1. Show that the circles $x^2+y^2-2x-4y-20=0$, $x^2+y^2+6x+2y-90=0$ Touch each other internally. Find the point of contact and eq"n of tangent at point of contact.

Sol: eq"n s of the given circles

$S \equiv x^2+y^2-2x-4y-20=0 \ldots\ldots(1)$

$S' \equiv x^2+y^2+6x+2y-90=0 \ldots\ldots(2)$

centres $(-\mathbf{g},-\mathbf{f})$: $C_1\ (1,2),\ C_2(-3,-1)$

radius(r) $=\sqrt{\mathbf{g^2+f^2-c}}$

$r_1=\sqrt{1^2+2^2+20}=\sqrt{25}=5$

$r_2=\sqrt{3^2+1^2+90}=\sqrt{100}=10$

$C_1C_2=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$=\sqrt{(-3-1)^2+(-1-2)^2}$

$=\sqrt{(-4)^2+(-3)^2}=\sqrt{25}=5$

$\mathbf{C_1C_2=|r_1-r_2|}$

the circles touch each othe internally.

the point of contact p divides C_1C_2 externally in the ratio

$r_1:r_2=5:10=m:n=1:2$

$P=\left[\frac{mx_2-nx_1}{m-n},\frac{my_2-ny_1}{m-n}\right]$

$=\left(\frac{1(-3)-2(1)}{1-2},\frac{1(-1)-2(2)}{1-2}\right)$

$=\left(\frac{-3-2}{-1},\frac{-1-4}{-1}\right)$

$=(5,5)$

2. Show that the circles $x^2+y^2-8x-2y+8=0$, $x^2+y^2-2x+6y+6=0$ Touch each other. Find the point of contact.

Sol: eq"n s of the given circles

$S \equiv x^2+y^2-8x-2y+8=0 \ldots\ldots(1)$

$S' \equiv x^2+y^2-2y+6y+6=0 \ldots\ldots(2)$

centres $(-\mathbf{g},-\mathbf{f})$: $C_1\ (4,1),\ C_2(1,-3)$

radius(r) $=\sqrt{\mathbf{g^2+f^2-c}}$

$r_1=\sqrt{4^2+1^2-8}=\sqrt{17-8}=\sqrt{9}=3$

$r_2=\sqrt{1^2+3^2-6}=\sqrt{10-6}=\sqrt{4}=2$

$C_1C_2=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$=\sqrt{(1-4)^2+(-3-1)^2}$

$=\sqrt{(-3)^2+(-4)^2}=\sqrt{25}=5$

$\mathbf{C_1C_2=r_1+r_2}$

the circles touch each othe externally.

the point of contact p divides C_1C_2 Internally in the ratio

$r_1:r_2=3:2=m:n$

$P=\left[\frac{mx_2+nx_1}{m+n},\frac{my_2+ny_1}{m+n}\right]$

$=\left(\frac{3(1)+2(4)}{3+2},\frac{3(-3)+2(1)}{3+2}\right)$

$=\left(\frac{3+8}{5},\frac{-9+2}{5}\right)$

$=\left(\frac{11}{5},\frac{-7}{5}\right)$

3. Show that the circles $x^2+y^2+2ax+c=0$ and $x^2+y^2+2by+c=0$ touch each other if $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{c}$

Sol: Given eq''ns of the circles

$x^2+y^2+2ax+c=0$ ..... (1)

⇨$C_1(-a,0)$and $r_1=\sqrt{a^2-c}$

$x^2+y^2+2by+c=0$.... (2)

⇨$C_1(0,-b)$and $r_1=\sqrt{b^2-c}$

$C_1C_2=\sqrt{(0+a)^2+(-b-0)^2}=\sqrt{a^2+b^2}$

Given circles touch each other $|r_1+r_2|=C_1C_2$

⇨ $\sqrt{a^2-c}+\sqrt{b^2-c}=\sqrt{a^2+b^2}$

S. O. B

$\left[\sqrt{a^2-c}+\sqrt{b^2-c}\right]^2=a^2+b^2$

⇨ $\cancel{a^2}-c+\cancel{b^2}-c+2\sqrt{a^2-c}\sqrt{b^2-c}=\cancel{a^2}+\cancel{b^2}$

⇨ $\cancel{2}\sqrt{a^2-c}\sqrt{b^2-c}=\cancel{2}c$

⇨ $\sqrt{a^2-c}\sqrt{b^2-c}=c$ S.O.B ⇨ $(a^2-c)(b^2-c)=c^2$

⇨ $a^2b^2-a^2c-b^2c+\cancel{c^2}=\cancel{c^2}$

⇨ $a^2b^2-a^2c-b^2c=0$

⇨ $a^2b^2=c(a^2+b^2)$

⇨ $\frac{1}{c}=\frac{a^2+b^2}{a^2b^2}$ ⇨ $\frac{1}{c}=\frac{a^2}{a^2b^2}+\frac{b^2}{a^2b^2}$

$\therefore \frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{c}$

4. If the two circles $x^2+y^2+2gx+2fy=0$ and $x^2+y^2+2g'x+2f'y=0$ Touch each other then show that $f'g=fg'$.

Sol: Sol: Given eq''n of the circles

$S\equiv x^2+y^2+2gx+2fy=0$ ... (1)

centre $C_1(-g,-f)$

$S'\equiv x^2+y^2+2g'x+2f'y=0$ ... (2)

centre $C_2(-g',-f')$

above circles passes through $(0,0)$

If the circles touch each other, then

OC_1C_2 are collinear ⇨ Area of $OC_1C_2=0$

$O(0,0)$, $C_1(-g,-f)$ and $C_2(-g',-f')$

Area $=\frac{1}{2}|x_1y_2-x_2y_1|=0$⇨ $|(-g)(-f')-(-g')(-f)|=0$

⇨ $gf'-g'f=0$ $\therefore$ $\mathbf{f'g=fg'}$.

5. Prove that the radical axis of the circles $x^2+y^2+2gx+2fy+c=0$ *and* $x^2+y^2+2g'x+2f'y+c'=0$ is the diameter of the latter circle if $2g'(g-g')+2f'(f-f')=c-c'$.

Sol: Given eq''n of the circles

$S\equiv x^2+y^2+2gx+2fy+c=0$ ... (1)

$S'\equiv x^2+y^2+2g'x+2f'y+c'=0$ ... (2)

Radical eq''n of the given circles is S-S'=0

⇨$2x(g-g')+2y(f-f')+(c-c')=0$... (3)

Eq''n (3) becomes diameter of eq''n (2),

if Centre (-g', -f)'lies on (3)

⇨$2(-g')(g-g')+2(-f')(f-f')+(c-c')=0$

$\therefore$ $2g'(g-g')+2f'(f-f')=c-c'$.

6. If $x + y = 3$ *is the equation of the* chord of the circle $x^2 + y^2 - 2x + 4y - 8 = 0$, find the equation of the circle having AB as diameter.

Sol:

Given eq''n of the circle

$S \equiv x^2 + y^2 - 2x + 4y - 8 = 0 \ldots (1)$

$line L \equiv x + y - 3 = 0 \ldots . (2)$

Eq''n of the circle passing through the point of intersection of S=0 and L=0 is $S + \lambda L = 0$

$\Rightarrow (x^2 + y^2 - 2x + 4y - 8) + \lambda(x + y - 3) = 0 \ldots (3)$

$\Rightarrow (x^2 + y^2 + x(\lambda - 2) + (\lambda + 4)y - (3\lambda + 8) = 0.$

$centre \left[\frac{(2-\lambda)}{2}, -\frac{(4+\lambda)}{2}\right]$

$if\ AB\ a\ is\ diameter\ of\ circle\ (1), then\ centre$

$lies\ on\ x + y - 3 = 0$

$\Rightarrow \left(\frac{2-\lambda}{2}\right) - \frac{(4+\lambda)}{2} = 3$

$\Rightarrow -2 - 2\lambda = 6$

$\Rightarrow -2\lambda = 8 \Rightarrow \lambda = -4$

The required eq''n of the circle is from (3)

$(x^2 + y^2 + x(-4 - 2) + (-4 + 4)y - (3(-4) + 8) = 0$

$\therefore x^2 + y^2 - 6x + 4 = 0$

7. If the straight line $2x + 3y = 1$ intersects the circle $x^2 + y^2 = 4$ at the points A and B, find the equation of the circle having AB as diameter.

Sol: Given eq''n of the circle $S \equiv x^2 + y^2 = 4 \ldots (1)$

$line L \equiv 2x + 3y - 1 = 0 \ldots . (2)$

Eq''n of the circle passing through the point of intersection of S=0 and L=0 is $S + \lambda L = 0$

$\Rightarrow (x^2 + y^2 - 4) + \lambda(2x + 3y - 1) = 0 \ldots (3)$

$\Rightarrow (x^2 + y^2 + (2\lambda)x + (3\lambda)y - (\lambda + 4) = 0.$

$centre \left[-\lambda, -\frac{3\lambda}{2}\right]$

$if\ AB\ a\ is\ diameter\ of\ circle\ (1), then\ centre$

$lies\ on\ 2x + 3y - 1 = 0 \Rightarrow 2(-\lambda) + 3(3\lambda/2) = 1$

$\Rightarrow -4\lambda - 9\lambda = 2 \Rightarrow -13\lambda = 2 \Rightarrow \lambda = -\frac{2}{13}$

The required eq''n of the circle is from (3)

$(x^2 + y^2 - 4) - \frac{2}{13}(2x + 3y - 1) = 0$

$13(x^2 + y^2) - 52 - 4x - 6y + 2 = 0$

$\therefore \mathbf{13(x^2 + y^2) - 4x - 6y - 50 = 0}$

8. Find the equation of the circle passing through the point of intersection of the circles $x^2 + y^2 - 8x - 6y + 21 = 0$ and $x^2 + y^2 - 2x - 15 = 0$ *and* $(1, 2)$.

Sol:

Given eq''n of the circle

$S \equiv x^2 + y^2 - 8x - 6y + 21 = 0 \ldots (1)$

$S' \equiv x^2 + y^2 - 2y - 15 = 0 \ldots . (2)$

Eq''n of the circle passing through the point of intersection of S=0 and S'=0 is $S + \lambda S' = 0$

$\Rightarrow \mathbf{(x^2 + y^2 - 8x - 6y + 21) + \lambda(x^2 + y^2 - 2x - 15) = 0 \ldots (3)}$

Above eq''n passing through (1, 2)

$\Rightarrow (1^2 + 2^2 - 8(1) - 6(2) + 21) + \lambda(1^2 + 2^2 - 2(1) - 15) = 0$

$\Rightarrow (1 + 4 - 8 - 12 + 21) + \lambda(1 + 4 - 2 - 15) = 0$

$\Rightarrow (6) + \lambda(-12) = 0$

$\Rightarrow 6 = 12\lambda$

$\Rightarrow \lambda = \frac{1}{2}$ Sub in (3)

$\Rightarrow \mathbf{(x^2 + y^2 - 8x - 6y + 21) + \frac{1}{2}(x^2 + y^2 - 2x - 15) = 0}$

$\Rightarrow \mathbf{(2x^2 + 2y^2 - 16x - 12y + 42) + (x^2 + y^2 - 2x - 15) = 0}$

$\Rightarrow \mathbf{(3x^2 + 3y^2 - 18x - 12y + 27) = 0\ (\div by\ 3)}$

$\mathbf{x^2 + y^2 - 6x - 4y + 9 = 0}$.

9. Find the equation of the circle whose diameter is the common chord of the circles $x^2+y^2+2x+3y+1=0$ and $x^2+y^2+4x+3y+2=0$.

Sol: $2x^2+2y^2+2x+6y+1=0$.$(S+\lambda S'=0)$

10. If the angle between the circles $x^2+y^2-12x-6y+41=0, x^2+y^2+kx+6y-59=0, find\ k.$

Sol: Given circles $S=x^2+y^2-12x-6y+41=0$ ... (1)

$S'=x^2+y^2+kx+6y-59=0$ ... ... (2)

$\theta=45° \Rightarrow cos45=\frac{1}{\sqrt{2}}$

$g=-6; f=-3; c=41 \qquad g'=\frac{k}{2}; f'=3; c'=-59$

$r_1=\sqrt{g^2+f^2-c}=\sqrt{6^2+3^2-41}=2$

$r_2=\sqrt{\left(\frac{k}{2}\right)^2+(3)^2+59}=\sqrt{\frac{k^2+36+236}{4}}=\frac{\sqrt{k^2+272}}{2}$

$cos\theta=\frac{|c+c'-2gg'-2ff'|}{2r_1r_2}$

$\Rightarrow cos45°=\frac{\left|41-59-(6)\left(\frac{k}{2}\right)-2(-3)(3)\right|}{2.2.\frac{\sqrt{k^2+272}}{2}}$

$\Rightarrow \frac{1}{\sqrt{2}}=\frac{|-18+6k+18|}{2.\sqrt{k^2+272}} \Rightarrow \frac{1}{\sqrt{2}}=\frac{|6k|}{2.\sqrt{k^2+272}}$

$\Rightarrow \frac{1}{\sqrt{2}}=\frac{|3k|}{\sqrt{k^2+272}}$ S.O.B$\Rightarrow \frac{1}{2}=\frac{9k^2}{k^2+272}$

$\Rightarrow k^2+272=18k^2$

$\Rightarrow 17k^2=272 \Rightarrow k^2=\frac{272}{17}$

$\Rightarrow k^2=16 \quad \therefore k=\pm 4$

11. Find the radical centre of the circles $x^2+y^2+4x-7=0$, $2x^2+2y^2+3x+5y-9=0$ and $x^2+y^2+y=0$.

Sol:

Given circles

$S\equiv x^2+y^2+4x-7=0$ .....(1)

$S'\equiv 2x^2+2y^2+3x+5y-9=0$ ... ... (2)

$S''\equiv x^2+y^2+y=0$ .....(3)

$radical\ \ axis\ of$ (1)& (2)

$2x^2+2y^2+8x-14=0$

$2x^2+2y^2+3x+5y-9=0$

$5x-5y-5=0$..... (4)

$radical\ \ axis\ of$ (2)& (3)

$2x^2+2y^2+3x+5y-9=0$

$2x^2+2y^2+\ 0+2y=0$

$3x+3y-9=0$.... (5)

Solving (4) & (5)

5	-5	-5	5
3	**3**	**-9**	**3**

$$(x,y)=\left[\frac{45+15}{15+15},\frac{-15+45}{15+15}\right]=\left[\frac{60}{30},\frac{30}{30}\right]=(2,1)$$

12. Find the equation of the circle which cuts the circles
$x^2 + y^2 + 2x + 4y + 1 = 0$
$2x^2 + 2y^2 + 6x + 8y - 3 = 0$ *and*
$x^2 + y^2 - 2x + 6y - 3 = 0$ *orthogonally.*

Sol:
Given circles
$S \equiv x^2 + y^2 + 2x + 4y + 1 = 0 \ldots\ldots (1)$
$S' \equiv 2x^2 + 2y^2 + 6x + 8y - 3 = 0 \ldots\ldots (2)$
$S'' \equiv x^2 + y^2 - 2x + 6y - 3 = 0 \ldots\ldots (3)$

radical axis of (1)& (2)
$2x^2 + 2y^2 + 4x + 8y + 2 = 0$
$2x^2 + 2y^2 + 6x + 8y - 3 = 0$
$-2x + 0 + 5 = 0 \ldots\ldots (4)$

radical axis of (2)& (3)
$2x^2 + 2y^2 + 6x + 8y - 3 = 0$
$2x^2 + 2y^2 - 4x + 12y - 6 = 0$
$10x - 4y + 3 = 0 \ldots (5)$

Solving (4) & (5)

-2	0	5	-2
10	-4	3	10

$(x, y) = \left[\frac{0+2}{8-0}, \frac{50+6}{8-0}\right]$
$= \left[\frac{20}{8}, \frac{56}{8}\right] = \left(\frac{5}{2}, 7\right)$
$radius = \sqrt{S_{11}}$ *from* $\left(\frac{5}{2}, 7\right)$ to S=0

$r^2 = \sqrt{S_{11}}^2 = \left(\left(\frac{5}{2}\right)^2 + 7^2 + 2\left(\frac{5}{2}\right) + 4(7) + 2\right)$
$r^2 = \left(\frac{25}{4} + 83\right) = \frac{357}{4}$
eq''nof the circle with centre
$(a, b) = \left(\frac{5}{2}, 7\right)$ *and radius r is*
$\left[x - \frac{5}{2}\right]^2 + [y - 7]^2 = \frac{357}{4}$
$\Rightarrow x^2 - 5x + \frac{25}{4} + y^2 + 49 - 14y = \frac{357}{4}$
$\Rightarrow 4x^2 - 20x + 25 + 4y^2 + 196 - 56y - 357 = 0$
$\Rightarrow 4(x^2 + y^2 - 5x - 14y - 34) = 0$

$\therefore x^2 + y^2 - 5x - 14y - 34 = 0$

13. Find the eq''n of the circle which cuts orthogonally the circle $x^2 + y^2 - 4x + 2y - 7 = 0$, at Centre at (2, 3).

Sol: Given circle $\boldsymbol{x^2 + y^2 - 4x + 2y - 7 = 0 \ldots\ldots (1)}$

Let the required equation of the circle with Centre at

(2, 3)= (-g, -f) is $\boldsymbol{x^2 + y^2 - 4x - 6y + c = 0 \ldots\ldots (2)}$

Equation (1) and (2) are orthogonal

$\boldsymbol{2gg' + 2ff; = c + c'}$

$\boldsymbol{\Rightarrow 2(-2)(-2) + 2(1)(-3) = (-7) + (c)}$

$\boldsymbol{\Rightarrow 8 - 6 = (-7) + (c) \Rightarrow c = 9}$

$\boldsymbol{\therefore\ x^2 + y^2 - 4x - 6y + 9 = 0}$

14. Find the eq"n of the circle which passes through the point (0, -3) and intersects the circles $x^2+y^2-6x+3y+5=0$, $x^2+y^2-x-7y=0$ Orthogonally.

Sol: let the required circle

$\boldsymbol{S \equiv x^2+y^2+2gx+2fy+c=0}$ …..(∗)

$\boldsymbol{S' \equiv x^2+y^2-6x+3y+5=0}$,

$\boldsymbol{S'' \equiv x^2+y^2-x-7y=0}$

$(0,-3)$ *lies on* (∗) ⇨ $0+9-6f+c=0$

⇨ $\boldsymbol{0-6f+c+9=0}$ …..(**1**)

Given $S=0$ *and* $S'=0$ Are orthogonal

$\boldsymbol{2gg'+2ff'=c+c'}$ Condition for orthogonal

⇨ $2g(-3)+2f\left(\frac{3}{2}\right)=c+5$

⇨ $\boldsymbol{-6g+3f-c-5=0}$ ……(**2**)

Given $S=0$ *and* $S''=0$ Are orthogonal

$\boldsymbol{2gg'+2ff'=c+c'}$ Condition for orthogonal

⇨ $2g\left(-\frac{1}{2}\right)+2f\left(-\frac{7}{2}\right)=c+0$

⇨ $\boldsymbol{-g-7f-c=0}$ ……(**3**)

Eq"n (1) + (2)

$$\begin{array}{r} 0-6f+c+9=0 \\ -6g+3f-c-5=0 \\ \hline \boldsymbol{-6g-3f+4=0} \end{array}$$ ……(**4**)

Eq"n (1) + (3)

$$\begin{array}{r} 0-6f+c+9=0 \\ -g-7f-c+0=0 \\ \hline \boldsymbol{-g-13f+9=0} \end{array}$$ ……(**5**)

Solving (4) & (5)

6	3	-4	6
1	13	-9	1

$(x,y)=\left[\frac{-27}{78-3},\frac{-4+54}{78-3}\right]=\left[\frac{25}{75},\frac{50}{75}\right]$

$\left(\frac{1}{3},\frac{2}{3}\right)$ Sub in (1)⇨$-6\left(\frac{2}{3}\right)+9+c=0$

⇨$c=-9+4=-5$

$\therefore\ x^2+y^2+2\left(\frac{1}{3}\right)x+2\left(\frac{2}{3}\right)y-5=0$

$3x^2+3y^2+2x+3y-15=0$

15. Find the eq"n of the circle which cuts the circles $x^2+y^2-4x-6y+11=0$, $x^2+y^2-10x-4y+21=0$ Orthogonally, and has the diameter along the st line $2x+3y=7$.

Sol: Let the required circle

$\boldsymbol{S \equiv x^2+y^2+2gx+2fy+c=0}$ …..(∗)

$\boldsymbol{S \equiv x^2+y^2-4x-6y+11=0}$,

$\boldsymbol{S'' \equiv x^2+y^2-10x-4y+21=0}$

given line $\boldsymbol{L \equiv 2x+3y-7=0}$

$(-g,-f)$ *lies on* (L) ⇨ $-2g-3f-7=0$

⇨ $\boldsymbol{2g+3f+7=0}$ …..(**1**)

Given $S=0$ *and* $S'=0$ are orthogonal

$\boldsymbol{2gg'+2ff'=c+c'}$ condition for orthogonal

⇨ $2g(-2)+2f(-3)=c+11$

⇨ $\boldsymbol{-4g-6f-c-11=0}$ ……(**2**)

Given $S=0$ *and* $S''=0$ Are orthogonal

$\boldsymbol{2gg'+2ff'=c+c'}$ Condition for orthogonal

⇨ $2g(-5)+2f(-2)=c+21$

$\Rightarrow \mathbf{-10g - 4f - c - 21 = 0} \ldots\ldots (\mathbf{3})$

$Eq''n\ (2) - (3)$

$-4g - 6f - c - 11 = 0$

$-10g - 4f - c - 21 = 0$

$\mathbf{6g - 2f + 10 = 0} \ldots\ldots\ldots\ldots (\mathbf{4})$

Solving (1) & (4)

2	3	7	2
6	-2	10	6

$(x, y) = \left[\frac{30+14}{-4-1}, \frac{42-20}{-4-18}\right] = \left[\frac{44}{-2}, \frac{22}{-2}\right]$

$(-2, -1)$ Sub in (2) $\Rightarrow -4(-2) - 6(-1) = c + 11$

$\Rightarrow c = 14 - 11 = 3$

$\therefore\ x^2 + y^2 + 2(-2)x + 2(-1)y + 3 = 0$

$x^2 + y^2 - 4x - 2y + 3 = 0$

16. Find the eq''n of the circle which passes through the point (2, 0)(0, 2) and intersects the circles $2x^2 + 2y^2 + 5x - 6y + 4 = 0$, Orthogonally.

Sol: Let the required circle

$S \equiv x^2 + y^2 + 2gx + 2fy + c = 0 \ldots\ldots (*)$

$S' \equiv x^2 + y^2 + \frac{5}{2}x - 3y + 2 = 0,$

$(2, 0) lies\ on\ (*) \Rightarrow 4 + 0 + 4g + 0 + c = 0$

$\Rightarrow \mathbf{4g + 0 + c + 4 = 0} \ldots\ldots (\mathbf{1})$

$(0, 2) lies\ on\ (*) \Rightarrow 0 + 4 + 0 + 4f + c = 0$

$\Rightarrow \mathbf{0 + 4f + c + 4 = 0} \ldots\ldots (\mathbf{2})$

$Given\ S = 0\ and\ S' = 0$ Are orthogonal

$\mathbf{2gg' + 2ff' = c + c'}$ Condition for orthogonal

$\Rightarrow 2g\left(\frac{5}{4}\right) + 2f\left(-\frac{3}{2}\right) = c + 2$

$\Rightarrow \mathbf{\frac{5}{2}g - 3f - c - 2 = 0} \ldots\ldots (\mathbf{3})$

$Eq''n\ (1) + (3)$

$4g + 0 + c + 4 = 0$

$\frac{5}{2}g - 3f - c - 2 = 0$

$\mathbf{\frac{13}{2}g - 3f + 2 = 0 \Rightarrow 13g - 6f + 4 = 0} \ldots\ldots (\mathbf{4})$

$Eq''n\ (2) + (3)$

$0 + 4f + c + 4 = 0$

$\frac{5}{2}g - 3f - c - 2 = 0$

$\mathbf{\frac{5}{2}g + f + 2 = 0 \Rightarrow 5g + 2f + 4 = 0} \ldots\ldots (\mathbf{4})$

Ans: $\mathbf{7x^2 + 7y^2 - 8x - 8y - 12 = 0,}$

17. Find the eq"n and length of the common chord of the two circles $x^2 + y^2 + 3x + 5y + 4 = 0$,
$x^2 + y^2 + 5x + 3y + 4 = 0$.

Sol: Given circles

$S \equiv x^2 + y^2 + 3x + 5y + 4 = 0$,

$S' \equiv x^2 + y^2 + 5x + 3y + 4 = 0$.

$Equation\ of\ common\ chord\ is\ S - S' = 0$

$\Rightarrow -2x + 2y = 0 \Rightarrow x - y = 0 \ldots\ldots (1)$

$Centre\ \left(-\frac{3}{2}, -\frac{5}{2}\right)\ and$

$radius(r) = \sqrt{\left(\frac{3}{2}\right)^2 + \left(\frac{5}{2}\right)^2 - 4} = \sqrt{\frac{9+25-}{4}}$

$\sqrt{\frac{18}{4}} = \sqrt{\frac{9}{2}}\ \ of\ S = 0$

d $is\ \perp^{lar}\ distance\ from\ \left(-\frac{3}{2}, -\frac{5}{2}\right)\ to\ (1)$

$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}$

$$\boldsymbol{d} = \frac{\left|-\frac{3}{2}+\frac{5}{2}\right|}{\sqrt{1^2+1^2}} = \frac{\left|\frac{2}{2}\right|}{\sqrt{2}} = \frac{1}{\sqrt{2}}$$

Length of chord is $=2\sqrt{r^2 - d^2}$

$= 2\sqrt{\frac{9}{2} - \frac{1}{2}}$

$= 2\sqrt{\frac{8}{2}}$

$= 2.\sqrt{4}$

$= 2.2$

$= 4$

AIMS TUTORIAL MATHEMAITCS -2B

Q.No: 12 SYSTEM OF CIRCLE