DEFINITIONS, CONCEPTS AND FORMULAE

1. Permutation : An arrangement that can be formed by taking some or all of a finite set of things is called 'permutation'.
2. A permutation is said to be a 'linear permutation' if the objects are arranged in a line.
3. Fundamental principle or counting principle : If an operation can be done in 'm' ways and second operation can be done is 'n' ways then the two operations can be done in 'mn' ways.
4. The number of permutations of 'n' dissimilar things taken r($r \le n$) things at a time is
   ${}^nP_r = n(n-1)(n-2) \ldots (n-r+1) = \frac{n!}{(n-r)!}$
5. ${}^nP_r = r.\ {}^{n-1}P_{r-1} + {}^{n-1}P_r$.
6. The number of permutations of 'n' dissimilar things taken all at a time = ${}^np_n = n!$ (when repetitions are not allowed).
7. The number of permutations of 'n' things taken 'r' at a time, when repetition of things is allowed any number of times is n^r.
8. The number of permutations of n dissimilar things taken 'r' at a time with atleast one repetition is $n^r - {}^nP_r$.
9. The number of permutations of 'n' things taken all at a time when p of them are alike and rest all are different is $\frac{n!}{P!}$.
10. If p things are alike of first kind, q things are alike of second kind, r things are alike of third kind, then the number of permutations formed with p+q+r things is $\frac{(p+q+r)!}{p!.q!.r!}$.
11. A permutation is said to be a 'circular permutation' if the objects are arranged in the form a circle (or a closed curve).
12. The number of circular permutations formed with 'n' objects taken 'r' at a time is $\frac{{}^nP_r}{r}$.
13. The number of circular permutations formed with 'n' objects (persons) is (n-1)!.
14. The number of circular permutations with 'n' dissimilar flowers (beads etc.) is $\frac{1}{2}$ (n-1)!.
15. The number of functions that can be defined from set A into set B is $n(B)^{n(A)}$.
16. The number of one-one functions that can be defined from set A into set B is ${}^{n(B)}P_{n(A)}$.
17. The number of onto functions that can be defined from set A onto set B if n(A) = n and n(B) = 2, is 2^n-2.
18. The number of bijections that can be defined from set A onto A is [n(A)]!.
19. Combination : A selection that can be formed by taking some or all of a finite set of things is called a combination.
20. ${}^nC_r = \frac{n!}{(n-r)!\,r!}$.
21. ${}^nC_o = 1$, ${}^nC_n = 1$, ${}^nC_1 = n$, ${}^nC_r = {}^nC_{n-r}$.
22. $r! = \frac{{}^nP_r}{{}^nC_r}$, $\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r}$
23. ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$.
24. If ${}^nC_r = {}^nC_s$ then r = s or n = r + s.

AIMS

25. The number of ways in which (m+n) things can be divided into two different groups of m and n things respectively is $\frac{(m+n)!}{m!\,n!}$
26. The number of ways in which 2n things can be divided
    i) into two equal groups of n things each is $\frac{(2n)!}{2!(n!)^2}$.
    ii) among two persons equally is $\frac{(2n)!}{(n!)^2}$
27. The number of ways in which (m+n+p) things can be divided into three different groups of m, n and p things respectively is $\frac{(m+n+p)!}{m!n!p!}$
28. The number of ways in which 3n things can be divided
    i) into three equal groups of n things each is $\frac{(3n)!}{3!(n!)^3}$
    ii) among three persons equally is $\frac{(3n)!}{(n!)^3}$.
29. The total number of combinations of p+q things taken any number of things at a time when p things are alike of one kind and q things are alike of

second kind is (p+1) (q+1) -1.

30. The total number of combinations with 'n' dissimilar things taken
i) one or more at a time is 2^n-1
ii) two or more at a time is $2^n-1-{}^nC_1$.

31. If a polygon has n sides, then the number of diagonals is $\frac{n(n-3)}{2}$.

32. In a plane there are n points and no three of which are collinear except k points which lie on a line then
i) number of straight lines that can be formed by joining them is ${}^nC_2-{}^kC_2+1$.
ii) number of triangles that can be formed by joining them = ${}^nC_3-{}^kC_3$.

PERMUTATIONS

LEVEL I (VSAQ)

1. If ${}^nP_4$ = 1680, then find n.

A: ${}^nP_4 = 1680$

$\Rightarrow n(n-1)(n-2)(n-3) = 168 \times 10$
$= 8 \times 21 \times 10$
$= 8 \times 7 \times 3 \times 5 \times 2$
$= 8 \times 7 \times 6 \times 5$
$\therefore$ n = 8.

2. If ${}^nP_7 = 42 . {}^nP_5$, then find n.

A: Given that ${}^nP_7 = 42. {}^nP_5$

$\Rightarrow n(n-1)(n-2)(n-3)(n-4)(n-5)(n-6)$
$= 42. n(n-1)(n-2)(n-3)(n-4)$
$\Rightarrow (n-5)(n-6) = 42$
$\Rightarrow (n-5)(n-6) = (7)(6)$
$\Rightarrow n - 5 = 7$
$\therefore$ n = 12.

3. If ${}^{n=1}P_5 : {}^nP_6 = 2 : 7$, find n

A: $\frac{{}^{n+1}P_5}{{}^nP_6} = \frac{2}{7}$

$\Rightarrow 7.{}^{n+1}P_5 = 2.{}^nP_6$
$\Rightarrow 7.(n+1) n(n-1)(n-2)(n-3)$
$= 2. n(n-1)(n-2)(n-3)(n-4)(n-5)$
$\Rightarrow 7(n+1) = 2(n-4)(n-5)$
$\Rightarrow 7n + 7 = 2(n^2 - 9n + 20)$
$\Rightarrow 2n^2 - 25n + 33 = 0$
$\Rightarrow 2n^2 - 3n - 22n + 33 = 0$
$\Rightarrow n(2n-3) - 11(2n-3) = 0$
$\Rightarrow (n-11)(2n-3) = 0$
$\Rightarrow$ n = 11 or n = 3/2 is not possible
$\therefore$ n = 11.

4. If ${}^{18}P_{r-1} : {}^{17}P_{r-1} = 9 : 7$, find r

A: $\frac{{}^{18}P_{r-1}}{{}^{17}P_{r-1}} = \frac{9}{7}$

$\Rightarrow 7.{}^{18}P_{r-1} = 9.{}^{17}P_{r-1}$
$\Rightarrow 7.\frac{18!}{[18-(r-1)]!} = 9.\frac{17!}{[17-(r-1)]!}$
$\Rightarrow \frac{7.(18)(17!)}{(19-r)(18-r)!} = \frac{9(17!)}{(18-r)!}$
$\Rightarrow 14 = 19 - r$
$\therefore$ r = 5.

5. If ${}^{12}P_r$ = 1320, find r.

A: ${}^{12}P_r = 1320$
$= 132 \times 10$
$= 12 \times 11 \times 10$
$= {}^{12}P_3$
$\therefore$ r = 3.

6. If ${}^{12}P_5 + 5.{}^{12}P_4 = {}^{13}P_r$, find r.

A: Given that ${}^{12}P_5 + 5.{}^{12}P_4 = {}^{13}P_r$
Comparing this with ${}^{n-1}P_r + r.{}^{n-1}P_{r-1} = {}^nP_r$
Here r = 5.

7. Find the number of 5 letter words that can be formed using the letters of the word "NATURE' that begin with 'N" when repetition is allowed.

A: Number of 5 letter words that can be formed using the letters of the word 'NATURE' that begin with 'N' when repetition is allowed = $1 \times 6^4 = 1296$.

8. Find the number of 4 letter words that can be formed using the letters of the word 'PISTON' in which atleast one letter is repeated.

A: Number of 4 letter words that can be formed using the letters of the word 'PISTON' in which atleast one letter is repeated = $n^r - {}^nP_r$
$= 6^4 - {}^6P_4$
$= 1296 - 360$
$= 936.$

9. Find the number of ways of arranging 7 persons around a circle.

A: Number of ways of arranging 7 persons around a circle = (n - 1)!
= (7 - 1)!
= 6!
= 720!

10. Find the number of chains that can be prepared using 7 different coloured beads.

A: Number of chains that can be prepared using 7 differnt coloured beads

$= \frac{1}{2}(n-1)!$

$= \frac{1}{2}(7-1)!$

$= \frac{1}{2} \times 720$

$= 360.$

11. Find the number of ways of arranging the letters of the word 'MATHEMATICS'.

A: Number of ways of arranging the letters of the word 'MATHEMATICS'

$= \frac{11!}{2!2!2!}$ since it contains M's - 2, A's - 2, T's - 2.

12. Find the number of ways of arranging letters of the word 'ASSOCIATIONS'.

A: Given word 'ASSOCIATIONS' contains

A's - 2, S's - 3, O's - 2, I's - 2.

Number of ways of arranging the letters of the given word $= \frac{12!}{2!3!2!2!}$

LEVEL I (SAQ)

1. Find the sum of 4 digited numbers that can be formed using the digits 1, 2, 4, 5, 6 without repetition.

A: The number of 4 digited numbers formed by using the given 5 digits $= {}^5P_4 = 120$.

Now, we find the sum of these 120 numbers.

If we fill the units place with 1, then the remaining 3 places can be filled with the left over 4 digits in ${}^4P_3$ ways.

This means ${}^4P_3$ numbers contain 1 in the units place .

Similarly ${}^4P_3$ numbers contain the digits 2, 4, 5, 6 in the units place.

$\therefore$ Sum of the given digits in units place of all 120 numbers

$= {}^4P_3(1) + {}^4P_3(2) + {}^4P_3(4) + {}^4P_3(5) + {}^4P_3(6)$

$= {}^4P_3(1 + 2 + 4 + 5 + 6)$

$= {}^4P_3(18)(1)$

Similarly, the value of sum of digits in ten's place

$= {}^4P_3(18)(10)$

The values of sums of the digits in 100's place and 1000's place $= {}^4P_3(18)(100)$ and ${}^4P_3(18)(1000)$

Hence the sum of all four digit numbers formed by using the digits 1, 2, 4, 5, 6

$= {}^4P_3(18)(1) + {}^4P_3\ 18)(10) + {}^4P_3(18)(100) + {}^4P_3(18)(1000)$

$= {}^4P_3(18)(1 + 10 + 100 + 1000)$

$= {}^4P_3(18)(1111)$

$= (24)(18)(1111)$

$= 4, 79, 952.$

2. Find the number of four digit numbers that can be formed using the digits 1, 2,5 6, 7. How many of them are divisible by (i) 2 (ii) 3 (iii) 4 (iv) 25.

A: Number of four digit numbers formed using 1, 2, 5, 6, 7 $= {}^5P_4 = 120$.

i) A number is divisible by 2 when its units place must be filled with an even digit from among the given digits. This can be done in 2ways.

Now , the remaining 3 places can be filled with remaining 4 digits in ${}^4P_3$ ways.

II I

$\therefore$ The number of 4 digited numbers divisible by 2

$= 2 \times 24 = 48.$

ii) A number is divisible by 3 when the sum of the digits in that number is a multiple of 3.

The possible cases are 1256, 1257, 1267, 1567, 2567.

The 4 digits such that their sum is a multiple of 3 from the given digits are 1, 2, 5, 7.

They can be arranged in 4! ways.

$\therefore$ The number of 4 digited numbers divisible by 3 = 24.

iii) A number is divisible by 4 only when the last two places (tens and units places) of it is a multiple of 4.

Here, the last two places can be filled with one of the following: 12, 16, 52, 56, 72, 76. Thus the last two places can be filled in 6 ways. The remaining two places can be filled by the left over 3 digits in $^3P_2$ ways.

II I

$\therefore$ The number of 4 digited numbers divisible by 4
= 6 x 6 = 36.

iv) A number is divisible by 25 when its last two places are filled with either 25 or 75.
Thus the last two places can be filled in 2 ways. The remaining 2 places from the remaining 3 digits can be filled in $^3P_2$ ways.

$\therefore$ The number of 4 digits numbers divisible by 25
= 2 x 6 = 12.

II I

3. Prove that $^nP_r = \dfrac{n!}{(n-r)!}$.

A: Clearly ^nP_r is equal to the number of ways of filling r blank places which are arranged in a row by n dissimilar things.
The first blank place can be filled by any one of hte n things and hence it can be done in n ways. To fill the second blank place, can be filled in n - 1 ways.
Similarly the third blank place can be filled in n - 2 ways and and so on. Proceeding in this way, the r^{th} blank place can be filled in n - r + 1 ways. By counting principle, the r blank places can be filled in

$n(n-1)(n-2)\,........\,(n-r+1)$ ways

$\therefore {}^nP_r = n(n-1)(n-2)\,.......\,(n-r+1)$

$$= \frac{n(n-1)(n-2)......(n-4+1)(n-r).......3.2.1}{1.2.3........(n-r)}$$

$$= \frac{n!}{(n-r)!}.$$

4. Prove that $^nP_r = r\,.\,{}^{n-1}P_{r-1} + {}^{n-1}P_r$.

A: Clearly ^nP_r denotes the number of permutation of n dissimilar things taken r at a times these permutations can be divided into two parts.
1) The permutations that contain a particular thing and 2) The permutations that do not contain the particular thing.

To find the number of permutations that contain the particular thing, first fill that particular thing in any one of the r blank places arranged in a row and hten fill the other r - 1 things. The particular thing can be filled in r ways and the remaining things can be filled in $^{n-1}P_{r-1}$ ways. By counting principle, the number of permutations = $r\,.\,{}^{n-1}P_{r-1}$.

To find the number of permutations that do not contain the particular thing, fill the remaining n - 1 things in the r blank places arranged in a row. It can be done in $^{n-1}P_r$ ways.

$\therefore {}^nP_r = r\,.\,{}^{n-1}P_{r-1} + {}^{n-1}P_r$.

5. Find the number of 4 letter words that can be formed using the letters of the word 'MIXTURE' which (i) contain the letter X (ii) do not contain the letter X.

A: We have fill up 4 blanks using 7 letters of the word 'MIXTURE'. Take 4 blanks.
i) First we put x in one of the 4 blanks. This can be done in 4 ways. Now we can fill the remaining 3 places with the remaining 6 letters in $^6P_3$ ways. Thus the number of 4 letter words that do not contain the letter X = $^6P_4$ = 360.

6. Find the number of ways of arranging 6 boys and 6 girls in a row so that (i) all the girls sit together (ii) no two girls sit together (iii) boys and girls sit together .
No. of boys = 6
No.of girls = 6.

A: i) Given condition : All the girls sit together in a row. Treat the 6 girls as one unit. So the number of units = 1 + 6 = 7. These 7 units can be arranged in a row in 7! ways. Now 6 girls can be arranged among themselves in 6! ways. Hence, by the counting principle, the number of arrangements in which all 6 girls are together is 7! x 6! = 36, 28, 800.

ii) Given condition : No two girls sit together. First of all, we shall arrange 6 boys in a row in 6! ways. The girls can be arranged in the 7 gaps in $^7P_6$ ways.

✓ B ✓ B ✓ B ✓ B ✓ B ✓ B

Hence, by the counting principle, the number of arrangements in which no two girls sit together is 6! x $^7P_6$ = 36, 28, 800.

iii) Given condition : Boys and girls sit alternatively.

✓ B ✓ B ✓ B ✓ B ✓ B ✓ B
B ✓ B ✓ B ✓ B ✓ B ✓ B ✓

First of all six boys can be arranged in a row in 6! ways. Then by considering only one end gap, in 6 continuous spaces, 6 girls can be arranged in 6!

as shown in the above figure. Hence total number of arrangements in which boys and girls set alternately

= 6! 6! + 6! 6!
= 2(6!) (6!)
= 2(720) (720)
= 10, 36, 800.

7. Find the number of ways of arranging 5 different mathematics books, 4 different physics books and 3 different chemistry such that the books of the same subject are together.

A: Number of mathematics books = 5
Number of physics books = 4
Number of chemistry books = 3

Given condition : Books of same subject are together.
Treate 5 mathematics books as 1st unit, 4 physics books as 2nd unit, 3 chemistry books as 3rd unit. Now the number of units is 3. These 3 units can be arranged in a row in 3! ways. Then 5 mathematics books can be shuffled internally in 5! ways, 4 physics books can be shuffled internally in 4! ways, 3 chemistry books can be shuffled internally in 3! ways. By the counting principle, total number of arrangements such that books of same subject are together
= 3! x 5! x 4! x 3!
= 6 x 120 x 24 x 6
= 720 x 144
= 103, 680.

8. Find the rank of the word 'MASTER'.

A: We shall find the rank of the word 'MASTER'. Alphabatical order of letters of the word is A, E, M, R, S, T.

No.of 6 letter words formed which begin with 'A' = 5!

No.of 6letter words formed which begin with 'E' = 5!

No.of 6 letter words formed which begin with 'MAE' = 3!

No.of 6letter words formed which begin with 'MAR' = 3!

No.of 6letter words formed which begin with 'MASE' = 2!

No.of 6letter words formed which begin with 'MASR' = 2!

Next word formed is MASTER = 1
$\therefore$ Rank of the word 'MASTER'
= 2(5!) + 2(3!) + 2(2!) + 1
= 2(120) + 2(6) + 2(2) + 1
= 240 + 12 + 4 + 1
= 257.

9. Find the rank of the word 'REMAST'.

A: We shall find the rank of the word 'REMAST'. Alphabatical order of letters of the word is A, E, M, R, S, T.

No.of 6letter words formed which begin with 'A' = 5!

No.of 6letter words formed which begin with 'E' = 5!

No.of 6letter words formed which begin with 'M' = 5!

No.of 6letter words formed which begin with 'RA' = 4!

No.of 6letter words formed which begin with 'REA' = 3!

Next word formed is REMAST = 1
$\therefore$ Rank of the word 'REMAST'
= 3(5!) + 4! + 3! + 1
= 3(120) + 24 + 6 + 1
= 391.

AIMS

10. Find the number of 4 digit telephone numbers that can be formed using the digits 1, 2, 3, 4, 5, 6 with atleast one digit repeated.

A: Given digits are 1, 2, 3, 4, 5, 6.
Number of 4 digited numbers with repetition = 6^4.
Number of 4 digited numbers without repetition = ${}^6P_4$.
$\therefore$ Number of 4 digited numbers formed with atleast one digit repeated.
$= 6^4 - {}^6P_4 = 1296 - 360 = 936.$

11. Find the number of numbers that are greater than 4000 which can be formed using the digits 0, 2, 4, 6, 8 without repetetion.

A: Given digits are 0, 2, 4, 6, 8.
Number of 5 digited numbers formed
= 4 x 4! = 4 x 24 = 96.

To get the number of 4 digited numbers greater than 4000, first of all 1000'2 can be filled by 4, 6, 8 in 3 ways.

Then the remaining 3 places can be filled with the 4 other digits is ${}^4P_3$ ways.

$\therefore$ No.of 4 digited numbers greater than 4000
$= 3 \times {}^4P_3 = 72.$

$\therefore$ The number of numbers greater than 4000
= 96 + 72 = 168.

12. Find the number of ways of arranging 7 gents and 4 ladies around a circular table if no two

ladies wish to sit together.

A: First of all 7 gents are arranged around a circular table in (7 - 1)! = 6! ways. Now between the gents there are 7 places in which 4 ladies are to be seated in $^7P_4$ ways.

By the counting principle, number of circular permutations such that no two ladies sit together

$= 6! \times {}^7P_4$

= 720 x 840

= 6, 04, 800.

13. Find the number of different ways of preparing a garland using 7 distinct red roses and 4 distinct yellow roses such that no two yellow roses come together.

A: First of all, we arrange 7 red roses in the galand form in (7 - 1) ! = 6! ways.

Now, there are 7 gaps in between the red roses and we can arrange 4 yellow roses in 7 gaps in $^7P_4$ ways. Thus total number of garlands formed is $6! \times {}^7P_4$.

= 720 x 840

= 6, 04, 800.

14. Find the number of ways of arranging the letters of the word 'SINGING' so that

i) they begin and end with I.

ii) the two G's come together.

A: Given word is SINGING.

It contains I's - 2, G's = 2, N's - 2.

i) Given condition : Arrangement begin and end with I. First of all, we have to fill the first and last places with I's in $\frac{2!}{2!}$ = 1 ways

as shown in the above figure.

Now, we fill the remaining 5 places with the remaining 5 letters S, N, G, N, G in $\frac{5!}{2!2}$ = 30 ways.

Hence, the number required arrangements

= 1 x 30 = 30.

ii) Given condition : Two G's come together. Treat the two G's as one unit. Then we have 6 letters in which there are 2I's and 2N's. Hence they can be arranged in $\frac{5!}{2!2!}$ = 180 ways.

Then, the two G's among themselves can be arranged in $\frac{2!}{2!}$ = 1 way.

Hence, the number of required permutations

= 180 x 1

= 180.

15. Find the rank of the word 'EAMCET'.

A: We shall find the rank of the word 'EAMCET'.

Alphabetical order of letters of the word is

A, C, E, E, M, T

No.of 6 letter words formed which begin with 'A' = $\frac{5!}{2!}$

No.of 6 letter words formed which begin with 'C' = $\frac{5!}{2!}$

No.of 6 letter words formed which begin with 'EAC' = 3!

No.of 6 letter words formed which begin with 'EAE' = 3!

Next word formed is EAMCET = 1

Hence, the rank of the word 'EAMCET'

$= = \cancel{2}\left(\frac{120}{\cancel{2}}\right) + 2(6) + 1$

= 133.

AIMS

LEVEL II (VSAQ)

1. Find the number of ways of forming a committee of 5 members, out of 6 Indians and 5 Americans so that always Indians will be in majority in the committee.

A: No.of Indians = 6

No.of Americans = 5

The committee should contian 5 members with majority for Indians

6 Indians	5 Americans
3	2
4	1
5	0

Total number of ways of forming committee

$= {}^6C_3 . {}^5C_2 + {}^6C_4 . {}^5C_1 + {}^6C_5 . {}^5C_0$

= 20(10 + 15(5) + 6(1)

= 200 + 75 + 6

= 281.

2. Find the number of ways of selecting a Cricket team of 11 players from 7 bats men and 6 bowlers such that there will be atleast 5 bowlers in the team.

A: Number of batsmen = 7

Number of bowlers = 6

We shall form a team of 11 players wiht atleast 5 bowlers

7 Batsmen	6 Bowlers
6	5
5	6

Total number of teams formed

$= {}^7C_6 \,.\, {}^6C_5 + {}^7C_5 \,.\, {}^6C_6$

$= 7(6) + 21(1)$

$= 42 + 21$

$= 63.$

3. For $1 \le 4 \le n$, with usual notation, if ${}^nC_{r-1} + {}^nC_r = {}^{n+1}C_{r-1}$, find r.

A: Given ${}^nC_{r-1} + {}^nC_r = {}^{n+1}C_{r-1}$

$\Rightarrow {}^{n+1}C_r = {}^{n+1}C_{r-1}$

it ${}^nC_r = {}^nC_s$ then r = s or n = r + s.

Here $n + 1 = r + r - 1$ $\quad r = r - 1$

$n + 2 = 2r$ $\quad$ It is impossible

$r = \dfrac{n+2}{2}.$

4. Prove that for $3 \le r \le n$, ${}^{n-3}C_r + 3\,.\,{}^{n-3}C_{r-1} + 3\,.\,{}^{n-3}C_{r-2} + {}^{n-3}C_{r-3} = {}^nC_r$.

A: Now ${}^{n-3}C_r + 3\,.\,{}^{n-3}C_{r-1} + 3\,.\,{}^{n-3}C_{r-2} + {}^{n-3}C_{r-3}$

$= \{{}^{n-3}C_r + {}^{n-3}C_{r-1}\} + 2\{{}^{n-3}C_{r-1} + {}^{n-3}C_{r-2}\} + \{{}^{n-3}C_{r-2} + {}^{n-3}C_{r-3}\}$

$= {}^{n-2}C_r + 2\,.\,{}^{n-2}C_{r-1} + {}^{n-2}C_{r-2}$

$= \{{}^{n-2}C_r + {}^{n-2}C_{r-1}\} + \{{}^{n-2}C_{r-1} + {}^{n-2}C_{r-2}\}$

$\because\ {}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

$= {}^{n-1}C_r + {}^{n-1}C_{r-1}$

$= {}^nC_r.$

5. Prove that ${}^{25}C_4 + \sum\limits_{r=0}^{4} {}^{29-r}C_3$.

A: Now ${}^{25}C_4 + \sum\limits_{r=0}^{4} {}^{29-r}C_3$

$= {}^{29}C_3 + {}^{28}C_3 + {}^{27}C_3 + {}^{26}C_3 + \{{}^{25}C_3 + {}^{25}C_4\}$

$\because\ {}^nC_{r-1} + {}^nC_r = {}^{n+1}C_r$

$= {}^{29}C_3 + {}^{28}C_3 + {}^{27}C_3 + \{{}^{26}C_3 + {}^{26}C_4\}$

$= {}^{29}C_3 + {}^{28}C_3 + \{{}^{27}C_3 + {}^{27}C_4\}$

$= {}^{29}C_3 + \{{}^{28}C_3 + {}^{28}C_4\}$

$= {}^{29}C_3 + {}^{29}C_4$

$= {}^{30}C_4.$

6. Find the number of subsets of A having 12 elements (i) atleast 3 elements (ii) atmost 3 elements.

A: Number of elements in A = 12.

i) Number of subsets of A having atleast 3 elements

$= {}^{12}C_3 + {}^{12}C_4 + \ldots\ldots. + {}^{12}C_{12}$

$= \{{}^{12}C_0 + {}^{12}C_1 + {}^{12}C_2 + {}^{12}C_3 + \ldots\ldots.. + {}^{12}C_{12}\}$

$- \{{}^{12}C_0 + {}^{12}C_1 + {}^{12}C_2\}$

$\because\ {}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots.. + {}^nC_n = 2^n$

$= 2^{12} - \{1 + 12 + 66\}$

$= 4096 - 79$

$= 4017.$

ii) Number of subsets of A having atmost 3 elements

AIMS

$= {}^{12}C_0 + {}^{12}C_1 + {}^{12}C_2 + {}^{12}C_3$

$= 1 + 12 + 66 + 220$

$= 299.$

7. Prove that ${}^nC_r = \dfrac{n!}{(n-r)!\,r!}$.

A: ${}^nC_r$ means the number of combinations of n dissimilar things taken r at a time. Consider one of the ${}^nC_r$ combinations. this combination consists of r dissimilar things. If we permute the r things, we get r! permutations. Thus each combination gives rise to r! permutation and hence ${}^nC_r$ combinations give rise to ${}^nC_r$. r! permutations. But the number of permutations of n dissimilar things taken r at a time is ${}^nP_r$.

$\therefore {}^nC_r \,.\, r! = {}^nP_r$

$\Rightarrow {}^nC_r \,.\, r! = \dfrac{n!}{(n-r)!}$

$\Rightarrow {}^nC_r = \dfrac{n!}{(n-r)!\,r!}.$

8. If ${}^nC_r = {}^nC_s$, then prove that r = s or n = r + s.

A: Now ${}^nC_r = {}^nC_s$

then $r = s$ or $r \ne s$

Suppose that $r > s$

$\Rightarrow n - r < n - s$

Now ${}^nC_r = {}^nC_s$

$\Rightarrow \dfrac{n!}{(n-r)!r!} = \dfrac{n!}{(n-s)!s!}$

$\Rightarrow (n-r)!\, r! = (n-s)!\, s!$

$\Rightarrow (n-r)!\, s!\, (s+1)(s+2) \ldots\ldots. r$

$= (n-r)!\,(n-r+1)(n-r+2) \ldots\ldots.. (n-s)\, s!$

$\Rightarrow (s+1)(s+2) \ldots\ldots r = (n-r+1)(n-r+2) \ldots (n-s)$

Since each side of the above relation is a product of r - s consecutive positive integers, we get

$r = n - s$

$\Rightarrow n = r + s$

Similarly if $r < s$, then also we can prove that $n = r + s$.

$\therefore$ If ${}^nC_r = {}^nC_s$, then $r = s$ or $n = r + s$.

9.Prove that ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$.

A: Now ${}^nC_r + {}^nC_{r-1}$

$= \dfrac{n!}{(n-r)!r!} + \dfrac{n!}{[n-(r-1)]!\,(r-1)!}$

$= \dfrac{n!(n+1-r)+n!r}{r!\,(n+1-r)!}$

$= \dfrac{n!(n+1-r+r)}{r!\,(n+1-r)!}$

$= \dfrac{(n+1)!}{(n+1-r)!r!}$

$= {}^{n+1}C_r$.

10.Show that $\dfrac{{}^{4n}C_{2n}}{{}^{2n}C_n} = \dfrac{1.3.5 \ldots\ldots. (4n-1)}{\{1.3. \ldots\ldots..(2n-1)\}^2}$.

A: Now $\dfrac{{}^{4n}C_{2n}}{{}^{2n}C_n} = \dfrac{\left\{\dfrac{(4n)!}{(2n)!\,(2n)!}\right\}}{\left\{\dfrac{(2n)!}{n!\;n!}\right\}}$

$= \dfrac{(4n)!}{(2n)!} \text{ x } \left\{\dfrac{n!}{(2n)!}\right\}^2$

$= \dfrac{1.2.3.4 \ldots\ldots.(4n-2)(4n-1)(4n)}{1.2 \ldots\ldots..(2n-1)(2n)!} \left\{\dfrac{1.2 \ldots\ldots.(n-1)n!}{1.2.3.4 \ldots\ldots.(2n-2)(2n-1)(2n)}\right\}^2$

$= \dfrac{1.3.5 \ldots\ldots.(4n-1)}{\{1.3 \ldots..(2n-1)\}^2} \text{ x } \dfrac{2^{2n}}{(2n)^2}$

$= \dfrac{1.3.5 \ldots\ldots.(4n-1)}{\{1.3 \ldots..(2n-1)\}^2}$.

11.How many 5 letter words can be formed using 3 consonants and 2 vowels from the letters of the word 'MIXTURE'.

A: Given word 'MIXTURE' contains 3 vowels and 4 consonants.

Number of ways of selecting 3 consonants = ${}^4C_3$

Number of ways of selecting 2 vowels = ${}^3C_2$

Total number of selections = ${}^4C_3 . {}^3C_2$.

Number of arrangements of 5 letter words with 3 consonants, 2 vowels

$= {}^4C_3 . {}^3C_2 . 5!$

$= (4)(3)(120)$

$= 1440.$

LEVEL II (VSAQ)

1. If ${}^nP_3 = 1320$ then find 'n'.

A: Given that ${}^nP_3 = 1320 = 132 \text{ x } 10 = 12 \text{ x } 11 \text{ x } 10$

$\Rightarrow {}^nP_3 = {}^{12}P_3 \Rightarrow \boxed{n = 12}$.

AIMS

2. If ${}^{(n+1)}P_5 : {}^nP_6 = 2 : 7$ then find 'n'.

A: Given that ${}^{(n+1)}P_5 : {}^nP_6 = 2 : 7$

$\Rightarrow \dfrac{{}^{(n+1)}P_5}{{}^nP_6} = \dfrac{2}{7} \Rightarrow \dfrac{\dfrac{(n+1)!}{(n+1-5)!}}{\dfrac{n!}{(n-6)!}} =$

$\Rightarrow \dfrac{(n+1)!}{(n-4)!} \text{x} \dfrac{(n-6)!}{n!} = \dfrac{2}{7}$

$\Rightarrow \dfrac{(n+1)n!}{(n-4)(n-5)\cancel{(n-6)!}} \text{x} \dfrac{\cancel{(n-6)!}}{n!} = \dfrac{2}{7}$

$\Rightarrow 7(n+1) = 2(n^2 - 5n - 4n + 20)$

$\Rightarrow 7n + 7 = 2(n^2 - 9n + 20) \Rightarrow 7n + 7 = 2n^2 - 18n + 40$

$\Rightarrow 2n^2 - 25n + 33 = 0 \Rightarrow 2n^2 - 22n - 3n + 33 = 0$

$\Rightarrow 2n(n-11) - 3(n-11) = 0 \Rightarrow (n-11)(2n-3) = 0.$

$n = 1, \dfrac{3}{2} \Rightarrow \boxed{n = 11} \left[\because n \neq \dfrac{3}{2}, \text{a non integer}\right]$.

3. If ${}^{56}P_{r+6} : {}^{54}P_{r+3} = 30800 : 1$, find r.

A: Given that ${}^{56}P_{r+6} : {}^{54}P_{r+3} = 30800 : 1$

$\Rightarrow \dfrac{{}^{56}P_{r+6}}{{}^{54}P_{r+3}} = \dfrac{30800}{1}$

$\Rightarrow \dfrac{(56)!}{[56-(r+6)]!} \text{ x } \dfrac{[54-(r+3)]!}{(54)!} = 30800$

$$\Rightarrow \frac{(56)!}{(50-r)!} \text{ x } \frac{(51-r)!}{(54)!} = 30800$$

$$\Rightarrow \frac{(56)(55)\cancel{(54)}!}{\cancel{(50-r)}!} \text{x} \frac{(51-r)\cancel{(50-r)}!}{\cancel{(54)}!} = 30800$$

$$\Rightarrow (56)(55)(51-r) = 30800$$

$$\Rightarrow 51-4 = \frac{30800}{56 \text{x} 55} \Rightarrow 51-r = 10 \Rightarrow \boxed{r = 41}.$$

4. In a class there are 30 students. On the New Year day, every student posts a greeting card to all his/her classmates. Find the total number of greeting cards posted by them.

A: Total number of greeting cards posted by the students = $^{30}P_2$ = 30 x 29 = 870.

5. If there are 25 railways stations on a railway line, how many types of single second class tickets must be printed, so as to enable a passenger to travel from one station to another.

A: The number of single second class tickets must be printed = $^{25}P_2$ = 25 x 24 = 600.

6. Find the number of ways in which 4 letters can be put in 4 addressed envelopes so that no letter goes into the envelop meant for it.

A: Formula : The number of dearrangements of 'n' distinct things is

$$n!\left[\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} + \frac{1}{5!} + \text{..........}(-1)^n \frac{1}{n!}\right]$$

$\therefore$ Required number of ways = $= 4!\left[\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!}\right]$

$$= 24\left[\frac{1}{2} - \frac{1}{6} + \frac{1}{24}\right] = 24\left[\frac{12-4+1}{24}\right] = 9.$$

7. Find th number of palindromes with 6 digits that can be formed using the digits
(i) 0, 2, 4, 6, 8 (ii) 1, 3, 5, 7, 9

A: (i) The number of palindromes formed
= 4 x $5^{4/2}$ = 4 x 5^2 = 100.

(ii) The number of palindromes formed
= $5^{6/2}$ = 5^3 = 125.

8. Find the number of functions from a set A containing 5 element s into a set B containing 4 elements.

A: Required number of functions = $[n(B]^{n(A)}$
= 4^5 = 1024.

9. Find the number of injections from a set A containing 4 elements into a set B containing 6 elements.

A: Required number of injections (one - one functions).
= $^{n(B)}P_{n(A)}$ = $^6P_4$ = 6.5.4.3 = 360.

10. Find the number of surjections from a set A containing 6 elements onto a set B containing 2 elements.

A: Required no. of surjections (onto functions)
= $2^{n(A)}$ - 2 - 2^6 - 2 = 64 - 2 = 62.

11. Find the number of bijections from a set A containing 7 elements into itself.

A: Required number of bijections - n(A)! = 7!..

12. Find the number of ways of arranging 7 gents, 4 ladies around a circular table if no two ladies wish to sit together.

A: First we arrange 7 gents around a circular table as (7 - 1)! = 6!.

In between there are 7 gaps, these 7 gaps can be filled with 4 ladies as $^7P_4$ ways.

AIMS

$\therefore$ Total number of ways = 6! x $^7P_4$.

13. Find the number of ways of arranging the letters of the word
i) INDEPENDENCE
ii) INTERMEDIATE
iii) PERMUTATION
iv) COMBINATION

A: Given word contains 12 letters

In which there are 3N's, 2D's, 4E's

$\therefore$ The number of required arrangements = $\frac{12!}{3!2!4!}$

ii) The no. of letters in the word 'INTERMEDIATE' = 12

In this there are 2I's, 2T's, 3E's.

$\therefore$ The number of required arrangements = $\frac{12!}{2!2!3!}$.

iii) Given word 'PERMUTATION' contains 11 letters in which there are 2T's

The number of required arrangements = $\frac{(11)!}{2!}$.

iv) Given word 'COMBINATION' contains 11 letters.

In which there are 2O's, 2I's, 2N's

$\therefore$ The number of required arrangements =

$\frac{(11)!}{2!2!2!} = \frac{(11)!}{(2!)^3}$.

14. In a class there are 30 students. If each student plays a chess game with each of the other student, then find the total number of chess games played by them.

A: No of students in the class = 30.

$\therefore$ Total no. of chess games played by them

$$= {}^{30}C_2 = \frac{30 \times 29}{2 \times 1} = 435.$$

15. Find the number of ways of selecting 4 boys and 3 girls from a group of 8 boys and 5 girls.

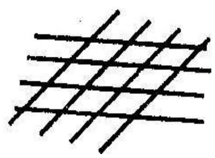
A: The no. of ways of selecting 4 boys from 8 boys is ${}^8C_4$.

The no. of ways of selecting 3 girls from 5 girls is ${}^5C_3$.

$\therefore$ Total no. of selection = ${}^8C_4 \times {}^5C_3$

$$= \frac{8 \times 7 \times 6 \times 5}{4 \times 3 \times 2 \times 1} = \frac{5 \times 4 \times 3}{3 \times 2 \times 1} = 70 \times 10 = 700.$$

16. Find the number of parallelograms formed by the set of 'm' parallel lines intersects an other set of 'n' parallel lines.



A: A parallelogram is formed with 2 lines from the first set of 'm' lines and 2 lines from the second set of 'n' lines.

$\therefore$ The number of required parallelograms = ${}^mC_2 \times {}^nC_2$.

17. If there are 5 alike pens, 6 alike pencils and 7 alike erasers, find the no. of ways of selecting any number of (one or more) things out of them.

A: Formula: If 'p; things are alike of one kind 'q' things are alike of second kind and 'r' things are alike of third kind then the no. of ways of selecting any number of things (one or more) is (p + 1) (q + 1) (r + 1) -1.

$\therefore$ The required number of ways

= (5 + 1) (6 + 1) (7 + 1) - 1

= 6.7.8 - 1 = 336 - 1 = 335.

18. Find the no. of zeros in 100!

A: $100! = 2^{\alpha}3^{\beta}5^{\gamma}7^{\delta}$................ $\boxed{100 = 2^2 \times 5^2}$

where

$$\alpha = \left[\frac{100}{2}\right] + \left[\frac{100}{2^2}\right] + \left[\frac{100}{2^3}\right] + \left[\frac{100}{2^4}\right] + \left[\frac{100}{2^5}\right] + \left[\frac{100}{2^6}\right]$$

= 50 + 25 + 12 + 6 + 3 + 1 = 97

and $\gamma = \left[\frac{100}{5}\right] + \left[\frac{100}{5^2}\right] = 20 + 4 = 24$

Now, the number of zero's in 100! is the power of 10(2 x 5) in 100! which is 24.

19. Find te numbr of ways in which 12 things can be (i) divided into 4 equal groups (ii) distributed to 4 persons equally.

A: i) The number of ways of dividing 12 things into 4 equal groups = $\frac{12!}{(3!)^4.4!}$.

ii) The no. of ways of distributing 12 things to 4 persons = $\frac{12!}{(3!)^4}$.

LEVEL II (SAQ)

1. If the letters of the word PRISON are permuted in all posible ways and the words thus formed are arranged in dictionary order, find the rank of the word 'PRISON'.

AIMS

A: We shall find the rank of the word 'PRISON'.

Alphabetical order of the letters is INOPRS.

No. of 6 letter words formed which being with 'I' = 5!

No. of 6 letter words formed which being with 'N' = 5!

No. of 6 letter words formed which being with 'O' = 5!

No. of 6 letter words formed which being with 'PI' = 4!

No. of 6 letter words formed which being with 'PN' = 4!

No. of 6 letter words formed which being with'PO' = 4!

No. of 6 letter words formed which being with 'PRIN' = 2!

No. of 6 letter words formed which being with 'PRIO' = 2!

No. of 6 letter words formed which being with 'PRISN' = 1!

Next word formed is PRISON = 1.

$\therefore$ Rank of 'PRISON'.

= 3(5!) + 3(4!) + 2(2!) + 1 + 1.

= 3(120) + 3(24) + 2(2) + 1 + 1.

= 360 + 72 + 4 + 2

= 438.

2. There are 9 objects and 9 boxes. Out of 9 objects, 5 cannot fit in three small boxes. How many arrangements can be made such that each object can be put in one box only.

A: 5 big objects can be put in 6 big boxes in ${}^6P_5$ ways.

Then 4 small objects can be put 1 big box and 3 small boxes in $^4P_4$ ways.

By the counting principle, total number of required arrangements = $^6P_5 \times 4!$

= 720 x 24

= 17280.

3. A candidate is required to answer 6 out of 10 questions which are divided into two groups A and B each containing 5 questions. He is not permitted to attempt more than 4 questions from either group. Find the number of ways in which the candidate can choose six questions.

A: The candidate can answer the questions from group A and B as follows:

No. of questions from group A(5)	No. of questions from group B(5)	No.of ways
4	2	$^5C_4 . ^5C_2$
3	3	$^5C_3 . ^5C_3$
2	4	$^5C_2 . ^5C_4$

Total number of ways = 5(10) + 10(10) + 10(5)

= 50 + 100 + 50

= 200

$\therefore$ Thus the number of ways of selecting 6 questions out of 10 questions is 200.

COMBINATIONS

LEVEL I (VSAQ)

1. If $^nC_5 = {^nC_6}$, then find $^{13}C_n$.

A: Given that $^nC_5 = {^nC_6}$

$\Rightarrow$ r = s or n = r + s

Here 5 $\neq$ 6, n = 5 + 6 = 11

$\therefore {^{13}C_n} = {^{13}C_{11}} = {^{13}C_2} = \frac{13.12}{2} = 78.$

2. If $^{12}C_{r+1} = {^{12}C_{3r-5}}$, find r.

A: Now $^{12}C_{r+1} = {^{12}C_{3r-5}}$

$\Rightarrow$ r = s or n = r + s.

$\Rightarrow$ r + 1 = 3r - 5   12 = r + 1 + 3r - 5

$\Rightarrow$ 2r = 6   4r = 16

$\Rightarrow$ r = 3   r = 4

$\therefore$ r = 3 or 4

3. If $10.{^nC_2} = 3.{^{n+1}C_3}$, find n.

A: Given that $10.{^nC_2} = 3.{^{n+1}C_3}$

$$\Rightarrow 10\frac{n(n-1)}{2} = \frac{3(n+1)n(n-1)}{6}$$

$\Rightarrow$ 10 = n + 1

$\therefore$ n = 9.

4. If $^nC_4 = 210$, find n.

A: $$^nC_4 = \frac{210x24}{24} = \frac{21x10 \times 8 \times 3}{4!} = \frac{7x3 \times 10 \times 8 \times 3}{4!} = \frac{10 \times 9 \times 8 \times 7}{4!} = {^{10}C_4}$$

$\therefore$ n = 10.

5. If $^{12}C_r = 495$, find the possible values of r.

A: $$^{12}C_r = \frac{495 \times 24}{24} = \frac{5 \times 99 \times 12 \times 2}{4!} = \frac{12 \times 11 \times 10 \times 9}{4!} = {^{12}C_4} \text{ or } {^{12}C_8}$$

$\therefore$ r = 4 or 8.

6. If $^nP_r = 5040$ and $^nC_r = 210$, find n and r.

A: We know that $r! = \frac{^nP_r}{^nC_r} = \frac{5040}{210} = 24 = 4!$

$\therefore$ r = 4.

Also $^nP_4 = 5040$

= 10 x 504

= 10 x 9 x 56

= 10 x 9 x 8 x 7

= $^{10}P_4$

$\therefore$ n = 10.

7. Find the value of $^{10}C_5 + 2.^{10}C_4 + ^{10}C_3$

A: $^{10}C_5 + 2.^{10}C_4 + ^{10}C_3$

$= \{^{10}C_5 + ^{10}C_4\} + \{^{10}C_4 + ^{10}C_3\} \because {}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

$= {}^{11}C_5 + {}^{11}C_4$

$= {}^{12}C_5$

8. If a set A has 12 elements, then find the number of subsets of A having 4 elements.

A: Number of elements in set A = 12.

Number of 4 elements subsets formed

$= {}^{12}C_4$

$= \frac{12 \times 11 \times 10 \times 9}{24} = 495$

9. In a class there are 30 students. If each student plays a chess game with each of the other students, then find the total number of chess games played by them.

A: Numbr of students = 30

Total number of chess games played

$= {}^nC_2$

$= {}^{30}C_2$

$= \frac{30 \times 29}{2}$

= 435.

10. Find the number of ways of selecting 3 vowels and 2 consonants from the letters of the word 'EQUATION'.

A: Given word 'EQUATION' contains 5 vowels and 3 consonants.

Number of ways of selecting 3 vowels and 2 consonants from 5 vowels and 3 consonants

$= {}^5C_3 \times {}^3C_2$

= 10 x 3

= 30.

11. Find the number of diagonals of a polygon with 12 sides.

A: Number of sides = 12

No. of diagonals $= \frac{n(n-3)}{2}$

$= \frac{12 \times 9}{2}$

= 54.

12. Find the number of positive divisiors of 1080.

A: 1080 = 10 x 108

= 10 x 9 x 12

$= 2 \times 5 \times 3^2 \times 2^2 \times 3$

$= 2^3 \times 3^3 \times 5^1$

Number of positive divisors of 1080

= (3 + 1) (3 + 1) (1 + 1)

= 32.

LEVEL I (SAQ)

1. There are 'm' points in a plane out of which 'p' points are collinear and no three of the points are collinear unless all the three are from these p points. Find the number of different

i) Straight lines passing thorugh pair of distinct points.

ii) Triangles formed by joining these points.

A: i) From the given 'm' points, by drawing straight lines passing through 2 distinct points at a time, we are supposed to get $^mC_2$ number of lines. But, since 'p' out of these 'm' points are collinear, by forming lines passing through these 'p' points 2 at a time we get only one line instead of getting $^pC_2$. Therefore, the number of different lines formed = $^mC_2 - {}^pC_2 + 1$.

ii) From the given 'm' points, by joining 3 at a time, we are supposed to get $^mC_3$ number of triangles. Since 'p' out of these 'm' points are collinear, by joining these 'p' points 3 at a time we do not get any triangle when as we are supposed to get $^pC_3$ number of triangles. Hence the number of triangles formed by joining the given 'm' points is $^mC_3 - {}^pC_3$.

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