FIITJEE

KUKATPALLY CENTRE

IMPORTANT QUESTIONS
FOR
INTERMEDIATE PUBLIC EXAMINATIONS
IN

MATHS-IB

FIITJEE KUKATPALLY CENTRE: # 22-97, Plot No.1, Opp. Patel Kunta Huda Park, Vijaynagar Colony, Hyderabad - 500 072. Ph.: 040-64601123
Regd. Off.: 29A, ICES House, Kalu Sarai, Sarvapriya Vihar, New Delhi - 110 016. Ph: 011 - 2651 5949, 2656 9493, Fax: 2651 3942

Points to Remember

- **Mathematics** question paper contains 24 questions divided into three sections A, B and C. In Section A 10 very short answer questions of 2 marks each and you have to answer all questions. In Section B, 7 short answer questions of 4 marks each and you have to answer 5 questions. In Section C 7 long answer questions of 7 marks each and you have to answer 5 questions.
- **Mathematics** is a subject where you learn by doing things. The more you practice the better you will be able to understand the concepts.
- It's not just the answer that counts in **Mathematics**. Much of your scoring is based on the intermediate steps in getting to the answer. So show all your steps. Your objective should be to convince your examiner that you know how to solve the problem.
- Don't just aim for 90 percent. Aim high and go for 100 percent. This is a subject where you can score cent percent.
- No additional sheet provided except the main answer booklet which contains 24 ruled pages, you can use the last few pages in main answer book to do the rough work. Cancel those rough pages at last
- If you are not in a position to solve any particular question, then it is better not to waste time on that question. Skip and head to the next question, then solve those questions afterwards
- You must write the formula whenever it is required as marks for the formula may be given in the marking scheme and chances of errors are less
- Underline important points in the answer to make them more prominent.
- Mark the attempted questions so that you can come back to the unattempted at the end of the paper
- Always attempt all the questions including choice because you may get full marks if question is mis-printed or wrong.
- Clarity in each step and error free calculations is the key to success in the mathematics exam.
- As far as possible, maintain the sequence of answers.
- Bright students should ideally look at attempting the paper from Section A and then Section B and then Section C
- If you have finished the paper it is better not to submit the answer sheet. Utilize the remaining time for revision.
- First complete booklet which contains all the important questions and then do all examples and exercise problems in Telugu Academy Text book.

All the best

Mathematics Dept.

Points to Remember

For all those who wish to score 150 out of 150, the time to start the preparation has come. The wise man finishes the things well before the D-day while the fool wastes the plenty of time available and rushes up the things in the end, and achieves nothing. So better you make a well thought out plan of study. A good plan is the road-map to success, it shows you the final destination. Reduce your plan to writing. The moment you complete this, you will have definitely given a concrete shape to your desire. The reason that most people never reach this goal is that they don't define them, learn about them, or seriously consider them as achievable. Remember this famous quote: "you were born to win, but to be a winner, you must plan to win, prepare to win, and expect to win." Once you have put a plan of study, stick to it completely. Initially you will find it somewhat difficult to follow, but it would become a routine after sometime paving the way of success for you.

We, at **FIITJEE** will always help you, motivate you, guide you in achieving your dream of scoring 100%. In this direction we are providing you with important questions and answers booklet-1 which will ensure you at least 50 marks out of 75 if you are thorough with all these questions and answers. In this series we will provide booklet-2 which will ensure you at least 70 if you do everything in this booklet-2.To score 75 out of 75 you need to do the entire telugu academy text book which will help to achieve good ranks in JEE (Mains), BITSAT and other engineering entrance exams.

For your reference, we are providing March 2014 Intermediate public examination paper in the next page.

INTERMEDIATE PUBLIC EXAMINATION, MAY 2014

Total No. of Questions - 24 | **Reg. No.**
Total No. of Printed Pages - 2

Part - III
MATHEMATICS, Paper-I (B)
(English Version)

Time : 3 Hours] **[Max. Marks : 75**

SECTION - A $10 \times 2 = 20$ M

I. Very Short Answer Type questions:

1. Transform the equation $4x - 3y + 12 = 0$ into

   (i) Slope-Intercept form, (ii) Intercept form

2. Find the value of 'P', if the lines $4x - 3y - 7 = 0, 2x + Py + 2 = 0$ and $6x + 5y - 1 = 0$ are concurrent.

3. Find the ratio in which the XZ-plane divides the line joining $A(-2,3,4)$ and $B(1,2,3)$.

4. Find the equation of the plane whose intercepts on X,Y,Z-axes are 1,2,4 respectively

5. Compute $\underset{x \to 0}{Lim} \dfrac{x\left(e^x - 1\right)}{1 - \cos x}$.

6. Compute $\underset{x \to \infty}{Lim} \dfrac{x^2 + 5x + 2}{2x^2 - 5x + 1}$

7. If $f(x) = 1 + x + x^2 + ... + x^{100}$, then find $f^{'}(1)$

8. If $y = ae^{nx} + be^{-nx}$, then prove that $y^{''} = n^2 y$.

9. If the increase in the side of a square is 4%, then find the approximate percentage of increase in the area of the square.

10. Define the strictly increasing function and strictly decreasing function on an interval.

SECTION – B $5 \times 4 = 20$ M

II. Short Answer Type questions:
(i) Attempt any **five** questions
(ii) Each question carries **four** marks

11. If the distance from P to the points $(2,3)$ and $(2,-3)$ are in the ratio 2 : 3, then find the equation of the locus of P.

12. When the axes are rotated through an angle $\frac{\pi}{4}$, find the transformed equation of $3x^2 + 10xy + 3y^2 = 9$.

13. If $Q(h,k)$ is the foot of the perpendicular from $P(x_1, y_1)$ on the straight line $ax+by+c=0$, then show that $\frac{h-x_1}{a} = \frac{k-y_1}{b} = -\frac{(ax_1+by_1+c)}{a^2+b^2}$

14. Show that $f(x) = \begin{cases} \frac{\cos ax - \cos bx}{x^2}, & if\ x \neq 0 \\ \frac{1}{2}(b^2-a^2), & if\ x = 0 \end{cases}$

where a and b are real constants, is continuous at 0.

15. Find the derivative of $\cos ax$ from the first principle.

16. Find the equations of tangent and normal to the curve $y = x^3 + 4x^2$ at $(-1,3)$

17. Find the lengths of sub-tangent, sub-normal at any point on the curve $x = a(\cos t + t\sin t), y = a(\sin t - t\cos t)$

SECTION – C

5 × 7 = 35 M

III. Long Answer Type questions:
(i) Attempt any **five** questions
(ii) Each question carries **seven** marks

18. Find the ortho-centre of the triangle formed by the lines $x+2y=0$, $4x+3y-5=0$ and $3x+y=0$.

19. Show that the product of the perpendicular distances from a point (α, β) to the pair of straight lines $ax^2+2hxy+by^2=0$ is $\left|\frac{a\alpha^2+2h\alpha\beta+b\beta^2}{\sqrt{(a-b)^2+4h^2}}\right|$

20. Find the angle between the lines joining the origin to the points of intersection of the curve $x^2+2xy+y^2+2x+2y-5=0$ and the line $3x-y+1=0$.

21. Find the angle between the lines, whose direction cosines satisfy the equations $l+m+n=0$ and $l^2+m^2-n^2=0$.

22. If $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$, then show that $\frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$.

23. Show that the condition for the orthogonality of the curves $ax^2+by^2=1$ and $a_1x^2+b_1y^2=1$ is $\frac{1}{a}-\frac{1}{b}=\frac{1}{a_1}-\frac{1}{b_1}$.

24. Find two positive integers x and y such that $x+y=60$ and xy^3 is maximum.

VERY SHORT ANSWER QUESTIONS

1.

A. Find the equation of line which makes an angle of 150^0 with positive x-axis and passing through (-2, -1).

B. Find the equation of straight line passing through origin and making equal angles with coordinate axes.

C. Find the equation of the straight line passing through (-2, 4) and making non zero intercepts whose sum is zero.

D. Find the equation of the straight line making an angle of $Tan^{-1}\left(\frac{2}{3}\right)$ with the positive x-axis and has y-intercept 3.

E. Find the equation of the straight line passing through (3, -4) and making X and Y-intercepts which are in the ratio 2 : 3.

F. Find the equations of the straight line passing through the following points :

a) (2, 5), (2, 8) b) (3, -3), (7, -3) c) (1, -2), (-2, 3) d) $\left(at_1^{\,2}, 2at_1\right)\left(at_2^{\,2}, 2at_2\right)$

G. Find the equations of the straight lines passing through the point (4, -3) and (i) parallel (ii) perpendicular to the line passing through the points (1, 1) and (2, 3).

H. Prove that the points (-5, 1), (5, 5), (10, 7) are collinear and find the equation containing the lines.

I. If the portion of a straight line intercepted between the axes of coordinates is bisected at (2p, 2q). Find the equation of the straight line.

J. A straight line passing through A (-2, 1) makes an angle of 30^0 with $\overleftrightarrow{OX}$ in the positive direction. Find the points on the straight line whose distance from A is 4 units.

K. Find the points on the line $3x-4y-1=0$ which are at a distance of 5 units from the point (3, 2).

L. If the area of the triangle formed by the straight lines $x=0$, $y=0$ and $3x+4y=a$ $(a>0)$ is 6. Find the value of a.

M. Transform the equation $x+y+1=0$ into normal form.

N. Find the ratio in which the straight line $2x+3y-20=0$ divides the join of the points (2, 3) and (2, 10).

O. State whether (3, 2) and (-4, -3) are on the same side or on opposite side of the straight line $2x-3y+4=0$

2.

A. Find the equation of the straight line passing through the point of intersection of the lines $x+y+1=0$ and $2x-y+5=0$ and containing the point (5, -2).

B. If a, b, c are in A.P, show that $ax+by+c=0$ represents a family of concurrent lines and find the concurrency.

C. Find the point of concurrency of the lines represented by $(2+5k)x-3(1+2k)y+(2-k)=0$

D. Find the area of the triangle formed by the coordinate axes and the line $3x-4y+12=0$

E. Find the set of values of 'a' if the points (1, 2) and (3, 4) lie on the same side of the straight line $3x-5y+a=0$

F. Find the value of k, if the angle between the straight lines $4x-y+7=0$ and $kx-5y-9=0$ is 45^0.

G. If $2x-3y-5=0$ is the perpendicular bisector of the line segment joining (3, -4) and (α, β). Find $\alpha+\beta$.

H. Find the length of the perpendicular drawn from (3, 4) to the line $3x-4y+10=0$.

I. Find the distance between the straight lines $5x-3y-4=0$ and $10x-6y-9=0$.

J. Find the circumcentre of the triangle formed by the lines $x=1, y=1$ and $x+y=1$.

K. Find the value of 'k' if the straight lines $y-3kx+4=0$ & $(2k-1)x-(8k-1)y-6=0$ are perpendicular.

L. Find the equation of the line perpendicular to the line $3x+4y+6=0$ and making an intercept -4 on the x-axis.

M. Find the orthocenter of the triangle whose sides are given by $x+y+10=0$, $x-y-2=0$ and $2x+y-7=0$.

N. If (-2, 6) is the image of the point (4, 2) w.r.t the line L, then find the equation of L.

O. Find the angle which the straight line $y=\sqrt{3x}-4$ makes with y-axis.

3.

A. Find x if the distance between $(5,-1,7)$ and $(x,5,1)$ is 9 units.

B. Show that the points $(2,3,5)$, $(-1,5,-1)$ and $(4,-3,2)$ form a right angled isosceles triangle.

C. Show that the points $(1,2,3)$,$(2,3,1)$ and $(3,1,2)$ form an equilateral triangle.

D. P is a variable point which moves such that 3PA = 2PB. If A = $(-2,2,3)$ and $B=(13,-3,13)$, prove that P satisfies the equation $x^2+y^2+z^2+28x-12y+10z-247=0$

E. Show that the points $(1,2,3)$,$(7,0,1)$ and $(-2,3,4)$ are collinear.

F. Show that ABCD is a square where A, B, C, D are the points $(0,4,1)$,$(2,3,-1)$,$(4,5,0)$ and $(2,6,2)$ respectively.

4.

A. Find the equation of the plane passing through $(2,0,1)$ and $(3,-3,4)$ and perpendicular to $x-2y+z=6$.

B. Find the equation of the plane through $(4,4,0)$ and perpendicular to the planes $2x+y+2z+3=0$ and $3x+3y+2z-8=0$.

C. Find the equation of the plane through the points $(2,2,-1)$,$(3,4,2)$,$(7,0,6)$.

D. A plane meets the coordinate axes in A,B,C. If centroid of the ΔABC is (a,b,c). Show that the equation to the plane is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=3$.

E. Find the equation of the plane if the foot of the perpendicular from origin to the plane is $(2,3,-5)$.

F. Find the equation to the plane parallel to the ZX-plane and passing through $(0,4,4)$.

G. Find the equation of the plane if the foot of the perpendicular from origin to the plane is $(1,3,-5)$.

H. Reduce the equation $x+2y-3z-6=0$ of the plane to the normal form.

I. Find the equation of the plane passing through the point $(-2,1,3)$ and having $(3,-5,4)$ as d.r.'s of its normal.

J. Find the angle between the planes $x+2y+2z-5=0$ and $3x+3y+2z-8=0$.

Find the equation of the plane passing through the point $(1,1,1)$ and parallel to the plane $x+2y+3z-7=0$

5.

A. Evaluate $\underset{x\to 0}{Lt}\ x^2\cos\frac{2}{x}$

B. Compute $\underset{x\to 3}{Lt}\ \frac{x^2-8x+15}{x^2-9}$

C. If $f(x)=\begin{cases}x^2 & , x\le 1\\ 2x-1, & x>1\end{cases}$ then find $\underset{x\to 1+}{Lt}\ f(x)$ and $\underset{x\to 1-}{Lt}\ f(x)$

D. Compute $\underset{x\to 2+}{Lt}\ [x]+x$ and $\underset{x\to 2-}{Lt}\ [x]+x$

E. Find $\underset{x\to 0}{Lt}\ f(x)$ where $f(x)=\begin{cases}x-1 & if\ x<0\\ 0 & if\ x=0\\ x+1 & if\ x>0\end{cases}$

F. Find $\underset{x\to 0}{Lt}\ \frac{\sqrt{1+x}-1}{x}$

G. Compute $\underset{x\to 0}{Lt}\ \frac{e^x-1}{\sqrt{1+x}-1}$

H. Compute $\underset{x\to 0}{Lt}\ \frac{a^x-1}{b^x-1}$,$(a>0, b>0, b\ne 1)$

I. Compute $\underset{x\to 0}{Lt}\ \frac{\sin ax}{\sin bx}$, $b\ne 0$, $a\ne b$

J. Compute $\underset{x\to 0}{Lt}\ \frac{e^{3x}-1}{x}$

K. Compute $\underset{x\to 0}{Lt}\ \frac{e^x-\sin x-1}{x}$

L. Evaluate $\underset{x\to 1}{Lt}\ \frac{\log_e x}{x-1}$

M. Compute $\underset{x\to \pi/2}{Lt}\ \frac{\cos x}{x-\pi/2}$

N. Compute $\underset{x\to 1}{Lt}\ \frac{\sin(x-1)}{x^2-1}$

O. Compute $\underset{x\to 0}{Lt}\ \frac{\sin(a+bx)-\sin(a-bx)}{x}$

P. Compute $\underset{x\to 0}{Lt}\ \frac{\tan(x-a)}{x^2-a^2}$ $(a\ne 0)$

Q. Compute $\underset{x\to 3}{Lt}\ \frac{e^x-e^3}{x-3}$

6.

A. Compute $\underset{x\to 0}{Lt}\ \dfrac{1-\cos mx}{1-\cos nx}\ \ n\neq 0$

B. Compute $\underset{x\to 0}{Lt}\ \dfrac{x\left(e^{x}-1\right)}{1-\cos x}$

C. Compute $\underset{x\to\infty}{Lt}\ \dfrac{\sqrt{x^{2}+6}}{2x^{2}-1}$

D. Compute $\underset{x\to\infty}{Lt}\ \dfrac{8|x|+3x}{3|x|-2x}$

E. Compute $\underset{x\to-\infty}{Lt}\ \dfrac{5x^{3}+4}{\sqrt{2x^{4}+1}}$

F. Compute $\underset{x\to\infty}{Lt}\ \sqrt{x^{2}+x}$

G. Compute $\underset{x\to-\infty}{Lt}\ \dfrac{2x+3}{\sqrt{x^{2}-1}}$

H. Compute $\underset{x\to\infty}{Lt}\ \dfrac{2+\sin x}{x^{2}+3}$

I. Compute $\underset{x\to\infty}{Lt}\ \dfrac{2+\cos^{2}x}{x+2007}$

J. Compute $\underset{x\to\infty}{Lt}\ \dfrac{\cos x+\sin^{2}x}{x+1}$

K. Is f defined by $f(x)=\begin{cases}\dfrac{\sin 2x}{x}, & if\ x\neq 0\\ 1, & if\ x=0\end{cases}$ continuous at $x=0$

L. Show that $f(x)=\begin{cases}\dfrac{\cos ax-\cos bx}{x^{2}} & if\ x\neq 0\\ \dfrac{1}{2}\left(b^{2}-a^{2}\right) & if\ x=0\end{cases}$ where a and b are real constants, is continuous at 0

7.

A. If $f(x)=7^{x^{2}+3x}\ (x>0)$ then find $f'(x)$

B. If $f(x)=x\,e^{x}\sin x$, then find $f'(x)$

C. If $f(x)=\sin(\log x)\,,(x>0)$ find $f'(x)$

D. If $f(x)=\left(x^{3}+6x^{2}+12x-13\right)^{100}$ find $f'(x)$

E. If $f(x)=\log_{7}(\log x)\,, x>0$ find $\dfrac{dy}{dx}$

F. If $f(x)=1+x+x^2+.......+x^{100}$ then find $f'(1)$

G. If $f(x)=2x^2+3x-5$ then prove that $f'(0)+3f'(-1)=0$

H. If $f(x)=\log(\sec x+\tan x)$ find $f'(x)$

8.

A. If $y=\sin^{-1}\sqrt{x}$, find $\frac{dy}{dx}$

B. If $y=\log(\cosh 2x)$, find $\frac{dy}{dx}$

C. If $y=\log\left(\sin(\log x)\right)$, find $\frac{dy}{dx}$

D. Find the derivative of $\log\left(\frac{x^2+x+2}{x^2-x+2}\right)$

E. If $y=\frac{2x+3}{4x+5}$ then find y^{11}

F. If $y=a\,e^{nx}+b\,e^{-nx}$ then prove that $y^{11}=n^2y$

H. Find the second order derivative of $f(x)=\log\left(4x^2-9\right)$

9.

A. Find dy and Δy of $y=f(x)=x^2+x$ at $x=10$ when $\Delta x=0.1$

B. The side of a square is increased from 3 cm to 3.01cm. Find the approximate increase in the area of the square .

C. If the radius of a sphere is increased from 7 cm to 7.02 cm then find the approximate increase in the volume of the sphere.

D. If the increase in the side of a square is 2% then find the approximate percentage of increase in its area.

E. Find dy and Δy if $y=\frac{1}{x+2}$, $x=8$ and $\Delta x=0.02$

F. Find the approximate value of $\sqrt[3]{999}$

G. The diameter of a sphere is measured to be 40 cm. If the error of 0.02 cm is made in it, then find the approximate errors of o.02cm is made in it, then find the approximate errors in volume and surface area of the sphere.

H. Find the slope of the tangent to the curve $y=\frac{x-1}{x-2}(x\neq 2)$ at $x=10$

I. Show that the length of the subnormal at any point on the curve $y^2=4ax$ is constant

J. Show that the length of the subtangent at nay point on the curve $y=a^x$ $(a>0)$ is a constant

10.

A. Find the value of k, so that the length of the subnormal at any point on the curve $y = a^{1-k}.x^k$ is a constant

B. Show that the length of the subnormal at any point on the curve $xy = a^2$ varies as the cube of the ordinate of the point.

C. Find the average rate of the change of $S = f(t) = 2t^2 + 3$ between $t = 2$ and $t = 4$

D. Find the rate of change of area of a circle w.r.t radius when $r = 5cm$

E. The distance – time formula for the motion of a particle along a straight line is $S = t^3 - 9t^2 + 24t - 18$. Find when and where the velocity is zero.

F. Verify Rolle's theorem for the function $y = f(x) = x^2 + 4$ in $[-3,3]$

G. Verify Rolle's theorem for the function $f(x) = x(x+3)e^{-x/2}$ in $[-3,0]$

H. Find the intervals on which $f(x) = x^2 - 3x + 8$ is increasing or decreasing

I. Find the intervals on which the function $f(x) = x^3 + 5x^2 - 8x + 1$ is a strictly increasing function

J. Let $f(x) = (x-1)(x-2)(x-3)$. Prove that there is more than one $'c'$ in $(1,3)$ such that $f'(c) = 0$

SHORT ANSWER QUESTIONS

11.

A. Find the equation of the locus of P, if the ratio of the distances from P to A (5, -4) and B (7, 6) is 2 : 3.

B. A (2, 3), B (-3, 4) be two given points. Find the equation of locus of P so that the area of the triangle PAB is 8.5

C. Find the equation of locus of a point P such that the distance of P from origin is twice the distance of P from A (1, 2)

D. Find the equation of locus of P, if the line segment joining (2, 3) and (-1, 5) subtends a right angle at P.

E. Find the equation of locus of a point, the difference of whose distances from (-5, 0) and (5, 0) is 8.

F. Find the equation of locus of P, if A(2, 3), B (2, -3) and PA + PB = 8.

G. The ends of the hypotenuse of a right angled triangle are (0,6) and (6,0). Find the equation of the locus of its third vertex.

H. A(5, 3) and B(3, -2) are two fixed points. Find the equation of the locus of P, so that the area of triangle PAB is 9.

I. A(1, 2), B(2, -3) and C(-2, 3) are three points. A point P moves such that $\mathrm{PA}^2 + \mathrm{PB}^2 = 2\mathrm{PC}^2$ P. Show that the equation to the locus of P is $7x - 7y + 4 = 0$.

12.

A. Find the point to which the origin is to be shifted by the translation of axes so as to remove the first degree terms from the eq. $ax^2+2hxy+by^2+2gx+2fy+c=0$, where $h^2 \neq ab$.

B. Show that the axes are to be rotated through an angle of $\frac{1}{2}Tan^{-1}\left(\frac{2h}{a-b}\right)$ so as to remove the XY term from the equation $ax^2+2hxy+by^2=0$ if $a \neq b$, and through an angle $\frac{\pi}{4}$ if a = b.

C. When the origin is shifted to the point (2, 3), the transferred equation of a curve is $x^2+3xy-2y^2+17x-7y-11=0$. Find the original equation of the curve.

D. When the axes are rotated through an angle 45^0, the transformed equation of a curve is $17x^2-16xy+17y^2=225$. Find the original equation of the curve.

E. When the axes are rotated through an angle $\frac{\pi}{6}$, find the transformed equation of $x^2+2\sqrt{3}xy-y^2=2a^2$.

F. When the axes are rotated through an angle $\frac{\pi}{4}$, find the transformed equation of $3x^2+10xy+3y^2=9$.

G. When the origin is shifted to the point (2, 3), the transformed equation of a curve is $x^2+3xy-2y^2+17x-7y-11=0$. Find the original equation of the curve.

13.

A. A straight line parallel to the line $y=\sqrt{3}x$ passes through Q (2, 3) and cuts the line $2x+4y-27=0$ at P. Find the length PQ.

B. Transform the equation $3x+4y+12=0$ into (i) slope-intercept form (ii) intercept form (iii) normal form.

C. A straight line Q (2, 3) makes an angle $\frac{3\pi}{4}$ with the negative direction of the X-axis. If the straight line intersects the line $x+y-7=0$ at P, find the distance PQ.

D. A straight line L with negative slope passes through the point (8, 2) and cuts positive coordinate axes at the points P & Q. Find the minimum value of OP + OQ as L varies, where O is the origin.

E. A straight line L is drawn through the point A (2, 1) such that its point of intersection with the straight line $x+y=9$ is at a distance of $3\sqrt{2}$ from A. Find the angle which the line L makes with the positive direction of the X-axis.

F. Find the value of k, if the lines $2x-3y+k=0, 3x-4y-13=0$ and $8x-11y-33=0$ are concurrent.

G. A variable straight line drawn through the point of intersection of the straight lines $\frac{x}{a}+\frac{y}{b}=1$ and $\frac{x}{b}+\frac{y}{a}=1$ meets the coordinate axes at A and B. Show that the locus of the midpoint of $\overline{AB}$ is $2(a+b)xy=ab(x+y)$.

H. Show that the origin is within the triangle whose angular points are (2, 1) (3, -2) and (-4, -1).

I. Find the equations of the straight lines passing through the point (-3, 2) and making an angle of 45^0 with the straight line $3x-y+4=0$.

J. Each side of a square is of length 4 units. The centre of the square is (3, 7) and one of its diagonals is parallel to $y=x$. Find the coordinates of its vertices.

K. If P and Q are the lengths of the perpendiculars from the origin to the straight lines $x\sec\alpha+y\,\mathrm{cosec}\,\alpha=a$ and $x\cos\alpha-y\sin\alpha=a\cos 2\alpha$. Prove that $4P^2+Q^2=a^2$.

L. Find the area of the rhombus enclosed by the four straight lines $ax\pm by\pm c=0$.

M. Find the equations of the straight lines passing through (1, 1) and which are at a distance of 3 units from (-2, 3).

N. Transform the equation $\frac{x}{a}+\frac{y}{b}=1$ into the normal form when a > 0, b > 0. If the perpendicular distance of the straight line from the origin is P, deduce that $\frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}$.

O. If the straight lines $ax+by+c=0, bx+cy+a=0$ and $cx+ay+b=0$ are concurrent, then prove that $a^3+b^3+c^3=3abc$.

P. Find the point on the straight line $3x+y+4=0$, which is equidistant from the point (-5,6)and (3, 2).

Q. Find the area of the triangle formed by the straight lines $2x-y-5=0, x-5y+11=0$ and $x+y-1=0$.

R. If the four straight lines $ax+by+p=0, ax+by+q=0, cx+dy+r=0$ and $cx+dy+s=0$ form a parallelogram, show that the area of the parallelogram so formed is $\left|\frac{(p-q)(r-s)}{bc-ad}\right|$

S. The base of an equilateral triangle is $x+y-2=0$ and the opposite vertex is (2, -1). Find the equations of the remaining sides.

T. If the opposite vertices of a square are (-2, 3) and (8, 5). Find the equations of the sides of the square.

14.

A. Compute $\underset{x\to 1}{Lt}\dfrac{(2x-1)\left(\sqrt{x}-1\right)}{2x^2+x-3}$

B. Compute $\underset{x\to a}{Lt}\dfrac{x\sin a-a\sin x}{x-a}$

C. Compute $\underset{x\to 0}{Lt}\dfrac{\cos ax-\cos bx}{x^2}$

D. Compute $\underset{x\to 0}{Lt}\dfrac{(1+x)^{1/8}-(1-x)^{1/8}}{x}$

E. Compute $\underset{x\to\infty}{Lt}\dfrac{x^2-\sin x}{x^2-2}$

F. Check the continuity of the function f given below at 1 and 2

$$f(x)=\begin{cases} x+1 & if\ \ x\le 1 \\ 2x & if\ \ 1<x<2 \\ 1+x^2 & if\ \ x\ge 2 \end{cases}$$

G. Check the continuity of the following function at '2'

$$f(x)=\begin{cases} \frac{1}{2}\left(x^2-4\right) & if\ \ 0<x<2 \\ 0 & if\ \ x=2 \\ 2-8x^{-3} & if\ \ x>2 \end{cases}$$

H. Check the continuity of f given by

$$f(x)=\begin{cases} \dfrac{x^2-9}{x^2-2x-3} & if\ 0<x<5\ \ and\ x\ne 3 \\ 1.5 & if\ \ x=3 \end{cases}$$ at the point 3

I. If f, given by $f(x)=\begin{cases} kx^2-k & ,\ if\ x\ge 1 \\ 2 & ,\ if\ x<1 \end{cases}$ is a continuous function on $\mathbb{R}$, then find the value so k.

J. Check the continuity of f given by $f(x)=\begin{cases} 4-x^2 & if\ \ x\le 0 \\ x-5 & if\ \ 0<x\le 1 \\ 4x^2-9 & if\ \ 1<x<2 \\ 3x+4 & if\ \ x\ge 2 \end{cases}$

K. Find real constants a,b so that the function f given by

$$f(x)=\begin{cases}\sin x & if\ \ x\le 0\\ x^2+a & if\ \ 0<x<1\\ bx+3 & if\ \ 1\le x\le 3\\ -3 & if\ \ x>3\end{cases}$$ is continuous on $\mathbb{R}$

15.

A. Find the derivative of the following functions from first principles

(i) x^3 (ii) ax^2+bx+c (iii) $\sin 2x$ (iv) $\cos ax$ (v) $x\sin x$

B. Show that the function $f(x)=|x|+|x-1|$, $x\in\mathbb{R}$ is differentiable for all real numbers except for 0 & 1

C. Find the derivative of $\boldsymbol{x}=\tanh^2\boldsymbol{y}$

D. If $y=\tan^{-1}\sqrt{\dfrac{1-x}{1+x}}\ (|x|<1)$ find $\dfrac{dy}{dx}$

E. If $y=\tan^{-1}\left(\dfrac{2x}{1-x^2}\right)(|x|<1)$ find $\dfrac{dy}{dx}$

F. If $x^y=e^{x-y}$ then show that $\dfrac{dy}{dx}=\dfrac{\log x}{(1+\log x)^2}$

G. If $\sin y=x\sin(a+y)$, then show that $\dfrac{dy}{dx}=\dfrac{\sin^2(a+y)}{\sin a}(a\ne n\pi)$

H. Find the derivative of $\tan^{-1}\left(\dfrac{\sqrt{1+x^2}-1}{x}\right)$

I. Find $\dfrac{dy}{dx}$ if $x=a\left(\dfrac{1-t^2}{1+t^2}\right)$, $y=\dfrac{2bt}{1+t^2}$

J. Find $\dfrac{dy}{dx}$ if $x=a(\cos t+t\sin t)$, $y=a(\sin t-t\cos t)$

K. Differentiate $f(x)$ with respect to $g(x)$ for $f(x)=\sec^{-1}\left(\dfrac{1}{2x^2-1}\right)$, $g(x)=\sqrt{1-x^2}$

L. Show that $y=x+\tan x$ satisfies $\cos^2 x\dfrac{d^2y}{dx^2}+2x=2y$

M. If $y=ax^{n+1}+bx^{-n}$ then prove that $x^2y''=n(n+1)y$

N. If $ay^4=(x+b)^5$ then prove that $5yy''=(y')^2$

O. If $y=ae^{-bx}\cos(cx+d)$ then prove that $y''+2by'+\left(b^2+c^2\right)y=0$

16.

A. Find the equations of the tangent and normal to the curve $y^4 = ax^3$ at (a,a)

B. Show that the equation of the tangent to the curve $\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 2$ $(a \neq 0, b \neq 0)$ at the point (a,b) is $\frac{x}{a} + \frac{y}{b} = 2$

C. If the slope of the tangent to the curve $x^2 - 2xy + 4y = 0$ at a point on it is $-3/2$, then find the equations of the tangent and normal at that point.

D. Find the value of K so that the length of the subnormal at any point on the curve $xy^k = a^{k+1}$ is a constant

E. Find the angle between the curve $2y = e^{-x/2}$ and $y - axis$

17.

A. The volume of a cube is increasing at a rate of 9 cubic centimetres per second. How fast is the surface area increasing when the length of the edge is 10 centimeters .

B. A container in the shape of an inverted cone has height 12 cm and radius 6 cm at the top. If it is filled with water at the rate of $12\, cm^3/\text{sec},$ what is the rate of change in the height of water level when the tank is filled $8cm$

C. A stone is dropped intro a quiet lake and ripples move in circles at the speed of $5cm/\sec$. At the instant when the radius of circular ripple is $8cm$, how fast is the enclosed arc increases.

D. A container is in the shape of an inverted cone has height $8m$ and radius $6m$ at the top. If it is filled with water at the rate of $2\,m^3/\min ute$, how fast is the height of water changing when the level is $4m$?

E. A point P is moving on the curve $y = 2x^2$. The x coordinate of P is increasing at the rate of 4 units per second. Find the rate at which the y coordinate is increasing when the point is at $(2,8)$.

LONG ANSWER QUESTIONS

18.

A. If Q(h, k) is the foot of the perpendicular from $p(x_1, y_1)$ on the straight line $ax + by + c = 0$, then show that $\frac{h - x_1}{a} = \frac{k - y_1}{b} = \frac{-(ax_1 + by_1 + c)}{a^2 + b^2}$.

And hence find the foot of the perpendicular from (-1, 3) on the straight line $5x - y - 18 = 0$.

B. If Q(h, k) is the image of the point $p(x_1, y_1)$ w.r.t the straight line $ax+by+c=0$ then show that $\frac{h-x_1}{a}=\frac{k-y_1}{b}=\frac{-2(ax_1+by_1+c)}{a^2+b^2}$. And hence find the image of (1, -2) w.r.t the straight line $2x-3y+5=0$.

C. Find the orthocenter of the triangle with the vertices (-2, -1) (6, -1) and (2, 5).

D. Find the circumcenter of the triangle whose vertices are (1, 3), (0, -2) and (-3, 1).

E. If the equations of the sides of a triangle are $7x+y-10=0, x-2y+5=0$ and $x+y+2=0$, find the orthocenter of the triangle.

F. Find the circumcenter of the triangle whose sides are $3x-y-5=0, x+2y-4=0$ and $5x+3y+1=0$.

G. Find the incentre of the triangle whose sides are $x+y-7=0, x-y+1=0$ and $x-3y+5=0$.

H. Two adjacent sides of a parallelogram are given by $4x+5y=0$ and $7x+2y=0$ and one diagonal is $11x+7y=9$. Find the equations of the remaining sides and the other diagonal.

I. Find the equations of the straight lines passing through $(1,1)$ and which are at a distance of 3 units from $(-2,3)$

J. A line is such that its segment between the lines $5x-y+4=0$ and $3x+4y-4=0$ is bisected at the point $(1,5)$. Obtain its equation.

19.

A. If the equation $ax^2+2hxy+by^2=0$ represents a pair of intersecting lines, then the combined equation of the pair of bisectors of the angles between these lines is $h(x^2-y^2)=(a-b)xy$

B. Show that the product of perpendicular distances from a point (α,β) to the pair of straight lines $ax^2+2hxy+by^2=0$ is $\frac{|a\alpha^2+2h\alpha\beta+b\beta^2|}{\sqrt{(a-b)^2+4h^2}}$

C. Show that the area of the triangle formed by the lines $ax^2+2hxy+by^2=0$ and $lx+my+n=0$ is $\left|\frac{n^2\sqrt{h^2-ab}}{am^2-2hlm+bl^2}\right|$

D. Two equal sides of an isosceles triangle are $7x-y+3=0$ and $x+y-3=0$ and its third side passes through the point $(1,0)$. Find the equation of the third side

E. Show that the straight lines represented by $3x^2+48xy+23y^2=0$ and $3x-2y+13=0$ form an equilateral triangles of area $\frac{13}{\sqrt{3}}$ sq. units

F. If (α,β) is the centroid of the triangle formed by the lines $ax^2+2hxy+by^2=0$ and $lx+my=1$, Prove that $\dfrac{\alpha}{bl-hm}=\dfrac{\beta}{am-hl}=\dfrac{2}{3\left(bl^2-2hlm+am^2\right)}$

G. If the equation $S\equiv ax^2+2hxy+by^2+2gx+2fy+c=0$ represents a pair of parallel straight lines then prove that (i) $h^2=ab$ (ii) $af^2=bg^2$ and

(iii) the distance between the parallel lines $=2\sqrt{\dfrac{g^2-ac}{a(a+b)}}=2\sqrt{\dfrac{f^2-bc}{b(a+b)}}$

H. If the pairs of lines represented by $ax^2+2hxy+by^2=0$ and $ax^2+2hxy+by^2+2gx+2fy+c=0$ form a rhombus, prove that $(a-b)fg+h\left(f^2-g^2\right)=0$

I. If two of the sides of a parallelogram are represented by $ax^2+2hxy+by^2=0$ and $px+qy=1$ is one of its diagonals. Prove that other diagonal is $y(bp-hq)=x(aq-hp)$

J. Show that the product of the perpendicular distances from the origin to the pair of straight lines represented by $ax^2+2hxy+by^2+2gx+2fy+c=0$ is $\dfrac{|c|}{\sqrt{(a-b)^2+4h^2}}$

K. If the equation $ax^2+2hxy+by^2+2gx+2fy+c=0$ represents a pair of intersecting lines then show that the square of the distance of their point of intersection from the origin is $\dfrac{c(a+b)-f^2-g^2}{ab-h^2}$. Also show that the square of this distance is $\dfrac{f^2+g^2}{h^2+b^2}$ if the given lines are perpendicular

20.

A. Show that the lines joining the origin to the points of intersection of the curve $x^2-xy+y^2+3x+3y-2=0$ and the straight line $x-y-\sqrt{2}=0$ are mutually perpendicular

B. Find the values of k, if the lines joining the origin to the point of intersection of the curve $2x^2-2xy+3y^2+2x-y-1=0$ and the line $x+2y=k$ are mutually perpendicular

C. Find the angle between the lines joining the origin to the points of intersection of the curve $x^2+2xy+y^2+2x+2y-5=0$ and the line $3x-y+1=0$

D. Find the condition for the chord $lx+my=1$ of the circle $x^2+y^2=a^2$ to subtend a right angle at the origin

E. Find the condition for the lines joining the origin to the points of intersection of the circle $x^2+y^2=a^2$ and the line $lx+my=1$ to coincide

F. Write down the equation of the pair of straight lines joining the origin to the points of intersection of the line $6x-y+8=0$ with the pair of straight lines $3x^2+4xy-4y^2-11x+2y+6=0$. Show that the lines so obtained make equal angles with coordinate axes

21.

A. Find the direction cosines of two lines which are connected by the relations $l+m+n=0$ and $mn-2nl-21m=0$

B. Show that the lines whose d.c's are given by $l+m+n=0$, $2mn+3nl-5lm=0$ are perpendicular to each other.

C. Find the angle between the lines whose direction cosines satisfy the equations $l+m+n=0$, $l^2+m^2-n^2=0$.

D. If a ray makes angles $\alpha,\beta,\gamma,\delta$ with the four diagonals of a cube find $\cos^2\alpha+\cos^2\beta+\cos^2\gamma+\cos^2\delta$.

E. Find the angle between the lines whose direction cosines are given by the equations $3l+m+5n=0$ and $6mn-2nl+5lm=0$.

F. If a variable line in two adjacent positions has direction cosines (l,m,n) and $(l+\delta l,m+\delta m,n+\delta n)$, show that the small angle $\delta\theta$ between the two positions is given by $(\delta\theta)^2=(\delta l)^2+(\delta m)^2+(\delta n)^2$.

22.

A. If $y=\tan^{-1}\left[\dfrac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right]$ for $0<|x|<1$, find $\dfrac{dy}{dx}$

B. If $y=x^{\tan x}+(\sin x)^{\cos x}$, find $\dfrac{dy}{dx}$

C. If $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$ then prove that $\dfrac{dy}{dx}=\sqrt{\dfrac{1-y^2}{1-x^2}}$

D. If $y=x\sqrt{a^2+x^2}+a^2\log\left(x+\sqrt{a^2+x^2}\right)$ then prove that $\dfrac{dy}{dx}=2\sqrt{a^2+x^2}$

E. Find $\dfrac{dy}{dx}$ if $y=\dfrac{(1-2x)^{2/3}(1+3x)^{-3/4}}{(1-6x)^{5/6}(1+7x)^{-6/7}}$

F. If $x^y+y^x=a^b$ then prove that $\dfrac{dy}{dx}=-\left[\dfrac{y\,x^{y-1}+y^x\log y}{x^y\log x+x\,y^{x-1}}\right]$

G. If $f(x)=\sin^{-1}\sqrt{\dfrac{x-\beta}{\alpha-\beta}}$ and $g(x)=\tan^{-1}\sqrt{\dfrac{x-\beta}{\alpha-x}}$ then $f'(x)=g'(x)$ $(\beta<x<\alpha)$

H. If $a > b > 0$ and $0 < x < \pi$, $f(x) = \left(a^2 - b^2\right)^{-1/2} \cos^{-1}\left(\frac{a\cos x + b}{a = b\cos x}\right)$ then $f'(x) = (a + b\cos x)^{-1}$

I. If $x = a(t - \sin t)$, $y = a(1 + \cos t)$ find $\frac{d^2y}{dx^2}$

J. If $y = e^{-\frac{k}{2}x}(a\cos nx + b\sin nx)$ then prove that $y'' + ky' + \left(n^2 + \frac{k^2}{4}\right)y = 0$

23.

A. Find the equations of the tangents to the curve $y = 3x^2 - x^3$ where it meets the $x - axis$

B. If the tangent at any point on the curve $x^{2/3} + y^{2/3} = a^{2/3}$ intersects the coordinates axes in A and B, then show that the length AB is constant.

C. If the tangent at any point P on the curve $x^m y^n = a^{m+n}\ (mn \neq 0)$ meets the coordinate axes in A,B then show that $AP : BP$ is constant

D. Show that the square of the length of subtangent at any point on the curve $by^2 = (x + a)^3\ \ (b \neq 0)$ varies with the length of the subnormal at that point

E. At any point t on the curve $x = a(t + \sin t)$, $y = a(1 - \cos t)$. Find the lengths of tangent, normal, subangent and sub normal.

F. Find the angle between the curves $xy = 2$ and $x^2 + 4y = 0$

G. Show that the condition for orthogonality of the curves $ax^2 + by^2 = 1$ and $a_1x^2 + b_1y^2 = 1$ is $\frac{1}{a} - \frac{1}{b} = \frac{1}{a_1} - \frac{1}{b_1}$

H. Show that the curves $y^2 = 4(x + 1)$ and $y^2 = 36(9 - x)$ intersect orthogonally.

I. The volume of a cube is increasing at a rate of 9 cubic centimetres per second. How fast is the surface area increasing when the length of the edge is 10 centimeters ?

J. A container in the shape of an inverted cone has height 12cm and radius 6cm at the top. If it is filled with water at the rate of $12\,cm^3/\sec$, What is the rate of change in the height of water level when the tank is filled 8cm ?

24.

A. Find two positive numbers whose sum is 15 so that the sum of their squares is minimum

B. Find the minimum area of the rectangle that can be formed with fixed perimeter 20

C. Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.

D. Find two positive integers x and y such that $x + y = 60$ and xy^3 is maximum

E. From a rectangular sheet of dimensions $30cm \times 80cm$, four equal squares of side $x\,cm$ are removed at the corners and the sides are then turned up so as to form an open rectangular box. Find the value of x, so that the volume of the box is the greatest.

F. A window is in the shape of a rectangle surmounted by a semi circle. If the perimeter of the window is 20 ft, find the maximum area.

G. If the curved surface of a right circular cylinder inscribed in a sphere of radius $'r'$ is maximum, show that the height of the cylinder is $\sqrt{2}r$

H. A wire of length L is cut into two parts which are bent respectively in the form of a square and a circle. What are the lengths of the pieces of the wire respectively so that the sum of the areas is the least.

I. Find the absolute maximum and absolute minimum of $f(x) = 8x^3 + 81x^2 - 42x - 8$ on $[-8,2]$

J. The profit function $p(x)$ of a company selling $'x'$ items per day is given by $P(x) = (150 - x)x - 1000$. Find the number of items that the company should manufacture to get maximum profit. Also find the maximum profit.

wish you all the best

FIITJEE

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