DEFINITIONS, CONCEPTS AND FORMULAE

1. **Random Experiment:-** An Experiment that can be repeated any number of times under identical conditions in which
(i) all possible out-comes of the experiment are known in advance.
(ii) The actual outcome in a particular case is not known in advance, is called a random experiment.
Eg:- 1. Tossing a coin.
2. Rolling a die.
2. **Elementary Event or Simple Event:-** Any possible out-come of a random experiment is called an elementary or simple event.
3. **Sample Space:-** The set of all elementary events (possible out-comes) of a random experiment is called the sample space S.
4. **Mutually Exclusive Events:-** Two or more events are said to be mutually exclusive if the occurrence of one of the events prevents the occurrence of any of the remaining events.
A, B are mutually exclusive events if $A \cap B = \phi$
A, B, C mutually exclusive events if $A \cap B = \phi$, $B \cap C = \phi$ and $C \cap A = \phi$.
5. **Equally Likely Events:-** Elementary events are said to be equally likely if they have the same chance of happening.
6. **Exhaustive Events:-** The list of all elementary events in a trial is called list of exhaustive events.
7. If A is an event in a sample space S, then the ratio $P(A) : P(\bar{A})$ is called the "odds in favour of A" and $P(\bar{A}) : P(A)$ is called the "odds against to A".
8. Any subset of the sample space is called an event.
9. $P(A) = \frac{n(A)}{n(S)} = \frac{n(A)}{n(S)}$
10. For any event A, $O \le P(A) \le 1$.
11. for impossible $P(\phi) = 0$, For sure events $P(S) = 1$.
12. If A, B are mutually exclusive, then $A \cap B = \phi$.
13. If A, B are exhaustive events, then $A \cup B = S$.
14. $P(\bar{A}) = 1 - P(A)$
15. i) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
if A, B are any two events.
ii) $P(A \cup B) = P(A) + P(B)$
if A, B are mutualy exclusive events
iii) $P(A \cup B) = P(A) + P(B) - P(A).P(B)$
if A, B are independent events
16. i) $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$
if A,B,C are any three events.
ii) $P(A \cup B \cup C) = P(A) + P(B) + P(C)$
if A,B,C are mutually exclusive events.
iii) $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A)P(B) - P(B)P(C) - P(C)P(A) + P(A)P(B)P(C)$
if A,B,C are independent events.
17. i) $P(A \cap B) = P(A) . P\left(\frac{B}{A}\right) = P(B)\,P\left(\frac{A}{B}\right)$
ii) $P(A \cap B) = P(A) . P(B)$
if A, B are independent events.
iii) $P(A \cap B) = 0$ if A, B are mututally exclusive events
18. i) $P(A \cap B \cap C) = P(A)P\left(\frac{B}{A}\right) . P\left(\frac{C}{A \cap B}\right)$
ii) $P(A \cap B \cap C) = P(A)\,P(B)\,P(C)$
if A, B, C are independent events
iii) $P(A \cap B \cap C) = 0$
if A, B, C are mutually exclusive events
19. i) $P(A \cup B) = 1 - P(\bar{A})\,P(\bar{B})$
if A, B are independent events
ii) $P(A \cup B \cup C) = 1 - P(\bar{A})\,P(\bar{B})\,P(\bar{C})$
20. i) $P(A \cap \bar{B}) = P(A) - P(A \cap B) = P(A - B)$.
ii) $P(\bar{A} \cap B) = P(B) - P(A \cap B) = P(B - A)$.
21. If A, B are two events in a sample space, then the event of happening B after the event A happening is called conditional event. It is denoted by $\left(\frac{B}{A}\right)$.
22. If A, B are two events in a sample space S and $P(A) \neq 0$, then the probability of B after the event A has occured is called conditional probability of B

given A. It is denoted by $P\left(\frac{B}{A}\right)$.

23. Two events A and B are 'independent' of each other if the occurence or non - occurence of one of them does not influence the occurrence or non - occurrence of the other. Event B is independent of A if $P(B) = P\left(\frac{B}{A}\right)$.

24. i) $P\left(\frac{B}{A}\right) = \frac{n(A \cap B)}{n(A)} = \frac{P(A \cap B)}{P(A)}$

ii) $P\left(\frac{A}{B}\right) = \frac{n(A \cap B)}{n(B)} = \frac{P(A \cap B)}{P(B)}$

25. If A, B are two independant events in a sample space S, then

i) $\bar{A}$, B are independent

ii) A, $\bar{B}$ are independent.

iii) $\bar{A}$, $\bar{B}$ are independent

26. If A_1, A_2 are two mutually exlusive and exhaustive events and E is any event, then

1) $P(E) = P(A_1) P\left(\frac{E}{A_1}\right) + P(A_2)\ P\left(\frac{E}{A_2}\right)$

27. If A_1, A_2, A_3 are three mutually exclusive and exhaustive events and E is any event, then

$$P(E) = P(A_1)P\left(\frac{E}{A_1}\right) + P(A_2)P\left(\frac{E}{A_2}\right) + P(A_3)P\left(\frac{E}{A_3}\right)$$

28. Bayes' theorem: If $A_1, A_2, \ldots. A_n$ are mutually exclusive and exhaustive events in a sample space S such that $P(A_i) > O$ for i = 1, 2, ..., n and E is any event with P(E) > O then

$$P\left(\frac{A_k}{E}\right) = \frac{P(A_k)\ P\left(\frac{E}{A_k}\right)}{\sum_{i=1}^{n} P(A_i) P\left(\frac{E}{A_i}\right)}$$

LEVEL I (SAQ)

1. In a class of 60 boys and 20 girls half of the boys and half of the girls know cricket. Find the probability of the event that a person selected from the class is either 'a boy' or ' a girl who knows cricket'.

A: Number of boys = 60

Number of girls = 20.

Let A be the event that the selected person is a boy P(A)=60/80

and B be the event that the selected person is a girl who knows cricket. P(B)=10/80

Clearly A, B are mutually exclusive events. i.e. $A \cap B \neq \phi$.

By addition theorem,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{60}{80} + \frac{10}{80} - 0$

$= \frac{70}{80}$

$= \frac{7}{8}$.

2. A problem in calculus is given to two students A and B whose chances of solving it are $\frac{1}{3}$ and $\frac{1}{4}$. What is the probability that the problem will be solved if both of them try independently.

A: Let A and B denote the events that the problem is solved by A and B respectively.

Here $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{4}$

Required probability

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= P(A) + P(B) - P(A)\,P(B)$

$\because$ A, B are independent

$= \frac{1}{3} + \frac{1}{4} - \frac{1}{3} \cdot \frac{1}{4}$

$= \frac{4+3-1}{12} = \frac{6}{12}$

$= \frac{1}{12}$.

3. If two numbers are selected randomly from 20 consecutive natural numbers, find the probability that the sum of two numbers is (i) an even number (ii) an odd number

A: Given set is { 1, 2, 3, 4 .... , 19, 20}

Consider the sets {2, 4, 6, ..., 20}, {1, 3, 5, ...19}.

Let A be event that the sum of two number is even and B be the event that sum of two numbers is odd when two numbers are selected from {1, 2, 3, ..., 20}.

Sum of two numbers is even if both of them are even or both are odd.

$$P(B)=\frac{n(A)}{n(S)}=\frac{{}^{10}C_2+{}^{10}C_2}{{}^{20}C_2}=\frac{45+45}{\frac{20\times19}{2}}=\frac{90}{190}=\frac{9}{19}$$

Sum of two numbers is odd if one number is even, one is odd.

$$P(B)=\frac{n(A)}{n(S)}=\frac{{}^{10}C_1+{}^{10}C_1}{{}^{20}C_2}=\frac{10.10}{190}=\frac{100}{190}=\frac{10}{19}.$$

4. If one ticket is randomly selected from tickets numbered 1 to 30, then find the probability that the number on the ticket is a multiple of 3 or 5.

A: Let A, B be the events that the number on the ticket is a multiple of 3, 5 respectively, when a ticket is selected from 1 to 30.

A = { 3, 6, 9, 12, 15, 18, 21, 24, 27, 30}

B = {5, 10, 15, 20, 25, 30}

$A \cap B$ = {15, 30}.

n (A) = 10, n (B) = 6, n $(A \cap B)$ = 2

Required probability $P(A \cup B)$

$$= P(A) + P(B) - P(A \cap B)$$

$$=\frac{10}{30}+\frac{6}{30}-\frac{2}{30}$$

$$=\frac{14}{30}$$

$$=\frac{7}{15}.$$

5.Find the probability of drawing an ace or spade from a well shuffled pack of spade from a well shuffled pack of 52 cards.

A. A= Event of drawing a spade

$\Rightarrow$ n(A) = 13

B = Event of drawing an ace

$\Rightarrow$ n(B) = 4

and $n(A \cap B) = 1$

Required probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$=\frac{13}{52}+\frac{4}{52}-\frac{1}{52}$$

$$=\frac{17-1}{52}=\frac{16}{52}=4/13$$

6. In an experiment of drawing a card from a pack, the event of getting a space is denoted by A and getting a pictured card (king, queen or jack) is denoted by B. Find the probability of A, B, $A \cap B$ and $A \cup B$.

A: Total number of cards in a pack = 52

$\therefore n(S) = {}^{52}C_1 = 52$.

Let A be the event of getting a spade an B be the event of getting a pictured card, when a card is drawn from a pack.

$$\therefore P(A)=\frac{n(A)}{n(S)}=\frac{{}^{13}C_1}{{}^{52}C_1}=\frac{13}{52}=\frac{1}{4}$$

$$P(A\cap B)=\frac{(A\cap B)}{n(S)}=\frac{{}^{3}C_1}{{}^{52}C_1}=\frac{3}{52}$$

By addition theorem,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$=\frac{13}{52}+\frac{12}{52}-\frac{3}{52}$$

$$=\frac{22}{52}$$

$$=\frac{11}{26}.$$

6. In a committee of 25 members, each member is proficient either in mathematics or in statistics or in both. If 19 of these are proficient in mathematics, 16 in statistics, find the probability that a person selected from the committee is proficient in both.

A: Let A be the event that the selected member is proficient in mathematics and B be the event that the selected member is proficient in statistics.

Given that out of 25 members, each member is proficient either in mathematics or in satistics or in both.

So A, B are exhanstive events

$\Rightarrow A \cup B = S$

$\Rightarrow P(A \cup B) = P(S)$

$\Rightarrow P(A) + P(B) - P(A \cap B) = 1$

by the axiom of certainity

$\Rightarrow P(A \cap B) = P(A) + P(B) - 1$

$= \frac{19}{25} + \frac{16}{25} - 1$

$= \frac{19+16-25}{25}$

$= \frac{10}{25}$

$= \frac{2}{25}$.

7. If A, B, C are three events in a sample space S, then show that $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A)\ P(A \cap B \cap C)$.

A: Given that A, B, C are three events in a sample space S.

$P(A \cup B \cup C) = P[A \cup (B \cup C)]$

$= P(A) + P(B \cup C) - P[A \cap (B \cup C)]$

$= P(A) + P(B) + P(C) - P(B \cap C) - \{P(A \cap B) \cup P(A \cap C)\}$

$= P(A) + P(B) + P(C) - P(B \cap C) - \{P(A \cap B) + P(A \cap C) - P[(A \cap B) \cap (A \cap C)]\}$

$= P(A) + P(B) + P(C) - P(B \cap C) - P(A \cap B) - P(A \cap C) + P(A \cap B \cap C)$

$= P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$.

8. State and Prove multiplication theorem on probability.

A: Multiplication theorem on probability : Let A, B be two events in a sample space S such that $P(A) \neq 0$, $P(B) \neq 0$. Then (i) $P(A \cap B) = P(A)\, P\left(\frac{B}{A}\right)$.

(ii) $P(A \cap B) = P(B)\, P\left(\frac{A}{B}\right)$

Let n(A), n(B), $n(A \cap B)$, n(S) be the number of sample points in A, B, $A \cap B$, S respectively.

then $P(A) = \frac{n(A)}{n(S)}$, $P(B) = \frac{n(B)}{n(S)}$,

$P(A \cap B) = \frac{n(A \cap B)}{n(S)}$

$P\left(\frac{B}{A}\right) = \frac{n(A \cap B)}{n(A)}$, $P\left(\frac{A}{B}\right) = \frac{n(A \cap B)}{n(B)}$

i) $P(A \cap B) = \frac{n(A \cap B)}{n(S)}$

$= \frac{n(A)}{n(S)} \cdot \frac{n(A \cap B)}{n(A)}$

$= P(A)\, P\left(\frac{B}{A}\right)$

ii) $P(A \cap B) = \frac{n(A \cap B)}{n(S)}$

$= \frac{n(B)}{n(S)} \cdot \frac{n(A \cap B)}{n(B)}$

$= P(B) \cdot P\left(\frac{A}{B}\right)$.

9. The probability that Australia wins a match against India in a cricket game is given to be $\frac{1}{3}$. If India and Austrlia play 3 matches, what is the probability that
i) Australia will loose all the three matches?
ii) Australia will win atleast one match?

A: Let A_1, A_2, A_3 be the events that Australia wins a match against India in the 1st, 2nd, 3rd games respectively.

Given that $P(A_1) = P(A_2) = P(A_3) = \frac{1}{3}$

i) Probability that Australia will loose all the three matches

$= P(\overline{A_1} \cap \overline{A_2} \cap \overline{A_3})$

$= P(\overline{A_1})\, P(\overline{A_2})\, P(\overline{A_3})$

$= \left(1-\frac{1}{3}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{3}\right)$

$= \left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)$

$= \frac{8}{9}.$

ii) Probability that Australia will win atleast one match

= 1 - probability that Australia will loose all the three matches

$= 1 - \frac{8}{27}$

$= \frac{27-8}{27}$

$= \frac{19}{27}.$

10. If A and B are independent events with P(A) = 0.6, P(B) = 0.7 then compute

i) $P(A \cap B)$ ii) $P(A \cup B)$ iii) $P\left(\frac{B}{A}\right)$

iv) $P(\bar{A} \cap \bar{B})$

A: Given that A, B are independent events with

P(A) = 0.6, P(B) = 0.7

i) $P(A \cap B) = P(A)\,P(B)$

$= (0.6)(0.7)$

$= 0.42$

ii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= P(A) + P(B) - P(A)\,P(B)$

$= 0.6 + 0.7 - (0.6)(0.7)$

$= 1.30 - 0.42$

$= 0.88.$

iii) $P\left(\frac{B}{A}\right) = P(B) = 0.7$

iv) $P(\bar{A} \cap \bar{B}) = P(\bar{A})\,P(\bar{B})$

$\because$ A, B are independent

$= (1 - 0.6)(1 - 0.7)$

$= (0.4)(0.3)$

$= 0.12.$

11. A, B are twoindependent events such that the probability of both the events to occur is 1/6 and the probability of both the events do not occur is 1/3. Find P(A).

A: Given that A, B are independent events

Let P(A) = x and P(B) = y

Given that $P(A \cap B) = \frac{1}{6}$

$\Rightarrow P(A)\,P(B) = \frac{1}{6}$

$\Rightarrow xy = \frac{1}{6}$

Also $P(\bar{A} \cap \bar{B}) = \frac{1}{3}$

$\Rightarrow P(\bar{A})\,P(\bar{B}) = \frac{1}{3}$

$\Rightarrow [1 - P(A)]\ [1 - P(B)] = \frac{1}{3}$

$\Rightarrow (1 - x)(1 - y) = \frac{1}{3}$

$\Rightarrow 1 - x - y + xy = \frac{1}{3}$

$\Rightarrow 1 - (x + 6) + \frac{1}{6} = \frac{1}{3}$

$\Rightarrow \frac{7}{6} - \frac{1}{3} = x + y$

$\Rightarrow x + y = \frac{5}{6}$

Now $x + y = \frac{5}{6}$ and $xy = \frac{1}{6}$

$\Rightarrow x + \frac{1}{6x} = \frac{5}{6}$

$\Rightarrow \frac{6x^2 + 1}{6x} = \frac{5}{6}$

$\Rightarrow 6x^2 + 1 = 5x$

$\Rightarrow 6x^2 - 5x + 1 = 0$

$\Rightarrow 6x^2 - 3x - 2x + 1 = 0$

$\Rightarrow 3x(2x - 1) -1 (2x - 1) = 0$

$\Rightarrow (3x - 1) (2 - 1) = 0$

$\Rightarrow x = \frac{1}{3}$ or $\frac{1}{2}$

Hence $P(A) = \frac{1}{3}$ or $\frac{1}{2}$.

12. A speaks truth in 75% of the cases and B in 80% of the cases. Find the percentage of the cases of which they likely to contradict each other in stating the same fact.

A: Let A and B be the events that the persons A, B respectively to speak truth about an incident.

Given that $P(A) = \frac{75}{100} = \frac{3}{4}$

$P(B) = \frac{80}{100} = \frac{4}{5}$

Clearly A, B are independent events.

Now probability that their statements about an incident contradict each other

$= P[(A \cap \bar{B}) \cap (\bar{A} \cap B)]$

$= P(A \cap \bar{B}) + P(\bar{A} \cap B)$

$= P(A)\, P(\bar{B}) + P(\bar{A})\, P(B)$

$= \frac{3}{4}\left(1 - \frac{4}{5}\right) + \left(1 - \frac{3}{4}\right)\left(\frac{4}{5}\right)$

$= \frac{3}{4}\left(\frac{4}{5}\right) + \frac{1}{4}\left(\frac{4}{5}\right)$

$= \frac{7}{20}$

Hence the percentage of the cases of which they likely to contradict each other

$= \frac{7}{20} \times 100 = 35\%.$

13. A bag B_1 contains 4 white and 2 black balls. Bag B_2 contains 3 white and 4 black balls. A bag is drawn at random and a ball is chosen at random from it. Then what is the probability that the ball is white?

A: Let A_1, A_2 be the events of choosing bags B_1, B_2 respectively.

Here A_1, A_2 are equally likely events

Then $P(A_1) = \frac{1}{2}$, $P(A_2) = \frac{1}{2}$

Let E be the event of drawing a white ball from the seelcted bag.

$P\left(\frac{E}{A_1}\right)$ = Probability of drawing a white ball from bag B_1

$= \frac{4}{6}$

$= \frac{2}{3}$

$P\left(\frac{E}{A_2}\right)$ = Probability of drawing a white ball from bag B_2

$= \frac{3}{7}$

By total probability theorem,

$P(E) = P(A_1)\, P\left(\frac{E}{A_1}\right) + P(A_2)\left(\frac{E}{A_2}\right)$

$= \frac{1}{2} . \frac{2}{3} + \frac{1}{2} . \frac{3}{7}$

$= \frac{14 + 9}{42}$

$= \frac{23}{42}.$

14. Two persons A and B are rolling a die on the condition that the person who gets 3 will win the game. If A starts the game, then find the probabilities of A and B respectively to win the game.

A. Let p be the probabilities of getting 3 on a dice = 1/6

q be the probability of not getting 3

$\Rightarrow$ q = 1 - p

$= 1-\frac{1}{6}$

$q = \frac{5}{6}$

A, B be the events that A, B will win the game respectively.

A starts the game

Then A will win in 1^{st} or 3^{rd} or 5^{th} ... chances

The probabilities of A will win the game is

P(A) = p + qqp + qqqqp + ...

$\Rightarrow$ p(1 + q^2 + q^4 + ...)

$\Rightarrow p\left(\frac{1}{1-q^2}\right)$ $\left[\because S_\infty = \frac{a}{1-r}\right]$

$= \frac{1}{6}\left[\frac{1}{1-\left(\frac{5}{6}\right)^2}\right]$

$= \frac{1}{6}\left[\frac{1}{\frac{36-25}{36}}\right]$

$p(A) = \frac{6}{11}$

Probability of B will win game is

$p(B) = 1 - p(A)$

$= 1-\frac{6}{11}$

$\therefore\ p(B) = \frac{5}{11}.$

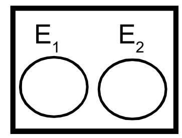
LEVEL I (LAQ)

1. State and explain the axioms that define 'Probability function'. Prove addition theorem on probability.

Addition theorem : If A, B are any two events in a sample space S, then

P (A $\cup$ B) = P (A) + P (B) - P (A $\cap$ B)

Proof :



Case 1 : Suppose that A $\cap$B = ϕ.

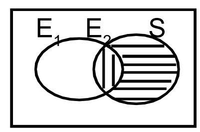
P(A $\cap$B) =P(ϕ.)=0

P (A $\cup$ B) = P (A) + P (B)

by the axiom of union

= P(A) + P(B) - 0

P (A $\cup$ B) = P (A) + P (B) - P (A $\cap$ B).....(1)



Case 2 : Suppose that A $\cap$B$\neq$ ϕ.

From the adjacent venn diagram

P (A $\cup$ B) = P (A U (B-A))

= P(A) + P(B - A)

B - A= B **- (A $\cap$ B)**

= P(A) + P(B - (A $\cap$ B))

= P(A) + P(B) - P(A $\cap$ B)

P (A $\cup$ B) = P (A) + P (B) - P (A $\cap$ B).....(2)

from (1) & (2)

P (A $\cup$ B) = P (A) + P (B) - P (A $\cap$ B)

2. State and prove Baye's theorem.

A: **Baye's theorem :** If A_1, A_2, A_3 .... A_n are mutually exclusive and exhaustive events in a sample space S such that $P(A_i) > 0$ for i = 1, 2, 3, ..., n and E is any event with P(E) > 0, then

$$P\left(\frac{A_k}{E}\right) = \frac{P(A_k)\,P\left(\frac{E}{A_k}\right)}{\sum_{i=1}^{n} P(A_i)P\left(\frac{E}{A_i}\right)} \text{ for } k = 1, 2, \ldots, n.$$

Proof :

Given that $A_1, A_2, A_3, \ldots A_n$ are mutually exclusive and exhanstive events in a sample space S.

$\Rightarrow A_1 \cup A_2 \cup \ldots \cup A_n = S$

$\Rightarrow \bigcup_{i=1}^{n} A_i = S$

Since $A_1, A_2, A_3, \ldots, A_n$ are mutually exclusive, and for any event t the events $E \cap A_1, E \cap A_2, E \cap A_n$ are also mutually exclusive.

Now $P(E) = P(E \cap S)$

$$= P\left(E \cap \bigcup_{i=1}^{n} A_i\right)$$

$$= P\left(\bigcup_{i=1}^{n} (E \cap A_i)\right)$$

$$= \sum_{i=1}^{n} P(E \cap A_i)$$

$$\therefore P(E) = \sum_{i=1}^{n} P(A_i)P\left(\frac{E}{A_i}\right) \text{ -------- (1)}$$

For k = 1, 2, ..., n

$$P(E \cap A_k) = P(E)\,P\left(\frac{A_k}{E}\right)$$

$$\Rightarrow P\left(\frac{A_k}{E}\right) = \frac{P(A_k)\,P\left(\frac{E}{A_k}\right)}{\sum_{i=1}^{n} P(A_i)P\left(\frac{E}{A_i}\right)} \text{ for } i = 1, 2, 3, \ldots, n.$$

3. A, B, C are 3 newspapers from a city. 20% of the population read A, 16% read B, 14% read C, 8% read both A and B, 5% read both A and C, 4% read both B and C and 2% read all the three. Find the percentage of population who read atleast one newspaper and also find the percentage of population who read the newspaper A only.

A: Let A, B, C be the events that a person selected from the city reads newspapers A, B, C respectively

Given that $P(A) = \frac{20}{100}, P(B) = \frac{16}{100}, P(C) = \frac{14}{100}$

$P(A \cap B) = \frac{8}{100}, P(A \cap C) = \frac{5}{100}, P(B \cap C) = \frac{4}{100}$

$P(A \cap B \cap C) = \frac{2}{100}, P(A \cup B \cup C) = ?$

Probability that a person selected from the city reads atleast one newspaper

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

$$= \frac{20}{100} + \frac{16}{100} + \frac{14}{100} - \frac{8}{100} - \frac{4}{100} - \frac{5}{100} + \frac{2}{100}$$

$$= \frac{20 + 16 + 14 - 8 - 4 - 5 + 2}{100}$$

$$= \frac{52 - 17}{100}$$

$$= \frac{35}{100}$$

$\therefore$ Required percentage of population who read atleast one newspaper

$= P(A \cup B \cup C) \times 100$

$= \frac{35}{100} \times 100$

= 35%

Probability that the selected person read the newspaper A only

$= P(A) - P(A \cap B) - P(A \cap C) + P(A \cap B \cap C)$

$$= \frac{20}{100} - \frac{8}{100} - \frac{4}{100} + \frac{2}{100}$$

$$= \frac{9}{100}$$

$\therefore$ Percentage of population who read the newspaper A only

$= \frac{9}{100} \times 100$

= 9%.

4. The probabilities of three events A, B, C are such that P(A) = 0.3, P(B) = 0.4, P(C) = 0.8, P(A ∩ B) = 0.08, P(A ∩ C) = 0.28, P(A ∩ B ∩ C) = 0.09 and P(A ∪ B ∪ C) ≥ 0.75. Show that P(B ∩C) lies in the interval [0.23, 0.48].

A: Given that
P(A) = 0.3,
P(B) = 0.4,
P(C) = 0.8

P(A ∩ B) = 0.08,
P(A ∩ C) = 0.28,
P(A ∩ B ∩ C) = 0.09

P(A ∪ B ∪ C) ≥ 0.75

Now
$0.75 \le P(A \cup B \cup C) \le 1$

$\Rightarrow 0.75 \le P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C) \le 1$

$\Rightarrow 0.75 \le 0.3 + 0.4 + 0.8 - 0.08 - (B \cap C) - 0.28 + 0.09 \le 1$

$\Rightarrow 0.75 \le 1.23 - P(B \cap C) \le 1$

$\Rightarrow 0.75-1-23 \le 1.23-1.23 - P(B \cap C) \le 1-1.23$

$\Rightarrow -0.48 \le -P(B \cap C) \le -0.23$

$\Rightarrow 0.48 \ge P(B \cap C) \ge 0.23$

$\Rightarrow P(B \cap C) \in [0, 23, 0.48]$.

P(B ∩C) lies in the interval [0.23, 0.48].

5. A, B, C are three horses running in a race. The probability of A to win the race is twice that of B, and probability of B is twice that of C. What are the probabilities of A, B, C to win the race. Also find the probability that horse A loses in the race.

A: Let A, B, C be the events of winning in the race by the horses A, B, C respectively.

Given that P(A) = 2P(B) and P(B) = 2P(C)

$\Rightarrow P(A) = 2P(B)$

$= 2(2P(C))$

$= 4P(C)$

Clearly the events A, B, C are mutually exclusive and exhanstive.

$\Rightarrow A \cup B \cup C = S$

$\Rightarrow P(A \cup B \cup C) = P(S)$

$\Rightarrow P(A) + P(B) + P(C) = 1$ by the axiom of certainity

$\Rightarrow 4P(C) + 2P(C) + P(C) = 1$

$\Rightarrow 7P(C) = 1$

$\Rightarrow P(C) = \frac{1}{7}$

$\therefore P(B) = 2P(C)$

$= 2\left(\frac{1}{7}\right)$

$= \frac{2}{7}$

$P(A) = 4P(C)$

$= 4\left(\frac{1}{7}\right)$

$= \frac{4}{7}$

$\therefore P(A) = \frac{4}{7}, P(B) = \frac{2}{7}, P(C) = \frac{1}{7}$

Probability that horse A loses in the race

$= P(\bar{A})$

$= 1 - P(A)$

$= 1 - \frac{4}{7}$

$= \frac{3}{7}$.

7. If A, B, C are three independent events of an experiment such that $P(A \cap \bar{B} \cap \bar{C}) = \frac{1}{4}$, $P(\bar{A} \cap B \cap \bar{C}) = \frac{1}{8}$, $P(\bar{A} \cap \bar{B} \cap \bar{C}) = \frac{1}{4}$ then find P(A), P(B) and P(C).

A: Given that A, B, C are independent events and

$P(A \cap \bar{B} \cap \bar{C}) = \frac{1}{4} \Rightarrow P(A)\,P(\bar{B})\,P(\bar{C}) = \frac{1}{4}$ ----- (1)

$P(\bar{A} \cap B \cap \bar{C}) = \frac{1}{8} \Rightarrow P(\bar{A})\,P(B)\,P(\bar{C}) = \frac{1}{8}$ ----- (2)

$P(\bar{A} \cap \bar{B} \cap \bar{C}) = \frac{1}{4} \Rightarrow P(\bar{A})\,P(\bar{B})\,P(\bar{C}) = \frac{1}{4}$ ----- (3)

$\frac{(1)}{(3)} \Rightarrow \frac{P(\bar{A})P(\bar{B})P(\bar{C})}{P(\bar{A})P(\bar{B})P(\bar{C})} = \frac{\left(\frac{1}{4}\right)}{\left(\frac{1}{4}\right)}$

$\Rightarrow \frac{P(A)}{1-P(A)} = 1$

$\Rightarrow P(A) = 1 - P(A)$

$\Rightarrow 2P(A) = 1$

$\Rightarrow P(A) = \frac{1}{2}$

$\frac{(2)}{(3)} \Rightarrow \frac{P(\bar{A})P(B)P(\bar{C})}{P(\bar{A})P(\bar{B})P(\bar{C})} = \frac{\left(\frac{1}{8}\right)}{\left(\frac{1}{4}\right)}$

$\Rightarrow \frac{P(B)}{1-P(B)} = \frac{1}{2}$

$\Rightarrow 2P(B) = 1 - P(B)$

$\Rightarrow 3P(B) = 1$

$\Rightarrow P(B) = \frac{1}{3}$

From (1), $P(A)\,P(\bar{B})\,P(\bar{C}) = \frac{1}{4}$

$\Rightarrow \frac{1}{2}\left(1-\frac{1}{3}\right)P(\bar{C}) = \frac{1}{4}$

$\Rightarrow \frac{1}{2}\left(\frac{2}{3}\right)P(\bar{C}) = \frac{1}{4}$

$\Rightarrow P(\bar{C}) = \frac{3}{4}$

$\Rightarrow P(C) = 1 - P(\bar{C})$

$= 1 - \frac{3}{4}$

$= \frac{1}{4}$

$\therefore P(A) = \frac{1}{2}, P(B) = \frac{1}{3}, P(C) = \frac{1}{4}$.

8. A, B, C are aiming to shoot a balloon. a will succeed 4 times out of 5 attempts. The chance of B to shoot the balloon is 3 out of 4 and that of C is 2 out of 3. If the three aim the balloon simultaneously, then find the probability that atleast two of them hit the balloon.

A: Let A, B, C, be the events that the shooters A, B, C suceed in shooting the balloon.

Given that $P(A) = \frac{4}{5}$, $P(B) = \frac{3}{4}$, $P(C) = \frac{2}{3}$

Clearly A, B, C are indepedent events.

Probability that atleast two of them hit the balloon

$= P(A \cap B \cap \bar{C}) + P(A \cap \bar{B} \cap C) + P(\bar{A} \cap B \cap C) + P(A \cap B \cap C)$

$= P(A)\,P(B)\,P(\bar{C}) + P(A)\,P(\bar{B})\,P(C) + P(\bar{A})\,P(B)\,P(C) + P(A)\,P(B)\,P(C)$

$= \frac{4}{5}\cdot\frac{3}{4}\left(1-\frac{2}{3}\right) + \frac{4}{5}\left(1-\frac{3}{4}\right)\left(\frac{2}{3}\right) + \left(1-\frac{4}{5}\right)\left(\frac{3}{4}\right)\left(\frac{2}{3}\right) + \left(\frac{4}{5}\right)\left(\frac{3}{4}\right)\left(\frac{2}{3}\right)$

$= \frac{4}{5}\cdot\frac{3}{4}\cdot\frac{1}{3} + \frac{4}{5}\cdot\frac{1}{4}\cdot\frac{2}{3} + \frac{1}{5}\cdot\frac{3}{4}\cdot\frac{2}{3} + \frac{4}{5}\cdot\frac{3}{4}\cdot\frac{2}{3}$

$= \frac{12+8+6+24}{60}$

$= \frac{50}{60}$

$= \frac{5}{6}$.

9. i) For any two events, show that

$P(\bar{A} \cap \bar{B}) = 1 + P(A \cap B) - P(A) - P(B)$

ii) If A, B are two events with $P(A \cup B) = 0.65$, $P(A \cap B) = 0.15$ then find $P(\bar{A}) + P(\bar{B})$

A: i) We know that

$\bar{A} \cap \bar{B} = \overline{A \cup B}$

$\Rightarrow P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B})$

$= 1 - P(A \cup B)$

$= - \{P(A) + P(B) - P(A \cap B)\}$

$= 1 + P(A \cap B) - P(A) + P(B)$

ii) Given that $P(A \cup B) = 0.65$, $P(A \cap B) = 0.15$

By addition theorem,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(A \cup B) + P(A \cap B) = P(A) + P(B)$ ------ (1)

Now $P(\bar{A}) + P(\bar{B})$

$= 1 - P(A) + 1 - P(B)$

$= 2 - \{P(A) + P(B)$

$= 2 - \{P(A \cup B) + P(A \cap B)\}$ from (1)

$= 2 - \{0.65 + 0.15\}$

$= 2 - 0.8$

$= 1.2.$

10. In a shooting test the probability of A, B, C hitting the targets are $\frac{1}{2}, \frac{2}{3}$ and $\frac{3}{4}$ respectively. If all of them fire at the same target, find the probability that
i) Only one of them hits the target
ii) Atleast one of them hits the target

A: Let A, B, C be the events that the three persons A, B, C respectively hitting the target.

Given that $P(A) = \frac{1}{2}$, $P(B) = \frac{2}{3}$, $P(C) = \frac{3}{4}$

Clearly the events are independent.

i) Probability that only one of them hits the target.

$= P(A \cap \bar{B} \cap \bar{C}) + P(\bar{A} \cap B \cap \bar{C}) + P(\bar{A} \cap \bar{B} \cap C)$

$= P(A)\ P(\bar{B})\ P(\bar{C}) + P(\bar{A})\ P(B) P(\bar{C}) + P(\bar{A})\ P(\bar{B}) P(C)$

$= \frac{1}{2}\left(1-\frac{2}{3}\right)\left(1-\frac{3}{4}\right)+\left(1-\frac{1}{2}\right)\left(\frac{2}{3}\right)\left(1-\frac{3}{4}\right) + \left(1-\frac{1}{2}\right)\left(1-\frac{2}{3}\right)+\left(\frac{3}{4}\right)$

$= \frac{1}{2}\cdot\frac{1}{3}\cdot\frac{1}{4} + \frac{1}{2}\cdot\frac{2}{3}\cdot\frac{1}{4} + \frac{1}{2}\cdot\frac{1}{3}\cdot\frac{3}{4}$

$= \frac{1+2+3}{24}$

$= \frac{6}{24}$

$= \frac{1}{4}.$

ii) Probability that atleast one of them hits the target = 1 - probabilities that all the three fail to hit the target

$= 1 - P(\bar{A} \cap \bar{B} \cap \bar{C})$

$= 1 - P(\bar{A})\ P(\bar{B})\ P(\bar{C})$

$= 1 - \left(1-\frac{1}{2}\right)\left(1-\frac{2}{3}\right)\left(1-\frac{3}{4}\right)$

$= 1 - \left(\frac{1}{2}\right)\left(\frac{1}{3}\right)\left(\frac{1}{4}\right)$

$= 1 - \frac{1}{24}$

$= \frac{23}{24}.$

11. Define conditional probability. There are 3 black and 4 white balls in first bag; 4 black and 3 white balls in the second bag. A die is rolled and the first bag is selected if it is 1 or 3, and the second bag for the rest. Find the probability of drawing a black ball from the selected bag.

A: Conditional probability :

If A, B are two events in a sample space S and $P(A) \neq 0$, then the probability of B after the event A has occured is called conditional probability of B given A. It is denoted by $P\left(\frac{B}{A}\right)$.

$$P\left(\frac{B}{A}\right) = \frac{n(A \cap B)}{n(A)} = \frac{P(A \cap B)}{P(A)}$$

Let A_1, A_2 be the events of selecting first and seond bags respectively

Let E be the events of drawing a black ball from the selected bag.

Now $P(A_1) = \frac{2}{6} = \frac{1}{3}$

$P(A_2) = \frac{4}{6} = \frac{2}{3}$

Probability of drawing a black ball from the first bag

$P\left(\frac{E}{A_1}\right) = \frac{^3C_1}{^7C_1} = \frac{3}{7}$

Probability of drawing a black ball from the second bag $P\left(\frac{E}{A_2}\right) = \frac{^4C_1}{^7C_1} = \frac{4}{7}$

By total probability theorem,

$P(E) = P(A_1)\ P\left(\frac{E}{A_1}\right) + P(A_2)\ P\left(\frac{E}{A_2}\right)$

$= \frac{1}{3}\left(\frac{3}{7}\right) + \frac{2}{3}\left(\frac{4}{7}\right)$

$= \frac{3+8}{21} = \frac{11}{21}.$

12. Three boxes numbered I, II, III contain 1 white, 2 black and 3 red balls; 2 white, 1 black and 1 red ball; 4 white, 5 black and 3 red balls respectively. One box is randomly selected and a ball is drawn from it. If the ball is red, then find the probability that it is from box II.

A:

Box	White	Black	Red
I	1	2	3
II	2	1	1
III	4	5	3

Let A_1, A_2, A_3 be the events of selecting the boxes I, II, III respectively

Clearly the events A_1, A_2, A_3 are equally likely

$\therefore P(A_1) = P(A_2) = P(A_3) = \frac{1}{3}$

Let E be the event of drawing a red ball from the selected bag

Now $P\left(\frac{E}{A_1}\right)$ = Probability of drawing a red ball from box I

$$= \frac{{}^3C_1}{{}^6C_1} = \frac{3}{6} = \frac{1}{2}$$

$P\left(\frac{E}{A_2}\right)$ = Probability of drawing a red ball from box II

$$= \frac{{}^1C_1}{{}^4C_1} = \frac{1}{4}$$

$P\left(\frac{E}{A_3}\right)$ = Probability of drawing a red ball from box II

$$= \frac{{}^3C_1}{{}^{12}C_1} = \frac{3}{12} = \frac{1}{4}.$$

By Baye's theorem, required probability

$$P\left(\frac{A_2}{E}\right) = \frac{P(A_2)P\left(\frac{E}{A_2}\right)}{P(A_1)P\left(\frac{E}{A_1}\right)+P(A_2)P\left(\frac{E}{A_2}\right)+P(A_3)P\left(\frac{E}{A_3}\right)}$$

$$= \frac{\frac{1}{3}\cdot\frac{1}{4}}{\frac{1}{3}\cdot\frac{1}{2}+\frac{1}{3}\cdot\frac{1}{4}+\frac{1}{3}\cdot\frac{1}{4}}$$

$$= \frac{\frac{1}{3}\cdot\frac{1}{4}}{\frac{1}{3}\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{4}\right)}$$

$$= \frac{1}{4}.$$

13. In a certain college, 25% of the boys and 10% of the girls are studying mathematics. The girls constitute 60% of the student strength. If a student selected at random is found studying mathematics, find the probability that the student is a girl.

A: Let A_1 be the event that the selected student is a boy and A_2 be the event that the selected student is a girl. Let E be the event that the selected student is studying mathematics.

$P(A_1) = \frac{40}{100} = \frac{2}{5}$

$P(A_2) = \frac{60}{100} = \frac{3}{5}$

$P\left(\frac{E}{A_1}\right)$ = Probability that the selected student is a boy who is studying mathematics

$$= \frac{25}{100}$$

$= \frac{1}{4}$

$P\left(\frac{E}{A_2}\right)$ = Probability that the selected student is a girl who is studying mathematics.

$= \frac{10}{100}$

$= \frac{1}{10}$

By Baye's stheorem, required probability

$$P\left(\frac{A_2}{E}\right) = \frac{P(A_2)P\left(\frac{E}{A_2}\right)}{P(A_1)P\left(\frac{E}{A_1}\right)+P(A_2)P\left(\frac{E}{A_2}\right)}$$

$$= \frac{\frac{3}{5}\left(\frac{1}{10}\right)}{\frac{2}{5}\left(\frac{1}{4}\right)+\frac{3}{5}\left(\frac{1}{10}\right)}$$

$$= \frac{\left(\frac{3}{50}\right)}{\left(\frac{10+6}{100}\right)}$$

$$= \frac{3}{8}.$$

14.In a box containing 15 bulbs, 5 are defective. If 5 bulbs are selected at random from the box, find the probability of the event, that
(i) None of them is defective
(ii) Atleast one of them is defective
(iii) Only one of them is defective

A: Total number of bulbs = 15
Number of defective bulbs = 5
$\therefore$ Number of good bulbs = 10
(i) Probability that none of them is defective, when 5 bulbs are defective

$$= \frac{^{10}C_5}{^{15}C_5}$$

$$= \frac{10.9.8.7.6}{5!} \times \frac{5!}{15.14.13.12.11}$$

$$= \frac{12}{143}$$

(ii) Probability that atleast one bulb is defective
= 1 - probability that none of them is defective

$$= 1 - \frac{12}{143}$$

$$= \frac{143-12}{143}$$

$$= \frac{131}{143}$$

(iii) Probability that only one of them is defective

$$= \frac{^5C_1.^{10}C_4}{^{15}C_5}$$

$$= \frac{5.10.9.8.7}{24} \times \frac{120}{15.14.13.12.11}$$

$$= \frac{50}{143}.$$

15.A bag contains 12 two rupee coins, 7 one rupee coins and 4 half a rupee coins. If three of them are selected at random, then find the probability that (i) the sum of three coins is maximum
(ii) the sum of three coins is minimum
(iii) each coin is of different value.

A: Number of two rupee coins = 12
Number of one rupee coins = 7
Number of half a rupee coins = 4
Total number of coins = 23
(i) To have sum of three coins as maximum, we shall select all the three coins are from two rupee coins.
$\therefore$ probability that the sum of three coins is

maximum $= \frac{^{12}C_3}{^{23}C_3}$

$$= \frac{12.11.10}{6} \times \frac{6}{23.22.21}$$

$$= \frac{20}{161}$$

(ii) To have sum of three coins is minimum, we shall select all the three coins are from half a rupee coins.
$\therefore$ probability that the sum of three coins is

minimum $= \frac{^4C_3}{^{23}C_3}$

$= \frac{4.3.2}{6} \times \frac{6}{23.22.21}$

$= \frac{4}{1771}$

(iii) Probability that each coin is of different value when three coins are selected

$= \frac{{}^{12}C_1 . {}^{7}C_1 . {}^{4}C_1}{{}^{23}C_3}$

$= \frac{12.7.4}{1} \times \frac{6}{23.22.21}$

$= \frac{48}{253}$.

16. Three screws are drawn at random from a lot of 50 screws, 5 of which are defective. Find the probability of the event that all 3 screws are non - defective, assuming that the drawing is i) with replacement ii) without replacement.

A: Number of defective screws = 5

Number of good screws = 45

Total number of screws = 50.

i) Consider the drawing of 3 screws with replacement.

Let A, B, C be the events of drawing 1^{st}, 2^{nd}, 3^{rd} screws as non - defective. when three screws are drawn with replacement.

Now $P(A \cap B \cap C) = P(A)\, P\left(\frac{B}{A}\right) P\left(\frac{C}{A \cap B}\right)$

$= \frac{{}^{45}C_1}{{}^{50}C_1} \cdot \frac{{}^{45}C_1}{{}^{50}C_1} \cdot \frac{{}^{45}C_1}{{}^{50}C_1}$

$= \frac{9^3}{10^3} = \frac{729}{1000}$.

ii) Consider the drawing of 3 screws without replacement.

Let A, B, C be the events of drawing 1^{st}, 2^{nd}, 3^{rd} screws as non defective, when three screws are drawn without replacement.

Required probability,

$P(A \cap B \cap C) = P(A)\, P\left(\frac{B}{A}\right) P\left(\frac{C}{A \cap B}\right)$

$= \frac{{}^{45}C_1}{{}^{50}C_1} \cdot \frac{{}^{44}C_1}{{}^{49}C_1} \cdot \frac{{}^{43}C_1}{{}^{48}C_1}$

$= \frac{45}{50} \cdot \frac{44}{49} \cdot \frac{43}{48}$

$= \frac{1419}{1960}$.

17. If one card is drawn at random from a pack of cards then show that the event of getting an ace and getting a heart are independent events.

A. Let A, B be the events of getting an ace, a heart card respectively.

And S be the sample space

n(A) = 4, n(B) = 13, n(S) = 52

$\therefore \; P(A) = \frac{n(A)}{n(S)} = \frac{4}{52} = \frac{1}{13}$

$P(B) = \frac{n(B)}{n(S)} = \frac{13}{52} = \frac{1}{4}$

$n(A \cap B) = 1,$ i.e. a heart of ace

$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{1}{52}$

$= \frac{1}{13} \times \frac{1}{4}$

$P(A \cap B) = P(A)P(B)$

$\therefore$ A, B are independent events.

18. If A and B are independent events with P(A) = 0.2, P(B) = 0.5 then find i) $P\left(\frac{A}{B}\right)$ ii) $P\left(\frac{B}{A}\right)$

iii) $P(A \cap B)$ **iv)** $P(A \cup B)$

A. Given A, B are independent events

$\Rightarrow$ P(A) = 0.2

P(B) = 0.5

i) $P(A\cap B)=P(A)P(B)$

$= (0.2)(0.5)$

$= 0.1$

ii) $P\left(\frac{A}{B}\right)=\frac{P(A\cap B)}{P(B)}=\frac{0.1}{0.5}=\frac{1}{5}=0.2$

iii) $P\left(\frac{B}{A}\right)=\frac{P(B\cap A)}{P(A)}=\frac{0.1}{0.2}=\frac{1}{2}=0.5$

iv) $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$= 0.2 + 0.5 - 0.1 = 0.6$

LEVEL II (LAQ)

1. The probabilities of three mutually exclusive events are respectively given as $\frac{1+3p}{3},\frac{1-p}{4},\frac{1-2p}{2}$. Prove that $\frac{1}{3}\le p\le\frac{1}{2}$.

A. Let A, B, C be the given three mutually exclusive events.

$P(A)=\frac{1+3p}{3},\ P(B)=\frac{1-p}{4},\ P(C)=\frac{1-2p}{2}$

$0\le P(A)\le 1 \quad 0\le P(B)\le 1 \quad 0\le P(C)\le 1$

$\Rightarrow 0\le\frac{1+3p}{3}\le \quad 0\le\frac{1-p}{4}\le 1 \quad 0\le\frac{1-2p}{2}\le 1$

$\Rightarrow 0\le 1+3p\le 3 \quad 0\le 1-p\le 4 \quad 0\le 1-2p\le 2$

$-1\le 3p\le 2 \quad -1\le -p\le 3 \quad -1\le -2p\le 1$

$-\frac{1}{3}\le p\le\frac{2}{3} \quad -3\le p\le 1 \quad -\frac{1}{2}\le p\le\frac{1}{2}$

$\downarrow \quad \downarrow \quad \downarrow$

(1) (2) (3)

Also $0\le P(A\cup B\cup C)\le 1$

$\Rightarrow 0\le P(A)+P(B)+P(C)\le 1$

$\Rightarrow 0\le\frac{1+3p}{3}+\frac{1-p}{4}+\frac{1-2p}{2}\le 1$