1. **If the abscissa of points A, B are the roots of the equation $x^2+2ax-b^2=0$ and the ordinates of A, B are the roots of $x^2+2px-q^2=0$, then find the equation of a circle for which AB as a diameter.**

Sol: Let $A(x_1, y_1)$ and $B(x_2, y_2)$

Given that

x_1, x_2 be the roots of $x^2+2ax-b^2=0$

$\Rightarrow (x-x_1)(x-x_2)=x^2+2ax-b^2$

And

Given that

y_1, y_2 be the roots of $y^2+2py-q^2=0$

$\Rightarrow (y-y_1)(y-y_2)=y^2+2py-q^2$

Equation of the circle with AB as a diameter is

$\mathbf{(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0}$

$\Rightarrow (x^2+2ax-b^2)+(y^2+2py-q^2)=0$

$\Rightarrow x^2+y^2+2ax+2py-b^2-q^2=0$

Is the required eq''n of the circle.

2. **If a point P is moving such that the lengths of tangents from P to the circle $x^2+y^2-6x-4y-12=0$, $x^2+y^2+6x+18y+26=0$ are in the ratio 2:3 the find the equation of the locus of p.**

Sol: let $P(x_1, y_1)$ be any point on the locus

Given that $\frac{\sqrt{S_{11}}}{\sqrt{S'_{11}}}=\frac{2}{3} \Rightarrow 3\sqrt{S_{11}}=2\sqrt{S'_{11}}$

$\Rightarrow 3\sqrt{{x_1}^2+{y_1}^2-6x_1-4y_1-12}$

$=2\sqrt{{x_1}^2+{y_1}^2+6x_1+18y_1+26}$ S.O.B

$\Rightarrow 9({x_1}^2+{y_1}^2-6x_1-4y_1-12)$

$=4({x_1}^2+{y_1}^2+6x_1+18y_1+26)$

$\Rightarrow (9{x_1}^2+9{y_1}^2-54x_1-36y_1-108)-4{x_1}^2-4{y_1}^2-24x_1-72y_1-104=0$

$\Rightarrow 5{x_1}^2+5{y_1}^2-78x_1-108y_1-212=0$

$\therefore$ the equation of locus of p is

$5\mathbf{x^2+5y^2-78x-108y-212=0}$

3. **Find the pole of $3x+4y-45=0$ with respect to $x^2+y^2-6x-8y+5=0$**

sol: given equation of the circle

$x^2+y^2-6x-8y+5=0$ .....(1)

Centre (3, 4) and $r=\sqrt{(-3)^2+(-4)^2-5}=\sqrt{20}$

Given line $3x+4y-45=0$ here $l=3, m=4$ & $n=-45$

The pole $=\left(-g+\frac{lr^2}{lg+mf-n}, -f+\frac{mr^2}{lg+mf-n}\right)$

$=\left(3+\frac{3(20)}{3(-3)+4(-4)+45}, 4+\frac{4(20)}{3(-3)+4(-4)+45}\right)$

$=\left(3+\frac{3(20)}{20}, 4+\frac{4(20)}{20}\right)=(3+3, 4+4)=(6.8)$

Find the pole of $x+y+2=0$ with respect to $x^2+y^2-4x-6y-12=0$

4. **Find the value of k if $kx+3y-1=0$, $2x+y+5=0$ are conjugate lines with respect to the circle $x^2+y^2-2x-4y-4=0$.**

Sol: Given equation of the circle

$x^2+y^2-2x-4y-4=0$ .....(1)

Centre (1, 2) and $r=\sqrt{(1)^2+(2)^2+4}=\sqrt{9}=3$

Given line $2x+y+5=0$ here $l=2, m=1$ and $n=5$

The pole $=\left(-g+\frac{lr^2}{lg+mf-n}, -f+\frac{mr^2}{lg+mf-n}\right)$

$=\left(1+\frac{2(9)}{2(-1)+1(-2)-5}, 2+\frac{1(9)}{2(-1)+1(-2)-5}\right)$

$=\left(1+\frac{2(9)}{-9}, 2+\frac{1(9)}{-9}\right)$

$=(1-2, 2-1)=(-1, 1)$

$(-1, 1)$ lies on $kx+3y-1=0$

$\Rightarrow -k+3-1=0 \Rightarrow k=2$

5. **Find the equations of the tangents to the circle $x^2+y^2-4x+6y-12=0$ which are parallel to $x+y-8=0$.**

Sol: given equation of the circle

$\mathbf{S \equiv x^2+y^2-4x+6y-12=0}$

Centre (2, -3) and radius (r) $=\sqrt{(-2)^2+(3)^2+12}$

$=\sqrt{4+9+12}=\sqrt{25}=5$

The given line $x+y-8=0$ .....(1)

Slope(m)$=-\frac{a}{b}=-\frac{1}{1}=-1$

Eq''n of tangent to S=0 & parallel to (1)

is $\mathbf{(y-y_1)=m(x-x_1)\pm r\sqrt{1+m^2}}$

$\Rightarrow (y+3)=-1(x-2)\pm 5\sqrt{1+1}$

$\Rightarrow x-2+y+3\pm 5\sqrt{2}=0$

Hence required eq''n of tangents are $x+y+1\pm 5\sqrt{2}=0$.

6. **Find the equations of the tangents to the circle $x^2+y^2+2x-2y-3=0$ which are perpendicular to $3x-y+4=0$.**

Sol: given equation of the circle

$x^2+y^2+2x-2y-3=0$ .....(1)

Centre $(-1, 1)$ and radius (r) $=\sqrt{(1)^2+(-1)^2+3}=\sqrt{5}$

The given line $3x-y+4=0$ .....(2)

Slope(m)$=-\frac{a}{b}=-\frac{3}{-1}=3$ and $\perp^{lar}$ slope(m) $=-\frac{1}{3} \Rightarrow m^2=\frac{1}{9}$

Eq''n of tangent to S=0 & $\perp^{lar}$ to (2)

is $\mathbf{(y-y_1)=m(x-x_1)\pm r\sqrt{1+m^2}}$

$\Rightarrow (y-1)=-\frac{1}{3}(x+1)\pm\sqrt{5}\sqrt{1+\left(\frac{1}{3}\right)^2}$

$\Rightarrow (y-1)=-\frac{(x+1)}{3}\pm\frac{\sqrt{5}\sqrt{10}}{3}$

$\Rightarrow 3(y-1)=-(x+1)\pm 5\sqrt{2}$

$\Rightarrow x+1+3y-3\pm 5\sqrt{2}=0$

Hence required eq''n of tangents are

$x+3y-2\pm 5\sqrt{2}=0$.

7. **Find the equation of the tangent to $x^2+y^2-2x+4y=0$ at $(3,-1)$. Also find the equation of tangent parallel to it.**

Sol: given equation of the circle

$x^2+y^2-2x+4y=0$ .....(1)

Centre $(1,-2)$ and radius $(r) = \sqrt{(-1)^2+(2)^2+0}=\sqrt{5}$

The equation of tangent at $(3,-1)$ is

$\mathbf{S_1 = xx_1+yy_1+g(x+x_1)+f(y+y_1)+c=0}$

$\Rightarrow x(3)+y(-1)-1(x+3)+2(y-1)=0$

$\Rightarrow 3x-y-x-3+2y-2=0$

$\Rightarrow 2x+y-5=0$ ....(2)

here slope $(m)=-2$

Required eq''n of the tangent to (1) and it is parallel to (2) is

$(y-y_1) = m(x-x_1)\pm r\sqrt{1+m^2}$

$\Rightarrow (y+2)=-2(x-1)\pm\sqrt{5}\sqrt{1+(-2)^2}$

$\Rightarrow (y+2)=-2(x-1)\pm\sqrt{5}\sqrt{5}$

$\Rightarrow (y+2)=-2(x-1)\pm 5$

$\Rightarrow y+2=-2x+2\pm 5$

$\therefore 2x+y\pm 5=0.$

8. **Show that the tangent at $(-1,2)$ of the circle $x^2+y^2-4x-8y+7=0$ touches the circle $x^2+y^2+4x+6y=0$ and also find its point of contact.**

Sol: equation of the tangent at $(-1,2)$ to the circle $x^2+y^2-4x-8y+7=0$ is

$\mathbf{S_1 = xx_1+yy_1+\frac{2g(x+x_1)}{2}+\frac{2f(y+y_1)}{2}+c=0}$

$\Rightarrow x(-1)+y(2)-2(x-1)-4(y+2)+7=0$

$\Rightarrow -3x-2y+1=0$

$\Rightarrow 3x+2y-1=0$ ....(1)

For the circle $\mathbf{x^2+y^2+4x+6y=0}$ centre $(-2,-3)$,

$r=\sqrt{(2)^2+(3)^2-0}=\sqrt{13}$

$\perp$ Distance from centre $(-2,-3)$ to given line (1)

$=\frac{|3(-2)+2(-3)-1|}{\sqrt{(3)^2+(2)^2}}=\frac{|-6-6-1|}{\sqrt{13}}=\frac{13}{\sqrt{13}} = \sqrt{13}$ so the line (1) also touches the 2nd circle.

let (h,k) be the required point of contact.

so it is the foot of the $\perp$ from the centre $(-2,-3)$

$\frac{h-x_1}{a}=\frac{k-y_1}{b}=-\frac{(ax_1+by_1+c)}{a^2+b^2}$

$\Rightarrow \frac{h+2}{3}=\frac{k+3}{2}=-\frac{[3(-2)+2(-3)-1)]}{3^2+2^2}$

$\Rightarrow \frac{h+2}{3}=\frac{k+3}{2}=-\frac{(-13)}{13}=1$

$\Rightarrow \frac{h+2}{3}=1$ and $\Rightarrow \frac{k+3}{2}=1$

$h+2=3$ and $k+3=2$

$h=3-2$ and $k=2-3$

$h=1, k=-1$

Coordinate of point of contact $= (1,-1.)$

9. **Find the equations of normal to the circle $x^2+y^2-4x+6y+11=0$ at $(3,2)$ also find the other point where normal meets the circle.**

Sol: given equation of the circle

$x^2+y^2-4x-6y+11=0$ .....(1)

Centre C $(2,3) = (-g,-f)$

Given point A $(3,2) = (x_1,y_1)$

The equation of the normal is

$\mathbf{(x-x_1)(y_1+f)-(y-y_1)(x_1+g)=0}$

$\Rightarrow (x-3)(2-3)-(y-2)(3-2)=0$

$\Rightarrow -x+3-y+2=0 \Rightarrow x+y-5=0.$

centre of the circle is mid point of A and B

$\left[\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right]=\mathbf{(2,3)}$

$\Rightarrow \left[\frac{3+a}{2},\frac{2+b}{2}\right]=(2,3)$

$\Rightarrow \frac{3+a}{2}=2$ and $\frac{2+b}{2}=3$

$\Rightarrow 3+a=4$ and $2+b=6$

$\Rightarrow a=4-3$ and $b=6-2$

B $(a,b)=(1,4)$

10. **Find the mid point of the chord intercepted by $x^2+y^2-2x-10y+1=0$ on the line $x-2y+7=0$, Also find the length of the chord.**

Sol: circle $\mathbf{x^2+y^2-2x-10y+1=0}$ centre $(1,5)$,

$r=\sqrt{(1)^2+(5)^2-1}=5$

$\perp$ Distance from centre $(1,5)$ to given line $\mathbf{x-2y+7=0}$

$=\frac{|1(1)-2(5)+7|}{\sqrt{(1)^2+(2)^2}}=\frac{|1-10+7|}{\sqrt{5}}=\frac{2}{\sqrt{5}}$

length of chord intercepted by the circle is

$\mathbf{2\sqrt{r^2-d^2}} = 2\sqrt{25-\frac{4}{5}}=2\sqrt{\frac{125-4}{5}}$

$=2\sqrt{\frac{121}{5}}=\frac{2(11)}{\sqrt{5}}=\frac{22}{\sqrt{5}}$units

let (h,k) be the required mid point.

so it is the foot of the $\perp$ from the centre $(1,5)$

$\frac{h-x_1}{a}=\frac{k-y_1}{b}=-\frac{(ax_1+by_1+c)}{a^2+b^2}$

$\Rightarrow \frac{h-1}{1}=\frac{k-5}{-2}=-\frac{[1(1)-2(5)+7]}{1^2+2^2}$

$\Rightarrow \frac{h-1}{1}=\frac{k-5}{-2}=-\frac{(-2)}{5}=\frac{2}{5}$

$\Rightarrow \frac{h-1}{1}=\frac{2}{5}$ and $\Rightarrow \frac{k-5}{-2}=\frac{2}{5}$

$5h-5=2$ and $5k-25=-4$

$5h=2+5=7$ and $5k=-4+25$

$h=\frac{7}{5}, k=\frac{21}{5}$

11. **Find the length of chord intercepted by the circle**
$x^2+y^2-8x-2y-8=0$ on the line $x+y+1=0$.

Sol: given equation of the circle

$x^2+y^2-8x-2y-8=0$ .....(1)

Centre $(4,1)$ and $r=\sqrt{(-4)^2+(-1)^2+8}=\sqrt{25}=5$

Given line $x+y+1=0$

$\perp$ Distance from centre $(-2,-3)$ to given line (1)

$=\frac{|1(4)+1(1)+1|}{\sqrt{(1)^2+(1)^2}}$ $\qquad \frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$

$=\frac{|6|}{\sqrt{2}}=3\sqrt{2}=\sqrt{18}$

length of chord intercepted by the circle is

$2\sqrt{r^2-d^2}\ =\ 2\sqrt{25-18}=2\sqrt{7}$units

Find the length of chord intercepted by the circle

$x^2+y^2-x+3y-2=0$ on the line $y=x-3$. $\boxed{\text{Ans: } 2\sqrt{26}}$

12. **Find the equation of the circle with centre $(-2,3)$cutting a chord length 2 units on $3x+4y+4=0$.**

Sol: given centre C $(-2,3)$

Given equation of the chord $3x+4y+4=0$ ....(1)

$d=\perp$ Distance from centre C $(-2,3)$ to given line (1)

$d\ =\frac{|3(-2)+4(3)+4|}{\sqrt{(3)^2+(4)^2}}=\frac{|-6+12+4|}{\sqrt{25}}=\frac{10}{5}=2$

Given length of chord $2\sqrt{r^2-d^2}=\ 2$

$\Rightarrow\sqrt{r^2-d^2}=\ 1$

$\Rightarrow r^2-d^2=1\ (d=2)$

$\Rightarrow r^2-4=1$

$\therefore\ r^2=5$

Required eq''n of the circle is

$(x-a)^2+(y-b)^2=r^2$

$\Rightarrow(x+2)^2+(y-3)^2=5$

$x^2+y^2+4x-6y+8=0$

13. **Find the equation of pair of tangents drawn from $(0,0)$ to $x^2+y^2+10x+10y+40=0$**

Sol: given equation of the circle

$x^2+y^2+10x+10y+40=0$ .....(1), $P(x_1,y_1)=\ (0,0)$

$S_1=xx_1+yy_1+\frac{2g(x+x_1)}{2}+\frac{2f(y+y_1)}{2}+c=0$

$S_1\equiv x(0)+y(0)+5(x+0)+5(y+0)+40$

$S_1\equiv 5x+5y+40$

$S_{11}\equiv\ 02+02+10(0)\ +10(0)\ +40=40$

eq''nof the pair of tangents $S_1{}^2=SS_{11}$

$(5x+5y+40\ )^2=(\ x2+y2+10x+10y+40)(40)$

$25(x+y+8)^2=(\ x2+y2+10x+10y+40)(40)$

$\Rightarrow 5\{x^2+y^2+64+2xy+16y+16x\}$

$=\{8x^2+8y^2+80x+80y+320\}$

$\Rightarrow\{8x^2+8y^2+80x+80y+320$

$-5x^2-5y^2-320-10xy-80y-80x=0$

$\Rightarrow 3x^2-10xy+3y^2=0$

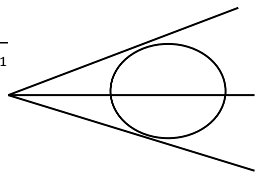
13. **Find the condition that the tangents drawn from the exterior point $(0,0)$to the circle $S=x^2+y^2+2gx+2fy+c=0$ are perpendicular to each other.**

Sol: given equation of the circle

$x^2+y^2+2gx+2fy+c=0$...(1)

$r=\sqrt{g^2+f^2-c}$, length of tangent=$\sqrt{S_{11}}$

if θ is angle $\frac{b}{w}$ the



pair of tangents drawn from

(0, 0) to S=0 is

$S_{11}=0^2+0^2+2g(0)+2f(0)+c=c$

Then $\tan\frac{\theta}{2}=\frac{r}{\sqrt{S_{11}}}$ $\qquad[\theta=90°]$

$\Rightarrow\tan\frac{90°}{2}=\frac{\sqrt{g^2+f^2-c}}{\sqrt{c}}$ $\Rightarrow\tan 45°=\frac{\sqrt{g^2+f^2-c}}{\sqrt{c}}$

$1=\frac{\sqrt{g^2+f^2-c}}{\sqrt{c}}$ S.O.B and cross multiplying$\Rightarrow c=g^2+f^2-c$

$\therefore 2c=g^2+f^2$

14. **Find the area of the triangle formed by the normal at $(3,-4)$to the circle $x^2+y^2-22x-4y+25=0$ with the coordinate axes.**

Sol: given equation of the circle

$x^2+y^2-22x-4y+25=0$ .....(1)

Centre C $(11,2)\ =\ (-g,-f)$

Given point A $(3,-4)\ =\ (x_1,y_1)$

The equation of the normal is

$\mathbf{(x-x_1)(y_1+f)-(y-y_1)(x_1+g)=0}$

$\Rightarrow(x-3)(-4-2)-(y+4)(3-11)=0$

$\Rightarrow 3x-4y-25=0.$

Area of the triangle formed by the normal with the

coordinate axes $=\frac{1}{2}\left|\frac{c^2}{a.b}\right|=\frac{1}{2}\left|\frac{(-25)^2}{3.(-4)}\right|$

$=\frac{625}{24}$ sq. units

15. **Find the inverse point of (-2, 3) w.r.t the circle $x^2+y^2-4x-6y+9=0$.**

Sol: given equation of the circle

$x^2+y^2-4x-6y+9=0$ .....(1)

Centre C $(2,3)\ =(x_1,y_1)$, given point P (-2, 3)= (x_2,y_2)

eq''n of CP is $(y-y_1)=m(x-x_1)$

$\Rightarrow(y-2)=\frac{3-3}{-2-2}(x-2)$

$\Rightarrow y-2=0$ ....(1)

eq''n of polar of $p(-2,3)$ is $S_1=0$

$S_1=xx_1+yy_1+\frac{2g(x+x_1)}{2}+\frac{2f(y+y_1)}{2}+c=0$

$\Rightarrow x(-2)+y(3)-2(x-2)-3(y+3)+9=0$

$\Rightarrow -2x+3y-2x+4-3y-9+9=0$

$\Rightarrow -4x=-4\Rightarrow x=1$ ....(2)

Solving (1) & (2) $\Rightarrow(x,y)=(1,3)$

$\therefore$ The inverse point of p is (1, 3)