

DEFINITIONS, CONCEPTS AND FORMULAE

Random variable:

- Let S be the sample space of a random experiment. A real valued function $X : S \rightarrow R$ is called a random variable.
- Let $X : S \rightarrow R$ be a discrete random variable with range $\{x_1, x_2, x_3, \dots\}$. If $\sum x_i P(X = x_i)$ exists, then $\sum x_i P(X = x_i)$ is called the 'mean' of the random variable X. It is denoted by μ or \bar{x} . If $\sum (x_i - \mu)^2 P(X = x_i)$ exists, then $\sum (x_i - \mu)^2 P(X = x_i)$ is called variance of the random variable X. It is denoted by σ^2 .
- $\sum_{i=1}^n P(X = X_i) = 1$.
- $\sum_{i=1}^{\infty} P(X = X_i) = 1$.
- Mean of the random variable $\mu = \sum_{i=1}^n x_i P(X = x_i)$.
- Variance $\sigma^2 = \sum_{i=1}^n x_i^2 P(X = x_i) - \mu^2$.
- Standard deviation = $\sqrt{\text{Variance}}$.

Binomial distribution:

- A discrete random variable X is said to follow a binomial distribution with parameters n and p where $0 < p < 1$ if $P(X = r) = {}^n C_r q^{n-r} p^r$ where $r = 0, 1, 2, \dots, n$.
- If X is a binomial variate with parameters n, p, then it is described by writing $X \sim B(n, p)$.
- n, p are the parameters of the binomial distribution.
- $P(X = r) = {}^n C_r q^{n-r} p^r$ where $p + q = 1$
- B.D is $(q + p)^n$
- Mean of X is np.
- Variance of X is npq.

15. Standard deviation = \sqrt{npq} .

Poisson distribution:

16. Let $\lambda > 0$ be a real number. A random variable X with range $\{0, 1, 2, \dots\}$ is said to follow poisson distribution with parameter λ if

$$P(X = r) = \frac{e^{-\lambda} \lambda^r}{r!} \text{ for } r = 0, 1, 2, \dots$$



- λ is the only parameter of the poisson distribution.
- $P(X = r) = \frac{e^{-\lambda} \lambda^r}{r!}$ where $r = 0, 1, 2, \dots, \infty$.
- Mean = λ .
- Variance = λ .
- Standard deviation = $\sqrt{\lambda}$.

LEVEL - I (VSAQ)

1. The probability distribution of a random variable X is

X = x	0	1	2	3
P(X = x)	k	3k	3k	k

Find the value of k and the mean of X.

A: Given that X is a random variable.

$$\begin{aligned} \Rightarrow \sum_{i=1}^3 P(X = x_i) &= 1 \\ \Rightarrow k + 3k + 3k + k &= 1 \\ \Rightarrow 8k &= 1 \\ \Rightarrow k &= 1/8 \end{aligned}$$

Let μ be the mean of X.

$$\begin{aligned} \therefore \mu &= \sum_{i=1}^3 x_i P(x = x_i) \\ &= 0(k) + 1(3k) + 2(3k) + 3(k) \\ &= 12k \\ &= \frac{12}{8} = \frac{3}{2} \\ \therefore k &= 1/8, \mu = 3/2 \end{aligned}$$

2. A random variable X has the range

$\{1, 2, 3, \dots\}$ If $P(X = r) = \frac{C^r}{r!}$ for $r = 1, 2, 3, \dots$, then find C.

A: Given that X is a common variable

$$\Rightarrow \sum_{r=1}^{\infty} P(x = r) = 1$$

$$\Rightarrow \sum_{r=1}^{\infty} \frac{C^r}{r!} = 1$$

$$\frac{C}{1!} + \frac{C^2}{2!} + \frac{C^3}{3!} + \dots = 1$$

Adding 1 on bothsides,

$$1 + \frac{C}{1!} + \frac{C^2}{2!} + \dots = 1 + 1$$

$$\Rightarrow e^C = 2$$

$$\Rightarrow C = \log_e 2.$$

3. Find the constant C, so that

$$f(x) = C \left(\frac{2}{3}\right)^x, \quad x = 1, 2, 3, \dots \text{ is the}$$

probability distribution function of a discrete random variable X.

A: Given that X is a discrete random variable

$$\Rightarrow \sum_{x=1}^{\infty} f(x) = 1$$

$$\Rightarrow \sum_{x=1}^{\infty} C \left(\frac{2}{3}\right)^x = 1$$

$$\Rightarrow C \left[\frac{2}{3} + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^3 + \dots = 1 \right]$$

$$\Rightarrow C \left[\frac{\frac{2}{3}}{1 - \frac{2}{3}} \right] = 1 \quad \because S_{\infty} = \frac{a}{1-r}$$

$$\Rightarrow 2C = 1$$

$$\therefore C = \frac{1}{2}.$$

4. For a binomial distribution with mean 6 and variance 2, Find the first two terms of hte distribution..

A: Let n, p be the paremeters of the binomial distribution.

$$np = 6, \quad npq = 2.$$

$$q = \frac{npq}{np} = \frac{2}{6} = \frac{1}{3}.$$

$$\Rightarrow p = 1 - q = 1 - 1/3 = 2/3.$$

$$\text{Also } np = 6$$

$$\Rightarrow n \times 2/3 = 6$$

$$\Rightarrow n = 9$$

First two terms of the binomial distribution are

$$P(X = 0), P(X = 1)$$

$$= {}^9C_0 \left(\frac{1}{3}\right)^9 \left(\frac{2}{3}\right)^0, {}^9C_1 \left(\frac{1}{3}\right)^8 \left(\frac{2}{3}\right)^1$$

$$= \frac{1}{3^9}, \frac{2}{3^7}.$$



5. The mean and variance of a binomial distribution are 4 and 3 respectively. Find P(X ≥ 1)

A: Let n, p be the parameters of the binomial distribution.

$$np = 4, \quad npq = 3$$

$$q = \frac{npq}{np} = \frac{3}{4}$$

$$\Rightarrow p = 1 - q = 1 - \frac{3}{4} = \frac{1}{4}$$

$$\text{Also } np = 4$$

$$n \cdot 1/4 = 4$$

$$n = 16$$

Required probability P(X ≥ 1)

$$= \sum_{r=1}^{16} P(X = r)$$

$$= \sum_{r=1}^{16} {}^{16}C_r \left(\frac{3}{4}\right)^{16-r} \left(\frac{1}{4}\right)^r.$$

6. If the mean and variance of a binomial variate X are 2.4 and 1.44 respectively, then find p and n.

A: Let n, p be the parameter of the binomial distribution.

$$\text{Given that } np = 2.4, \quad npq = 1.44$$

$$q = \frac{npq}{np} = \frac{1.44}{2.4} = \frac{144}{240} = \frac{12}{20} = \frac{3}{5}$$

$$\Rightarrow p = 1 - q = 1 - \frac{3}{5} = \frac{2}{5}$$

$$\text{Also } np = 2.4$$

$$\Rightarrow n \left(\frac{2}{5}\right) = 2.4$$

$$\Rightarrow n = 6 \quad \because n = 6, \quad p = 2/5$$

7. X follows Poisson distribution such that $P(X = 1) = 3P(X = 2)$. Find the variance of X.

A: Let λ be the parameter of the poisson distribution.
Given that $P(X = 1) = 3P(X = 2)$

$$\Rightarrow \frac{e^{-\lambda}\lambda^1}{1!} = 3 \frac{e^{-\lambda}\lambda^2}{2!}$$

$$\Rightarrow 1 = \frac{3\lambda}{2}$$

$$\Rightarrow \lambda = \frac{2}{3}$$

$$\text{Variance of X} = \frac{2}{3}$$

8. A poisson variate X satisfies $P(X = 1) = P(X = 2)$. Find $P(X = 5)$.

A: Let λ be the parameter of the poisson distribution.

Given that $P(X = 1) = P(X = 2)$

$$\Rightarrow \frac{e^{-\lambda}\lambda^1}{1!} = \frac{e^{-\lambda}\lambda^2}{2!}$$

$$\Rightarrow 1 = \frac{\lambda}{2}$$

$$\Rightarrow \lambda = 2$$

$$\text{Now } P(X = 5) = \frac{e^{-\lambda}\lambda^5}{5!}$$

$$= \frac{e^{-2} \cdot 2^5}{5!}$$

$$= \frac{32e^{-2}}{120}$$

$$= \frac{4e^{-2}}{15}$$

LEVEL - I (LAQ)

1. If $X : S \rightarrow R$ is a discrete random variable with range $\{x_1, x_2, x_3, \dots\}$; μ is mean and σ^2 is variance of X, then prove that $\sigma^2 + \mu^2 = \sum x_i^2 P(X = x_i)$.

A: Proof:

Given that $X : S \rightarrow R$ is a discrete random variable with range $\{x_1, x_2, x_3, \dots\}$. Since X is a discrete

random variable, $\sum_{i=1}^{\infty} P(X = x_i) = 1$

By the definition, mean $\mu = \sum_{i=1}^{\infty} x_i P(X = x_i)$

By the definition, variance

$$\sigma^2 = \sum_{i=1}^{\infty} (x_i - \mu)^2 P(X = x_i)$$

$$\sigma^2 = \sum_{i=1}^{\infty} (x_i^2 - 2\mu x_i + \mu^2) P(X = x_i)$$

$$= \sum_{i=1}^{\infty} x_i^2 P(X = x_i) - 2\mu \sum_{i=1}^{\infty} x_i P(X = x_i)$$

$$+ \mu^2 \sum_{i=1}^{\infty} P(X = x_i)$$

$$= \sum_{i=1}^{\infty} x_i^2 P(X = x_i) - 2\mu(\mu) + \mu^2(1)$$

$$= \sum_{i=1}^{\infty} x_i^2 P(X = x_i) - \mu^2$$

$$\therefore \sigma^2 + \mu^2 = \sum_{i=1}^{\infty} x_i^2 P(X = x_i)$$

2. $X = x_i$	-2	-1	0	1	2	3
$P(X = x_i)$	0.1	k	0.2	2k	0.3	k

is the probability distribution of a random variable X. Find the value of k and the variance of X.

A: Given that X is a random variable.

Let μ be the mean and σ^2 be the variance of X.

$$\text{Here } \sum_{i=-2}^3 P(X = x_i) = 1$$

$$\Rightarrow 0.1 + k + 0.2 + 2k + 0.3 + k = 1$$

$$\Rightarrow 4k + 0.6 = 1$$

$$\Rightarrow 4k = 1 - 0.6 = 0.4$$

$$\Rightarrow k = 0.1$$

$$\text{Mean } \mu = \sum_{i=-2}^3 P(X = x_i)$$

$$= (-2)(0.1) - 1(0.1) + 0(0.2) + 1(0.2) + 2(0.3) + 3(0.1)$$

$$= 0.8$$

We know that

$$\sigma^2 + \mu^2 = \sum_{i=-2}^3 P(X = x_i)$$

$$\sigma^2 + 0.8^2 = (-2)^2(0.1) + (-1)^2(0.1) + 0^2(0.2) + (1^2)(0.2) + 2^2(0.3) + 3^2(0.1)$$

$$\sigma^2 = 2.80 - 0.64$$

$$\sigma^2 = 2.16$$

Hence $k = 0.1$, $\sigma^2 = 2.16$.

3. The Probability distribution of a random variable X is given below.

$X = x_i$	1	2	3	4	5
$P(X = x_i)$	k	2k	3k	4k	5k

Find the value of k and the mean and variance of X.

A: Given that X is a random variable

$$\Rightarrow \sum_{i=1}^5 P(X = x_i) = 1$$

$$\Rightarrow k + 2k + 3k + 4k + 5k = 1$$

$$\Rightarrow 15k = 1$$

$$\Rightarrow k = \frac{1}{15}$$

Let μ be the mean and σ^2 be the variance of X

$$\begin{aligned} \mu &= \sum_{i=1}^5 P(X = x_i) \\ &= 1(k) + 2(2k) + 3(3k) + 4(4k) + 5(5k) \\ &= k + 4k + 9k + 16k + 25k \\ &= 55k \\ &= \frac{55}{15} \\ &= \frac{11}{3} \end{aligned}$$

$$\begin{aligned} \sigma^2 &= \sum_{i=1}^5 x_i^2 P(X = x_i) - \mu^2 \\ &= 1(k) + 4(2k) + 9(3k) + 16(4k) + 25(5k) - \left(\frac{11}{3}\right)^2 \\ &= 225k - \frac{121}{9} \\ &= 225\left(\frac{1}{15}\right) - \frac{121}{9} \\ &= 15 - \frac{121}{9} \\ &= \frac{135 - 121}{9} \\ &= \frac{14}{9} \end{aligned}$$

4. A random variable X has the following probability distribution.

$X = x_i$	0	1	2	3	4	5	6	7
$P(X = x_i)$	0	k	2k	2k	3k	k ²	2k ²	7k ² + k

Find (i) k (ii) the mean (iii) $P(0 < X < 5)$

A: Given that X is a random variable

$$\begin{aligned} \sum_{i=0}^7 P(X = x_i) &= 1 \\ \Rightarrow 0 + k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k &= 1 \end{aligned}$$

$$\Rightarrow 10k^2 + 9k - 1 = 0$$

$$\Rightarrow 10k^2 + 10k - k - 1 = 0$$

$$\Rightarrow 10k(k + 1) - 1(k + 1) = 0$$

$$\Rightarrow (k + 1)(10k - 1) = 0$$

$$k = -1 \text{ is false}$$

$$\Rightarrow k = \frac{1}{10}$$

Let μ be the mean of X.

$$\begin{aligned} \mu &= \sum_{i=0}^7 x_i P(X = x_i) \\ &= 0(0) + 1(0.1) + 2(0.2) + 3(0.2) + 4(0.3) + 5(0.01) \\ &\quad + 6(0.02) + 7(0.17) \\ &= 0.1 + 0.4 + 0.6 + 1.2 + 0.05 + 0.12 + 1.19 \\ &= 3.66 \end{aligned}$$

$$P(0 < X < 5)$$

$$= P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4)$$

$$= k + 2k + 2k + 3k$$

$$= 8k$$

$$= 8(0.1)$$

$$= 0.8$$

5. The range of random variable X is {0, 1, 2}. Given that $P(X = 0) = 3C^3$, $P(X = 1) = 4C - 10C^2$, $P(X = 2) = 5C - 1$.

Find (i) the value of C,

(ii) $P(X < 1)$

(iii) $P(1 < X \leq 2)$

(iv) $P(0 < X \leq 3)$

A: Given that range of random variable X is {0, 1, 2}.

$$P(X = 0) = 3C^3, P(X = 1) = 4C - 10C^2, P(X = 2) = 5C - 1$$

Since X is a random variable,

$$\begin{aligned} \sum_{i=0}^2 P(X = x_i) &= 1 \\ \Rightarrow P(X = 0) + P(X = 1) + P(X = 2) &= 1 \\ \Rightarrow 3C^3 + 4C - 10C^2 + 5C - 1 &= 1 \\ \Rightarrow 3C^3 - 10C^2 + 9C - 2 &= 0 \end{aligned}$$

Clearly C = 1 satisfies this equation



By Synthetic division,

$$\begin{array}{r|rrrr} & 3 & -10 & 9 & -2 \\ 1 & 0 & 3 & -7 & 2 \\ \hline & 3 & -7 & 2 & \underline{0} \end{array}$$

Now the above equation becomes,

$$\begin{aligned} (C - 1)(3C^2 - 7C + 2) &= 0 \\ \Rightarrow C = 1, 3C^2 - 6C - C + 2 &= 0 \\ &\Rightarrow 3C(C - 2) - 1(C - 2) = 0 \\ &\Rightarrow (C - 2)(3(-1)) = 0 \end{aligned}$$

$$\therefore C = 1, 2, \frac{1}{3}.$$

C = 1, 2 are not possible.

SO, the value of C = $\frac{1}{3}$.

$$\begin{aligned} \text{ii) } P(X < 1) &= P(X = 0) \\ &= 3C^3 \\ &= 3\left(\frac{1}{3}\right)^3 \\ &= \frac{1}{9}. \end{aligned}$$

$$\begin{aligned} \text{iii) } P(1 < X \leq 2) &= P(X = 2) \\ &= 5C - 1 \\ &= \frac{5}{3} - 1 \\ &= \frac{2}{3}. \end{aligned}$$

$$\begin{aligned} \text{iv) } P(0 < X \leq 3) &= P(X = 1) + P(X = 2) \\ &= \frac{2}{9} + \frac{2}{3} \\ &= \frac{8}{9}. \end{aligned}$$

6. If X is a random variable with the probability

$$\text{distribution } P(X = k) = \frac{(k + 1)C}{2^k} \quad (k = 0, 1, 2, \dots)$$

then find C ?

A: Here, we use the following :

Sum of the infinite A.G.P ($|r| < 1$).

$$\begin{aligned} a(1) + (a + d)r + (a + 2d)r^2 + \dots \infty \\ = \frac{a}{1-r} + \frac{dr}{(1-r)^2} \end{aligned}$$

Given that X is a random variable with

$$P(X = k) = \frac{(k + 1)C}{2^k}, \quad k = 0, 1, 2, \dots$$

$$\begin{aligned} \Rightarrow \sum_{k=0}^{\infty} P(X = k) \\ \Rightarrow \sum_{k=0}^{\infty} \frac{(k + 1)C}{2^k} = 1. \end{aligned}$$

$$\Rightarrow C \sum_{k=0}^{\infty} (k + 1) \cdot \frac{1}{2^k} = 1$$

$$\Rightarrow C \left[1 \cdot 1 + 2 \cdot \frac{1}{2} + 3 \cdot \frac{1}{2^2} + 4 \cdot \frac{1}{2^3} + \dots \infty \right] = 1 \quad \text{(AIMS)}$$

Here a = 1, d = 1, r = $\frac{1}{2}$

$$\Rightarrow C \left[\frac{a}{1-r} + \frac{dr}{(1-r)^2} \right] = 1$$

$$\Rightarrow C \left[\frac{1}{1-\frac{1}{2}} + \frac{1 \cdot \frac{1}{2}}{\left(1-\frac{1}{2}\right)^2} \right] = 1$$

$$\Rightarrow C \left[\frac{1}{\frac{1}{2}} + \frac{\frac{1}{2}}{\frac{1}{4}} \right] = 1$$

$$\begin{aligned} \Rightarrow C [2 + 2] &= 1 \\ \Rightarrow 4C &= 1 \end{aligned}$$

$$C = \frac{1}{4}.$$

7. If the mean and variance of a binomial variate X are 2.4 and 1.44 respectively find P(1 < X ≤ 1.4).

A: Let n, p be the parameters of the binomial distribution.

Given that np = 2.4, npq = 1.44

$$\text{Now } q = \frac{npq}{np} = \frac{1.44}{2.4} = \frac{144}{240} = \frac{12}{20} = \frac{3}{5}$$

$$\therefore p = 1 - q = 1 - \frac{3}{5} = \frac{2}{5}$$

Also np = 2.4

$$\Rightarrow n \left(\frac{2}{5}\right) = 2.4$$

$$\Rightarrow n = 6$$

Required probability P(1 < X ≤ 4)

$$= P(X = 2) + P(X = 3) + P(X = 4)$$

$$\begin{aligned}
 &= {}^6C_2 \left(\frac{3}{5}\right)^4 \left(\frac{2}{5}\right)^2 + {}^6C_3 \left(\frac{3}{5}\right)^3 \left(\frac{2}{5}\right)^3 + {}^6C_4 \left(\frac{3}{5}\right)^2 \left(\frac{2}{5}\right)^4 \\
 &= \frac{15(81)4}{5^6} + \frac{20(27)(8)}{5^6} + \frac{15(9)(16)}{5^6} \\
 &= \frac{972 + 864 + 432}{5^5} \\
 &= \frac{2268}{3125}
 \end{aligned}$$

8. If the difference between the mean and the variance of a binomial distribution is $\frac{5}{9}$, then find the probability for the event of 2 successes, when the experiment is conducted 5 times.

A: Let n, p be the parameters of the binomial distribution.

Given that n = 5,

$$np - npq = \frac{5}{9}$$

$$5p(1 - q) = \frac{5}{9}$$

$$p^2 = \frac{1}{9}$$

$$p = \frac{1}{3}$$

Required probability is

$$P(X = 2) = {}^5C_2 \left(\frac{2}{3}\right)^3 \left(\frac{1}{3}\right)^2$$

$$= 10 \left(\frac{8}{27}\right) \left(\frac{1}{9}\right)$$

$$= \frac{80}{243}$$

9. One in nine ships is likely to be wrecked when they set on sail. When 6 ships are set on sail, find the probability for

- (i) at least one will arrive safely,
- (ii) exactly three will arrive safely.

A: Let X be the number of ships arriving safely, when six ships are set on sail. X follows binomial distribution,

Let n, p be the parameter.

$$\text{the } n = 6, q = \frac{1}{9}$$

$$\Rightarrow p = 1 - q = 1 - \frac{1}{9} = \frac{8}{9}$$

(i) Probability that at least one ship will arrive safely
 $= P(X \geq 1)$

$$= 1 - P(X = 0)$$

$$= 1 - {}^6C_0 \left(\frac{1}{9}\right)^6 \left(\frac{8}{9}\right)^0$$

$$= 1 - \frac{1}{9^6}$$

(ii) Probability that exactly three ships will arrive safely

$$= P(X = 3)$$

$$= {}^6C_3 \left(\frac{1}{9}\right)^3 \left(\frac{8}{9}\right)^3$$

$$= 20 \left(\frac{8^3}{9^6}\right)$$



10. In the experiment of tossing a coin n times, if the variable X denotes the number of heads and P(X = 4), P(X = 5), P(X = 6) are in A.P, then find n.

A: Let X denotes the number of heads, when a coin is tossed for n times.

Clearly X follows binomial distribution.

Let n, p be the parameters of binomial distribution.

$$\text{Here } p = \frac{1}{2}$$

$$q = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}$$

Given that P(X = 4), P(X = 5), P(X = 6) are in A.P

$$\Rightarrow {}^nC_4 \left(\frac{1}{2}\right)^{n-4} \left(\frac{1}{2}\right)^4, {}^nC_5 \left(\frac{1}{2}\right)^{n-5} \left(\frac{1}{2}\right)^5,$$

$${}^nC_6 \left(\frac{1}{2}\right)^{n-6} \left(\frac{1}{2}\right)^6 \text{ are in A.P.}$$

$$\Rightarrow {}^nC_4 \cdot \frac{1}{2^n}, {}^nC_5 \cdot \frac{1}{2^n}, {}^nC_6 \cdot \frac{1}{2^n} \text{ are in A.P}$$

$$\Rightarrow {}^nC_4, {}^nC_5, {}^nC_6, \text{ are in A.P}$$

$$\Rightarrow 2 \cdot {}^nC_5 = {}^nC_4 + {}^nC_6$$

$$\Rightarrow 2 \cdot \frac{n!}{(n-5)!5!} = \frac{n!}{(n-4)!4!} + \frac{n!}{(n-6)!6!}$$

$$\Rightarrow \frac{2}{(n-5)(n-6)!5(4!)} = \frac{1}{(n-4)(n-5)(n-6)!4!}$$

$$+ \frac{1}{(n-6)!6(5)(4!)}$$

$$\Rightarrow \frac{2}{5(n-5)} = \frac{1}{(n-4)(n-5)} + \frac{1}{30}$$

$$\Rightarrow \frac{2}{5(n-5)} = \frac{30 + n^2 - 9n + 20}{(n-4)(n-5)}$$

$$\Rightarrow 12(n-4) = 30 + n^2 - 9n + 20$$

$$\Rightarrow 12n - 48 = 30 + n^2 - 9n + 20$$

$$\Rightarrow n^2 - 21n + 98 = 0$$

$$\Rightarrow n^2 - 7n - 14n + 98 = 0$$

$$\Rightarrow n(n-7) - 14(n-7) = 0$$

$$\Rightarrow n = 7 \text{ or } 14.$$

11.

X = x	-3	-2	-1	0	1	2	3
P(X = x)	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$

is the probability distribution of a random variable X. Find the variance of X.

A: Let μ be the mean and σ^2 be the variance of random variable X.

$$\text{Mean } \mu = \sum_{i=-3}^3 x_i P(X = x_i)$$

$$= (-3)\left(\frac{1}{9}\right) + (-2)\left(\frac{1}{9}\right) + (-1)\left(\frac{1}{9}\right) + 0\left(\frac{1}{3}\right) + 1\left(\frac{1}{9}\right) + 2\left(\frac{1}{9}\right) + 3\left(\frac{1}{9}\right)$$

$$= 0.$$

$$\therefore \text{Mean } \mu = 0$$

We know that

$$\sigma^2 = \sum_{i=-3}^3 x_i^2 P(X = x_i) - \mu^2$$

$$= (-3)^2 \left(\frac{1}{9}\right) + (-2)^2 \left(\frac{1}{9}\right) + (-1)^2 \left(\frac{1}{9}\right) + 0^2 \left(\frac{1}{3}\right)$$

$$+ (1^2) \left(\frac{1}{9}\right) + (2^2) \left(\frac{1}{9}\right) + 3^2 \left(\frac{1}{9}\right) - 0^2$$

$$= 9 \cdot \frac{1}{9} + 4 \cdot \frac{1}{9} + 1 \cdot \frac{1}{9} + 0 + 1 \cdot \frac{1}{9} + 4 \cdot \frac{1}{9} + 9 \cdot \frac{1}{9} - 0$$

$$= \frac{1}{9}(9 + 4 + 1 + 1 + 4 + 9)$$

$$= \frac{28}{9}.$$

$$\text{Hence variance of X is } \frac{28}{9}$$

LEVEL - II (VSAQ)

1. The probability that a person chosen at random is left handed (in hand writing) is 0.1. What is the probability that in a group of 10 people, there is one who is left handed.

A: Here $n = 10$, $p = 0.1 = \frac{1}{10}$, $q = 0.9 = \frac{9}{10}$.

\therefore The required probability that exactly one out of 10 is left handed is $P(X=1) = {}^{10}C_1 p^1 q^{10-1}$.

AIMS

$$= 10 \cdot \left(\frac{1}{10}\right)^1 \cdot \left(\frac{9}{10}\right)^9 = \left(\frac{9}{10}\right)^9.$$

2. It is given that 10% of the electric bulbs manufactured by a company are defective. In a sample of 20 bulbs, find the probability that more than 2 are defective.

A: Let X be number of defective bulbs in the sample of 20 bulbs.

The probability that a bulb will be defective is

$$p = \frac{10}{100} = \frac{1}{10}, \text{ Hence } q = 1 - \frac{1}{10} = \frac{9}{10}$$

Now, X follows the binomial distribution with

$$\text{parameters } n = 20, p = \frac{1}{10}, q = \frac{9}{10}.$$

\therefore The required probability is

$$P(X > 2) = \sum_{k=3}^{20} {}^{20}C_k \left(\frac{1}{10}\right)^k \left(\frac{9}{10}\right)^{20-k}$$

$$= \sum_{k=3}^{20} {}^{20}C_k \cdot \frac{9^{20-k}}{10^{20}}.$$

3. On an average, rain falls on 12 days in every 30 days, find the probability that, rain will fall on just 3 days of a given week.

A: Let X be the number of days rain falls in a week.

The probability that rain will fall on a day.

$$p = \frac{12}{30} = \frac{2}{5}, \text{ Hence } q = 1 - \frac{2}{5} = \frac{3}{5}.$$

Now, X follows the binomial distribution with

$$\text{parameters } n = 7, p = \frac{2}{5}, q = \frac{3}{5}$$

∴ The required probability is

$$P(X = 3) = {}^7C_3 \left(\frac{2}{5}\right)^3 \left(\frac{3}{5}\right)^4 = 35 \cdot \frac{2^3 \cdot 3^4}{5^7}$$

4. In a book of 450 pages, there are 400 typographical errors. Assuming that the number of errors per page follow the Poisson law, find the probability that a random sample of 5 pages will contain no typographical error.

A: Let the average number of errors per page in the

$$\text{book is } \lambda = \frac{400}{450} = \frac{8}{9}$$

The probability that a page contain 'r' errors is

$$P(X = r) = \frac{e^{-\lambda} \lambda^r}{r!}$$

The probability that a page contain no errors is

$$P(X = 0) = \frac{e^{-\lambda} \lambda^0}{0!} = e^{-8/9}$$

∴ The required probability that a random sample of 5 pages will contain no error is

$$[P(X = 0)]^5 = [e^{-8/9}]^5$$

5. Deficiency of red cells in the blood cells is determined by examining a specimen of blood under a microscope, Suppose a small volume contains on an average 20 red cells for normal persons. Using the poisson distribution, find the probability that a specimen of blood taken from a normal person will contain less than 15 red cells.

A: Here $\lambda = 20$.

Let $P(X = r)$ denote the probability that a specimen taken from a normal person will contain 'r' red cells.

∴ The required probability is

$$P(X < 15) = \sum_{r=0}^{14} P(X = r) = \sum_{r=0}^{14} \frac{e^{-20} 20^r}{r!}$$

6. In a city, 10 accidents take place in a span of 50 days. Assuming that the number of accidents follows the Poisson distribution, find the probability that there will be 3 or more accidents in a day.

A: Here $\lambda = \frac{10}{50} = 0.2$

The required probability is

$$P(X \geq 3) = 1 - [P(X = 0) + P(X = 1) + P(X = 2)]$$

$$= 1 - \left[\frac{e^{-0.2} (0.2)^0}{0!} + \frac{e^{-0.2} (0.2)^1}{1!} + \frac{e^{-0.2} (0.2)^2}{2!} \right]$$

$$= 1 - \left[\frac{1}{e^{0.2}} + \frac{1}{5e^{0.2}} + \frac{1}{50e^{0.2}} \right] = 1 - \frac{61}{50e^{0.2}}$$



LEVEL - II (LAQ)

1. The range of a random variable X is {1, 2, 3,...} and $P(X = k) = \frac{C^k}{k!}$, $k = 1, 2, 3, \dots$. Find the value of C and $P(0 < X < 3)$.

A. Given that X is a random variable

$$\Rightarrow \sum_{k=1}^{\infty} P(X = k) = 1$$

$$\Rightarrow \sum_{k=1}^{\infty} \frac{C^k}{k!} = 1$$

$$\Rightarrow \frac{C}{1!} + \frac{C^2}{2!} + \frac{C^3}{3!} + \dots = 1$$

Adding 1 on both sides

$$\Rightarrow 1 + \frac{C}{1!} + \frac{C^2}{2!} + \frac{C^3}{3!} + \dots = 1 + 1$$

$$\Rightarrow e^C = 2$$

$$\Rightarrow C = \log_e 2$$

ii) $P(0 < X < 3)$

$$\Rightarrow P(X = 1) + P(X = 2)$$

$$= \frac{C}{1!} + \frac{C^2}{2!}$$

$$= \log_e 2 + (\log_e 2)^2$$

2. A cubical die is thrown. Find the mean and variance of X , giving the number on the face that shows up.

A: Let X be the number on the face that shows up when a die is thrown. Here X is a random variable. Probability distribution of X is shown below.

$X = x_i$	1	2	3	4	5	6
$P(X = x_i)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

Let μ be the mean and σ^2 be the variance of X

$$\begin{aligned}\mu &= \sum_{i=1}^6 x_i P(X = x_i) \\ &= 1\left(\frac{1}{6}\right) + 2\left(\frac{1}{6}\right) + 3\left(\frac{1}{6}\right) + 4\left(\frac{1}{6}\right) + 5\left(\frac{1}{6}\right) + 6\left(\frac{1}{6}\right) \\ &= \frac{21}{6} \\ &= \frac{7}{2}.\end{aligned}$$

$$\begin{aligned}\sigma^2 &= \sum_{i=1}^6 x_i^2 P(X = x_i) - \mu^2 \\ &= 1^2\left(\frac{1}{6}\right) + 2^2\left(\frac{1}{6}\right) + 3^2\left(\frac{1}{6}\right) + 4^2\left(\frac{1}{6}\right) + 5^2\left(\frac{1}{6}\right) \\ &\quad + 6^2\left(\frac{1}{6}\right) - \left(\frac{7}{2}\right)^2 \\ &= \frac{91}{6} - \frac{49}{4} \\ &= \frac{182 - 147}{12} \\ &= \frac{35}{12}.\end{aligned}$$

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AIMS