}{\sqrt{x^2 - a^2}}$	$(-\infty, -a) \cup (a, \infty)$	$(0, \infty)$
10) $\frac{x^2 - a^2}{x - a}$	$R - \{a\}$	$R - \{2a\}$
11) $\lvert x - a \rvert + \lvert x - b \rvert$	R	$[\lvert a - b \rvert, \infty)$

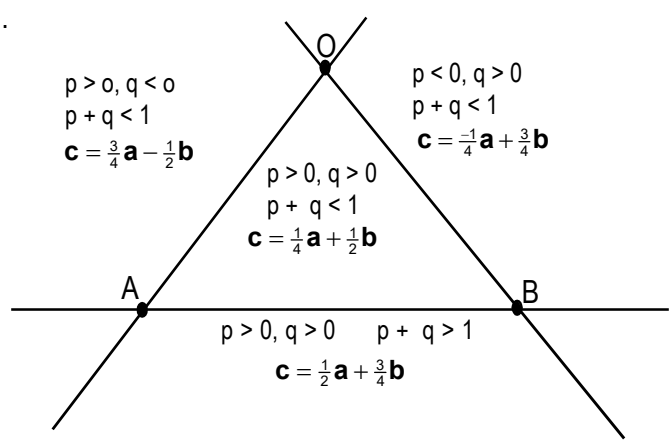
2. MATHEMATICAL INDUCTION

The following checks are to be carried:

1. To identify n^{th} term of the series : $S_1 = t_1$, $S_2 = t_2$.
2. To find the sum of n terms of the series : $S_1 = t_1$, $S_2 = t_1 + t_2$.
3. To find the sum of n - 1 terms of the series : $S_2 = t_1$, $S_3 = t_1 + t_2$.
4. To find the sum of n - 2 terms of the series : $S_3 = t_1$, $S_4 = t_1 + t_2$.
5. To find the sum of 2n terms of the series : $S_1 = t_1 + t_2$, $S_2 = t_1 + t_2 + t_3 + t_4$.
6. To find the sum of 3n terms of the series: $S_1 = t_1 + t_2 + t_3$, $S_2 = t_1 + t_2 + t_3 + t_4 + t_5 + t_6$.
7. To find the product of n terms of the series : $P_1 = t_1$, $P_2 = t_1 t_2$.
8. To find the product of n terms of the series ($n \geq 2$) $P_2 = t_1$, $P_3 = t_1 t_2$.
9. Divisiblity : Find S_1 and S_2
   i) If $S_1 = 0$, Ans is S_2 ii) If $S_1 \neq 0$, Ans is G.C.D of S_1, S_2 iii) If $S_1 \neq 0$, and is a prime number, Ans is S_1.
10. The product of r consecutive natural numbers is divided by r !.
11. The sum of the cubes of the three consecutive natural numbers is divisible by 9.
12. Sum of the numbers in the nth set of the series {1} + {2,3} + {4,5,6} + ... is $t_n = \frac{n}{2}(n^2 + 1)$.
13. Sum of the numbers in the nth set of the series {1} + {2 + 3 + 4} + {5 + 6 + 7 + 8 + 9} + ... is $(n - 1)^3 + n^3$.
14. $\frac{1}{a(a+d)} + \frac{1}{(a+d)(a+2d)} + \ldots\ldots\ldots\ldots$ n terms $= \frac{n}{a(a+nd)}$.

3. PROPERTIES OF VECTORS

1. Δ ABC, then the length of the median through A is $\left|\frac{\overline{AB} + \overline{AC}}{2}\right|$.
2. Circumcentre of Δ ABC with vertices A (a_1, b_1, c_1) B (a_1, b_2, c_1) C (a_2, b_1, c_1) is $\left(\frac{a_1 + a_2}{2}, \frac{b_1 + b_2}{2}, c_1\right)$
3.



Let O be the origin. $\overline{OA}$ = **a**, $\overline{OB}$ = **b** , then the point C with position vector = p**a** + q**b** lies

i) Inside ΔOAB, if p > 0, q > 0 and p + q < 1.

ii) Outside ΔOAB, but inside $\underline{|AOB}$ if p > 0, q > 0, p + q > 1.

iii) Outside ΔOAB, but inside $\underline{|OAB}$, if p < 0, q > 0, p + q < 1

iv) Outside ΔOAB, but inside $\underline{|OBA}$, if p > 0, q < 0, p + q < 1

v) On the line AB, if p + q = 1 and p > 0, q > 0 and p < 0, q > 0 or p > 0, q < 0.

vi) Outside ΔOAB but none of the above regions if p < 0, q > o, p + q > 1 or p > 0, q < 0, p + q > 1.

4. If **a**, **b** represent the adjacent sides of a parallelogram, then the lengths of its diagonals of a parallelogram, then the lengths of its sides are |**a** + **b**| , |**a** - **b**|.

5. If **a**, **b** represent the diagonals of a parallelogram, then the lengths of its sides are $\frac{1}{2}$|**a** + **b**|, $\frac{1}{2}$ |**a** - **b**|.

6. In Δ ABC, if AP is internal angle bisector of $\underline{|A}$, then $\overline{AP} = \frac{|\overline{AB}|\,\overline{AC} + |\overline{AC}|\,\overline{AB}}{|\overline{AB}| + |\overline{AC}|}$.

7. If $\overline{OA}$ = **a**, $\overline{OB}$ = **b**, then a vector along.

   i) internal bisector of $\underline{|AOB}$ is t $(\hat{a} + \hat{b})$ ii) external bisector of $\underline{|AOB}$ is t $(\hat{a} - \hat{b})$.

4. DOT AND CROSS PRODUCTS

1. $\frac{\text{projection of } \bar{a} \text{ on } \bar{b}}{\text{projection of } \bar{b} \text{ on } \bar{a}} = \frac{|\bar{a}|}{|\bar{b}|}$.

2. Angle between two diagonals of a cube is $\cos^{-1}\frac{1}{3}$.

3. Angle between a diagonal and a face diagonal of a cube is $\cos^{-1}\sqrt{\frac{2}{3}}$ or $\cos^{-1}\frac{\sqrt{6}}{3}$ or $\tan^{-1}\frac{1}{\sqrt{2}}$.

4. If $\bar{a}, \bar{b}, \bar{c}$ are mutually perpendicular vectors, then $|\bar{a} + \bar{b} + \bar{c}| = \sqrt{|\bar{a}|^2 + |\bar{b}|^2 + |\bar{c}|^2}$.

5. If $\bar{a}, \bar{b}, \bar{c}$ are mutually perpendicular vectors of equal magnitude, then

   $(\bar{a} + \bar{b} + \bar{c}, \bar{a}) = (\bar{a} + \bar{b} + \bar{c}, \bar{b}) = (\bar{a} + \bar{b} + \bar{c}, \bar{c}) = \cos^{-1}\frac{1}{\sqrt{3}}$.

6. For any vector $\bar{a}$; $(\bar{a}\,.\,\bar{i})\,\bar{i} + (\bar{a}\,.\,\bar{j})\,\bar{j} + (\bar{a}\,.\,\bar{k})\,\bar{k} = \bar{a}$

7. If $\bar{a} = a_1\,\bar{i} + a_2\,\bar{j} + a_3\,\bar{k}, \bar{b} = b_1\,\bar{i} + b_2\,\bar{j} + b_3\,\bar{k}$ then $|\bar{a} \times \bar{b}| = \sum (a_2\,b_3 - a_3\,b_2)^2$

8. If $\bar{a}, \bar{b}, \bar{c}$ form a right handed system, then

i) $\bar{a}, \bar{c}, \bar{b}$ forn a L.H.S

ii) $\bar{a}, \bar{b}, \overline{-c}$, form a L.H.S

iii) $\bar{a}, \overline{-b}, \overline{-c}$ form a R.H.S

iv) $\bar{b}, \bar{c}, \bar{a}$ and $\bar{c}, \bar{a}, \bar{b}$ form a R.H.S

9. For any vector $\bar{a}$, $\left(\bar{a} \times \bar{i}\right)^2 + \left(\bar{a} \times \bar{j}\right)^2 + \left(\bar{a} \times \bar{k}\right)^2 = 2\left|\bar{a}\right|^2$.

10. $\left(\bar{a} \times \bar{b}\right)^2 + \left(\bar{a} . \bar{b}\right)^2 = \left|\bar{a}\right|^2 \left|\bar{b}\right|^2$.

5. TRIPLE PRODUCTS

1. If the vectors $\bar{a}\ \bar{b}\ \bar{c}$ form a RHS, then $\left[\bar{a}\ \bar{b}\ \bar{c}\right] > 0$.

2. If the vectors $\bar{a}\ \bar{b}\ \bar{c}$ form a LHS, then $\left[\bar{a}\ \bar{b}\ \bar{c}\right] < 0$.

3. If $\bar{u}$ is a unit vector, then the maximum value of $\left[\bar{u}\ \bar{v}\ \bar{w}\right]$ is $\left|\bar{v} \times \bar{w}\right|$.

4. If $\bar{a}, \bar{b}, \bar{c}$ are mutually perpendicular vectors, then $\left[\bar{a}\ \bar{b}\ \bar{c}\right] = \pm\left|\bar{a}\right|\left|\bar{b}\right|\left|\bar{c}\right|$.

5. $\left[\bar{a}+\bar{b}\ \ \bar{b}+\bar{c}\ \ \bar{c}+\bar{a}\right] = 2\left[\bar{a}\ \bar{b}\ \bar{c}\right]$.

6. $\left[\bar{a}-\bar{b}\ \ \bar{b}-\bar{c}\ \ \bar{c}-\bar{a}\right] = 0$.

7. $\left[\bar{a} \times \bar{b}\ \ \bar{b} \times \bar{c}\ \ \bar{c} \times \bar{a}\right] = \left[\bar{a}\ \bar{b}\ \bar{c}\right]^2$.

8. $\left(\bar{a} \times \bar{b}\right) \times \bar{c} = \bar{a} \times \left(\bar{b} \times \bar{c}\right)$ iff $\bar{a}, \bar{c}$ are collinear vectors.

6. TRIGONOMETRIC FUNCTIONS

1. The sides of a right angled triangle are 3, 4, 5 ; 5, 12, 13 ; 7, 24, 25 ; 8, 15, 17 ; 9, 40, 41 ; 11, 60, 61 .
2. $\sin^4\theta + \cos^4\theta = 1 - 2\sin^2\theta\cos^2\theta$.
3. $\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta$.
4. If $a\cos\theta + b\sin\theta = c$, then $a\sin\theta + b\cos\theta = \pm\sqrt{a^2 + b^2 + c^2}$.
5. If $\sin\theta + \operatorname{cosec}\theta = 2$, then $\sin^n\theta + \operatorname{cosec}^n\theta = 2$.
6. If $\sec\theta + \tan\theta = \frac{a}{b}$, then

   i) $\sin\theta = \frac{a^2 - b^2}{a^2 + b^2}$ ii) $\sec\theta = \frac{1}{2}\left(\frac{a}{b} + \frac{b}{a}\right)$ iii) $\tan\theta = \frac{1}{2}\left(\frac{a}{b} - \frac{b}{a}\right)$

7. If cosec θ + cot θ $= \frac{a}{b}$ then

i) $\cos\theta = \frac{a^2 - b^2}{a^2 + b^2}$ ii) $\text{cosec}\,\theta = \left(\frac{a}{b} + \frac{b}{a}\right)$ iii) $\cot\theta = \left(\frac{a}{b} - \frac{b}{a}\right)$

8. If A + B = 90^0, then
i) $\sin^2 A + \sin^2 B = 1$ ii) $\cos^2 A + \cos^2 B = 1$ iii) Tan A Tan B = 1.

9. Domain and Range table

Function	Domain	Range
1) sin x	R	[-1,1]
2) cos x	R	[-1,1]
3) tan x	$R - \left\{(2n+1)\frac{\pi}{2} : n \in Z\right\}$	R
4) cot x	$R - \{n\pi : n \in Z\}$	R
5) sec x	$R - \left\{(2n+1)\frac{\pi}{2} : n \in Z\right\}$	R - (-1,1)
6) cosec x	$R - \{n\pi : n \in Z\}$	R - (-1,1)

7. COMPOUND ANGLES

1. tan A + tan B + tan (A + B) tan A tan B = tan (A + B)
2. tan A - tan B - tan (A - B) tan A tan B = tan (A - B) where A > B.
3. If A + B = 45^0, then (1 + tan A) (1 + tan B) = 2.
4. If A + B = 135^0, then (1 + cot A) (1 + cot B) = 2 cot A cot B.
5. If A + B = C, then tan A tan B tan C = tan C - tan A - tan B
6. Cos θ + cos ($120^0 + \theta$) + cos ($120^0 + \theta$) = 0.
7. Sin θ + sin ($120^0 + \theta$) - sin ($120^0 + \theta$) = 0.
8. If A + B + C = 180^0, then
   i) tan A + tan B + tan C = tan A tan B tan C
   ii) $\tan\frac{A}{2}\tan\frac{B}{2} + \tan\frac{B}{2}\tan\frac{C}{2} + \tan\frac{C}{2}\tan\frac{A}{2} = 1.$
   iii) cot A cot B + cot B cot C + cot C cot A = 1.
   iv) $\cot\frac{A}{2} + \cot\frac{B}{2} + \cot\frac{C}{2} = \cot\frac{A}{2}\cot\frac{B}{2}\cot\frac{C}{2}$
9. If A + B + C = 90^0, then
   i) tan A tan B + tan B tan C + tan C tan A = 1
   ii) cot A + cot B + cot C = cot A cot B cot C
10. If A + B + C = n π , n $\in$ Z, then tan A + tan B + tan C = tan A tan B tan C

8. MULTIPLIES AND SUBMULTIPLES

1. $\sin\theta \sin(60^0 - \theta)\sin(60^0 + \theta) = \frac{1}{4}\sin 3\theta$.
2. $\cos\theta \cos(60^0 - \theta)\cos(60^0 + \theta) = \frac{1}{4}\cos 3\theta$.
3. $\tan\theta \tan(60^0 - \theta)\tan(60^0 + \theta) = \tan 3\theta$.
4. $\sin^2\theta + \sin^2(60^0 - \theta) + \sin^2(60^0 + \theta) = \frac{3}{2}$.
5. $\cos^2\theta + \cos^2(60^0 - \theta) + \cos^2(60^0 + \theta) = \frac{3}{2}$.
6. $\frac{1}{\sin 10^0} - \frac{\sqrt{3}}{\cos 10^0} = 4$.
7. $\cos x \cos 2x \cos 4x \ldots\ldots \cos 2^n x = \frac{\sin 2^{n+1}x}{2^{n+1}\sin\theta}$
8. $\cot\theta - \tan\theta = 2\cot 2\theta$.
9. $\tan 22\frac{1}{2}^0 = \sqrt{2} - 1 = \cot 67\frac{1}{2}^0$.
10. $\cot 22\frac{1}{2}^0 = \sqrt{2} + 1 = \tan 67\frac{1}{2}^0$.
11. $\tan 7\frac{1}{2}^0 = \sqrt{6} - \sqrt{4} - \sqrt{3} + \sqrt{2} = (\sqrt{3} - \sqrt{2})(\sqrt{2} - 1)$
12. $\cot 7\frac{1}{2}^0 = \sqrt{6} + \sqrt{4} + \sqrt{3} + \sqrt{2} = (\sqrt{3} + \sqrt{2})(\sqrt{2} + 1)$
13. $\cos^3\theta + \cos^3(120^0 - \theta) + \cos^3(120^0 + \theta) = \frac{3}{2}\cos 3\theta$.

9. TRANSFORMATIONS

1. In ΔABC, (i) $\sin(A + B) = \sin C$ (ii) $\cos(A + B) = \cos C$.
   iii) $\sin\left(\frac{A+B}{2}\right) = \cos\frac{C}{2}$ iv) $\cos\left(\frac{A+B}{2}\right) = \sin\frac{C}{2}$
2. If $A + B + C = 180$, then
   i) $\sin 2A + \sin 2B + \sin 2C = 4\sin A \sin B \sin C$.
   ii) $\sin A + \sin B + \sin C = 4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$
   iii) $\cos 2A + \cos 2B + \cos 2C = -1 - 4\cos A \cos B \cos C$
   iv) $\cos A + \cos B + \cos C = 1 + 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$
   v) $\sin^2 A + \sin^2 B + \sin^2 C = 2 + 2\cos A \cos B \cos C$.
   vi) $\cos^2 A + \cos^2 B + \cos^2 C = 1 - 2\cos A \cos B \cos C$.
3. If $A - B = 60^0$, then $\cos^2 A + \cos^2 B - \cos A \cos B = \frac{3}{4}$
4. $\left(\frac{\cos A + \cos B}{\sin A - \sin B}\right)^n + \left(\frac{\sin A + \sin B}{\cos A - \cos B}\right)^n = 2\cot^n\left(\frac{A - B}{2}\right)$ if n is an even integer
   $= 0$ if n is an odd integer.

10. PERIODICITY AND EXTREME VALUES

1. If the periods of f (x), g (x) are respectively m,n ; then the period of f(x) + g(x) or f(x) - g(x) or $\frac{f(x)}{g(x)}$ = LCM of {m,n}.
2. f (x) = [x] is not a periodic function.
3. Period of f (x) = x - [x] is 1.
4. Minimum value of $\cos^2 x \sin^2 x$ is 0.
5. Maximum value of $\cos^2 x \sin^2 x$ is $\frac{1}{4}$.
6. Range of $\sin^2 x$ or $\cos^2 x$ is [0,1].
7. Local minimum of a tan x + b cot x is $2\sqrt{ab}$.
8. Local minimum of a sec x + b tan x is $\sqrt{a^2 - b^2}$.
9. Minimum value of $a^2 \sin^2 x + b^2 \operatorname{cosec}^2 x$ or $a^2 \cos^2 x + b^2 \sec^2 x$ or $a^2 \tan^2 x + b^2 \cot^2 x$ is 2ab.
10. Minimum value of $a^2 \sec^2 x + b^2 \operatorname{cosec}^2 x$ is $(a+b)^2$.
11. Minimum value of a sec x + b cosec x is $(a^{2/3} + b^{2/3})^{3/2}$.
12. L.CM of fractions = $\frac{\text{L.C.M of numerators}}{\text{G.C.D of denominators}}$

11. TRIGONOMETRIC EQUATIONS

1. If α, β are different values of x satisfying a cos x + b sin x = c, $\tan\left(\frac{\alpha+\beta}{2}\right) = \frac{b}{a} = \frac{\text{Coe. of sin x}}{\text{Coe. of cos x}}$.
2. If $\sin\theta = \sin\alpha$, then $\theta = \alpha$ or $180^0 - \alpha$.
3. If $\cos\theta = \cos\alpha$, then $\theta = \alpha$ or $-\alpha$.
4. The trignometric equation a cos x + b sin x = c has no solution if $|c| > \sqrt{a^2 + b^2}$.

12. INVERSE TRIGONOMETRIC FUNCTIONS

1. Domain and Range table for inverse trigonometric functions:

Function	Domain	Range
1) $\sin^{-1} x$	[-1,1]	$\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$
2) $\cos^{-1} x$	[-1,1]	$[0, \pi]$
3) $\tan^{-1} x$	R	$\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$
4) $\cot^{-1} x$	R	$(0, \pi)$
5) $\sec^{-1} x$	R - (-1,1)	$\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$
6) $\operatorname{cosec}^{-1} x$	R - (-1,1)	$\left[\frac{-\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$

2. $\text{Tan}^{-1}\frac{m}{n} + \text{Tan}^{-1}\left(\frac{n-m}{n+m}\right) = \frac{\pi}{4}$

3. $\text{Tan}^{-1}\frac{m}{n} - \text{Tan}^{-1}\left(\frac{m-n}{m+n}\right) = \frac{\pi}{4}$.

4. $\text{Tan}\left[2\text{tan}^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)\right] = x$.

5. If a,b,c are positive, then $\text{Tan}^{-1}\left(\sqrt{\frac{a(a+b+c)}{bc}}\right) + \text{Tan}^{-1}\left(\sqrt{\frac{b(a+b+c)}{ca}}\right) + \text{Tan}^{-1}\left(\sqrt{\frac{c(a+b+c)}{ab}}\right) = \pi$.

6. If x,y,z are positive, then $\text{Tan}^{-1}\frac{yz}{x\sqrt{x^2+y^2+z^2}} + \text{Tan}^{-1}\frac{zx}{y\sqrt{x^2+y^2+z^2}} + \text{Tan}^{-1}\frac{xy}{z\sqrt{x^2+y^2+z^2}} = \frac{\pi}{2}$

7. If $\text{Tan}^{-1}x + \text{Tan}^{-1}y + \text{Tan}^{-1}z = \pi$, then $x + y + z = xyz$

8. If $\text{Tan}^{-1}x + \text{Tan}^{-1}y + \text{Tan}^{-1}z = \frac{\pi}{2}$, then $xy + yz + zx = 1$.

9. If $\text{Cos}^{-1}x + \text{Cos}^{-1}y + \text{Cos}^{-1}z = \pi$, then $x^2 + y^2 + z^2 + 2xyz = 1$

10. If $\text{Sin}^{-1}x + \text{Sin}^{-1}y + \text{Sin}^{-1}z = \frac{\pi}{2}$, then $x^2 + y^2 + z^2 + 2xyz = 1$.

11. If $\text{Sin}^{-1}x + \text{Sin}^{-1}y + \text{Sin}^{-1}z = \pi$, then $x\sqrt{1-x^2} + y\sqrt{1-y^2} + z\sqrt{1-z^2} = 2xyz$.

12. If $\text{Cos}^{-1}\left(\frac{x}{a}\right) + \text{Cos}^{-1}\left(\frac{y}{b}\right) = \theta$, then $\frac{x^2}{a^2} - \frac{2xy}{ab}\cos\theta + \frac{y^2}{b^2} = \sin^2\theta$.

13. If $\text{Sin}^{-1}x + \text{Sin}^{-1}y = \theta$, then $\text{Cos}^{-1}x + \text{Cos}^{-1}y = \pi - \theta$.

14. If $\text{Cos}^{-1}x + \text{Cos}^{-1}y = \theta$, then $\text{Sin}^{-1}x + \text{Sin}^{-1}y = \pi - \theta$.

15. If $\text{Sin}^{-1}\left(\frac{a}{x}\right) + \text{Sin}^{-1}\left(\frac{b}{x}\right) = \frac{\pi}{2}$, then $x = \sqrt{a^2+b^2}$.

16. If $\text{Cos}^{1}\left(\frac{a}{x}\right) + \text{Cos}^{-1}\left(\frac{b}{x}\right) = \frac{\pi}{2}$, then $x = \sqrt{a^2+b^2}$.

17. If $\text{Tan}^{-1}\left(\frac{a}{x}\right) + \text{Tan}^{-1}\left(\frac{b}{x}\right) = \frac{\pi}{2}$, then $x = \sqrt{ab}$

13. HYPERBOLIC FUNCTIONS

1. $\tanh^2 x = \dfrac{\cosh 2x - 1}{\cosh 2x + 1}$

2. Domain and Range table for hyperbolic functions:

Function	Domain	Range
1) sinh x	R	R
2) coshx	R	$[1, \infty)$
3) tanh x	R	(-1,1)
4) coth x	R -{ 0 }	$(-\infty, -1) \cup (1, \infty)$
5) sech x	R	(0,1]
6) cosech x	R- { 0 }	R - { 0 }

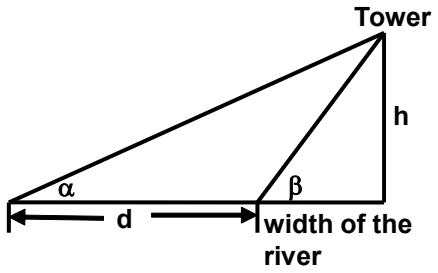
3. Domain and Range table for Inverse hyperbolic functions:

Function	Domain	Range
1) $\sinh^{-1} x$	R	R
2) $\cosh^{-1} x$	$[1, \infty)$	$[0, \infty)$
3) $\tanh^{-1} x$	(-1,1)	R
4) $\coth^{-1} x$	$(-\infty, -1) \cup (1, \infty)$	R - { 0 }
5) $\text{sech}^{-1} x$	(0,1]	$[0, \infty)$
6) $\text{cosech}^{-1} x$	R- { 0 }	R - { 0 }

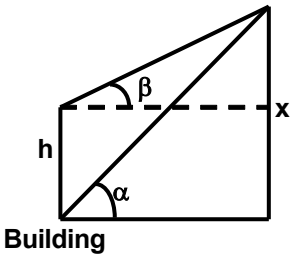
14. HEIGHTS AND DISTANCES

1. $h = \dfrac{d \sin\alpha \sin\beta}{\sin(\beta - \alpha)}$

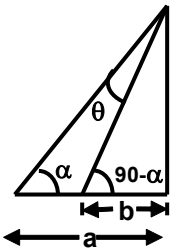
Note: If $\alpha = 15^0$, $\beta = 30^0$, then $h = \dfrac{d}{2}$



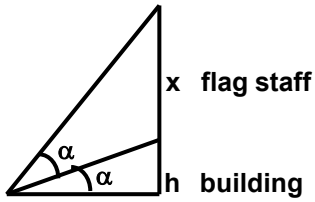
2. $x = \dfrac{h\sin\alpha\cos\beta}{\sin(\alpha-\beta)}$



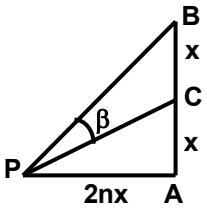
3. $h = \sqrt{ab}, \sin\theta = \dfrac{a-b}{a+b}$



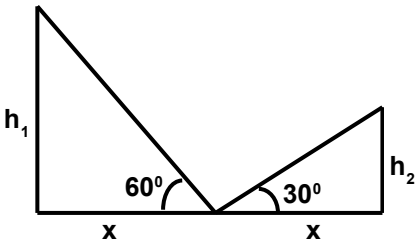
4. $x = \dfrac{h(d^2+h^2)}{d^2-h^2}$



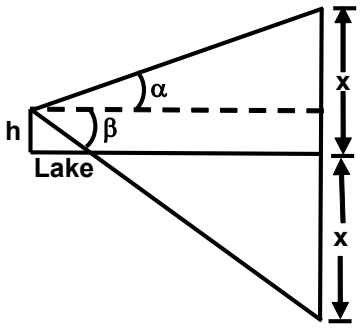
5. If AP = n.AB then $\tan\beta = \dfrac{n}{2n^2+1}$



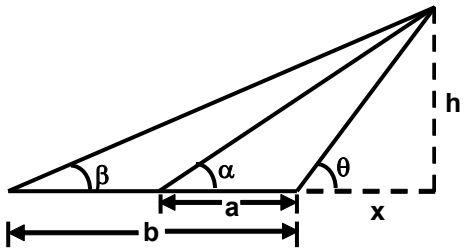
6. $h_1 : h_2 = 3 : 1$



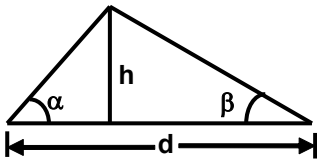
7. $x = \dfrac{h\sin(\beta+\alpha)}{\sin(\beta-\alpha)}$



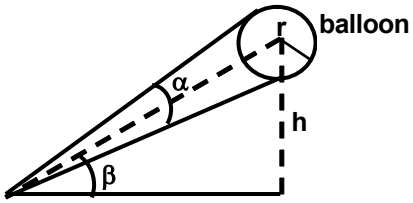
8. $\theta = \tan^{-1}\left(\dfrac{b-a}{b\cot\alpha - a\cot\beta}\right)$



9. $h = \dfrac{d\sin\alpha\ \sin\beta}{\sin(\alpha+\beta)}$

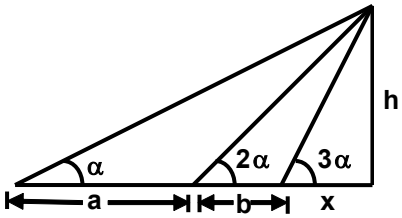


10. $h = r\sin\beta\ \text{cosec}\ \dfrac{\alpha}{2}$



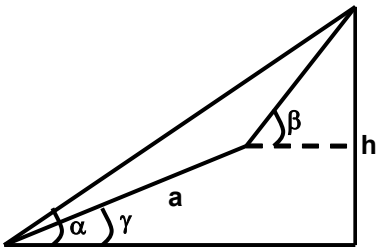
11. $h = \dfrac{a}{2b}\sqrt{(3b-a)(b+a)}$

If a = d, b = $\dfrac{3d}{4}$ then $36\,h^2 = 35d^2$

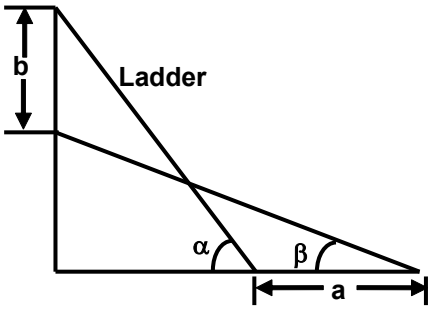


12. $\beta > \alpha > \gamma$

$h = \dfrac{a\sin\alpha\sin(\beta-\gamma)}{\sin(\beta-\alpha)}$



13. $\tan\left(\dfrac{\alpha+\beta}{2}\right) = \dfrac{a}{b}$



15. PROPERTIES OF TRIANGLES

1. For an acute angled triangle, circumcentre lies inside the triangle.
2. For an obtuse angled triangle, circumcentre lies outside the triangle.
3. For a right angled triangle, circumcentre lies on the hypotenuse and circumradius $R = \frac{1}{2}$ hypotenuse .
4. If $\cot\frac{A}{2} : \cot\frac{B}{2} : \cot\frac{C}{2} = x : y : z$ then $a : b : c = \sin A : \sin B : \sin C : = y + z : z + x : x + y$.
5. If $x^2 + x + 1$, $2x + 1$, $x^2 - 1$ are the sides of a triangle, then its largest angle is 120^0.
6. If the sides of a triangle are in the ratio $x : y : \sqrt{x^2 + xy + y^2}$, then the greatest angle is 120^0.
7. If the angles of a triangle Δ ABC are in A.P, then B is 60^0.
8. If a = 13, b = 14, c = 15 then s = 21 and $\Delta = 84$.
9. If $\cos A + \cos B + \cos C = \frac{3}{2}$, then the triangle is equilateral.
10. If $\sin A + \sin B + \sin C = \frac{3\sqrt{3}}{2}$, then the triangle is equilateral.
11. If $\tan A + \tan B + \tan C = 3\sqrt{3}$, then the triangle is equilateral.
12. If $\cot A + \cot B + \cot C = \sqrt{3}$, then the triangle is equilateral.
13. If $a^2 + b^2 + c^2 = 8R^2$, then the triangle is right angled.
14. If $a^2 + b^2 + c^2 = 9R^2$, then the triangle is right angled.
15. If $\cos^2 A + \cos^2 B + \cos^2 C = 1$, (or) $\sin^2 A + \sin^2 B + \sin^2 C = 2$, then the triangle is right angled.
16. If $a \cos B = b \cos A$, then the triangle is isosceles.
17. If $a \cos A = b \cos B$, then the triangle is isosceles or right angled .
18. If $\cos A \cos B + \sin A \sin B \sin C = 1$. then the triangle is right angled and isosceles with angles $45^0, 45^0, 90^0$.
19. $\sum a^3 \cos (B - C) = 3abc$
20. $\sum a^3 \sin (B - C) = 0$.
21. $\cot A + \cot B + \cot C = \frac{a^2 + b^2 + c^2}{4\Delta}$
22. $\cot\frac{A}{2} + \cot\frac{B}{2} + \cot\frac{C}{2} = \frac{(a+b+c)^2}{4\Delta} = \frac{s^2}{\Delta}$
23. $a^2 \sin 2B + b^2 \sin 2A = 4\Delta$.
24. If $\frac{a}{b+c} + \frac{b}{c+a} = 1, \Leftrightarrow C = 60^0$.
25. If $\frac{1}{a+b} + \frac{1}{a+c} = \frac{3}{a+b+c} \Leftrightarrow A = 60^0$.
26. If $\frac{b}{a^2 - c^2} + \frac{c}{a^2 - b^2} = 0 \Leftrightarrow A = 60^0$.

27. $r_1 + r_2 + r_3 - r = 4R$.

28. $r_1 + r_2 - r_3 + r = 4R \cos C$.

29. $r_1 - r = 4R \sin^2 \frac{A}{2}$.

30. $r_2 + r_3 = 4R \cos^2 \frac{A}{2}$.

31. $\frac{rr_1}{r_2 r_3} = \tan^2 \frac{A}{2}$

32. $a^2 = (r_1 - r)(r_2 + r_3)$; $b^2 = (r_3 - r)(r_1 + r_3)$; $c^2 = (r_3 - r)(r_1 + r_2)$.

33. $a = (r_2 + r_3)\sqrt{\frac{rr_1}{r_2 r_3}}$

34. $r_1r_2 + r_2r_3 + r_3r_1 = s^2$.

35. If $(r_2 - r_1)(r_3 - r_1) = 2 r_2r_3$ then $A = 90^0$.

36. If $\left(1-\frac{r_1}{r_2}\right)\left(1-\frac{r_1}{r_3}\right) = 2$ then $A = 90^0$.

37. In ΔABC (i) $\cos A + \cos B + \cos C = 1 + \frac{r}{R}$ ii) $\sin A + \sin B + \sin C = \frac{s}{R}$ iii) $\sum a^2 \cos^2 \frac{A}{2} = s + \frac{\Delta}{R}$

38. If $r_1 = s$ then $A = 90^0$.
39. If $r : R : r_1 = 2 : 5 : 12$ then the triangle is right angled at A.
40. If $r : R : r_1 = 1 : 2 : 3$ then the triranlge is equilateral.
41. If $r_1 = r_2 = r_3$, then the triangle is equilateral.
42. r_1, r_2, r_3 are in H.P $\Leftrightarrow$ a,b,c are in A.P.
43. If $(a - b)(s - c) = (b - c)(s - a)$ then r_1, r_2, r_3 are in A.P.

44. If A, A_1, A_2, A_3 are areas of incircle and excircles of a triangle, then $\frac{1}{\sqrt{A_1}} + \frac{1}{\sqrt{A_2}} + \frac{1}{\sqrt{A_3}} = \frac{1}{\sqrt{A}}$.

45. If $kr_1 = \ell r_2 = mr_3$ then $a : b : c = \ell + m : m + k : k + \ell$.

46. In Δ ABC, length of the internal angle bisector $AD = \frac{2bc}{b+c} \cos \frac{A}{2} = \frac{2 r_2r_3}{r_2 + r_3}$

47. In ΔABC, length of the median $AD = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$ or $\frac{1}{2}\sqrt{b^2 + c^2 + 2bc \cos A}$

48. If p_1, p_2, p_3 are the lengths of the altitudes through A,B,C of Δ ABC, then

i) $p_1 = \frac{2\Delta}{a}$, $p_2 = \frac{2\Delta}{b}$, $p_3 = \frac{2\Delta}{c}$ ii) $\frac{1}{p_1} + \frac{1}{p_2} + \frac{1}{p_3} = \frac{1}{r}$ iii) $\frac{1}{p_1} + \frac{1}{p_2} - \frac{1}{p_3} = \frac{1}{r_3}$

16. COMPLEX NUMBERS

1. $\sqrt{i} = \pm\left(\frac{1+i}{\sqrt{2}}\right)$
2. $\sqrt{-i} = \pm\left(\frac{1-i}{\sqrt{2}}\right)$
3. For any two complex numbers z_1, z_2 ; $|z_1 + z_2| < |z_1| + |z_2|$.
4. $\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$; $\sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$
5. Different forms of a complex number :
   i) General Form : a + ib or (a,b) ii) Polar form or modulus - amplitude form : $r(\cos\theta + i\sin\theta)$
   iii) Cis form : r cis θ iv) Exponential form : $re^{i\theta}$.
6. $1 = 1(\cos 0 + i\sin 0)$
   $-1 = 1(\cos\pi + i\sin\pi)$
   $i = 1(\cos\pi/2 + i\sin\pi/2)$
   $-i = 1[\cos(-\pi/2) + i\sin(-\pi/2]$
7. $\omega = 1[\cos 2\pi/3 + i\sin 2\pi/3]$
   $\omega^2 = 1[\cos 4\pi/3 + i\sin 4\pi/3]$
8. If z is any complex number, then
   i) $z\bar{z} = |z|^2$ ii) $z + \bar{z} = 2\,\text{Re}\,z$ iii) $z - \bar{z} = 2.\,\text{Im}\,z$.
9. Arg (iz) = $\pi/2$ + arg z.
10. The area of the triangle with vertices z, iz, z + iz is $\frac{1}{2}|z|^2$ square units
11. Area of the triangle with vertices z, ωz, $z + \omega z$ is $\frac{\sqrt{3}}{4}|z|^2$ square units.
12. $0, z_1, z_2$ form an equilateral triangle if $z_1^2 + z_2^2 + = z_1 z_2$.
13. If $a \in C$, $b \in R$, then $a\bar{z} + \bar{a}z + b = 0$ represents a straight line.
14. If $a \in C$, $b \in R$, then $z\bar{z} + a\bar{z} + \bar{a}z + b = 0$ represents a circle.
15. If $a, c \in R$ and $a \neq 0$, $b \in C$, then $az\bar{z} + b\bar{z} + \bar{b}z + c = 0$ represents a circle.

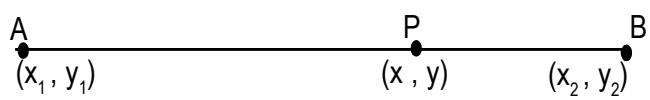
17. DEMOIVRE'S THEOREM & TRIGNOMETRIC EXPANSIONS

1. The nth roots of unity are cis $2k\pi/n$ or $e^{2k\pi i/n}$ where k = 0,1,2.....n - 1.
   i) Sum of the nth roots of unity is 0 ii) Product of nth roots of unity is $(-1)^{n-1}$.
   iii) The nth roots of unity are in G.P. with common ratio cis $2\pi/n$.
2. $(1 + \cos\theta + i\sin\theta)^n + (1 + \cos\theta - i\sin\theta)^n = 2^{n+1}\cos^n(\theta/2)\cos(n\theta/2)$.
3. $(1 + i)^n + (1 - i)^n = 2^{(n/2)+1}\cos n\pi/4$.
4. $(1 + i\sqrt{3})^n + (1 - i\sqrt{3})^n = 2^{n+1}\cos n\pi/3$.

5. $(\sqrt{3} + i)^n + (\sqrt{3} - i)^n = 2^{n+1} \cos n\pi/6$.
6. $(a + ib)^{m/n} + (a - ib)^{m/m} = 2 (a^2 + b^2)^{m/2n} \cos [m/n \operatorname{Tan}^{-1} (b/a)]$
7. If $\cos\alpha + \cos\beta + \cos\gamma = 0 = \sin\alpha + \sin\beta + \sin\gamma$ then
   i) $\sin3\alpha + \sin3\beta + \sin3\gamma = 3\sin(\alpha + \beta + \gamma)$
   ii) $\cos3\alpha + \cos3\beta + \cos3\gamma = 3\cos(\alpha + \beta + \gamma)$
   iii) $\cos(2\alpha - \beta - \gamma) + \cos(2\beta - \gamma - \alpha) + \cos(2\gamma - \alpha - \beta) = 3$.
   iv) $\sin(2\alpha - \beta - \gamma) + \sin(2\beta - \gamma - \alpha) + \sin(2\gamma - \alpha - \beta) = 3$.
   v) $\cos 2\alpha + \cos2\beta + \cos2\gamma = 0$.
   vi) $\sin 2\alpha + \sin2\beta + \sin2\gamma = 0$.
   vii) $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 3/2$
   viii) $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 3/2$
8. $\cot n\theta = \dfrac{{}^nC_0 \cot^n \theta - {}^nC_2 \cot^{n-2} \theta + {}^nC_4 \cot^{n-4} \theta - \ldots\ldots}{{}^nC_1 \cot^{n-1} \theta - {}^nC_3 \cot^{n-3} \theta + \ldots\ldots}$

18. 2D COORDINATE SYSTEM

1. If (x_1,y_1), (x_2,y_2) are two vertices of an equilateral triangle, then the third vertex is
   $$\left(\frac{(x_2 + x_2) \pm \sqrt{3}(y_1 - y_2)}{2}, \frac{(y_1 + y_2) \mp \sqrt{3}(x_1 - x_2)}{2}\right)$$
2. If (x_1,y_1), (x_2,y_2) are the ends of the hypotenuse of a right angled isosceles triangle, then the third vertex is
   $$\left(\frac{(x_1 + x_2) \pm (y_1 - y_2)}{2}, \frac{(y_1 + y_2) \mp (x_1 - x_2)}{2}\right).$$
3. AP : PB = $x_1 - x : x - x_2$ or $y_1 - y : y - y_2$



4. The ratio in which the line joining A (x_1,y_1) and B (x_2,y_2) is divided by i) X - axis is - $y_1 : y_2$ ii) Y - axis is - $x_1 : x_2$.
5. If P divides $\overline{AB}$ in the ratio l : m, then the point Q which divides $\overline{AB}$ in the ratio l : -m is called "harmonic conjugate" of P with respect to A,B.
6. If (x_1,y_1) (x_2,y_2) (x_3,y_3) are three consecutive vertices of a parallelogram or rectangle or rhombus or square then the fourth vertex is $(x_1 - x_2 + x_3, y_1 - y_2 + y_3)$.
7. If a right angled triangle is given, its
   i) circumcentre is the midpoint on the hypotenuse.
   ii) orthocentre is the vertex containing the right angle.
8. The triangle formed by (a,b) (a,c) (d,b) is right angled. It's
   i) orthocentre is (a,b).
   ii) circumcentre is $\left(\dfrac{a + d}{2}, \dfrac{b + c}{2}\right)$.
   iii) centre of the circle passing through them is $\left(\dfrac{a + d}{2}, \dfrac{b + c}{2}\right)$.

9. Area of the triangle formed by (x_1,y_1), (x_2,y_2) and (x_3,y_3) is $\frac{1}{2}\begin{vmatrix} x_1 - x_3 & x_2 - x_3 \\ y_1 - y_3 & y_2 - y_3 \end{vmatrix}$.

10. Area of the quadrilateral formed by (x_1,y_1), (x_2,y_2), (x_3,y_3) and (x_4, y_4) is $\frac{1}{2}\begin{vmatrix} x_1 - x_3 & x_2 - x_4 \\ y_1 - y_3 & y_2 - y_4 \end{vmatrix}$.

11. Centroid of the triangle divides each median in the ratio 2:1 from the vertex.

12. The incentre of the triangle formed by (0,0) (a,0) (0,b) is $\left(\frac{a|b|}{|a| + |b| + \sqrt{a^2 + b^2}}, \frac{|a|b}{|a| + |b| + \sqrt{a^2 + b^2}}\right)$.

13. If G is the centroid of ΔABC, then $AB^2 + BC^2 + CA^2 = 3 (AG^2 + BG^2 + CG^2)$.

14. If D (x_1,y_1), E(x_2,y_2), F(x_3,y_3) are the midpoints of the sides $\overline{BC}$, $\overline{CA}$, $\overline{AB}$ of Δ ABC, then
$A =(-x_1 + x_2 + x_3, -y_1 + y_2 + y_3)$; $B = (x_1 - x_2 + x_3, y_1 - y_2 + y_3)$; $C = (x_1 + x_2 - x_3, y_1 + y_2 - y_3)$.

15. If D,E,F are the midpoints of the sides $\overline{BC}$, $\overline{CA}$, $\overline{AB}$ respectively of ΔABC, then centroid of ΔABC coincides with the centroid of ΔDEF.

16. In ΔABC, if AP bisects its the angle A then BP: PC = AB : AC.

17. If O is the origin and A (x_1,y_1), B (x_2,y_2) are two points, then
i) OA. OB. cos $\underline{|AOB}$ = $x_1x_2 + y_1y_2$ ii) OA.OB sin $\underline{|AOB}$ = $|x_1y_2 - x_2y_1|$

19. LOCUS

1. Distance from P (x,y) to the (i) X - axis is |y| (ii) Y - axis |x|.
2. The ends of the hypotenuse of a right angled triangle are (x_1, y_1) and (x_2, y_2), then the equation of the locus of third vertex is $(x - x_1) (x - x_2) + (y - y_1) (y - y_2) = 0$.

20. CHANGE OF AXES

1. The point to which the origin has to be shifted to eliminate x and y terms in $ax^2 + by^2 + 2gx + 2fy + c = 0$ is (-g/a, -f/b)
2. The point to which the origin has to be shifted to eliminate x and y terms in the equation $a (x - \alpha)^2 + b (y - \beta)^2 = c$ is (α,β).
3. The point to which the origin has to be shifted to eliminate x,y terms in $f \equiv ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ is obtained by solving $\frac{\partial f}{\partial x} = 0$, $\frac{\partial f}{\partial y} = 0$.
4. If x,y terms are eliminated from $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ by the shifting the origin to (α , β) then the transformed equation is $aX^2 + 2hXY + bY^2 + g\alpha + f\beta + c = 0$.
5. The condition that the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ to take the form $aX^2 + 2hXY + bY^2 = 0$, when the axes are translated is $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$.
6. The angle of rotation of the axes to eliminate xy term in the equation $ax^2 + 2hxy + ay^2 + 2gx + 2fy + c = 0$ is $\pi/4$.
7. The angle of rotation of the axes so that the equation ax + by + c = 0 may be reduced to the form
i) X = constant is Tan^{-1} (b/a) ii) Y = constant is Tan^{-1} (-a/b).

21. STRAIGHT LINE

1. Area of the triangle formed by the line ax + by + c = 0 with the coordinate axes is $\frac{c^2}{2|ab|}$ square units.
2. The area of the rhombus formed by ax $\pm$ by $\pm$ c = 0 is $\frac{2c^2}{|ab|}$ square units.
3. The area of the parallelogram formed by the lines $a_1x + b_1y + c_1 = 0$ $a_2x + b_2y + c_2 = 0$, $a_1x + b_1y + d_1 = 0$, $a_2x + b_2y + d_2 = 0$ is $\left|\frac{(c_1 - d_1)(c_2 - d_2)}{a_1 b_2 - a_2 b_1}\right|$ square units.
4. Each side of a square is of length x units. The centre of the square is (α,β) and one of its diagonals is parallel to the line y = x. Then the coordinates of the vertices of the square are $(\alpha \pm \frac{x}{2}, \beta \pm \frac{x}{2})$.
5. If a line cuts the X, Y axes at A and B and (x_1,y_1) is the midpoint of AB, then the equation of the line is $\frac{x}{x_1} + \frac{y}{y_1} = 2$.
6. If a line cuts the axes at A and B and P (x_1,y_1) is a point on it dividing AB in the ratio ℓ : m, then the equation of the line is $\frac{mx}{x_1} + \frac{\ell y}{y_1} = \ell + m$.
7. The ends of a rod of length k move on the coordinate axes. The locus of the point on the rod which divides it in the ratio ℓ : m is $\frac{x^2}{m^2} + \frac{y^2}{\ell^2} = \frac{k^2}{(\ell + m)^2}$ or $\frac{x^2}{\ell^2} + \frac{y^2}{m^2} = \frac{k^2}{(\ell + m)^2}$.
8. If "h" is the length of the altitude in an equilateral triangle, then

   i) the length of its side is $\frac{2h}{\sqrt{3}}$ units.

   ii) area of the triangle is $\frac{h^2}{\sqrt{3}}$ square units.

   iii) circumradius R = $\frac{2h}{3}$.
9. In a square and rhombus, diagonals are perpendicular to each other.
10. Line L has intercepts a and b on the coordinate axes. When the axes are rotated through a given angle without changing the origin, the same line L has intercepts p and q on the transformed axes, then $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{p^2} + \frac{1}{q^2}$.
11. The algebraic sum of the perpendicular distances from A,B and C to a variable line is 0. Then the line passes through the centroid of Δ ABC.
12. The distance of the point (x_1, y_1) from the line ax + by + c = 0 measured along a line making an angle ' α' with x - axis is $\left|\frac{ax_1 + by_1 + c}{a\cos\alpha + b\sin\alpha}\right|$

13. If the given three straight lines form a triangle, then the number of circles that touch all the straight lines is 4 (one incircle and three excircle).
14. If a,b are in i) A.P then $2b = a + c$.
    ii) G.P. then $b^2 = ac$.
    iii) H.P. then $\frac{2}{b} = \frac{1}{a} + \frac{1}{c}$ or $b = \frac{2ac}{a+c}$.
15. Incentre of the triangle is equidistanct from the three sides of the triangle.
16. The area of the triangle formed by the lines $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$, $a_3x + b_3y + c_3 = 0$ is

$$\frac{\Delta^2}{2|C_1\, C_2\, C_3|} \text{ where } \Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix},\ C_1 = \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix},\ C_2 = \begin{vmatrix} a_1 & b_1 \\ a_3 & b_3 \end{vmatrix},\ C_3 = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}.$$

17. The equations of the altitudes of the triangle formed by the lines $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$, $a_3x + b_3y + c_3 = 0$ is given by $(a_2a_3 + b_2b_3)(a_1x + b_1y + c_1) = (a_1a_3 + b_1b_3)(a_2x + b_2y + c_2) = (a_1a_2 + b_1b_2)(a_3x + b_3y + c_3)$. Solving them in pairs, orthocentre can be computed.
18. When three vertices of a triangle are given
    i) To find circumcentre verify SA = SB = SC. ii) To find orthocentre, verify $m_1m_2 = -1$.
19. When equations of four sides of a quadrilateral are given and d_1, d_2 are the distances between parallel sides, m_1, m_2 are the slopes of two adjacent sides; θ is the angle between two adjacent sides, then the figure formed is

    i) Parallelogrram if $d_1 = d_2$ and $m_1 m_2 \neq -1$. Its area = $\frac{d_1d_2}{|\sin\theta|}$

    ii) Rectangle if $d_1 \neq d_2$ and $m_1 m_2 = -1$. Its area = d_1d_2.

    iii) Rhombus if $d_1 = d_2$ (= d) and $m_1m_2 = -1$. Its area $\frac{d^2}{|\sin\theta|}$.

    iv) Square if $d_1 = d_2$ and $m_1m_2 = -1$. Its area = d^2.
20. The straight lines $ax + by + c = 0$, $a_1x + b_1y + c_1 = 0$, $b_1x + a_1y + c_2 = 0$ always form an isosceles triangle.

* * * * * *

22. PAIR OF LINES

1. If θ is acute angle between the lines represented by $ax^2 + 2hxy + by^2 = 0$, then

   i) $\cos\theta = \dfrac{|a+b|}{\sqrt{(a-b)^2 + 4h^2}}$ ii) $\sin\theta = \dfrac{2\sqrt{h^2-ab}}{\sqrt{(a-b)^2 + 4h^2}}$ iii) $\tan\theta = \dfrac{2\sqrt{h^2-ab}}{|a+b|}$

2. The lines represented by $ax^2 + 2hxy + by^2 = 0$ are $ax + (h + \sqrt{h^2-ab})\,y = 0$ and $ax + (h - \sqrt{h^2-ab})\,y = 0$.
3. The equation of the pair of lines passing through (α, β) and parallel to $ax^2 + 2hxy + by^2 = 0$ is $a(x-\alpha)^2 + 2h(x-\alpha)(y-\beta) + b(y-\beta)^2 = 0$.
4. The equation of the pair of lines passing through (α,β) and perpendicular to $ax^2 + 2hxy + by^2 = 0$ is $b(x-\alpha)^2 - 2h(x-\alpha)(y-\beta) + a(y-\beta)^2 = 0$.
5. Angle bisectors of the lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are
   i) if $a_1a_2 + b_1b_2 > 0$, then ( ) + ( ) = 0 gives acute bisector.
   ( ) - ( ) = 0 gives obtuse bisector.
   ii) if $a_1a_2 + b_1b_2 < 0$, then ( ) - ( ) = 0 gives acute bisector.
   ( ) + ( ) = 0 gives obtuse bisector.
6. The difference of slopes of the lines represented by $ax^2 + 2hxy + by^2 = 0$ is $|m_1 - m_2| = \dfrac{2\sqrt{h^2-ab}}{|b|}$.
7. If one of the pair of lines $ax^2 + 2hxy + by^2 = 0$ bisect the angle between the coordinate axes, then $(a+b)^2 = 4h^2$.
8. The condition that one of the lines represented by $a_1x^2 + 2h_1xy + b_1y^2 = 0$ perpendicular to one of the lines represented by $a_2x^2 + 2h_2xy + b_2y^2 = 0$ is $(a_1a_2 - b_1b_2)^2 + 4(a_1h_2 + b_2h_1)(a_2h_1 + b_1h_2) = 0$.
9. If the pairs of lines $a_1x^2 + 2h_1xy + b_1y^2 = 0$, $a_2x^2 + 2h_2xy + b_2y^2 = 0$ have one line in common, then $(a_1b_2 - a_2b_1)^2 + 4(a_1h_2 - a_2h_1)(b_1h_2 - b_2h_1) = 0$.
10. A pair of lines is equally inclined to a line $L = 0$ if $L = 0$ is parallel to one of the angle bisectors.
11. Two pairs of lines $a_1x^2 + 2h_1xy + b_1y^2 = 0$, $a_2x^2 + 2h_2xy + b_2y^2 = 0$, are equally inclined to each other, if $\dfrac{h_1}{a_1 - b_1} = \dfrac{h_2}{a_2 - b_2}$
12. If the pair of lines $ax^2 + 2hxy + by^2 = 0$ are equally inclined with the coordinates axes, then $h = 0$, $ab < 0$.
13. The triangle formed by $ax^2 + 2hxy + by^2 = 0$ and $\ell x + my = 1$ is isosceles if $h(\ell^2 - m^2) = (a-b)\ell m$.
14. The centroid of the triangle formed by $ax^2 + 2hxy + by^2 = 0$ and $\ell x + my = 1$ is
    $$\left(\frac{2(b\ell - hm)}{3(am^2 - 2h\ell m + b\ell^2)}, \frac{2(am - h\ell)}{3(am^2 - 2h\ell m + b\ell^2)}\right).$$
15. If (α, β) is the centroid of the triangle formed by $ax^2 + 2hxy + by^2 = 0$ and $\ell x + my + n = 0$ then
    $$\ell\left(\frac{3\alpha}{2}\right) + m\left(\frac{3\beta}{2}\right) + n = 0.$$
16. The orthocentre of the triangle formed by the lines $ax^2 + 2hxy + by^2 = 0$ and $\ell x + my = 1$ is $(k\ell, km)$ where
    $$k = \frac{a+b}{am^2 - 2h\ell m + b\ell^2}.$$

17. If $ax^2 + 2hxy + by^2 = 0$ represents two sides of a triangle for which (c,d) is the orthocentre, then the equation of the third side is $(a + b)(cx + dy) = bc^2 - 2hcd + ad^2$.
18. The distance from a point (α, β) to a pair of lines passing through the origin is d, then the equation of the pair of lines is $(\beta x - \alpha y)^2 = d^2(x^2 + y^2)$.
19. If $ax^2 + 2hxy + by^2 = 0$ are two sides of a parallelogram and $\ell x + my = 1$ is one diagonal, then the equation of the other diagonal is $x(am - h\ell) = y(b\ell - hm)$.
20. The lines represented by $ax^2 + 2hxy + by^2 = 0$, $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ form a
    i) parallelogram if $a + b \neq 0$ and $(a - b) fg + h (f^2 - g^2) \neq 0$.
    ii) rectangle if $a + b = 0$ and $(a - b) fg + h (f^2 - g^2) \neq 0$.
    iii) rhombus if $a + b \neq 0$ and $(a - b) fg + h(f^2 - g^2) = 0$.
    iv) square if $a + b = 0$ and $(a - b) fg + h (f^2 - g^2) = 0$.
21. Length of the intercept on X - axis cut by the pair of lines $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ is $\dfrac{2\sqrt{g^2 - ac}}{|a|}$
22. Length of the intercept on Y - axis cut by the pair of lines $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ is $\dfrac{2\sqrt{f^2 - ac}}{|b|}$
23. The condition that the pair of lines represents by $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ to intersect on
    i) X - axis is $g^2 = ac$ or $2fgh - af^2 - ch^2 = 0$. ii) Y - axis is $f^2 = bc$ or $2fgh - bg^2 - ch^2 = 0$.
24. The product of the perpendiculars from the point (α, β) to the pair of lines $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ is $\dfrac{|a\alpha^2 + 2h\alpha\beta + b\beta^2 + 2g\alpha + 2f\beta + c|}{\sqrt{(a - b)^2 + 4h^2}}$.
25. The product of the perpendiculars from the origin to the pair of lines $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ is $\dfrac{|c|}{\sqrt{(a - b)^2 + 4h^2}}$.
26. If $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a pair of lines, then the square of the distance to their point of intersection from the origin is $\dfrac{c(a + b) - f^2 - g^2}{ab - h^2}$
27. The area of the parallelogram formed by $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, $ax^2 + 2hxy + by^2 = 0$ is $\dfrac{|c|}{2\sqrt{h^2 - ab}}$
28. The condition for the lines joining the origin to the points of intersection of the circle $x^2 + y^2 = a^2$ and the lines $\ell x + my = 1$ to
    i) coincide is $a^2 (\ell^2 + m^2) = 1$. ii) subtend a right angle at the origin is $a^2 (\ell^2 + m^2) = 2$.

23. 3D-COORDINATE SYSTEM

1. The ratio in which P (x, y, z) divides the line segment joining A (x_1, y_1, z_1), B $(x_2, y_2, z_2)$is $x_1 - x : x - x_2$ or $y_1 - y : y - y_2$ or $z_1 - z : z - z_2$.
2. The line segment joining (x_1, y_1, z_1), (x_2, y_2, z_2) is divided by
   i) XY - plane in the ratio - $z_1 : z_2$ ii) YZ - plane in the ratio - $x_1 : x_2$ iii) ZX - plane in the ratio - $y_1 : y_2$
3. If (α, β, γ) is the midpoint of $\overline{AB}$ where A = (x_1,y_1,z_1), then B = $(2\alpha - x_1, 2\beta - y_1, 2\gamma - z_1)$.
4. The centroid G of the tetrahedron ABCD divides the line joining any vertex to the centroid of its opposite face in the ratio 3:1.
5. In an equilateral triangle circumcentre, orthocentre, incentre, centroid coincide.
6. If d is the length of the diagonal of a square, then length of its side is $\frac{d}{\sqrt{2}}$.
7. Circumradius of a right angled triangle is $\frac{1}{2}$ (hypotenuse).
8. The points $A(x_1, y_1, z_1)$,B (x_2, y_2, z_2), $C(x_3, y_3, z_3)$ are collinear if $\frac{x_2 - x_1}{x_3 - x_1} = \frac{y_2 - y_1}{y_3 - y_1} = \frac{z_2 - z_1}{z_3 - z_1}$ or

$$\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix} = 0.$$

24. DIRECTION COSINES

1. If (ℓ,m) are direction cosines of a line $\overline{OP}$ and OP = r, then the coordinates of P are (ℓr, mr, nr).
2. If P = (x,y,z) and OP = r, then DC's of $\overline{OP}$ are $\left(\frac{x}{r}, \frac{y}{r}, \frac{z}{r}\right)$.
3. If (α, β, γ) are the angles made by a ray with the positive directions of the coordinate axes, then
   i) $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = 2$. ii) $\cos 2\alpha + \cos 2\beta + \cos 2\gamma + = -1$.
4. If (ℓ, m, n) are DC's of a line, then the maximum value of $\ell mn = \frac{1}{3\sqrt{3}}$.
5. The DC's of a line equally inclined to the coordinate axes are $\left(\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}\right)$.
6. The number of lines which are equally inclined to the coordinate axes is 4.
7. If θ is the acute angle between two lines whose DC's are (ℓ_1,m_1, n_1) and (ℓ_2,m_2, n_2), then
   i) cos = $|\ell_1, \ell_2 + m_1 m_2 + n_1 n_2|$ ii) $\sin \theta = \sqrt{\sum (m_1n_2 - m_2n_1)^2}$
8. If (a_1,b_1,c_1) (a_2, b_2, c_2) are DR's (or DC's) two lines, then the DR's of a line perpendicular to both the lines are $(b_1c_2 - b_2c_1, c_1a_2 - c_2a_1, a_1b_2 - a_2b_1)$.

9. If (ℓ_1,m_1, n_1) and (ℓ_2,m_2, n_2) are the DC's of two intersecting lines, then the DR's of the lines bisecting the angles between them are ($\ell_1 \pm \ell_2$ $m_1 \pm m_2$ $n_1 \pm n_2$).
10. If the DC's of two lines are connected by the relations $a\ell + bm + cn = 0$ and $fmn + gn\ell + h\ell m = 0$ then the two lines are i) perpendicular if $\frac{f}{a} + \frac{g}{b} + \frac{h}{c} = 0$ ii) parallel if $\sqrt{af} \pm \sqrt{bg} \pm \sqrt{ch} = 0$.
11. If the DC's of two lines are connected by the relations $a\ell + bm + cn = 0$ and $u\ell^2 + vm^2 + wn^2 = 0$, then the lines are i) perpendicular if $a^2 (v + w) + b^2 (w + u) + c^2 (u + v) = 0$., ii) parallel if $\frac{a^2}{u} + \frac{b^2}{v} + \frac{c^2}{w} = 0$.
12. The angle between any two diagonals of a cube is $\cos^{-1} \frac{1}{3}$.
13. The angle between a diagonal and a face diagonal of a cube is $\cos^{-1}\left(\sqrt{\frac{2}{3}}\right)$ or $\cos^{-1}\left(\frac{\sqrt{6}}{3}\right)$.
14. If the edges of a rectangular parallelopiped are a,b,c then the angles between the four diagonals are given by $\cos^{-1}\left(\frac{\pm a^2 \pm b^2 \pm c^2}{a^2 + b^2 + c^2}\right)$. Here in the numberator, the sings cannot be either all positive or all negative.
15. If a variable line in two adjacent positions has direction cosines (ℓ, m, n), ($\ell + \delta\ell$,m $+\delta$ m + m, n + δ n) and $\delta\theta$ is the angle between two positions, then $(\delta\ell)^2 + (\delta m)^2 + (\delta n)^2 = (\delta\theta)^2$.
16. The projection of the join of two points A (x_1, y_1, z_1) B(x_2, y_2, z_2) on a line whose DC's are (ℓ, m, n) is $|\ell(x_2 - x_1) + m(y_2 - y_1) + n(z_2 - z_1)|$.
17. If the projections of a line of length 'd' on the coordinate axes are d_1, d_2, d_3 respectively, then $d_1^2 + d_2^2 + d_3^2 = d^2$.
18. If the projections of a line of length 'd' on YZ, ZX, XY planes are d_1, d_2, d_3 respectively, then $d_1^2 + d_2^2 + d_3^2 = 2d^2$.
19. If a line makes angles α, β, γ, δ with the four diagonals of a cube, then $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = \frac{4}{3}$.

25. PLANE

1. Perpendicular distance from origin to the plane ax + by + cz + d = 0 is $\frac{|d|}{\sqrt{a^2 + b^2 + c^2}}$.
2. The ratio in which the plane $L \equiv ax + by + cz + d = 0$ divides the line segment joining A (x_1, y_1, z_1) B(x_2, y_2, z_2) is $\frac{-L_{11}}{L_{22}}$.
3. Let $L \equiv ax + by + cz + d = 0$ be a plane and A (x_1, y_1, z_1) be a point then
   i) A lies in the originside of the plane if $dL_{11} > 0$.
   ii) A lies in the opposite to origin side of the plane if $dL_{11} < 0$.
4. The plane ax + by + cz + d = 0 is parallel to
   i) X - axis if a = 0 ii) Y - axis if b = 0 iii) Z - axis if c = 0.
5. If a plane meets the coordinate axes in A,B,C such that the centroid of the triangle ABC is the point (p,q,r) then the equation of the plane is $\frac{x}{p} + \frac{y}{q} + \frac{z}{r} = 3$.

6. A variable plane is at a distance 'p' from the origin and meets the axes in A,B,C. The locus of the centroid of ΔABC is $x^{-2} + y^{-2} + z^{-2} = 9p^{-2}$.
7. A variable plane is at a distance '3p' from the origin and meets the axes in A,B,C. The locus of the centroid of the triangle ABC is $x^{-2} + y^{-2} + z^{-2} = p^{-2}$.
8. A variable plane is at a constant distance 'p' from the origin and meets the axes in A,B,C. The locus of the centroid of the tetrahedron OABC is $x^{-2} + y^{-2} + z^{-2} = 16p^{-2}$.
9. A variable plane is at a constant distance '4p' from the origin and meets the axes in A,B,C. The locus of the centroid of the tetrahedrol OABC is $x^{-2} + y^{-2} + z^{-2} = p^{-2}$.
10. Two systems of rectangular axes have the same origin.If a plane cuts them at distances a,b,c and a_1, b_1, c_1 respectively from the origin, then $a^{-2} + b^{-2} + c^{-2} = a_1^{-2} + b_1^{-2} + c_1^{-2}$.
11. Equation of any plane passing through the line of intersecting of the planes $L_1 = 0$ and $L_2 = 0$ is of the form $L_1 + \lambda L_2 = 0$.

26. LIMITS

1. If $\underset{x\to a}{Lt} \frac{f(x)}{g(x)}$ is of the form $\frac{0}{0}$ or $\frac{\infty}{\infty}$, then $\underset{x\to a}{Lt} \frac{f(x)}{g(x)} = \underset{x\to a}{Lt} \frac{f'(x)}{g'(x)} = \underset{x\to a}{Lt} \frac{f''(x)}{g''(x)}$ till indeterminate form is eliminated .

2. $\underset{x\to 0}{Lt} \frac{1-\cos x}{x} = 0.$

3. $\underset{x\to 0}{Lt} \frac{1-\cos ax}{x} = \frac{a^2}{2}$

4. $\underset{x\to 0}{Lt} \frac{1-\cos nx}{1-\cos mx} = \frac{n^2}{m^2}$

5. $\underset{x\to 0}{Lt} \frac{\cos ax - \cos bx}{x^2} = \frac{b^2 - a^2}{2}$

6. $\underset{x\to 0}{Lt} \frac{\cos ax - \cos bx}{\cos cx - \cos dx} = \frac{b^2 - a^2}{d^2 - c^2}$

7. If g(x) = x, sin x, Tanx, $Sin^{-1}x$, $Tan^{-1}x$, sinhx, Tanhx, $Sinh^{-1}x$ then

   i) $\underset{x\to 0}{Lt} \frac{\sqrt[m]{a+x^n} - \sqrt[m]{a-x^n}}{\{g(x)\}^n} = \frac{2}{m.a^{1-1/m}}$

   ii) $\underset{x\to 0}{Lt} \frac{\sqrt{a+x^n} + \sqrt{a-x^n}}{\{g(x)\}^n} = \frac{1}{\sqrt{a}}$

   iii) $\underset{x\to 0}{Lt} \frac{\sqrt{a+x^n} + \sqrt{a}}{\{g(x)\}^n} = \frac{1}{2\sqrt{a}}$.

8. $\underset{x\to \infty}{Lt} (\sqrt{x^2 + ax + b} - x) = \frac{a}{2}$

9. $\underset{x\to \infty}{Lt} (x - \sqrt{x^2 - ax + b}) = \frac{a}{2}$

10. If $\underset{x \to a}{Lt} f(x) = 0$ and g (x) is a bounded, then $\underset{x \to a}{Lt} f(x)\, g(x) = 0$.

11. $\underset{x \to \infty}{Lt} \dfrac{1^k + 2^k + 3^k + \ldots\ldots\ldots + n^k}{n^{k+1}} = \dfrac{1}{k+1}$

12. i) If $-1 < x < 1$, $\underset{x \to \infty}{Lt} x^n = 0$. ii) If $x > 1$, $\underset{x \to \infty}{Lt} x^n = \infty$.

13. $\underset{n \to \infty}{Lt} \left\{ \dfrac{1}{a(a+d)} + \dfrac{1}{(a+d)(a+2d)} + \ldots\ldots\ldots + \dfrac{1}{[a+(n-1)d]\,[a+nd]} \right\} = \dfrac{1}{ad}$

14. $\underset{x \to 0}{Lt} \dfrac{a^x - b^x}{x} = \log_e \left(\dfrac{a}{b} \right)$

15. $\underset{x \to 0}{Lt} \dfrac{a^x - 1}{b^x - 1} = \log_e a$

16. $\underset{x \to a}{Lt} [f(x)]^{g(x)} = e^k$ where $K = \underset{x \to a}{Lt} g(x) \log f(x)$, $f(a) \neq 1$.

$= \underset{x \to a}{Lt} g(x)\,[f(x) - 1]$, $f(a) = 1$.

17. i) $\underset{x \to 0}{Lt} (1 + ax)^{1/x} = e^a$

ii) $\underset{x \to \infty}{Lt} (1 + a/x)^x = e^a$.

18. $\underset{x \to 0}{Lt} \left[\dfrac{a_1^x + a_2^x + a_3^x + \ldots\ldots + a_n^x}{n} \right]^{1/x} = \sqrt[n]{a_1 a_2 a_3 \ldots a_n}$

19. $\underset{x \to \infty}{Lt} \left[\dfrac{a_1^{1/x} + a_2^{1/x} + a_3^{1/x} + \ldots\ldots + a_n^{1/x}}{n} \right]^x = \sqrt[n]{a_1 a_2 a_3 \ldots a_n}$

20. $\underset{x \to \infty}{Lt} \left(\dfrac{x+a}{x+b} \right)^{cx+d} = e^{c(a-b)}$

21. $\underset{x \to \infty}{Lt} \left(\dfrac{x^2 + ax + b}{x^2 + px + q} \right)^{cx+d} = e^{c(a-p)}$

22. $\underset{x \to \infty}{Lt} \dfrac{a^{x+\alpha} + b}{a^{x+\beta} + c} = a^{\alpha - \beta}$

23. If $a > 0$, then $\underset{x \to \infty}{Lt} \dfrac{[ax+b]}{x} = a$.

27. CONTINUITY

1. |x| is continous everywhere.
2. [x] and x - [x] are continuous on R - Z.
3. If f(x + y) = f(x) + f (y) $\forall$ x, y $\in$ R and f is continous at x = 0 then f is continuous for all x, y $\in$ R.

28. DERIVATIVES

1. $f'(x) = \underset{x \to 0}{Lt} \dfrac{f(x+h) - f(x)}{h}$

2. $f'(a) = \underset{x \to a}{Lt} \dfrac{f(x) - f(a)}{x-a}$

3. $(gof)'(x) = g'[f(x)]\ [f(x)]\ f'(x)$.

4. $\dfrac{d}{dx}\left[\dfrac{ax+b}{cx+d}\right] = \dfrac{ad-bc}{(cx+d)^2}$

5. If f(x,y) = c is an implicit function,then $\dfrac{dy}{dx} = \dfrac{-\partial f}{\partial x} / \dfrac{\partial f}{\partial y}$

6. If $x^m y^n = (x + y)^{m+n}$, then $\dfrac{dx}{dy} = \dfrac{y}{x}$.

7. If $x^m y^n = a^{m+n}$, then $\dfrac{dx}{dy} = \dfrac{-my}{nx}$.

8. If x = f (t), y = g (t), then $\dfrac{dx}{dy} = \dfrac{dy}{dt} / \dfrac{dx}{dt}$.

9. Derivative of f(x) with respect to g(x) is $f'(x) / g'(x)$.

10. If $y = \sqrt{f(x)\sqrt{f(x)\sqrt{f(x)\ldots\ldots\infty}}}$, then dy/dx = $f'(x)$.

11. If $y = \sqrt{f(x)+\sqrt{f(x)+\sqrt{f(x)+\ldots\ldots\infty}}}$, then dy/dx = $f'(x)$ / (2y - 1).

12. If y = f(x) + 1/y then $\dfrac{dx}{dy} = \dfrac{y^2 f'(x)}{y^2+1}$

13. If $y = f(x)^y$, then $\dfrac{dx}{dy} = \dfrac{y^2 f'(x)}{f(x)[1 - y \log f(x)]} = \dfrac{y^2 f'(x)}{f(x)[1 - \log y]}$

14. $\dfrac{d}{dx}\{f(x)^{g(x)}\} = f(x)^{g(x)} \left\{g(x)\dfrac{f'(x)}{f(x)} + g'(x)\log f(x)\right\}$

15. $\frac{d}{dx}\{ f(x)^{f(x)} \} = f(x)^{f(x)} [1+ \log (f(x))] f'(x)]$.

16. If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, then $\frac{dx}{dy} = \frac{\sqrt{1-x^2}}{\sqrt{1-y^2}}$

17. If $y = \text{Tan}^{-1}\left[\frac{a\cos f(x) + b\sin f(x)}{b\cos f(x) - a\sin f(x)}\right]$, then $\frac{dx}{dy} = f'(x)$.

18. If $y = \text{Tan}^{-1}\left[\frac{a\cos f(x) - b\sin f(x)}{b\cos f(x) + a\sin f(x)}\right]$, then $\frac{dx}{dy} = f'(x)$.

19. If $a^x + a^y = a^{x+y}$, then $\frac{dx}{dy} = -a^{y-x}$.

20. If f is even, then $f'(x)$ is odd.
21. If f is odd, then $f'(x)$ is even.
22. If f(xy) = k, then dy/dx = -y/x.
23. If $f(x + y) = f(x) f(y)\ \forall\ x,y \in R$ and $f(a) \neq (0)$ then $f'(a) = f'(0)\ f(a)$.
24. Every differentiable function is continuous but the converse need not be true. |x| is not differentiable at x = 0.

29. ERRORS AND APPROXIMATIONS

1. If $y = kx^n$, $k \in R$, then i) $\frac{\Delta y}{y} = n\frac{\Delta x}{x}$ ii) $\frac{\Delta y}{y} \times 100 = n\frac{\Delta x}{x} \times 100$
2. In Δ ABC, $\Delta A + \Delta B + \Delta C = 0$.
3. If a,b,c are the sides of ΔABC, then $\frac{\Delta a}{\cos A} + \frac{\Delta b}{\cos B} + \frac{\Delta c}{\cos C} = 0$.

30. RATE MEASURE

1. A particle attains maximum height if V = 0.
2. A particle attains maximum velocity if a = 0.

31. TANGENTS AND NORMALS

1. The equation of the tangent at (a,b) to the curve $\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 2$ is $\frac{x}{a} + \frac{y}{b} = 2$.
2. A tangent to the curve $x^{2/3} + y^{2/3} = a^{2/3}$ (or $x = a\cos^3\theta$, $y = a\sin^3\theta$) cuts the coordinate axes in A and B, then AB = a. Also equation of the tangent at 'θ' is $\frac{x}{a\cos\theta} + \frac{y}{a\sin\theta} = 1$.
3. If p,q are the lengths of perpendiculars from the origin to tangent and normal at a point respectively on the curves $x^{2/3} + y^{2/3} = a^{2/3}$, then $4p^2 + q^2 = a^2$.
4. A tangent to the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ (or $x = a\cos^4\theta$, $y = a\sin^4\theta$) cuts the axes in A and B, then OA + OB = a.
5. If the tangent at P on the curve $x^m y^n = a^{m+n}$ meets the coordinate axes at A and B, then AP :PB is n : m.
6. If the tangent at P to the curve $xy = a^2$ meets the axes at A,B then P divides AB in the ratio 1:1.
7. The area of the triangle formed by the tangent to the curve $xy = a^2$ at any point on the curve with the coordinate axes is $2a^2$.
8. At any point on the curve $by^2 = (x + a)^3$, $\frac{(L.S.T)^2}{L.S.N} = \frac{8b}{27}$
9. If the curve $xy = c^2$, $y^2 = 4ax$ cut each other orthogonally, then $c^4 = 32a^4$.
10. If the curves $a_1x^2 + b_1y^2 = 1$, $a_2x^2 + b_2y^2 = 1$ cut each other orthogonally, then $\frac{1}{a_1} - \frac{1}{b_1} = \frac{1}{a_2} - \frac{1}{b_2}$.
11. The angle between the curves $y^2 = 4ax$, $x^2 = 4ay$
    i) at (0,0) is 90^0. ii) at (4a, 4a) is $\tan^{-1} 3/4$.
12. The condition that the curves $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and $\frac{x^2}{\alpha^2} + \frac{y^2}{\beta^2} = 1$ may intersect orthogonally is $a^2 - b^2 = \alpha^2 - \beta^2$.
13. The angle of intersection of curves $\frac{x^2}{a^2 + k_1} + \frac{y^2}{b^2 + k_1} = 1$ and $\frac{x^2}{a^2 + k_2} + \frac{y^2}{b^2 + k_2}$ is 90^0.
14. To any curve y = f (x), length of subtangent, ordinate and length of sub normal are in G.P with common ratio m.

32. MAXIMA AND MINIMA

1. The minimum value of $(x - \alpha)(x - \beta)$ is $\frac{-(\alpha - \beta)^2}{4}$
2. The minimum value of a cot x + b tan x is $2\sqrt{ab}$.
3. The greatest value of $\sin^3x + \cos^3x$ is 1.
4. The minimum value of $a^2\sec^2\theta + b^2\operatorname{cosec}^2\theta$ is $(a + b)^2$.
5. The minimum value of a sec x + b cosec x is $(a^{2/3} + b^{2/3})^{3/2}$.
6. If x + y = k, then $x^2 + y^2$ is minimum at x = y = k/2.
7. If x + y = k, then xy is maximum at x = y = k/2.
8. If xy = k, then x + y is minimum at x = y = $\sqrt{k}$.
9. If x + y = k, then $x^3 + y^3$ is minimum at x = y = k/2.
10. If x + y = k, the x^3y^2 is maximum, then the required numbers are in the ratio 3:2.
11. The maximum rectangle inscribed in a circle is square.
12. The maximum area of a rectangle inscribed in a circle of radius r is $2r^2$.

13. The maximum triangle inscribed in a circle is equilateral.
14. The maximum area of a triangle inscribed in a circle of radius r is $\frac{3\sqrt{3}}{4} r^2$.
15. The perimeter of a sector is C cm. The maximum area of the sector is $\frac{C^2}{16}$ Sq.cm.
16. If the perimeter of a sector of a circle is constant and its area is maximum, then the angle of the sector is 2 radians.
17. The point on the parabola which is closest to the given line is the 'point of contact' of the tangent which is parallel to the given line.
18. For $x > 0$, $x + \frac{1}{x} \geq 2$.
19. If P and Q are two given points. Then the point A on x - axis for which PA + AQ is least is $\left(\frac{x_1y_2 + x_2y_1}{y_1 + y_2}, 0\right)$
20. The height of the cylinder of maximum volume which can be inscribed in a sphere of radius r is $\frac{2r}{\sqrt{3}}$.
21. The height of the cone of maximum volume which can be inscribed in a sphere of radius r is $\frac{4r}{3}$.
22. From four corners of a square sheet of metal of side 'a', four equal squares are cut off and the remaining edges are folded up to form a rectangular open box. If the volume of the box formed is to be maximum, the side of the square is a/6.
23. From the four corners of a rectangular sheet of metal of sides a,b; four equal squares are cut off and the remaining edges are folded up to form an open box. If the volume of the box is to be maximum, the side of the square is $\frac{a + b - \sqrt{a^2 + b^2 - ab}}{6}$.

33. PARTIAL DIFFERENTIATION

1. If $u = \log r$, $r^2 = x^2 + y^2 + z^2$ then $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z y^2} = \frac{1}{r^2}$.

2. If $u = f(r)$ and $r^2 = x^2 + y^2$ then $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f'(r) + \frac{1}{r} f'(r)$.

3. If $u = f(r)$ and $r^2 = x^2 + y^2 + x^2$ then $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z y^2} = f'(r) + \frac{2}{r} f'(r)$.

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