1. **If Q (h, k) is the foot of perpendicular from P (x_1, y_1) on the straight line $ax + by + c = 0$. Then P.T $\frac{h-x_1}{a} = \frac{k-y_1}{b} = -\frac{(ax_1+by_1+c)}{a^2+b^2}$.find the foot of perpendicular of (4, 1) w.r.t the line $3x - 4y + 12 = 0$.**

Sol: Q (h, k) is the foot of perpendicular from P (x_1, y_1) on the straight line

$ax + by + c = 0$ … …..(1)

$slope\ of\ the\ line\ (1)\ is\ (m_1) = -\frac{a}{b}$

$slope\ of\ PQ\ is\ (\boldsymbol{m_2}) = \frac{y_2-y_1}{x_2-x_1} = \frac{k-y_1}{h-x_1}$

$since\ PQ\ is\ \perp lar\ to\ (1) \Rightarrow m_1.m_2 = -1$

$(-\frac{a}{b}).(\frac{k-y_1}{h-x_1}) = -1$

$\frac{h-x_1}{a} = \frac{k-y_1}{b}\ (Applying\ ratio\ propotional)$

$\frac{h-x_1}{a} = \frac{k-y_1}{b} = \frac{a(h-x_1)+b(k-y_1)}{a.a+b.b}$

$\frac{h-x_1}{a} = \frac{k-y_1}{b} = \frac{(ah-ax_1+bk-by_1)}{a^2+b^2}$

$\frac{h-x_1}{a} = \frac{k-y_1}{b} = \frac{(-ax_1-by_1+ah+bk)}{a^2+b^2}$ … … … (2)

$since\ Q(h,k) lies\ on\ eq''n\ (1)$

$ah + bk + c = 0 \Rightarrow \boldsymbol{ah + bk = -c}$

$sub\ in\ eq''n(2)$

$\frac{h-x_1}{a} = \frac{k-y_1}{b} = \frac{(-ax_1-by_1-c)}{a^2+b^2}$

$\therefore \boldsymbol{\frac{h-x_1}{a} = \frac{k-y_1}{b} = -\frac{(ax_1+by_1+c)}{a^2+b^2}}$.

$let(h,k) be\ the\ required\ foot\ o\ f\ perpendicular\ from$ (4, 1) w.r.t the line $3x - 4y + 12 = 0$.

$\frac{h-4}{3} = \frac{k-1}{-4} = -\frac{(3(4)-4(1)+12)}{3^2+(-4)^2}$

$\frac{h-4}{3} = \frac{k-1}{-4} = -\frac{(20)}{25} \Rightarrow \left\langle \frac{h-4}{3} = -\frac{4}{5} \ \middle|\ \frac{k-1}{-4} = -\frac{4}{5} \right\rangle$

$\Rightarrow h - 4 = -\frac{12}{5}\ \&\ \ k - 1 = \frac{16}{5}$

$h = -\frac{12}{5} + 4\ \&\ \ k = \frac{16}{5} + 1$

$h = \frac{-12+20}{5} = \frac{8}{5}$, k= $\frac{16+5}{5} = \frac{21}{5}$

$\therefore (h,k) = (\frac{8}{5}, \frac{21}{5})$

2. **If Q (h, k) is image of P (x_1, y_1) on the straight line $ax + by + c = 0$. Then P.T $\frac{h-x_1}{a} = \frac{k-y_1}{b} = -\frac{2(ax_1+by_1+c)}{a^2+b^2}$ find the image of (1, 2) w.r.t the line $3x + 4y - 1 = 0$.**

Sol: Q (h, k) is image of P (x_1, y_1) on the straight line $ax + by + c = 0$ … …..(1)

$slope\ of\ the\ line\ (1)\ is\ (m_1) = -\frac{a}{b}$

$slope\ of\ PQ\ is\ (\boldsymbol{m_2}) = \frac{y_2-y_1}{x_2-x_1} = \frac{k-y_1}{h-x_1}$

$since\ PQ\ is\ \perp lar\ to\ (1) \Rightarrow m_1.m_2 = -1$

$(-\frac{a}{b}).(\frac{k-y_1}{h-x_1}) = -1$

$\frac{h-x_1}{a} = \frac{k-y_1}{b}\ (Applying\ ratio\ propotional)$

$\frac{h-x_1}{a} = \frac{k-y_1}{b} = \frac{a(h-x_1)+b(k-y_1)}{a.a+b.b}$

$\frac{h-x_1}{a} = \frac{k-y_1}{b} = \frac{(ah-ax_1+bk-by_1)}{a^2+b^2}$

$\frac{h-x_1}{a} = \frac{k-y_1}{b} = \frac{(-ax_1-by_1+ah+bk)}{a^2+b^2}$ … … … (2)

$M\ is\ the\ midpoint\ of\ PQ,$

$M\left[\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right] = \left[\frac{x_1+h}{2}, \frac{y_1+k}{2}\right]$

$since\ M\left[\frac{x_1+h}{2}, \frac{y_1+k}{2}\right] lies\ on\ eq''n\ (1)$

$a\left(\frac{x_1+h}{2}\right) + b\left(\frac{y_1+k}{2}\right) + c = 0$

$\Rightarrow ax_1 + ah + by_1 + bk + 2c = 0$

$\therefore ah + bk = -ax_1 - by_1 - 2c$

$sub\ in\ eq''n(2)$

$\frac{h-x_1}{a} = \frac{k-y_1}{b} = \frac{(-ax_1-by_1-ax_1-by_1-2c)}{a^2+b^2}$

$\therefore \boldsymbol{\frac{h-x_1}{a} = \frac{k-y_1}{b} = -\frac{2(ax_1+by_1+c)}{a^2+b^2}}$.

$let(h,k) be\ the\ image\ from$ (1, 2) w.r.t the line $3x + 4y - 1 = 0 \Rightarrow \frac{h-1}{3} = \frac{k-2}{-4} = -\frac{2[3(1)+4(2)-1]}{3^2+(-4)^2}$

$\frac{h-4}{3} = \frac{k-1}{4} = -\frac{2(10)}{25} \Rightarrow \left\langle \frac{h-1}{3} = -\frac{4}{5} \ \middle|\ \frac{k-2}{4} = -\frac{4}{5} \right\rangle$

$\Rightarrow h - 1 = -\frac{12}{5}\ \&\ \ k - 2 = \frac{-16}{5}$

$h = -\frac{12}{5} + 1\ \&\ \ k = \frac{-16}{5} + 2$

$h = \frac{-12+5}{5} = \frac{-7}{5}$, k= $\frac{-16+10}{5} = \frac{-6}{5}$

$\therefore (h,k) = \left(-\frac{7}{5}, -\frac{6}{5}\right)$.

3. **Find the circumcentre of the triangle whose vertices are (1, 3), (-3, 5) and (5, -1).**

Sol: Let the given vertices are
A(1, 3), B(-3, 5) and C(5, -1).
Let S(x, y) be the circumcentre of ΔABC *then*
$SA = SB = SC$ $\boxed{SA^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2}$

$(i) Consider \quad SA = SB \qquad S.O.B.S$

S(x, y)

A (1, 3) B (-3, 5)

$\Rightarrow (x-1)^2 + (y-3)^2 = (x+3)^2 + (y-5)^2$
$\Rightarrow x^2 + 1 - 2x + y^2 + 9 - 6y$
$= x^2 + 9 + 6x + y^2 + 25 - 10y$

$\Rightarrow -8 - 8x - 16 + 4y = 0$
$-8x + 4y - 24 = 0 \ (\div -4)$

$2x - y + 6 = 0 \ldots.(1)$

(ii) $Consider \quad SB = SC \qquad S.O.B.S$

S(x, y)

A (-3, 5) B (5, -1)

$\Rightarrow (x+3)^2 + (y-5)^2 = (x-5)^2 + (y+1)^2$
$\Rightarrow x^2 + 9 + 6x + y^2 + 25 - 10y$
$= x^2 + 25 - 10x + y^2 + 1 + 2y$

$\Rightarrow -16 + 16x + 24 - 12y = 0$
$16x - 12y + 8 = 0 \ (\div 4)$

$4x - 3y + 2 = 0 \ldots.(2)$

Solving (1) and (2) $\begin{matrix} 2 & -1 & 6 & 2 \\ 4 & -3 & 2 & 4 \end{matrix}$

$(x, y) = \left[\frac{-2+18}{-6+4}, \frac{24-4}{-6+4}\right] = \left[\frac{16}{-2}, \frac{20}{-2}\right]$

Hence coordinate of the circumcentre of ΔABC *are* $(-8, -10)$.
$[(-2,3), (2,-1), (4,0) \& (1,3), (0,-2), (-3,1)]$

4. **Find the circumcentre of the triangle whose vertices are (-2, 3), (2, -1) and (4, 0).**

Sol: Let the given vertices are
A(-2, 3), B(2, -1) and C(4, 0).
Let S(x, y) be the circumcentre of ΔABC *then*
$SA = SB = SC$ $\boxed{SA^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2}$

$(i) Consider \quad SA = SB \qquad S.O.B.S$

S(x, y)

A (-2, 3) B (2, -1)

$\Rightarrow (x+2)^2 + (y-3)^2 = (x-2)^2 + (y+1)^2$
$\Rightarrow x^2 + 4 + 4x + y^2 + 9 - 6y$
$= x^2 + 4 - 4x + y^2 + 1 + 2y$

$\Rightarrow 4x - 6y + 9 = -4x + 2y = 1$
$8x - 8y + 8 = 0 \ (\div 8)$

$x - y + 1 = 0 \ldots.(1)$

(ii) $Consider \quad SB = SC \qquad S.O.B.S$

S(x, y)

A (2, -1) B (4, 0)

$\Rightarrow (x-2)^2 + (y+1)^2 = (x-4)^2 + (y-0)^2$

$\Rightarrow x^2 + 4 - 4x + y^2 + 1 + 2y$
$= x^2 + 16 - 8x + y^2$

$\Rightarrow -4x + 2y + 5 = -8x + 16$
$8x - 4x + 2y + 5 - 16 = 0$

$4x + 2y - 11 = 0 \ldots.(2)$

Solving (1) and (2) $\begin{matrix} 1 & -1 & 1 & 1 \\ 4 & 2 & -11 & 4 \end{matrix}$

$(x, y) = \left[\frac{11-2}{2+4}, \frac{4+11}{2+4}\right] = \left[\frac{9}{6}, \frac{15}{6}\right]$

Hence coordinate of the circumcentre of ΔABC *are* $\left(\frac{3}{2}, \frac{5}{2}\right)$.

Aims tutorial

5. Find the circumcentre of the triangle whose sides are $x+y+2=0, 5x-y-2=0$ *and* $x-2y+5=0$.

Sol: Let ΔABC *whose sides are*

$x+y+2=0$ ..... (1)

$5x-y-2=0$ .... (2)

$x-2y+5=0 \ldots\ldots(3)$

solving eq''n (1) & (2)

1	1	2	1
5	-1	-2	5

$$A(x,y)=\left[\frac{-2+2}{-1-5},\ \frac{10+2}{-1-5}\right]=(0,-2)$$

solving eq''n(2) & (3)

5	-1	-2	5
1	-2	5	1

$$B(x,y)=\left[\frac{-5-4}{-10+1},\ \frac{-2-25}{-10+1}\right]=(1,3)$$

solving eq''n(1) & (3)

1	1	2	1
1	-2	5	1

$$C(x,y)=\left[\frac{5+4}{-2-1},\ \frac{2-5}{-2-1}\right]=(-3,1)$$

A(0, -2), B(1, 3) and C(-3, 1).

Let S(x, y) be the circumcentre of ΔABC *then* $SA=SB=SC$

$$\boxed{SA^2=(x_2-x_1)^2+(y_2-y_1)^2}$$

(i)*Consider* $SA=SB$ $S.O.B.S$

S(x, y)

A (0, -2) B (1, 3)

$\Rightarrow (x-0)^2+(y+2)^2=(x-1)^2+(y-3)^2$

$\Rightarrow x^2+0+0+y^2+4+4y$
$=x^2+1-2x+y^2+9-6y$

$\Rightarrow -1+2x-5+10y=0$
$2x+10y-6=0\ (\div 2)$

$1x+5y-3=0 \ldots.(4)$

(ii) *Consider* $SB=SC$ $S.O.B.S$

S(x, y)

A (1, 3) B (-3, 1)

$\Rightarrow (x-1)^2+(y-3)^2=(x+3)^2+(y-1)^2$
$\Rightarrow x^2+1-2x+y^2+9-6y$
$=x^2+9+6x+y^2+1-2y$

$\Rightarrow -8-8x+8-4y=0$
$-8x-4y=0\ (\div -4)$

$2x+y=0 \ldots.(5)$

Solving (1) and (2)

1	5	−3	1
2	1	0	2

$(x,y)=\left[\frac{0+3}{1-10},\frac{-6-0}{1-10}\right]=\left[\frac{3}{-9},\frac{-6}{-9}\right]$

Hence coordinate of the circumcentre of ΔABC *are* $\left(-\frac{1}{3},\frac{2}{3}\right)$.

6. Find the circumcentre of the triangle whose sides are $x+y=0, 2x+y+5=0$ *and* $x-y=2$

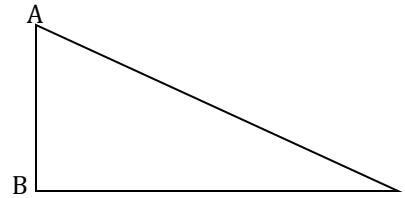
Sol: Let ΔABC *whose sides are*

$x+y=0$ ..... (1) Slope (m) $=-\frac{a}{b}=-1$

$2x+y+5=0$ .... (2) Slope (m) $=-\frac{a}{b}=-\frac{2}{1}$

$x-y=2 \ldots\ldots(3)$ Slope (m) $=-\frac{a}{b}=-\frac{1}{-1}=1$

since (1)& (3)*are perpendicular lines*
these lines form a right angled triangle.
Circumcentre is the mid point of hypoenuse.



solving eq''n (1) & (2)

1	1	0	1
2	1	5	2

$A(x,y) = \left[\frac{5-0}{1-2}, \frac{0-5}{1-2}\right] = (-5\ ,5)$

$solving\ eq''n(2)\ \&\ (3)$ 2 1 5 2
1 -1 -2 1

$C(x,y) = \left[\frac{-2+5}{-2-1}, \frac{5+4}{-2-1}\right] = (-1,-3)$

A (-5, 5) and C (-1, -3)

$Circumcentre\ is\ the\ mid\ point\ of\ hypoenuse.$

$=\left[\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right]=\left[\frac{-5-1}{2}, \frac{5-3}{2}\right] = (-3,1)$

7. Find the Orthocenter of the triangle whose vertices are (-5, -7), (13, 2) and (-5, 6).

Sol: Given vertices are (-5, -7), (13, 2) and (-5, 6)

Slope of B (13, 2), C (-5, 6)

$m = \frac{y_2-y_1}{x_2-x_1} = \frac{6-2}{-5-13} = \frac{4}{-18} = -\frac{2}{9}$

$Since\ AD\ \perp BC, slope\ of\ AD = \frac{9}{2}$

$eq''n\ of\ AD\ is\ (\boldsymbol{y-y_1}) = \boldsymbol{m(x-x_1)}$

$A(-5,-7)\ \ m = \frac{9}{2}$

$\Rightarrow (y+7) = \frac{9}{2}(x+5)$

$\Rightarrow 2y+14 = 9x+45$

$\Rightarrow 9x-2y+31 = 0 \ldots\ldots(1)$

Slope of A (-5, -7), C (-5, 6)

$m = \frac{y_2-y_1}{x_2-x_1} = \frac{6+7}{-5+5} = \frac{13}{0}$

$Since\ BE\ \perp AC, slope\ of\ BE = 0$

$eq''n\ of\ BE\ is\ (y-y_1) = m(x-x_1)$

$B(13,2)\ \ m = 0$

$\Rightarrow (y-2) = 0(x-13)$

$\Rightarrow y-2 = 0$

$\Rightarrow$y=2..... (2)

$sub\quad y = 2\ in\ (1)$

$\Rightarrow 9x-2(2)+31 = 0$

$\Rightarrow 9x = -27$

$\Rightarrow x = -\frac{27}{9} = -3$

$hence\ required\ orthocentre\ is\ (-3,2)$

8. Find the Orthocenter of the triangle whose vertices are (-2, -1), (6, -1) and (2, 5).

Sol: Given vertices are (-2, -1), (6, -1) and (2, 5)

Slope of B (6, -1), C (2, 5)

$m = \frac{y_2-y_1}{x_2-x_1} = \frac{5+1}{2-6} = \frac{6}{-4} = -\frac{3}{2}$

$Since\ AD\ \perp BC, slope\ of\ AD = \frac{2}{3}$

$eq''n\ of\ AD\ is\ (y-y_1) = m(x-x_1)$

$A(-2,-1)\ \ m = \frac{2}{3}$

$\Rightarrow (y+1) = \frac{2}{3}(x+2)$

$\Rightarrow 3y+3 = 2x+4$

$\Rightarrow 2x-3y+1 = 0 \ldots\ldots(1)$

Slope of A (-2, -1), C (2, 5)

$m = \frac{y_2-y_1}{x_2-x_1} = \frac{5+1}{2+2} = \frac{6}{4} = \frac{3}{2}$

$Since\ BE\ \perp AC, slope\ of\ BE = -\frac{2}{3}$

$eq''n\ of\ BE\ is\ (\boldsymbol{y-y_1}) = \boldsymbol{m(x-x_1)}$

$B(6,-1)\ \ m = -\frac{2}{3}$

$\Rightarrow (y+1) = -\frac{2}{3}(x-6)$

$\Rightarrow 3y+3 = -2x+12$

$\Rightarrow 2x+3y-9 = 0 \ldots\ldots(2)$

$solving\ eq''n\ (1)\ \&\ (2)$ 2 -3 1 2
2 3 -9 2

$A(x,y) = \left[\frac{27-3}{6+6}, \frac{2+18}{6+6}\right] = \left(\frac{24}{12}, \frac{20}{12}\right) = (2, \frac{5}{3})$

9. If the eq"ns of the sides of the triangle are $7x+y-10=0, x-2y+5=0$ and $x+y+2=0$. find the orthocentre.

sol:Let ΔABC whose sides are

$7x+y-10=0$ ..... (1)

$x-2y+5=0$ .... (2)

$x+y+2=0$ ..... (3)

solving eq"n (1) & (2)

7	1	-10	7
1	-2	5	1

$A(x,y)=\left[\frac{5-20}{-14-1}, \frac{-10-35}{-14-1}\right]=(1,3)$

solving eq"n(2) & (3)

1	-2	5	1
1	1	2	1

$B(x,y)=\left[\frac{-4-5}{1+2}, \frac{5-2}{1+2}\right]=(-3,1)$

solving eq"n(1) & (3)

7	1	-10	7
1	1	2	1

$C(x,y)=\left[\frac{2+10}{7-1}, \frac{-10-14}{7-1}\right]=(2,-4)$

A(1, 3), B(-3, 1) and C(2, -4).

Now

Slope of B (-3,1), C (2, -4)

$m=\frac{y_2-y_1}{x_2-x_1}=\frac{-4-1}{2+3}=\frac{-5}{5}=-1$

Since $AD \perp BC$, slope of $AD=1$

eq"n of AD is $(y-y_1)=m(x-x_1)$

$A(1,3)$ $m=1$

$\Rightarrow (y-3)=1(x-1) \Rightarrow y-3=x-1$

$\Rightarrow x-y+2=0$ ..... (4)

Slope of A (1, 3), C (2, -4)

$m=\frac{y_2-y_1}{x_2-x_1}=\frac{-4-3}{2-1}=\frac{-7}{1}$

Since $BE \perp AC$, slope of $BE=\frac{1}{7}$

eq"n of BE is $(y-y_1)=m(x-x_1)$

$B(-3,1)$ $m=\frac{1}{7}$

$\Rightarrow (y-1)=\frac{1}{7}(x+3) \Rightarrow 7y-7=x+3$

$\Rightarrow x-7y+10=0$..... (5)

solving eq"n (4) & (5)

1	-1	2	1
1	-7	10	1

$A(x,y)=\left[\frac{-10+14}{-7+1}, \frac{2-10}{-7+1}\right]=\left[\frac{4}{-6}, \frac{-8}{-6}\right]$

hence required orthocentre is $(-\frac{2}{3}, \frac{4}{3})$

10. If the eq"ns of the sides of the triangle are $x+y+10=0, x-y-2=0$ and $2x+y-7=0$. find the orthocentre.

sol:Let ΔABC whose sides are

$x+y+10=0$ ..... (1) (m) $=-\frac{a}{b}=-\frac{1}{1}=-1$

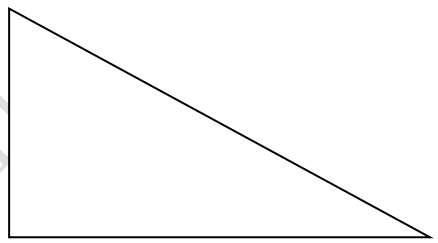
$x-y-2=0$ .... (2) (m) $=-\frac{a}{b}=-\frac{1}{-1}=1$

$2x+y-7=0$ ..... (3) (m) $=-\frac{a}{b}=-\frac{2}{1}=-2$

since (1)& (2)are perpendicular lines

these lines form a right angled triangle.

orthocentre is the point of intersection of (1)& (2)



solving eq"n (1) & (2)

1	1	10	1
1	-1	-2	1

$A(x,y)=\left[\frac{-2+10}{-1-1}, \frac{10+2}{-1-1}\right]$

$=\left[\frac{8}{-2}, \frac{12}{-2}\right]=[-4,-6]$

Hence the coordinates of Orthcentre ΔABC are $(-4,-6)$.

11. If P and q are the lengths of perpendiculars from the origin to the st lines
$\boldsymbol{x\sec\alpha + y\operatorname{cosec}\alpha = a}$
$\boldsymbol{and\ x\cos\alpha - y\sin\alpha = a\cos 2\alpha,}$
$\boldsymbol{P.T\ 4p^2 + q^2 = a^2}$ **.**

Sol: $x\sec\alpha + y\operatorname{cosec}\alpha = a$
⇨ $\frac{x}{\cos\alpha} + \frac{y}{\sin\alpha} = a$
⇨ $x\sin\alpha + y\cos\alpha = a\sin\alpha\cos\alpha$
⇨ $x\sin\alpha + y\cos\alpha - a\sin\alpha\cos\alpha = 0$
$length\ of\ \perp lar\ from\ (0,0) to\ above\ line$
is $p = \frac{|c|}{\sqrt{a^2+b^2}}$
⇨ $p = \frac{|-a\sin\alpha\cos\alpha|}{\sqrt{\sin^2\alpha+\cos^2\alpha}}$
⇨ $p = a\sin\alpha\cos\alpha$
⇨ $2p = a2\sin\alpha\cos\alpha$
⇨ $2p = a\sin 2\alpha$ S.O.B.S
⇨$4p^2 = a^2\sin^2 2\alpha \ldots..(1)$

$q\ is\ the\ length\ of\ \perp lar\ from\ (0,0)\ to\ line$
$x\cos\alpha - y\sin\alpha - a\cos 2\alpha = 0$
$q = \frac{|-a\cos 2\alpha|}{\sqrt{\cos^2\alpha+\sin^2\alpha}}$
⇨ $q = a\cos 2\alpha$ S.O.B.S
⇨ $q^2 = a^2\cos^2 2\alpha \ldots..(2)$

$now\ eq''n$ (1) & (2)

$4p^2 + q^2 = a^2\sin^2 2\alpha + a^2\cos^2 2\alpha$

$= a^2(\sin^2 2\alpha + \cos^2 2\alpha)$

$= a^2(1)$
$\boldsymbol{4p^2 + q^2 = a^2}$

12. Find the eq"n of the st lines passing through the point of intersection of the lines
$\boldsymbol{3x + 2y + 4 = 0, 2x + 5y = 1}$ **& whose distance from (2, -1) is 2.**

Sol: Given eq"ns
$3x + 2y + 4 = 0 \ldots.(1)$
$2x + 5y - 1 = 0 \ldots(2)$
$solving\ eq''n$ (1) & (2)

3	2	4	3
2	5	-1	2

$p(x,y) = \left[\frac{-2-20}{15-4}, \frac{8+3}{15-4}\right] = \left[\frac{-22}{11}, \frac{11}{11}\right]$
$= (-2, 1)$

Let 'm' be the slope of the line passing through P (-2, 1) is $(y - y_1) = m(x - x_1)$
⇨ $\boxed{(y-1) = m(x+2)}$

⇨$mx + 2m - y + 1 = 0$
⇨$mx - y + 2m + 1 = 0 \ldots.(3)$

Since distance from (2, -1) to (3) is 2
$\boldsymbol{d = \frac{(ax_1+by_1+c)}{\sqrt{a^2+b^2}}}$
⇨ $2 = \frac{(m(2)-1(-1)+2m+1)}{\sqrt{m^2+(-1)^2}}$
⇨ $2 = \frac{(4m+2)}{\sqrt{m^2+1}}$
⇨ $2 = \frac{2(2m+1)}{\sqrt{m^2+1}}$ S.O.B.S
⇨ $m^2 + 1 = (2m+1)^2$
⇨ $m^2 + 1 = 4m^2 + 4m + 1$
⇨ $4m^2 + 4m + 1 - m^2 - 1 = 0$
⇨ $3m^2 + 4m = 0$
⇨ $m(3m + 4) = 0$
$m = 0\ or\ m = -\frac{4}{3}$
$Required\ eq''n$
$case(i)\ if\ m = 0$ $case(ii)\ if\ m = -\frac{4}{3}$
$(y-1) = 0(x+2);$ ⇨ $(y-1) = -\frac{4}{3}(x+2)$
⇨ $y - 1 = 0.$ ⇨ $3y - 3 = -4x + 8$
⇨ $4x + 3y - 11 = 0.$

1. **If θ is the angle between the pair of lines $ax^2+2hxy+by^2=0$, then P.T $cos\theta = \frac{|a+b|}{\sqrt{(a-b)^2+4h^2}}$.**

Sol: let $ax^2+2hxy+by^2=0$ *represent the lines*

$l_1x+m_1y=0 \ldots\ldots(1)$

$l_2x+m_2y=0 \ldots\ldots(2)$

$\therefore ax^2+2hxy+by^2 \equiv [l_1x+m_1y][l_2x+m_2y]=0$

$\equiv l_1x[l_2x+m_2y]+m_1y[l_2x+m_2y]=0$

$\equiv l_1l_2x^2+l_1m_2xy+l_2m_1xy+m_1m_2y^2=0$

$\equiv l_1l_2x^2+(l_1m_2+l_2m_1)xy+m_1m_2y^2=0$

Comparing both sides x^2, y^2 & xy *coeff, we get*

$l_1l_2=a,\ m_1m_2=b$

& $l_1m_2+l_2m_1=2h$

now $\quad cos\theta = \frac{l_1l_2+m_1m_2}{\sqrt{{l_1}^2+{m_1}^2}\sqrt{{l_2}^2+{m_2}^2}}$

$\Rightarrow cos\theta = \frac{l_1l_2+m_1m_2}{\sqrt{(l_1l_2)^2+(l_1m_2)^2+(l_2m_1)^2+(m_1m_2)^2}}$

$= \frac{l_1l_2+m_1m_2}{\sqrt{(l_1l_2)^2+(m_1m_2)^2-2l_1l_2m_1m_2+(l_1m_2)^2+(l_2m_1)^2+2l_1l_2m_1m_2}}$

$= \frac{l_1l_2+m_1m_2}{\sqrt{(l_1l_2-m_1m_2)^2+(l_1m_2+l_2m_1)^2}}$

$= \frac{|a+b|}{\sqrt{(a-b)^2+(2h)^2}}$

$= \frac{|a+b|}{\sqrt{(a-b)^2+4h^2}}.$

2. **Prove that product of perpendiculars from a point (α,β) to the pair of st lines $ax^2+2hxy+by^2=0$ is $\frac{a\alpha^2+2h\alpha\beta+b\beta^2}{\sqrt{(a-b)^2+4h^2}}$**

Sol: let $ax^2+2hxy+by^2=0$ *represent the lines*

$l_1x+m_1y=0 \ldots\ldots(1)$

$l_2x+m_2y=0 \ldots\ldots(2)$

$\therefore ax^2+2hxy+by^2 \equiv [l_1x+m_1y][l_2x+m_2y]=0$

$\equiv l_1x[l_2x+m_2y]+m_1y[l_2x+m_2y]=0$

$\equiv l_1l_2x^2+l_1m_2xy+l_2m_1xy+m_1m_2y^2=0$

$\equiv l_1l_2x^2+(l_1m_2+l_2m_1)xy+m_1m_2y^2=0$

Comparing both sides x^2, y^2 & xy *coeff, we get*

$l_1l_2=a,\ m_1m_2=b$ & $l_1m_2+l_2m_1=2h$

w.k.t the length of the $\perp$ *larfrom* (x_1,y_1) to the line $ax+by+c=0$ *is* $\frac{(ax_1+by_1+c)}{\sqrt{a^2+b^2}}$

now product of the $\perp$ *larfrom* (α,β) to the lines (1) and (2) is $=\left(\frac{l_1\alpha+m_1\beta}{\sqrt{{l_1}^2+{m_1}^2}}\right)\left(\frac{l_2\alpha+m_2\beta}{\sqrt{{l_2}^2+{m_2}^2}}\right)$

$= \left|\frac{l_1\alpha[l_2\alpha+m_2\beta]+m_1\beta[l_2\alpha+m_2\beta]}{\sqrt{(l_1l_2)^2+(l_1m_2)^2+(l_2m_1)^2+(m_1m_2)^2}}\right|$

$\frac{l_1l_2\alpha^2+l_1m_2\alpha\beta+l_2m_1\alpha\beta+m_1m_2\beta^2}{\sqrt{(l_1l_2)^2+(m_1m_2)^2-2l_1l_2m_1m_2+(l_1m_2)^2+(l_2m_1)^2+2l_1l_2m_1m_2}}$

$= \left|\frac{l_1l_2\alpha^2+(l_1m_2+l_2m_1)\alpha\beta+m_1m_2\beta^2}{\sqrt{(l_1l_2-m_1m_2)^2+(l_1m_2+l_2m_1)^2}}\right|$

$= \left|\frac{a\alpha^2+2h\alpha\beta+b\beta^2}{\sqrt{(a-b)^2+(2h)^2}}\right|$

$= \left|\frac{a\alpha^2+2h\alpha\beta+b\beta^2}{\sqrt{(a-b)^2+4h^2}}.\right|$

Aims tutorial

3. If the eq"n $ax^2+2hxy+by^2=0$ represent a pair of lines, P.T the combined eq"n of the pair of bisectors bisecting the angle b/w these lines is $h(x^2-y^2)=(a-b)xy$.

Sol: let $ax^2+2hxy+by^2=0$ *represent the lines*

$l_1x+m_1y=0\ldots\ldots..(1)$

$l_2x+m_2y=0\ldots\ldots..(2)$

$\therefore ax^2+2hxy+by^2\equiv[l_1x+m_1y][l_2x+m_2y]=0$

$\equiv l_1x[l_2x+m_2y]+m_1y[l_2x+m_2y]=0$

$\equiv l_1l_2x^2+l_1m_2xy+l_2m_1xy+m_1m_2y^2=0$

$\equiv l_1l_2x^2+(l_1m_2+l_2m_1)xy+m_1m_2y^2=0$

Comparing both sides x^2, y^2 & xy *coeff, we get*

$l_1l_2=a,\ m_1m_2=b\ \&\ l_1m_2+l_2m_1=2h$

Now eq"ns of bisectors of angle b/w (1 & (2) are

$$\left(\frac{l_1x+m_1y}{\sqrt{{l_1}^2+{m_1}^2}}\right)=\pm\left(\frac{l_2x+m_2y}{\sqrt{{l_2}^2+{m_2}^2}}\right)$$

S.O.B.S and cross multiplying, we get

$\Rightarrow (l_1x+m_1y)^2({l_2}^2+{m_2}^2)$

$=(l_2x+m_2y)^2({l_1}^2+{m_1}^2)$

$\Rightarrow ({l_1}^2x^2+{m_1}^2y^2+2l_1m_1xy)({l_2}^2+{m_2}^2)$

$=({l_2}^2x^2+{m_2}^2y^2+2l_2m_2xy)({l_1}^2+{m_1}^2)$

$\Rightarrow (l_1l_2)^2x^2+(l_1m_2)^2x^2+(l_2m_1)^2y^2+(m_1m_2)^2x^2+2l_1m_1{l_2}^2xy+2l_1m_1{m_2}^2xy$

$=(l_1l_2)^2x^2+(l_2m_1)^2x^2+(l_1m_2)^2y^2+(m_1m_2)^2x^2+2l_2m_2{l_1}^2xy+2l_2m_2{m_1}^2xy$

$\Rightarrow x^2[(l_1m_2)^2-(l_2m_1)^2]$

$-y^2[(l_1m_2)^2-(l_2m_1)^2]$

$=2xy[l_1l_2(l_1m_2-l_2m_1)-m_1m_2((l_1m_2-l_2m_1)]$

$\Rightarrow[(l_1m_2+l_2m_1)(l_1m_2-l_2m_1)](x^2-y^2)$

$=2xy(l_1m_2-l_2m_1)[l_1l_2-m_1m_2]$

$\Rightarrow(l_1m_2+l_2m_1)(x^2-y^2)=2xy[l_1l_2-m_1m_2]$

$\Rightarrow 2h(x^2-y^2)=2xy(a-b)$

$\therefore h(x^2-y^2)=xy(a-b)$.

4. S.T the area of the triangle formed by the lines $ax^2+2hxy+by^2=0$ and $lx+my+n=0$ *is*

$$\frac{n^2\sqrt{h^2-ab}}{am^2-2hlm+bl^2}\ Sq.units.$$

Sol: let $ax^2+2hxy+by^2=0$ *represent the lines*

$l_1x+m_1y=0\ldots\ldots..(1)$

$l_2x+m_2y=0\ldots\ldots..(2)$

$lx+my+n=0\ldots.(3)$

$\therefore ax^2+2hxy+by^2\equiv[l_1x+m_1y][l_2x+m_2y]=0$

$\equiv l_1x[l_2x+m_2y]+m_1y[l_2x+m_2y]=0$

$\equiv l_1l_2x^2+l_1m_2xy+l_2m_1xy+m_1m_2y^2=0$

$\equiv l_1l_2x^2+(l_1m_2+l_2m_1)xy+m_1m_2y^2=0$

Comparing both sides x^2, y^2 & xy *coeff, we get*

$l_1l_2=a,\ m_1m_2=b\ \&\ l_1m_2+l_2m_1=2h$

Solving (1) & (2) we get, (0, 0)

Solving (1) & (3)

$$\begin{matrix} l_1 & m_1 & 0 & l_1 \\ l & m & n & l \end{matrix}$$

A $(x_1,y_1)=\left[\frac{nm_1-0}{l_1m-l\,m_1},\frac{0-nl_1}{l_1m-l\,m_1}\right]$

Similarly by solving (2) & (3) we get,

B $(x_2,y_2)=\left[\frac{nm_2}{l_2m-lm_2},\frac{nl_2}{l_2m-lm_2}\right]$

Now area of $\Delta OAB=\frac{1}{2}|x_1y_2-x_2y_1|$

$=\frac{1}{2}\left|\left(\frac{nm_1}{l_1m-l\,m_1}\right)\left(\frac{-nl_2}{l_2m-l\,m_2}\right)-\left(\frac{nm_2}{l_2m-l\,m_2}\right)\left(\frac{-nl_1}{l_1m-l\,m_1}\right)\right|$

$=\frac{1}{2}\left|\frac{n^2(l_1m_2-l_2m_1)}{(l_1m-l\,m_1)(l_2m-l\,m_2)}\right|$

$=\frac{1}{2}\left|\frac{n^2\sqrt{(l_1m_2+l_2m_1)^2-4l_1l_2\,m_1m_2}}{l_1\,l_2m^2-l_1m_2lm-l_2m_1lm+m_1\,m_2l^2}\right|$

$=\frac{1}{2}\left|\frac{n^2\sqrt{(2h)^2-4ab}}{am^2-(l_1m_2+l_2m_1)lm+bl^2}\right|$

$=\frac{1}{2}\left|\frac{n^2\sqrt{4h^2-4ab}}{am^2-2hlm+bl^2}\right|$

$=\frac{2}{2}\left|\frac{n^2\sqrt{h^2-ab}}{am^2-2hlm+bl^2}\right|=\left|\frac{n^2\sqrt{h^2-ab}}{am^2-2hlm+bl^2}\right|\ Sq\ .units$

5. If the eq"n

$\boldsymbol{ax^2+2hxy+by^2+2gx+2fy+c=0}$

Represent a pair of lines, P.T

$(i)\boldsymbol{abc+2fgh-af^2-bg^2-ch^2=0}$

$(ii)\boldsymbol{h^2\geq ab, g^2\geq ac, f^2\geq bc}$.

Sol:Let $ax^2+2hxy+by^2+2gx+2fy+\ c=0$

represent the lines

$l_1x+m_1y+n_1=0\ldots\ldots..(1)$

$l_2x+m_2y+n_2=0\ldots\ldots..(2)$

$\therefore ax^2+2hxy+by^2+2gx+2fy+\ c=0$

$\equiv[l_1x+m_1y+n_1][l_2x+m_2y+n_2]=0$

$\equiv l_1l_2x^2+l_1m_2xy+l_1n_2x+l_2m_1xy+m_1m_2y^2$
$+m_1n_2y+l_2n_1x+m_2n_1y+n_1n_2=0$

$\equiv l_1l_2x^2+(l_1m_2+l_2m_1)xy+m_1m_2y^2\ +\ (l_1n_2+$
$l_2n_1)x+(m_1n_2\ \ +m_2n_1)y+n_1n_2=0$

Comparing both sides

$x^2,\ xy,\ y^2, x, y-coeff$ & $constant$ *we get,*

$l_1l_2=a,\ \ m_1m_2=b\ ,\ \ l_1m_2+l_2m_1=2h$

$l_1n_2+l_2n_1=2g, m_1n_2+m_2n_1=2f$ And $n_1n_2=c$

now $(2g)(2f)(2h)$

$=(l_1n_2+l_2n_1)(l_1m_2+l_2m_1)\ (m_1n_2+m_2n_1)$

$=[l_1{}^2m_2n_2+l_1l_2m_1n_2+l_1l_2m_2n_1+l_2{}^2m_1n_1](m_1n_2$
$+m_2n_1\ )$

$=l_1{}^2m_1m_2n_2{}^2+l_1{}^2m_2{}^2n_1n_2+l_1l_2m_1{}^2n_2{}^2$
$+l_1l_2m_1m_2n_1n_2+l_1l_2m_1m_2n_1n_2+l_1l_2m_2{}^2n_1{}^2$
$+l_2{}^2m_1{}^2n_1n_2+l_2{}^2m_1m_2n_1{}^2$

$=l_1l_2[(m_1n_2)^2+(m_2n_1)^2]+m_1m_2[(l_1n_2)^2+(l_2n_1)^2]+$
$n_1n_2[(l_1m_2)^2+(l_2m_1)^2]\quad+2l_1l_2m_1m_2n_1n_2$

$=l_1l_2[(m_1n_2+m_2n_1)^2-2m_1m_2n_1n_2]$

$+m_1m_2[(l_1n_2+l_2n_1)^2-2l_1l_2n_1n_2]$

$+n_1n_2[(l_1m_2+l_2m_1)^2-2l_1l_2m_1m_2n_1n_2]$

$+2l_1l_2m_1m_2n_1n_2$

$=a[(2f)^2-2bc]+b[(2g)^2-2ac]$

$+c[(2h)^2-2ab]+2abc$

$\Rightarrow 8ghf=4af^2-2abc+4bg^2-2abc+4ch^2$
$-2abc+2abc$

$\Rightarrow 8ghf=4(af^2+bg^2+ch^2-abc)$

$\therefore\ abc\text{-}\ af^2-bg^2-ch^2=0$

$$now\ h^2-ab=\left[\frac{l_1m_2+l_2m_1}{2}\right]^2-l_1l_2m_1m_2$$

$$=\frac{(l_1m_2+l_2m_1)^2-4l_1l_2m_1m_2}{4}$$

$=\frac{(l_1m_2-l_2m_1)^2}{2}\geq 0\ \therefore h^2-ab\geq 0\Rightarrow h^2\geq ab$

Similarlly, we can show that $\boldsymbol{g^2\geq ac, f^2\geq bc}$.

6. If the eq"n

$\boldsymbol{ax^2+2hxy+by^2+2gx+2fy+c=0}$

Represent a pair of lines, P.T

$(i)\boldsymbol{h^2=ab}$, $(ii)\boldsymbol{af^2=bg^2}$ (iii) the distance b/w parallel lines is $2\sqrt{\frac{g^2-ac}{a(a+b)}}$ or $2\sqrt{\frac{f^2-bc}{b(a+b)}}$

Sol: Let $ax^2+2hxy+by^2+2gx+2fy+\ c=0$

represent the lines

$lx+my+n_1=0\ldots\ldots..(1)$

$lx+my+n_2=0\ldots\ldots..(2)$

$\therefore ax^2+2hxy+by^2+2gx+2fy+\ c=0$

$\equiv[lx+my+n_1][lx+my+n_2]=0$

$\equiv l^2x^2+lmxy+ln_2x+lmxy+m^2y^2$
$+mn_2y+ln_1x+mn_1y+n_1n_2=0$

$\equiv l^2x^2+2lmxy+m^2y^2$
$+l(n_1+n_2)x+m(n_1+n_2)y+n_1n_2=0$

Comparing both sides

$x^2,\ xy,\ y^2, x, y-coeff$ *and constant*

we get,

$l^2=a, m^2=b\ ,\ \ 2lm=2h$

$l(n_1+n_2)=2g, m(n_1+n_2)=2f$ & $n_1n_2=c$

now $h^2=(lm)^2=l^2m^2=ab\ \Rightarrow h^2=ab$

$af^2=l^2\left[\frac{m(n_1+n_2)}{2}\right]^2=m^2\left[\frac{l(n_1+n_2)}{2}\right]^2=bg^2$

$\Rightarrow af^2=bg^2$

distance b/w parellal lines (1) & (2)

$\frac{|C_1-C_2|}{\sqrt{a^2+b^2}}=\frac{|n_1-n_2|}{\sqrt{l^2+m^2}}=\frac{\sqrt{(n_1+n_2)^2-4n_1n_2}}{\sqrt{a+b}}$

$=\sqrt{\frac{\left(\frac{2g}{l}\right)^2-4c}{a+b}}=\sqrt{\frac{4g^2-4l^2c}{l^2(a+b)}}=2\sqrt{\frac{g^2-ac}{a(a+b)}}$

$or = \sqrt{\frac{\left(\frac{2f}{m}\right)^2 - 4c}{a+b}} = \sqrt{\frac{4f^2-4m^2c}{m^2(a+b)}} = 2\sqrt{\frac{f^2-bc}{b(a+b)}}$

7. S.T that the pair of st lines $6x^2 - 5xy - 6y^2 = 0$, $6x^2 - 5xy - 6y^2 + x + 5y - 1 = 0$ form a square.

Sol: Given pair of st lines

$6x^2 - 5xy - 6y^2 = 0$

$6x^2 - 5xy - 6y^2 + x + 5y - 1 = 0$

Now $6x^2 - 5xy - 6y^2 = 6x^2 - 9xy + 4xy - 6y^2$

$= 3x(2x - 3y) + 2y(2x - 3y)$

$= (2x - 3y)(3x + 2y)$

$\Rightarrow 2x - 3y = 0 \ldots (1) \Rightarrow m_1 = \frac{2}{3}$ $[m = -a/b]$

$3x + 2y = 0 \ldots\ldots (2) \Rightarrow m_2 = -\frac{3}{2}$

$[m_1 m_2 = -1]$

Let $6x^2 - 6xy - 6y^2 + x + 5y - 1$

$\equiv (2x - 3y + l)(3x + 2y + m)$

$\equiv \begin{bmatrix} 6x^2 + 4xy + 2mx - 9xy - 6y^2 - 3my \\ +3lx + 2ly + lm \end{bmatrix}$

$\equiv [6x^2 - 5xy - 6y^2(3l + 2m)x + (2l - 3m)y + lm]$

$equating\ the\ coefficients\ of\ x\ and\ y,$

$3l + 2m = 1 \ldots (a), 2l - 3m = 5 \ldots (b)$

Solving (a) & (b)

3 2 -1 3

2 -3 -5 2

$(l, m) = \left[\frac{-10-3}{-9-4}, \frac{-2+15}{-9-4}\right] = (1, -1)$

So the lines represented by

$6x^2 - 6xy - 6y^2 + x + 5y - 1 = 0\ are$

$2x - 3y + 1 = 0 \ldots (3) \Rightarrow m_3 = \frac{2}{3}$

$3x + 2y - 1 = 0 \ldots (4) \Rightarrow m_4 = -\frac{3}{2}$

$[m_3 m_4 = -1]$

Eq''n (1), (3) and (2), (4) are parallel lines

Eq''n (1), (2) and (3), (4) are perpendicular lines

so the figure form a rectangle.

Now the distance b/w Parallel lines (1), (3) is

$d_1 = \frac{|C_1 - C_2|}{\sqrt{a^2+b^2}} = \frac{|0+1|}{\sqrt{2^2+3^2}} = \frac{1}{\sqrt{13}}$

Also the distance b/w Parallel lines (2), (4) is

$d_2 = \frac{|C_1 - C_2|}{\sqrt{a^2+b^2}} = \frac{|0-1|}{\sqrt{3^2+2^2}} = \frac{1}{\sqrt{13}} \Rightarrow d_1 = d_2$ thus the figure formed is a square.

8. Find the angle b/w line joining the origin to the point of intersection of the curve $x^2 + 2xy + y^2 + 2x + 2y - 5 = 0$ And the line $3x - y + 1 = 0$.

Sol: Given curve

$x^2 + 2xy + y^2 + 2x + 2y - 5 = 0 \ldots (1)$

$Line\ 3x - y + 1 = 0 \ldots (2)$

$\Rightarrow 1 = -3x + y \ldots (3)$

$homogenising\ (1)\ using\ (3)$

$the\ combined\ eq''n\ is$

$(x^2 + 2xy + y^2) + (2x + 2y)(1) - 5(1)^2 = 0$

$\Rightarrow (x^2 + 2xy + y^2) + (2x + 2y)(-3x + y) - 5(-3x + y)^2 = 0$

$\Rightarrow (x^2 + 2xy + y^2) - 6x^2 + 2xy - 6xy + 2y^2 - 5(9x^2 + y^2 - 6xy) = 0$

$\Rightarrow x^2 + 2xy + y^2 - 6x^2 - 4xy + 4y^2 - 45x^2 - 5y^2 + 30xy = 0$

$\Rightarrow -50x^2 + 28xy - 2y^2 = 0 \quad (\div by - 2)$

$\Rightarrow 25x^2 - 14xy + y^2 = 0$

$a = 25, 2h = -14, b = 1 \begin{bmatrix} compare\ with \\ ax^2 + 2hxy + by^2 = 0 \end{bmatrix}$

$\boldsymbol{cos\theta} = \frac{|a+b|}{\sqrt{(a-b)^2+4h^2}} = \frac{25+1}{\sqrt{(25-1)^2+(-14)^2}}$

$= \frac{26}{\sqrt{24^2+14^2}}$

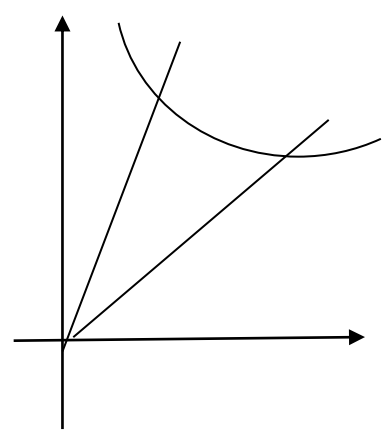
$= \frac{26}{\sqrt{576+196}}$

$= \frac{26}{\sqrt{772}}$

$= \frac{26}{\sqrt{4.193}}$

$= \frac{26}{2\sqrt{193}}$

$\theta = \cos^{-1}\left(\frac{13}{\sqrt{193}}\right)$



9. **Find the value of 'k' if line joining the origin to the point of intersection of the curve $2x^2 - 2xy + 3y^2 + 2x - y - 1 = 0$ And the line $x + 2y = k$ mutually perpendicular.**

SOL: Given curve

$2x^2 - 2xy + 3y^2 + 2x - y - 1 = 0$.... (1)

$Line\ x + 2y = k$ ... (2)

$\Rightarrow 1 = \frac{x+2y}{k}$ .... (3)

$homogenising\ (1)\ using\ (3)$

$the\ combined\ eq''n\ is$

$(2x^2 - 2xy + 3y^2) + (2x - y)(1) - 1(1)^2 = 0$

$\Rightarrow (2x^2 - 2xy + 3y^2) + (2x - y)(\frac{x+2y}{k}) - 1(\frac{x+2y}{k})^2 = 0$

$\Rightarrow (2x^2 - 2xy + 3y^2) + \frac{[2x^2+4xy-xy-2y^2]}{k}$

$-1\frac{[x^2-4xy+4y^2]}{k^2} = 0$

$\Rightarrow x^2\left[2 + \frac{2}{k} - \frac{1}{k^2}\right] + y\left[3 - \frac{2}{k} - \frac{4}{k^2}\right]$

$+2xy\left[-2 + \frac{3}{k} + \frac{4}{k^2}\right] = 0$

$a = 2 + \frac{2}{k} - \frac{1}{k^2},\ b = 3 - \frac{2}{k} - \frac{4}{k^2}$

$\left[\begin{array}{c} compare\ with \\ ax^2 + 2hxy + by^2 = 0 \end{array}\right]$

$since\ (1)\ \&\ (2)\ mutuallly\ perpendicular$

$coefficient\ of\ \ x^2 + coefficient\ of\ \ y^2 = 0$

$(a + b = 0)$

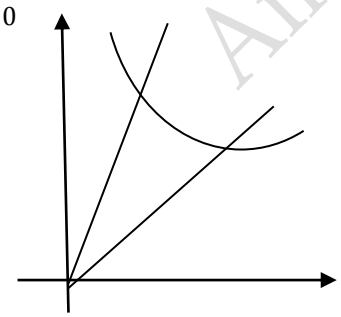
$\Rightarrow 2 + \frac{2}{k} - \frac{1}{k^2} + 3 - \frac{2}{k} - \frac{4}{k^2} = 0$

$\Rightarrow 5 - \frac{5}{k^2} = 0$

$\Rightarrow 5 = \frac{5}{k^2}$

$\Rightarrow k^2 = \frac{5}{5} = 1$

$k = \pm 1$



10. **Show that the lines joining the origin to the point of intersection of the curve $x^2 - xy + y^2 + 3x + 3y - 2 = 0$ And the line $x - y - \sqrt{2} = 0$. are mutually perpendicular.**

Sol: Given curve

$x^2 - xy + y^2 + 3x + 3y - 2 = 0$.... (1)

$Line\ x - y - \sqrt{2} = 0$ ... (2)

$\Rightarrow x - y = \sqrt{2}\ or\ \frac{x-y}{\sqrt{2}} = 1$ .... (3)

$homogenising\ (1)\ using\ (3)$

$the\ combined\ eq''n\ is$

$(x^2 - xy + y^2) + 3(x + y)(1) - 2(1)^2 = 0$

$\Rightarrow (x^2 - xy + y^2) + 3(x + y)(\frac{x-y}{\sqrt{2}}) - 5(\frac{x-y}{\sqrt{2}})^2 = 0$

$\Rightarrow (x^2 - xy + y^2) + \left[\frac{3x^2-3y^2}{\sqrt{2}}\right]$

$-\frac{2(x^2+y^2-2xy)}{2} = 0$

$\Rightarrow x^2 - xy + y^2 + \frac{3}{\sqrt{2}}x^2 - \frac{3}{\sqrt{2}}y^2$

$-x^2 - y^2 + 2xy = 0$

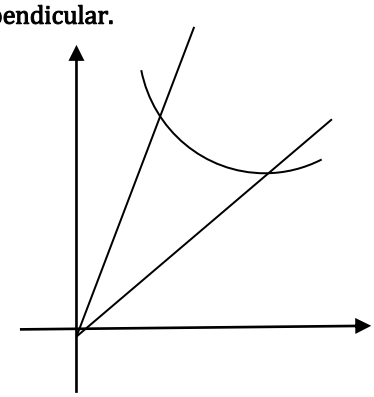
$\Rightarrow +\frac{3}{\sqrt{2}}x^2 + xy - \frac{3}{\sqrt{2}}y^2 = 0$

$a = \frac{3}{\sqrt{2}}, 2h = 1, b = \frac{3}{\sqrt{2}}\left[\begin{array}{c} compare\ with \\ ax^2 + 2hxy + by^2 = 0 \end{array}\right]$

$\therefore$coefficient of x^2 + coefficient of y^2

$$\frac{3}{\sqrt{2}} - \frac{3}{\sqrt{2}} = 0$$

$\therefore$ OA, OB are perpendicular.



11. Write down the equation of the pair of st lines joining the origin to the point of intersection of the curve $3x^2+4xy-4y^2-11x+2y+6=0$ And the line $6x-y+8=0$.

Sol: Given curve

$3x^2+4xy-4y^2-11x+2y+6=0$ .... (1)

$Line\ 6x-y+8=0$ ... (2)

⇨ $8=-6x+y$ $or\ 1=\left[\frac{-6x+y}{8}\right]$ .... (3)

$homogenising\ (1)\ using\ (3)$

$the\ combined\ eq''n\ is$

$(3x^2+4xy-4y^2)-(11x-2y)(1)+6(1)^2=0$

⇨ $(3x^2+4xy-4y^2)-(11x-2y)\left[\frac{-6x+y}{8}\right]$

$+6\left[\frac{-6x+y}{8}\right]^2=0$

⇨ $64(3x^2+4xy-4y^2)$

$-8[-66x^2+11xy+12xy-36y^2]$

$+6(36x^2+y^2-12xy)=0$

⇨ $936x^2-256xy+256xy-234y^2=0$

⇨ $-50x^2+28xy-2y^2=0$ $(\div by-2)$

⇨ $468x^2-117y^2=0$

⇨ $4x^2-y^2=0$ ... ... (3)

$a=4, 2h=0, b=-1\left[\begin{array}{c}compare\ with\\ ax^2+2hxy+by^2=0\end{array}\right]$

The equation of pair of angular bisectors of (3)

$h(x^2-y^2)=(a-b)xy$

⇨ $0(x^2-y^2)=(4+1)xy$

⇨ $0=5xy$

⇨$xy=0$ ⇨ $x=0\ or\ y=0$

Which equations are of coordinates axes.

∴the pair of lines are equally inclined to the coordinate axes.

12. Find the condition for the chord $lx+my=1$ of the circle $x^2+y^2=a^2$ to (i) subtend a right angle at the origin. (ii) to coincide.

Sol: Given circle $x^2++y^2=a^2$.... (1)

$chord\ lx+my=1$ ... ... (2)

$homogenising\ (1)\ using\ (2)$

$the\ combined\ eq''n\ is$

$(x^2+y^2)=a^2(1)^2$

⇨ $(x^2+y^2)=a^2(lx+my)^2$

⇨ $(x^2+y^2)=a^2[l^2x^2+m^2y^2+2lmxy]^2$

⇨ $(x^2+y^2)=a^2l^2x^2+a^2m^2y^2+a^22lmxy$

⇨ $x^2-a^2l^2x^2+y^2-a^2m^2y^2-a^22lmxy=0$

⇨ $(1-a^2l^2)\,x^2+(1-a^2m^2)y^2-a^22lmxy=0$

$\left[\begin{array}{c}compare\ with\\ ax^2+2hxy+by^2=0\end{array}\right]$

(i)∴coefficient of x^2 + coefficient of y^2=0

⇨$(1-a^2l^2)+(1-a^2m^2)=0$

⇨$2-a^2l^2-a^2m^2=0$

⇨$2=a^2l^2+a^2m^2$

∴$a^2(l^2+m^2)$=2

(ii) Condition for coincide $h^2=ab$

⇨ $(a^2lm)^2=(1-a^2l^2)(1-a^2m^2)$

⇨$a^4l^2m^2=1-a^2l^2-a^2m^2+a^4l^2m^2$

⇨$1=a^2l^2+a^2m^2$

$a^2(l^2+m^2)$=1

1. **If a ray makes angles α, β, γ and δ with four diagonals of a cube, then find $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = \frac{4}{3}$.**

Sol: let 'OABCDEFG' be the given cube with each edge '1' let 'O' be the origin and $\overline{OA}, \overline{OB}, \overline{OC}$ *as coordinate axes.*

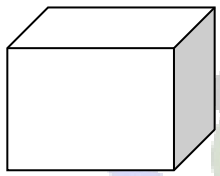
its vertices are

O(0, 0, 0)

A(**1**, 0, 0) D(1, 1, **0**)

B(0, **1**, 0) E(1, **0**, 1)

C(0, 0, **1**) F(**0**, 1, 1)



G (1, 1, 1)

D.r's of diagonals Dc's of diagonals

$[x_2 - x_1, y_2 - y_1, z_2 - z_1]$ $\left[\frac{x_2-x_1}{\sqrt{\sum(x_2-x_1)^2}}, ,, \quad ,,\right]$

Dr's of OG (1, 1, 1) Dc's of OG $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

Dr's of AF (−**1**, 1, 1) Dc's of AF $\left(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

Dr's of BE (1, −**1**, 1) Dc's of BE $\left(\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

Dr's of CD (1, 1, −**1**) Dc's of CD $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right)$

Let (l, m, n) be the DC's of the ray which make angles α, β, γ and δ with four diagonals of a cube OG, AF, BE and CD respectively.

Now $\cos\alpha = |l_1l_2 + m_1m_2 + n_1n_2|$

$\cos\alpha = \left|l\left(\frac{1}{\sqrt{3}}\right) + m\left(\frac{1}{\sqrt{3}}\right) + n\left(\frac{1}{\sqrt{3}}\right)\right| = \left|\frac{l+m+n}{\sqrt{3}}\right|$

Similarly we get, $\cos\beta = \left|\frac{-l+m+n}{\sqrt{3}}\right|$,

$\cos\gamma = \left|\frac{l-m+n}{\sqrt{3}}\right|$, $\cos\delta = \left|\frac{l+m-n}{\sqrt{3}}\right|$

Now $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta$

$= \left(\frac{l+m+n}{\sqrt{3}}\right)^2 + \left(\frac{-l+m+n}{\sqrt{3}}\right)^2 + \left(\frac{l-m+n}{\sqrt{3}}\right)^2 + \left(\frac{l+m-n}{\sqrt{3}}\right)^2$

$= \frac{4}{3}(l^2 + m^2 + n^2) = \frac{4}{3}(1) = \frac{4}{3}$

2. **Find the angle b/w the diagonals of a cube.**

Sol: Sol: let 'OABCDEFG' be the given cube with each edge '1' let 'O' be the origin and $\overline{OA}, \overline{OB}, \overline{OC}$ *as coordinate axes.*

its vertices are

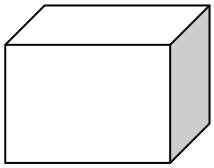
O(0, 0, 0)

A(**1**, 0, 0) D(1, 1, **0**)

B(0, **1**, 0) E(1, **0**, 1)

C(0, 0, **1**) F(**0**, 1, 1)

G (1, 1, 1)



Direction ratios $[x_2 - x_1, y_2 - y_1, z_2 - z_1,]$

D.r's of diagonals Dc's of diagonals

Dr's of OG (1, 1, 1) Dc's of OG $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

Dr's of AF (−**1**, 1, 1) Dc's of AF $\left(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

Dr's of BE (1, −**1**, 1) Dc's of BE $\left(\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

Dr's of CD (1, 1, −**1**) Dc's of CD $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right)$

If 'θ' is the angle between the diagonals OG and AF then

Cosθ $= |l_1l_2 + m_1m_2 + n_1n_2|$

$= \left|-\frac{1}{\sqrt{3}}\frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}}\frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}}\frac{1}{\sqrt{3}}\right|$

$= \left|-\frac{1}{3} + \frac{1}{3} + \frac{1}{3}\right|$

$\therefore \theta = \cos^{-1}\left(\frac{1}{3}\right)$

3. Find the angle b/w the lines whose direction cosines satisfy the eq"ns $l+m+n=0$, $l^2+m^2-n^2=0$.

Sol: Given eq"ns

$l+m+n=0 \ldots (1)$

$l^2+m^2-n^2=0. \ldots. (2)$

From (1) ⇨$l=-(m+n)$

Sub 'l' value in (2)

⇨ $(m+n)^2+m^2-n^2=0$

⇨ $m^2+n^2+2mn+m^2-n^2=0$

⇨ $2m^2+2mn=0$

⇨ $2m(m+n)=0$

⇨ $2m=0 \; and \; m+n=0$

$m=0 \ldots\ldots (3)$

$m+n=0 \ldots. (4).$

Solving (1) & (3)	Solving (1) & (4)
1 1 1 1 1 1 0 1 0 0 1 0	1 1 1 1 1 1 0 1 1 0 1 1
$\frac{l}{0-1}=\frac{m}{0-0}=\frac{n}{1-0}$	$\frac{l}{0-0}=\frac{m}{0-1}=\frac{n}{1-0}$
⇨$\frac{l}{-1}=\frac{m}{0}=\frac{n}{1}$	⇨$\frac{l}{0}=\frac{m}{-1}=\frac{n}{1}$
$Dr's \; of \; 1^{st} \; line \; (a_1,b_1,c_1)$ $=(-1,0,1)$	$Dr's \; of \; 2^{nd} \; line \; (a_2,b_2,c_2)$ $=(0,-1,1)$

$$\cos\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{{a_1}^2+{b_1}^2+{c_1}^2}\sqrt{{a_2}^2+{b_2}^2+{c_2}^2}}$$

$$⇨ \cos\theta=\frac{(-1)(0)+(0)(-1)+(1)(1)}{\sqrt{-1^2+0^2+1^2}\sqrt{0^2+-1^2+1^2}}$$

$$⇨ \cos\theta=\frac{0+0+1}{\sqrt{2}\sqrt{2}}$$

$$\cos\theta=\frac{1}{2}$$

$$\therefore \theta=60 \; or \; \frac{\pi}{3}$$

4. Find the angle b/w the lines whose direction cosines satisfy the eq"ns $3l+m+5n=0$, $6mn-2nl+5lm=0$.

Sol: Given eq"ns $3l+m+5n=0 \ldots (1)$

$6mn-2nl+5lm=0. \ldots. (2)$

From (1) ⇨$m=-(3l+5n)$

Sub 'm' value in (2)

⇨ $-6[3l+5n]n-2nl-5l[3l+5n]=0$

⇨ $-18ln-30n^2-2ln-15l^2-25ln=0$

⇨ $-15l^2-45ln-30n^2=0 \quad \div by -15$

⇨ $l^2+3ln+2n^2=0$

⇨ $l^2+2ln+ln+2n^2=0$

⇨$l(l+2n)+n(l+2n)=0$

⇨ $(l+2n)(l+n)=0$

$(l+2n)=0 \ldots\ldots (3)$

$(l+n)=0 \ldots. (4)$

Solving (1) & (3)	Solving (1) & (4)
3 1 5 3 1 5 1 0 2 1 0 2	3 1 5 3 1 5 1 0 1 1 0 1
$\frac{l}{2-0}=\frac{m}{5-6}=\frac{n}{0-1}$	$\frac{l}{1-0}=\frac{m}{5-3}=\frac{n}{0-1}$
⇨$\frac{l}{2}=\frac{m}{-1}=\frac{n}{-1}$	⇨$\frac{l}{1}=\frac{m}{2}=\frac{n}{-1}$
$Dr's \; of \; 1^{st} \; line \; (a_1,b_1,c_1)$ $=(2,-1,-1)$	$Dr's \; of \; 2^{nd} \; line \; (a_2,b_2,c_2)$ $=(1,2,-1)$

$$\cos\theta=\frac{|a_1a_2+b_1b_2+c_1c_2|}{\sqrt{{a_1}^2+{b_1}^2+{c_1}^2}\sqrt{{a_2}^2+{b_2}^2+{c_2}^2}}$$

$$⇨ \cos\theta=\frac{(2)(1)+(-1)(2)+(-1)(-1)}{\sqrt{2^2+1^2+1^2}\sqrt{1^2+2^2+1^2}}$$

$$⇨ \cos\theta=\frac{2-2+1}{\sqrt{6}\sqrt{6}} = \cos\theta=\frac{1}{6}$$

$$\therefore \theta=\cos^{-1}\left(\frac{1}{6}\right)$$

5. Show that the lines whose d.c's are given by $l + m + n = 0, 2mn + 3nl - 5lm = 0$.

Sol: given eq''ns

$l + m + n = 0 \ldots\ldots (1)$

$2mn + 3nl - 5lm = 0 \ldots\ldots (2)$

From (1) ⇨$l = -(m + n)$

Sub 'l' value in (2)

⇨$2mn - 3n(m + n) + 5(m + n)m = 0$

⇨$2mn - 3mn - 3n^2 + 5m^2 + 5mn = 0$

⇨$5m^2 + 4mn - 3n^2 = 0 \qquad \div by\ n^2$

⇨ $5\left[\frac{m}{n}\right]^2 + 4\left[\frac{m}{n}\right] - 3 = 0$

Let $\frac{m_1}{n_1}, \frac{m_2}{n_2}$ be the roots

Product of the roots $= \frac{c}{a}$

⇨ $\left[\frac{m_1}{n_1}\right]\left[\frac{m_2}{n_2}\right] = -\frac{3}{5}$ ⇨$\frac{m_1 m_2}{-3} = \frac{n_1 n_2}{5} \ldots\ldots (A)$

From (1) ⇨$m = -(l + n)$

Sub 'm' value in (2)

⇨$-2(l + n)n + 3nl + 5l(l + n) = 0$

⇨$-2ln - 2n^2 + 3nl + 5l^2 + 5ln = 0$

⇨$5l^2 + 6ln - 2n^2 = 0 \qquad \div by\ n^2$

⇨ $5\left[\frac{l}{n}\right]^2 + 6\left[\frac{l}{n}\right] - 2 = 0$

Let $\frac{l_1}{n_1}, \frac{l_2}{n_2}$ be the roots

Product of the roots $= \frac{c}{a}$

⇨ $\left[\frac{l_1}{n_1}\right]\left[\frac{l_2}{n_2}\right] = \frac{2}{5}$ ⇨$\frac{l_1 l_2}{-2} = \frac{n_1 n_2}{5} \ldots\ldots (B)$

From (A) & (B)

⇨$\frac{l_1 l_2}{-2} = \frac{m_1 m_2}{-3} = \frac{n_1 n_2}{5} = k(say)$

⇨$l_1 l_2 = -2k, m_1 m_2 = -3k, n_1 n_2 = 5k$

$\text{Cos}\theta = |l_1 l_2 + m_1 m_2 + n_1 n_2|$

$= -2k - 3k + 5k$

$= 0$

$\theta = 90°$

$\therefore$ *the two lines are perpendicular to each other.*

6. Find the direction cosines of the lines which are connected by the relation $l + m + n = 0$, $mn - 2nl - 2lm = 0$.

Sol: Given eq''ns

$l + m + n = 0 \ldots (1)$

$mn - 2nl - 2lm = 0. \ldots. (2)$

From (1) ⇨$l = -(m + n)$

Sub 'l' value in (2)

⇨ $mn + 2n(m + n) + 2(m + n)m = 0$

⇨ $mn + 2mn + 2n^2 + 2m^2 + 2mn = 0$

⇨ $2m^2 + 5mn + 2n^2 = 0$

⇨ $2m^2 + 4mn + mn + 2n^2 = 0$

⇨ $2m(m + 2n) + n(m + 2n) = 0$

⇨ $(m + 2n)(2\,m + n) = 0$

$m + 2n = 0 \ldots\ldots (3)$

$2m + n = 0 \ldots. (4)$

Solving (1) & (3)	Solving (1) & (4)
1 1 1 1 1 1 0 1 2 0 1 2	1 1 1 1 1 1 0 2 1 0 2 1
$\frac{l}{2-1} = \frac{m}{0-2} = \frac{n}{1-0}$	$\frac{l}{1-2} = \frac{m}{0-1} = \frac{n}{2-0}$
⇨$\frac{l}{1} = \frac{m}{-2} = \frac{n}{1}$	⇨$\frac{l}{-1} = \frac{m}{-1} = \frac{n}{2}$
$Dr's\ of\ 1^{st}\ line\ (a_1, b_1, c_1)$ $= (1, -2, 1)$	$Dr's\ of\ 2^{nd}\ line\ (a_2, b_2, c_2)$ $= (-1, -1, 2)$
$Dc's\ of\ 1^{st}\ line\ (l_1, m_1, n_1)$ $= (\frac{1}{\sqrt{6}}, -\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}})$	$Dc's\ of\ 2^{nd}\ line\ (l_2, m_2, n_2)$ $= (\frac{-1}{\sqrt{6}}, -\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}})$

7. Find the direction cosines of the lines which are connected by the relation $l - 5m + 3n = 0$, $7l^2 + 5m^2 - 3n^2 = 0$.

Sol: Given eq''ns

$l - 5m + 3n = 0 \ldots (1)$

$7l^2 + 5m^2 - 3n^2 = 0. \ldots. (2)$

From (1) ⇨$l = -(5m - 3n)$

Sub 'l' value in (2)

⇨ $7[5m - 3n]^2 + 5m^2 - 3n^2 = 0$

⇨ $7[25m^2 + 9n^2 - 30mn] + 5m^2 - 3n^2 = 0$

⇨ $175m^2 + 63n^2 - 210mn + 5m^2 - 3n^2 = 0$

⇨ $180m^2 - 210mn + 60n^2 = 0\ (\div by\ 30)$

⇨ $6m^2 - 7mn + 2n^2 = 0$

⇨ $6m^2 - 4mn - 3mn + 2n^2 = 0$

⇨ $2m(3m - 2n) — n(3m - 2n) = 0$

⇨ $(3m - 2n)(2m - n) = 0\ (3m - 2n) = 0 \quad or \quad (2m - n) = 0$

$(0 + 3m - 2n) = 0 \ldots\ldots (3)$

$(0 + 2m - n) = 0 \ldots\ldots (4)$

Solving (1) & (3)	Solving (1) & (4)
1 -5 3 1 -5 3 0 3 -2 0 3 -2	1 -5 3 1 -5 3 0 2 -1 0 2 -1
$\frac{l}{10-9} = \frac{m}{0+2} = \frac{n}{3+0}$	$\frac{l}{5-6} = \frac{m}{0+1} = \frac{n}{2-0}$
⇨$\frac{l}{1} = \frac{m}{2} = \frac{n}{3}$	⇨$\frac{l}{-1} = \frac{m}{1} = \frac{n}{2}$
$Dr's\ of\ 1^{st}\ line\ (a_1, b_1, c_1)$ $= (1, 2, 3)$	$Dr's\ of\ 2^{nd}\ line\ (a_2, b_2, c_2)$ $= (-1, 1, 2)$
$Dc's\ of\ 1^{st}\ line\ (l_1, m_1, n_1)$ $= (\frac{1}{\sqrt{14}}, -\frac{2}{\sqrt{14}}, \frac{1}{\sqrt{14}})$	$Dc's\ of\ 2^{nd}\ line\ (l_2, m_2, n_2)$ $= (\frac{-1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}})$

8. $S.T$ the points $(4, 7, 8), (2, 3, 4), (-1, -2, 1)$, $(1, 2, 5)$ are the vertices of a parallelogram.

Sol: *let the* Given *points*

$A(4, 7, 8), B(2, 3, 4), C(-1, -2, 1)$, D$(1, 2, 5)$

$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

$A(4, 7, 8) \quad B(2, 3, 4)$

$AB = \sqrt{(2 - 4)^2 + (3 - 7)^2 + (4 - 8)^2}$

$= \sqrt{(-2)^2 + (-4)^2 + (-4)^2}$

$= \sqrt{4 + 16 + 16} = \sqrt{36} = 6$

$B(2, 3, 4) \quad C(-1, -2, 1)$

$BC = \sqrt{(-1 - 2)^2 + (-2 - 3)^2 + (1 - 4)^2}$

$= \sqrt{(-3)^2 + (-5)^2 + (-3)^2}$

$BC = \sqrt{9 + 25 + 9} = \sqrt{43}$

$C(-1, -2, 1) \quad D(1, 2, 5)$

$CD = \sqrt{(1 + 1)^2 + (2 + 2)^2 + (5 - 1)^2}$

$CD = \sqrt{(2)^2 + (4)^2 + (4)^2}$

$CD = \sqrt{4 + 16 + 16} = \sqrt{36} = 6$

$D(1, 2, 5) \quad A(4, 7, 8)$

$DA = \sqrt{(4 - 1)^2 + (7 - 2)^2 + (8 - 5)^2}$

$DA = \sqrt{(3)^2 + (5)^2 + (5)^2}$

$DA = \sqrt{9 + 25 + 9} = \sqrt{43}$

$AB = CD, BC = DA$

$\therefore A, B, C, D$ *are the virtices of the parallelogram.*

9. $If\ (6,10,10),(1,0,-5),(6,-10,0)$ Are vertices of a triangle find the D.r's of its sides. Determine whether it is right angled or isosceles.

Sol: $let\ A(6,10,10), B(1,0,-5), C(6,-10,0)$

Are vertices of a triangle

Direction ratios $[x_2-x_1, y_2-y_1, z_2-z_1]$

$A(6,6,10)\quad B(1,0,-5)$

D.r's $of\ AB \Rightarrow [1-6, 0-10, -5-10]$

$= [-5,-10,-15] = [1,2,3]$

$B(1,0,-5), C(6,-10,0)$

D.r's $of\ BC \Rightarrow [6-1, -10-0, 0+5]$

$= [5,-10,5] = [1,-2,1]$

$A(6,10,10)\quad C(6,-10,0)$

D.r's $of\ AC \Rightarrow [6-6, -10-10, 0-10]$

$= [0,-20,-10] = [0,2,1]$

$\cos\angle ABC = \frac{|1(1+2(-2)+3(1)|}{\sqrt{1^2+2^2+3^2}\sqrt{1^2+(-2)^2+1^2}}$

$\cos\angle ABC = \frac{|1-4+3|}{\sqrt{1+4+9}\sqrt{1+4+1}}$

$\cos\angle ABC = \frac{0}{\sqrt{14}\sqrt{6}} = 0$

$\llcorner B = \frac{\pi}{2}$

$\therefore The\ given\ triangle\ is\ a\ right\ angled.$

1. ***If*** $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$, ***then P.T*** $\frac{dy}{dx}=\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$.

Sol: *let*

$x = sinA \Rightarrow A = \sin^{-1} x$

$y = sinB \Rightarrow B = \sin^{-1} y$

$\Rightarrow \sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$

$\Rightarrow \sqrt{1-sin^2A}+\sqrt{1-sin^2B}=a(sinA-sinB)$

$\Rightarrow \sqrt{cos^2A}+\sqrt{cos^2B}=a(sinA-sinB)$

$\Rightarrow cosA+cosB=a(sinA-sinB)$

$\Rightarrow 2cos\left[\frac{A+B}{2}\right]cos\left[\frac{A-B}{2}\right]=a2cos\left[\frac{A+B}{2}\right]sin\left[\frac{A-B}{2}\right]$

$\Rightarrow cos\left[\frac{A-B}{2}\right]=asin\left[\frac{A+B}{2}\right]$

$\Rightarrow \frac{cos\left[\frac{A-B}{2}\right]}{sin\left[\frac{A-B}{2}\right]}=a$

$\Rightarrow cot\left[\frac{A-B}{2}\right]=a$

$\Rightarrow \frac{A-B}{2}=\cot^{-1}(a)$

$\Rightarrow A-B=2\cot^{-1}(a)$

$\Rightarrow \sin^{-1}x-\sin^{-1}y=2\cot^{-1}(a)$

$diff\ w.r.t\ 'x'$

$\Rightarrow \frac{1}{\sqrt{1-x^2}}-\frac{1}{\sqrt{1-y^2}}\frac{dy}{dx}=0$

$\Rightarrow -\frac{1}{\sqrt{1-y^2}}\frac{dy}{dx}=-\frac{1}{\sqrt{1-x^2}}$

$\Rightarrow \frac{dy}{dx}=\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$

2. ***If*** $y=\tan^{-1}\left[\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right]$***,find*** $\frac{dy}{dx}$.

Sol:*let* $x^2=cos2\theta \Rightarrow \theta=\frac{1}{2}\cos^{-1}(x^2)$

$y=\tan^{-1}\left[\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right]$

$y=\tan^{-1}\left[\frac{\sqrt{1+cos2\theta}+\sqrt{1-cos2\theta}}{\sqrt{1+cos2\theta}-\sqrt{1-cos2\theta}}\right]$

$y=\tan^{-1}\left[\frac{\sqrt{2cos^2\theta}+\sqrt{2sin^2\theta}}{\sqrt{2cos^2\theta}-\sqrt{2sin^2\theta}}\right]$

$y=\tan^{-1}\left[\frac{\sqrt{2}(cos\theta+sin\theta)}{\sqrt{2}(cos\theta-sin\theta)}\right]$ $[\div\ by\ cos\theta]$

$y=\tan^{-1}\left[\frac{\frac{cos\theta}{cos\theta}+\frac{sin\theta}{cos\theta}}{\frac{cos\theta}{cos\theta}-\frac{sin\theta}{cos\theta}}\right]$

$y=\tan^{-1}\left[\frac{1+tan\theta}{1-tan\theta}\right]$

$y=\tan^{-1}\left[tan\left(\frac{\pi}{4}+\theta\right)\right]$

$y=\frac{\pi}{4}+\theta$

$y=\frac{\pi}{4}+\frac{1}{2}\cos^{-1}(x^2)$

$diff\ w.r.t\ 'x'$

$\frac{dy}{dx}=0+\frac{1}{2}\left[-\frac{1}{\sqrt{1-(x^2)^2}}\right]\frac{d}{dx}(x^2)$

$\frac{dy}{dx}=0+\frac{1}{2}\left[-\frac{1}{\sqrt{1-(x^2)^2}}\right](2x)$

$\frac{dy}{dx}=\left[\frac{-x}{\sqrt{1-x^4}}\right]$

3. $\boldsymbol{If\ y = x^{tanx} + (sinx)^{cosx}, find\ \frac{dy}{dx}}$.

Sol: $let\ y = P + Q \quad diff\ w.r.t\ 'x'$

$\frac{dy}{dx} = \frac{dP}{dx} + \frac{dQ}{dx} \ldots (1)$

$where\ P = x^{tanx}\ Applying \log\ \ on\ B.S$

$\Rightarrow \log P = \log x^{tanx}$

$\Rightarrow \log P = \tan x . \log x\ \ [(uv)' = uv' + vu']$

$diff\ w.r.t\ 'x'$

$\Rightarrow \frac{1}{P}\frac{dP}{dx} = \tan x\ \frac{d}{dx}(\log x) + \log x \frac{d}{dx}(\tan x)$

$\Rightarrow \frac{1}{P}\frac{dP}{dx} = \tan x . \frac{1}{x} + \log x . sec^2 x$

$\Rightarrow \frac{dP}{dx} = P\left[\frac{tanx}{x} + \log x . sec^2 x\right]$

$\Rightarrow \frac{dP}{dx} = x^{tanx}\left[\frac{tanx}{x} + \log x . sec^2 x\right] \ldots (2)$

$where\ Q = sinx^{cosx}\ Applying \log\ \ on\ B.S$

$\Rightarrow \log Q = \log sinx^{cosx}$

$\Rightarrow \log Q = \cos x . \log(\sin x)\ \ [(uv)' = uv' + vu']$

$diff\ w.r.t\ 'x'$

$\Rightarrow \frac{1}{Q}\frac{dQ}{dx} = \cos x\ \frac{d}{dx}[\log(sinx)] + \log(sinx)\frac{d}{dx}(cosx)$

$\Rightarrow \frac{1}{Q}\frac{dQ}{dx} = \cos x . \frac{1}{sinx} . cosx + \log sinx . (-sinx)$

$\Rightarrow \frac{dQ}{dx} = Q\left[\frac{cos^2 x}{sinx} - \log(sinx) . sinx\right]$

$\Rightarrow \frac{dQ}{dx} = sinx^{cosx}\left[\frac{cos^2 x}{sinx} - \log(sinx) . sinx\right] \ldots (3)$

$sub\ eq''n(2)\&\ (3) in\ (1)$

$\frac{dy}{dx} = \frac{dP}{dx} + \frac{dQ}{dx}$

$\frac{dy}{dx} = x^{tanx}\left[\frac{tanx}{x} + \log x . sec^2 x\right]$

$+ sinx^{cosx}\left[\frac{cos^2 x}{sinx} - \log(sinx) . sinx\right]$

4. $\boldsymbol{If\ y = (sinx)^{\log x} + x^{sinx}, find\ \frac{dy}{dx}}$.

Sol: $let\ y = P + Q \quad diff\ w.r.t\ 'x'$

$\frac{dy}{dx} = \frac{dP}{dx} + \frac{dQ}{dx} \ldots (1)$

$where\ P = sinx^{logx}\ Applying \log\ \ on\ B.S$

$\Rightarrow \log P = \log sinx^{logx}$

$\Rightarrow \log P = \log x . \log \sin x\ \ [(uv)' = uv' + vu']$

$diff\ w.r.t\ 'x'$

$\Rightarrow \frac{1}{P}\frac{dP}{dx} = \log x\ \frac{d}{dx}(\log sinx) + \log sinx \frac{d}{dx}(\log x)$

$\Rightarrow \frac{1}{P}\frac{dP}{dx} = \log x . \frac{1}{sinx} . cosx + \log sinx . \frac{1}{x}$

$\Rightarrow \frac{dP}{dx} = P\left[logx . cotx + (\log sinx)\frac{1}{x}\right]$

$\Rightarrow \frac{dP}{dx} = sinx^{logx}\left[logx . cotx + \frac{\log sinx}{x}\right] \ldots (2)$

$where\ Q = x^{sinx}\ Applying \log\ \ on\ B.S$

$\Rightarrow \log Q = \log x^{sinx}$

$\Rightarrow \log Q = \sin x . \log x\ \ [(uv)' = uv' + vu']$

$diff\ w.r.t\ 'x'$

$\Rightarrow \frac{1}{Q}\frac{dQ}{dx} = \sin x\ \frac{d}{dx}[\log x] + \log x \frac{d}{dx}(sinx)$

$\Rightarrow \frac{1}{Q}\frac{dQ}{dx} = \sin x . \frac{1}{x} + \log x . (cosx)$

$\Rightarrow \frac{dQ}{dx} = Q\left[\frac{sinx}{x} + \log x . cosx\right]$

$\Rightarrow \frac{dQ}{dx} = x^{sinx}\left[\frac{sinx}{x} + \log x . cosx\right] \ldots (3)$

$sub\ eq''n(2)\&\ (3) in\ (1)$

$\frac{dy}{dx} = \frac{dP}{dx} + \frac{dQ}{dx}$

$\frac{dy}{dx} = sinx^{logx}\left[logx . cotx + \frac{\log sinx}{x}\right]$

$+ x^{sinx}\left[\frac{sinx}{x} + \log x . cosx\right]$

Aims tutorial

5. $\boldsymbol{If\ x^y + y^x = a^b, S.T\ \frac{dy}{dx} = -\left[\frac{yx^{y-1}+y^x \log y}{x^y \log x + xy^{x-1}}\right]}$

$Sol: let\ P = x^y, Q = y^x$

$\Rightarrow P + Q = a^b\ \ diff\ w.r.t\ 'x'$

$\frac{dP}{dx} + \frac{dQ}{dx} = 0 \ldots (1)$

$where\ P = x^y\ Applying \log\ \ on\ B.S$

$\Rightarrow \log P = \log x^y$

$\Rightarrow \log P = \mathrm{y}.\log \mathrm{x}\ \ [(uv)' = uv' + vu']$

$diff\ w.r.t\ 'x'$

$\Rightarrow \frac{1}{P}\frac{dP}{dx} = \mathrm{y}\ \frac{d}{dx}(\log x) + \log x \frac{d}{dx}(y)$

$\Rightarrow \frac{1}{P}\frac{dP}{dx} = \mathrm{y}\ .\frac{1}{x} + \log x\,.\frac{dy}{dx}$

$\Rightarrow \frac{dP}{dx} = P\left[y.x^{-1} + \log x\,.\frac{dy}{dx}\right]$

$\Rightarrow \frac{dP}{dx} = x^y\left[y.x^{-1} + \log x\,.\frac{dy}{dx}\right]$

$\Rightarrow \frac{dP}{dx} = \left[y.x^{y-1} + x^y \log x\,.\frac{dy}{dx}\right] \ldots (2)$

$where\ Q = y^x\ Applying \log\ \ on\ B.S$

$\Rightarrow \log Q = \log y^x$

$\Rightarrow \log Q = \mathrm{x}.\log \mathrm{y}\ \ [(uv)' = uv' + vu']$

$diff\ w.r.t\ 'x'$

$\Rightarrow \frac{1}{Q}\frac{dQ}{dx} = \mathrm{x}\ \frac{d}{dx}(\log y) + \log y \frac{d}{dx}(x)$

$\Rightarrow \frac{1}{Q}\frac{dQ}{dx} = \mathrm{x}\ .\frac{1}{y}\frac{dy}{dx} + \log y\,.1$

$\Rightarrow \frac{dQ}{dx} = Q\left[\mathrm{x}\ .y^{-1}\frac{dy}{dx} + \log y\right]$

$\Rightarrow \frac{dQ}{dx} = y^x\left[\mathrm{x}\ .y^{-1}\frac{dy}{dx} + \log y\right]$

$\Rightarrow \frac{dQ}{dx} = \left[\mathrm{x}\ .y^{x-1}\frac{dy}{dx} + y^x \log y\right] \ldots (3)$

$sub\ eq''n(2)\&\ (3) in\ (1)$

$\frac{dP}{dx} + \frac{dQ}{dx} = 0$

$$y.x^{y-1} + x^y \log x\,.\frac{dy}{dx} + \mathrm{x}\ .y^{x-1}\frac{dy}{dx} + y^x \log y = 0$$

$$\frac{dy}{dx}(x^y \log x + xy^{x-1}) + (yx^{y-1} + y^x \log y) = 0$$

$$\frac{dy}{dx} = -\left[\frac{yx^{y-1}+y^x \log y}{x^y \log x + xy^{x-1}}\right]$$

6. $\boldsymbol{If\ y = x\sqrt{a^2+x^2} + a^2 \log\left[x + \sqrt{a^2+x^2}\right], find \frac{dy}{dx}.}$

$sol: y = x\sqrt{a^2+x^2} + a^2 \log\left[x + \sqrt{a^2+x^2}\right]$

$diff\ w.r.t\ 'x'$

$$\Rightarrow \frac{dy}{dx} = x.\frac{d}{dx}\left(\sqrt{a^2+x^2}\right) + \sqrt{a^2+x^2}.\frac{dx}{dx} + a^2.\frac{d}{dx}\left\{\log\left[x + \sqrt{a^2+x^2}\right]\right\}$$

$$\Rightarrow \frac{dy}{dx} = x.\frac{1}{2\sqrt{a^2+x^2}}\frac{d}{dx}(a^2+x^2) + \sqrt{a^2+x^2}.1 + a^2.\left\{\frac{1}{\left[x+\sqrt{a^2+x^2}\right]}\right\}\frac{d}{dx}\left[x+\sqrt{a^2+x^2}\right]$$

$$\Rightarrow \frac{dy}{dx} = x.\frac{1}{2\sqrt{a^2+x^2}}(0+2x) + \sqrt{a^2+x^2} + a^2.\left\{\frac{1}{\left[x+\sqrt{a^2+x^2}\right]}\right\}\left[1 + \frac{1}{2\sqrt{a^2+x^2}}(0+2x)\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{x^2}{\sqrt{a^2+x^2}} + \sqrt{a^2+x^2} + a^2.\left\{\frac{1}{\left[x+\sqrt{a^2+x^2}\right]}\right\}\left[\frac{\sqrt{a^2+x^2}+x}{\sqrt{a^2+x^2}}\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{x^2}{\sqrt{a^2+x^2}} + \sqrt{a^2+x^2} + \frac{a^2}{\sqrt{a^2+x^2}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{a^2+x^2}{\sqrt{a^2+x^2}} + \sqrt{a^2+x^2}$$

$$\Rightarrow \frac{dy}{dx} = \sqrt{a^2+x^2} + \sqrt{a^2+x^2}$$

$$\therefore \frac{dy}{dx} = 2\sqrt{a^2+x^2}$$

7. $\boldsymbol{If\ f(x) = \sin^{-1}\sqrt{\frac{x-\beta}{\alpha-\beta}}}$ **&**

$\boldsymbol{g(x) = \tan^{-1}\left[\sqrt{\frac{x-\beta}{\alpha-x}}\right], then\ P.T}$

$\boldsymbol{f'(x) = g'(x)}$

Sol: *let* $\theta = \sin^{-1}\sqrt{\frac{x-\beta}{\alpha-\beta}}$ ... (1)

$\Rightarrow \sin\theta = \sqrt{\frac{x-\beta}{\alpha-\beta}}$

and *we know that*

$\boxed{cos\theta = \sqrt{1-sin^2\theta}}$

$\Rightarrow cos\theta = \sqrt{1-\left[\sqrt{\frac{x-\beta}{\alpha-\beta}}\right]^2}$

$\Rightarrow cos\theta = \sqrt{1-\frac{x-\beta}{\alpha-\beta}}$

$\Rightarrow cos\theta = \sqrt{\frac{\alpha-\beta-x+\beta}{\alpha-\beta}}$

$\Rightarrow cos\theta = \sqrt{\frac{\alpha-x}{\alpha-\beta}}$

$Now\ \ tan\theta = \frac{sin\theta}{cos\theta}$

$\Rightarrow tan\theta = \frac{\sqrt{\frac{x-\beta}{\alpha-\beta}}}{\sqrt{\frac{\alpha-x}{\alpha-\beta}}}$

$\Rightarrow tan\theta = \sqrt{\frac{x-\beta}{\alpha-x}}$

$\Rightarrow \theta = \tan^{-1}\sqrt{\frac{x-\beta}{\alpha-x}} = g(x)$

$\Rightarrow \theta = g(x) \ldots.(2)$

$from\ eq''n(1)\&\ (2)$

$f(x) = g(x)$

$diff\ w.r.t\ 'x'$

$\Rightarrow f'(x) = g'(x)$

8. $\boldsymbol{If\ y = \frac{(1-2x)^{\frac{2}{3}}(1+3x)^{\frac{3}{4}}}{(1-6x)^{\frac{5}{6}}(1+7x)^{-\frac{6}{7}}}, find\ \frac{dy}{dx}}$.

$Sol: y = \frac{(1-2x)^{\frac{2}{3}}(1+3x)^{\frac{3}{4}}}{(1-6x)^{\frac{5}{6}}(1+7x)^{-\frac{6}{7}}}$

$applying\ \log on\ B.S$

$\boxed{\log\left[\frac{a.b}{c.d}\right] = \log a + \log b - \log c - \log d}$

$\Rightarrow \log y = \log\left[\frac{(1-2x)^{\frac{2}{3}}(1+3x)^{\frac{3}{4}}}{(1-6x)^{\frac{5}{6}}(1+7x)^{-\frac{6}{7}}}\right]$

$\Rightarrow \log y = \log(1-2x)^{\frac{2}{3}} + \log(1+3x)^{\frac{3}{4}}$ $\boxed{\log x^m = m\log x}$

$-\log(1-6x)^{\frac{5}{6}} - \log(1+7x)^{-\frac{6}{7}}$

$\Rightarrow \log y = \frac{2}{3}\log(1-2x) + \frac{3}{4}\log(1+3x)$ $\boxed{\frac{d}{dx}[\log x] = \frac{1}{x}}$

$-\frac{5}{6}\log(1-6x) + \frac{6}{7}\log(1+7x)$

$diff\ w.r.t\ 'x'$

$\Rightarrow \frac{1}{y}.\frac{dy}{dx} = \frac{2}{3}.\frac{1}{1-2x}(-2) + \frac{3}{4}.\frac{1}{1+3x}(3)$

$-\frac{5}{6}.\frac{1}{1-6x}(-6) + \frac{6}{7}.\frac{1}{1+7x}.(7)$

$\frac{dy}{dx} = y\left[-\frac{4}{3}\frac{1}{(1-2x)} + \frac{9}{4}\frac{1}{(1+3x)} + \frac{5}{(1-6x)} + \frac{1}{(1+7x)}\right]$

10. If $\boldsymbol{y = \tan^{-1}\left[\frac{2x}{1-x^2}\right] + \tan^{-1}\left[\frac{3x-x^3}{1-3x^2}\right] - \tan^{-1}\left[\frac{4x-4x^3}{1-6x^2+x^4}\right]}$ **then show that** $\boldsymbol{\frac{dy}{dx} = \frac{1}{1+x^2}}$.

Sol:

$put\ x = \tan\theta \quad \Rightarrow \theta = \tan^{-1}x$

$\Rightarrow y = \tan^{-1}\left[\frac{2x}{1-x^2}\right] + \tan^{-1}\left[\frac{3x-x^3}{1-3x^2}\right] - \tan^{-1}\left[\frac{4x-4x^3}{1-6x^2+x^4}\right]$

$\Rightarrow y = \tan^{-1}\left[\frac{2\tan\theta}{1-tan^2\theta}\right] + \tan^{-1}\left[\frac{3x-tan^3\theta}{1-3tan^2\theta}\right] - \tan^{-1}\left[\frac{4\tan\theta-4tan^3\theta}{1-6tan^2\theta+tan^4\theta}\right]$

$\Rightarrow y = \tan^{-1}[\tan 2\theta] + \tan^{-1}[\tan 3\theta] - \tan^{-1}[\tan 4\theta]$

$\Rightarrow y = 2\theta + 3\theta - 4\theta = \theta \Rightarrow \theta = \tan^{-1}x$

$diff\ w.r.t\ 'x'$

$\Rightarrow \frac{dy}{dx} = \frac{1}{1+x^2}$

Aims tutorial

1. If the tangent at any point P on the curve $x^m y^n = a^{m+n}$ meets the coordinate axes in A and B then show that AP:BP is a constant.

2. If the tangent at any point on the curve $x^{2/3} + y^{2/3} = a^{2/3}$ intersects the coordinate axes in A and B then show that length of AB is a constant.

3. Show that the curves $y^2 = 4(x+1)$ and $y^2 = 36(9-x)$ intersect orthogonally.

4. Find the angle b/w the curves $y^2 = 4x$ & $x^2 + y^2 = 5$.
5. Find the angle b/w the curves $y^2 = 8x$ & $4x^2 + y^2 = 8$.
6. Find the angle b/w the curves $x + y + 2 = 0$ & $x^2 + y^2 - 10y = 0$.
7. Find the angle b/w the curves $xy = 2$ & $x^2 + 4y = 0$

8. *show that the curves* $6x^2 - 5x + 2y = 0 = 5$ And $4x^2 + 8y^2 = 3$ touch each other at (1/2, 1/2).

9. At any point 't' on the curve $x = a(acost + tsint)$ $y = a(sin1 - tcost)$, find the length of tangent, normal, sub tangent and subnormal.

10. At any point 't' on the curve $x = a(t + sint)$ $y = a(1 - cost)$, find the length of tangent, normal, sub tangent and subnormal.

11. *show that the condition* for the orthogonality of the curves $ax^2 + by^2 = 1$ and $ax_1{}^2 + by_1{}^2 = 1$ Is $\frac{1}{a} - \frac{1}{b} = \frac{1}{a_1} - \frac{1}{b_1}$.
12. Show that square of the length of sub tangent at any point on the curve $by^2 = (x+a)^3$.

Q.NO.11

1. A (2, 3), B (-3, 4) are two points. Find the equation of locus of p so that the area of the triangle PAB is 8.5.

Sol: Let P(x, y) be any point on the locus.

Given two points A (2, 3), B (-3, 4)
Given geometric property: ΔPAB=8.5

$$\rightarrow \frac{1}{2}\begin{vmatrix} x & 2 & -3 & x \\ y & 3 & 4 & y \end{vmatrix}$$

$$\rightarrow \frac{1}{2}|3x - 2y + 8 + 9 - 3y - 4x| = 8.5$$

$$\rightarrow |-x - 5y + 17| = 17$$

$$\Rightarrow |x + 5y - 17| = 17$$
$(x + 5y - 17) = 17$ Or $(x + 5y - 17) = -17$
$(x + 5y - 17 - 17) = 0$ Or $(x + 5y - 17 + 17) = 0$
$(x + 5y - 34) = 0$ Or $(x + 5y) = 0$

∴ the locus of P $(x + 5y - 34)(x + 5y) = 0$

2. Find the equation of the locus of P, if the ratio of the distances from P to A(5, -4), and B(7, 6) is 2:3.
Sol: Let P(x, y) be any point on the locus.
Given two points A (5, -4), B (7, 6)
Given geometric property: AP: BP=2:3.
$\frac{AP}{PB} = \frac{2}{3}$
3AP=2PB S.O.B

$9AP^2 = 4PB^2$

$9[(x-5)^2+(y+4)^2] = 4[(x-7)^2+(y-6)^2]$

$\Rightarrow 9[x^2+25-10x+y^2+16+8y]$
$= 4[x^2+49-14x+y^2+36-12y]$

$\Rightarrow 9x^2+225-90x+9y^2+144+72y$
$= 4x^2+196-56x+4y^2+144-48y$

$\Rightarrow 5x^2+5y^2-34x+120y+120y+29 = 0$
∴ Locus of p is
$5x^2+5y^2-34x+120y+120y+29 = 0$

3. The ends of the hypotenuse of a right angled triangle are (0, 6), and (6, 0). Find the equation of its third vertex.

Sol: Let P(x, y) be the third vertex.
Given two points A (0, 6), B (6, 0)
Given geometric property: $\angle PAB=90^0$.
$AP^2+PB^2=AB^2$
$(x-0)^2+(y-6)^2+(x-6)^2+(y-0)^2$
$= (0-6)^2+(6-0)^2$
$\Rightarrow x^2+y^2+36-12y+x^2+36-12x+y^2$
$=36+36.$
$\Rightarrow 2x^2+2y^2-12x-12y=0.$
$\therefore$ Locus of p is $x^2+y^2-6x-6y=0$.

4. **A (1, 2), B (2, -3) and C (-2, 3) are three points. A point P moves such that $PA^2+PB^2=2PC^2$.show that the equation of the locus of P is 7x-7y +4=0.**

Sol: Let P(x, y) be any point on the locus.
A(1, 2), B(2, -3) and C(-2, 3)are three given points
Given geometric property: $PA^2+PB^2=2PC^2$
$[(x-1)^2+(y-2)^2+(x-2)^2+(y+3)^2]= [(x+2)^2+(y-3)^2]$
$\Rightarrow [x^2+1-2x+y^2+4-4y]+[x^2+4-4x+y^2+9+6y]$
$=2[x^2+4+4x+y^2+9-6y]$
$\Rightarrow 2x^2+2y^2-6x+2y+18=2x^2+2y^2+8x-12y+26$
$\Rightarrow -6x-8x+2y+12y+18-26=0.$
$\Rightarrow -14x+14y+8=0.$
$\Rightarrow 7x-7y+4=0.$
$\therefore$ Locus of p is $7x-7y+4=0$.

5. **Find the equation of locus of P, if A (4, 0), B (-4, 0) and |PA-PB|=4.**

Sol: Let P(x, y) be any point on the locus.
Given two points A(4, 0),B(-4, 0)
Given condition: |PA-PB|=4.
$\Rightarrow PA=4\pm PB$ S.O.B
$PA^2=(4\pm PB)^2$
$PA^2=(4^2+PB^2\pm 8PB).$
$PA^2-4^2-PB^2=\pm 8PB.$
$[(x-4)^2+(y-0)^2]-16-[(x+4)^2+(y-0)^2]=\pm 8PB.$
$\Rightarrow [X^2+16-8x+y^2]-16-[x^2+16+8x+y^2]=\pm 8PB.$
$-16x-16=\pm 8PB.$
$-8(2x+2)=\pm 8PB.$ S.O.B
$(2x+2)^2=PB^2$
$4x^2+4+8x=[(x+4)^2+(y-0)^2]$
$4x^2+4+8x=[x^2+16+8x+y^2]$
$3x^2-y^2=12$
$\therefore$ Locus of p is $3x^2-y^2=12$.

6. **Find the equation of locus of P, so that the distance from the origin is twice the distance of P from A (1, 2).**

Sol: Let P(x, y) be any point on the locus.
Given geometric property: OP=2AP

S.O.B $OP^2=4AP^2$

$\Rightarrow [(x-0)^2+(y-0)^2]=4[(x-1)^2+(y-2)^2]$

$\Rightarrow [x^2+y^2]=4[x^2+1+2.x.1+y^2+4+2.y.2]$

$\Rightarrow [x^2+y^2]=4x^2+4+8x+4y^2+16+16y]$

$\Rightarrow 3x^2+3y^2+8x+16y+20=0.$

7. **Find the equation of locus of P, if A (2, 3), B (2, -3) and |PA+PB|=8.**

Sol: Let P(x, y) be any point on the locus.

Given two points A (2, 3), B (2, -3)

Given condition: |PA+PB|=8.

$\Rightarrow PA = 8 \pm PB$ S.O.B

$PA^2 = (8 \pm PB)^2$

$PA^2 = (8^2 + PB^2 \pm 16PB)$.

$PA^2 - 8^2 - PB^2 = \pm 16PB$.

$\Rightarrow [(x-2)^2+(y-3)^2]-64-[(x-2)^2+(y+3)^2] = \pm 16PB$.

$\Rightarrow [X^2+4-4x+y^2+9-6y-64-[x^2+4-4x+y^2+9+6y]$
$= \pm 16PB$.

$\Rightarrow -12y-64 = \pm 16PB$.

$\Rightarrow -4(3y+16) = \pm 16PB$.
S.O.B
$\Rightarrow (3y+16)^2 = 16\ PB^2$

$\Rightarrow 9y^2+256+96y = 16[(x-2)^2+(y+3)^2]$

$\Rightarrow 9y^2+256+96y = 16[x^2+4-4x+y^2+9+6y]$

$\Rightarrow 16x^2-9y^2-64x-48=0$
$\therefore$ Locus of p is $16x^2-9y^2-64x-48=0$.

Q.No.12

1. **When the origin is shifted to the point (2, 3),the transformed equation of the curve is $X^2 + 3XY - 2Y^2 + 17X - 7Y - 11 = 0$. Find the original equation of the curve.**

sol: Transformed equation of the curve is
$$X^2 + 3XY - 2Y^2 + 17X - 7Y - 11 = 0$$
Here X=x-h=x-2 and Y=y-k=y-3
Original eq''n is

$\Rightarrow (x-2)^2 + 3(x-2)(y-3) - 2(y-3)^2 + 17(x-2) - 7(y-3) - 11 = 0$

$\Rightarrow x^2-4x+4+3xy-9x-6y+18-2y^2+12y-18+17x-34-7y+21-11=0$

$\Rightarrow x^2+3xy-2y^2+4x-y-20=0$.

2. **When the origin is shifted to the point (-1, 2), by the translation of axes. Find the transformed equation of $x^2 + y^2 + 2x - 4y + 1 = 0$.**

Sol: origin equation of the curve is
$x^2 + y^2 + 2x - 4y + 1 = 0$.
here x=X+h=X-1 and y=Y+k=Y+2
$(X-1)^2 + (Y+2)^2 + 2(X-1) - 4(Y+2) + 1 = 0$.
$\Rightarrow X^2+1-2X+Y^2+4+4Y+2X-2-4Y-8+1=0$.
$\therefore X^2 + Y^2 - 4 = 0$.

3. **When the axes are rotated through an angle 30^0,find the transformed equation of $x^2 + 2\sqrt{3}xy - y^2 = 2a^2$.**

sol: $x = X\cos\theta - Y\sin\theta$
and $y = X\sin\theta + Y\cos\theta$

$$given\ \theta = 3$$

Sin30 $= \frac{1}{2}$ and cos30$=\frac{\sqrt{3}}{2}$

$\Rightarrow$ x$=\frac{X\sqrt{3}}{2} - \frac{Y}{2} = \frac{\sqrt{3}X-Y}{2}$

$\Rightarrow$y$=\frac{X}{2} + \frac{y\sqrt{3}}{2} = \frac{X+\sqrt{3}Y}{2}$

Transformed equation is

$\Rightarrow [\frac{\sqrt{3}X-Y}{2}]^2+2\sqrt{3}[\frac{\sqrt{3}X-Y}{2}].[\frac{X+\sqrt{3}Y}{2}]-[\frac{X+\sqrt{3}Y}{2}]^2=2a^2$

$$\Rightarrow \frac{3X^2-2\sqrt{3}XY+Y^2}{4} + \frac{2\sqrt{3}[\sqrt{3}X^2-XY+3XY-\sqrt{3}Y^2]}{4} - [\frac{X^2+3Y^2+2\sqrt{3XY}}{4}] = 2a^2$$

$$\frac{3X^2-2\sqrt{3}XY+Y^2+6X^2+4\sqrt{3}XY-6Y^2-X^2-3Y^2-2\sqrt{3}XY}{4} = 2a^2$$

$\Rightarrow 8X^2-8Y^2=8a^2 \therefore X^2 - Y^2 = a^2$

4. **When the axes are rotated through an angle 45⁰,find the transformed equation of $3x^2+10xy+3y^2=9$.**

H/W

5. **When the axes are rotated through an angle 45⁰, the transformed equation of the curve is $17x^2-16xy+17y^2=225$.find the original equation of the curve.**

Sol: X=xcos$\theta + ysin\theta$ Y=-xsin$\theta + ycos\theta$

Given $\theta = 45$

Sin45 $= \frac{1}{\sqrt{2}}$ and cos30$=\frac{1}{\sqrt{2}}$

x$=x\frac{1}{\sqrt{2}} + y\frac{1}{\sqrt{2}} = \frac{x+y}{\sqrt{2}}$

$\Rightarrow$y$=\frac{-x}{\sqrt{2}} + \frac{y}{\sqrt{2}} = \frac{-x+y}{\sqrt{2}}$

the original equation of the curve is

$\Rightarrow 17X^2-16XY+17Y^2=0$

$\Rightarrow 17[\frac{x+y}{\sqrt{2}}]^2 - 16\left[\frac{x+y}{\sqrt{2}}\right]\left[\frac{-x+y}{\sqrt{2}}\right] + 17[\frac{-x+y}{\sqrt{2}}]^2 = 225$

$\Rightarrow \frac{17[x^2+y^2+2xy]}{2} - \frac{16[y^2-x^2]}{2} + \frac{17[x^2+y^2-2xy]}{2} = 225$

$\frac{\Rightarrow 17x^2+17y^2+34xy-16y^2+16x^2+17x^2+17y^2-34xy}{2}=225$

$\Rightarrow 50x^2+18y^2=450$

$\therefore 25x^2+9y^2=225$ Or $\frac{x^2}{9}+\frac{y^2}{25}=1$ is the original eq''n

6. **When the axes are rotated through an angle α, find the transformed equation of $x\cos\alpha + y\sin\alpha = p$.**

Sol: the given eq''n is xcos $\alpha + ysin\alpha = p$.

$\therefore$ the axes are rotated through an angle α

$\Rightarrow$x=Xcos $\alpha - Ysin\alpha$ $and\ y = Xsin\alpha + Ycos\alpha$

the transformed equation is

(Xcos $\alpha - Ysin\alpha$)cos α + $(Xsin\alpha + Ycos\alpha)sin\alpha = p$.

$\Rightarrow$Xcos² α -Ycos $\alpha sin\alpha$+Xsin² $\alpha + Ycos\alpha sin\alpha = p$

$\Rightarrow$ Xcos² α + Xsin² $\alpha = p$

$\Rightarrow$X (cos² α + sin² α) $= p$

$\therefore$ The transformed eq''n is X=p.

7. **Show that the axes are rotated through an angle of $\frac{1}{2}\tan^{-1}\left[\frac{2h}{a-b}\right]$ so as to remove the xy terms from the equation $ax^2+2hxy+by^2=0, if\ a \neq b$ and through the angle$\frac{\pi}{4}$, if a=b.**

Sol: If the axes are rotated through an angle $'\alpha, then$

$\Rightarrow$ x=Xcos $\alpha - Ysin\alpha$ $and\ y = Xsin\alpha + Ycos\alpha$

Therefore the given eq''n transformed as

$\Rightarrow$a(Xcos $\alpha - Ysin\alpha$)²
+ 2h [Xcos $\alpha - Ysin\alpha$][$Xsin\alpha + Ycos\alpha$]
+b$(Xsin\alpha + Ycos\alpha)^2$=0

To remove XY terms from the eq''n, the coeff of XY term must be zero.

-2XYcos$\alpha.sin\alpha$ +2hXYcos²α-2hXYsin²α+2bXycosα.sinα=0

$\Rightarrow$i.e, (b-a) sin$\alpha.cos\alpha$ + h(cos²$\alpha - sin2\alpha$)=0

$\Rightarrow$hcos2$\alpha = \frac{(a-b)}{2}sin2\alpha$

$\Rightarrow$tan2$\alpha = \frac{2h}{a-b}$

$\Rightarrow \alpha = \frac{1}{2}\tan^{-1}\frac{2h}{a-b}$

If a=b $\Rightarrow h\cos 2\alpha = 0$
$\Rightarrow 2\alpha = 90 \quad \therefore \alpha = 45$

Aims tutorial

Q.No.13

1. **Find the value of k, if the lines 2x-3y+k=0, 3x-4y-13=0 and 8x-11y-33=0 are concurrent.**

Sol: given lines are

L_1:2x-3y+k=0…… (1)

L_2: 3x-4y-13=0 …….. (2)

L_3:8x-11y-33=0……(3)

Solving (2) and (3)

3	-4	-13	3
8	-11	-33	8

$(X, y)=(\frac{132-143}{-33+32}, \frac{-104+99}{-33+32})$

= (11, 5)
Point of intersection of line (2) and (3) is (11, 5)
Since (1), (2) and (3) are concurrent (11, 5) ∈ (1)
⇒2x-3y+k=0
⇒2 (11)-3(5) =-k ⇒22-15=-k
∴k=-7

2. **If the straight lines ax + by + c =0, bx + cy + a=0 and cx + ay +b=0 are concurrent, then prove that $a^3+b^3+c^3=3abc$.**

Sol: given lines
ax + by + c =0
bx + cy + a=0
cx + ay +b=0 are concurrent

$$\begin{matrix} a & b & c \\ b & c & a \\ c & a & b \end{matrix} = 0$$

⇒a (bc-a^2)-b (b^2-ac) +c (ab-c^2) =0
⇒abc – a^3 – b^3 +abc + abc – c^3 =0
3abc – a^3 – b^3 – c^3 =0
∴ a^3 + b^3 + c^3 =3abc

3. **Find the value of k, if the angle between the straight lines**
4x – y+ 7=0 and kx -5y -9=0 is 45.

Sol: given lines4x – y+ 7=0 and kx -5y -9=0
Here a_1=4, b_1=-1, a_2=k and b_2=-5

$$\cos\theta=\frac{|a_1a_2+b_1b_2|}{\sqrt{{a_1}^2+{b_1}^2}.\sqrt{{a_2}^2+{b_2}^2}}$$

$$=\frac{|4k+5|}{\sqrt{4^2+(-1)^2}.\sqrt{k^2+(-5)^2}}$$

$$\Rightarrow\frac{|4k+5|}{\sqrt{17}.\sqrt{k^2+25}}=\frac{1}{\sqrt{2}} \qquad \{\therefore\cos 45 = \frac{1}{\sqrt{2}}\}$$

S.O.B
⇒2 $(4k+5)^2$=17(k^2 + 25)
⇒2($16k^2$+25+40k) =17(k^2 +25)
⇒$32k^2$-$17k^2$ +80k-50k+50-425=0
⇒$15k^2$ +80k - 375=0 (÷ 5)
⇒$13k^2$ +16k - 75=0
⇒ (k-3) (3k+25) =0
K=3 or k=-25/3.

4. **Find the equation of lines passing from point of intersection of lines 3x + 2y + 4 = 0; 2x + 5y = 1 and whose distance from (2, -1) is '2'.**

Sol:

5. Find the point on the line 3x – 4y -1 =0 which are at a constant distance of 5 units from the point(3, 2).

6. A straight line parallel to the line y=$\sqrt{3}x$ passes through Q (2, 3) and cuts the line 2x + 4y -27=0 at P. find the length of PQ.

7. A straight line through Q($\sqrt{3}$, 2) makes an angle 30 with the positive direction of the X-axis. If the straight line intersects the line $\sqrt{3x}-4y+8=0$ *at P*.find the distance of PQ.

8. Find the equation of the straight line parallel to the line $3x+4y=7$ and passing through the point of intersection of the lines $x-2y-3=0$ and $x+3y-6=0$

9. If 3a+2b+4c=0, then show that ax+ by + c=0 represents a family of lines and find point of concurrency .

10. Find the equation line perpendicular to the line 3x+4y+6=0 making an intercept -4 on the X-axis.

11. Find the equation lines passing from point (-3,2) and making an angle 45 with the line 3x-y+4=0 .

Aims tutorial

1. **S.T f(x) =$f(x)=\begin{cases}\frac{\cos ax-\cos bx}{x^2}, & x\neq 0\\ \frac{1}{2}(a^2-b^2), & x=0\end{cases}$ where a and b are real constants, is continuous at 0.**

Sol: $\lim_{x\to 0}[\frac{\cos ax-\cos bx}{x^2}]$

$=\lim_{x\to 0}[\frac{-2\sin(\frac{ax+bx}{2})\,2\sin(\frac{ax-bx}{2})}{x^2}]$

$=\lim_{x\to 0}\frac{-2sin\frac{(a+b)x}{2}}{x}.\lim_{x\to 0}\frac{2sin\frac{(a-b)x}{2}}{x}$ $\{\therefore \lim_{x\to 0}\frac{sinax}{x}=a\}$

=-2$\frac{(a+b)}{2}\frac{(a-b)}{2}$

$=\frac{1}{2}(a^2-b^2)$

Given f (0) $=\frac{1}{2}(a^2-b^2), \lim_{x\to 0}f(x)=f(0)$s

∴f(x) is continuous at x=0.

2. **Check the continuity of 'f' given by $f(x)=\begin{cases}\frac{(x^2-9)}{(x^2-2x-3)}, & if < x < 5\ and\ \neq 3\\ 1.5, if\ \ x=0\end{cases}$ $at\ x=3$.**

Sol: $\lim_{x\to 3}f(x)=\lim_{x\to 3}\frac{(x^2-9)}{(x^2-2x-3)}$

$=\lim_{x\to 3}\frac{(x-3)(x+3)}{(x-3)(x+1)}=\frac{3+3}{3+1}$

$=\frac{6}{4}=1.5=f(3)$

∴f(x) is continuous at x=3.

3. **Check the continuity of 'following function at 2'**

$$f(x)=\begin{cases}\frac{1}{2}(x^2-4) & if\ 0<x<2\\ 0 & if\ x=2\\ 2-8x^{-3} & if\ x>2\end{cases}$$

Sol: $\lim_{x\to 2-}f(x)=\lim_{x\to 2-}\frac{1}{2}(x^2-4)$

$=\frac{1}{2}(4-4)=0$

$\lim_{x\to 2+}f(x)=\lim_{x\to 2+}(2-\frac{8}{x^3})$

=2-$\frac{8}{8}$=2-1=1

$\lim_{x\to 2-}f(x)\neq\lim_{x\to 2+}f(x)$

$\lim_{x\to 2}f(x)\ does\ not\ exist$

∴f(x) is not continuous at x=2.

4. **Find real constants a,b so that the function f given by**

f(x)=$\begin{cases}sinx & if\ x\leq 0\\ x^2+a & if\ 0\leq x\leq 1\\ bx+3 & if\ 1\leq x\leq 3\\ -3 & if\ x>3\end{cases}$ **is** continuous on R

Sol: $\lim_{x\to 0+}f(x)=\lim_{x\to 0+}(x^2+a)$=0+a=a

: $\lim_{x\to 0-}f(x)=\lim_{x\to 0-}(sinx)=0$

Since f(x) is continuous on R

LHS=RHS ⇒a=0.

: $\lim_{x\to 3+}f(x)=\lim_{x\to 3+}(-3)=-3$

: $\lim_{x\to 3-}f(x)=\lim_{x\to 3-}(bx+3)$=3b+3

Since f(x) is continuous on R

LHS=RHS ⇒3b+3=-3

⇒3b=-6

b=-2

5. $\lim_{x\to a}\left(\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}\right)$,

Sol:$\lim_{x\to a}\left(\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}\right)\frac{\sqrt{a+2x}+\sqrt{3x}}{\sqrt{a+2x}+\sqrt{3x}}.\frac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{3a+x}+2\sqrt{x}}$

$=\lim_{x\to a}\left(\frac{(a+2x-3x)}{\sqrt{a+2x}-\sqrt{3x}}\right)(\frac{\sqrt{3a+x}+\sqrt{4x}}{3a+x-4x})$

$=\lim_{x\to a}\left(\frac{(a-x)}{\sqrt{a+2x}+\sqrt{3x}}\right)(\frac{\sqrt{3a+x}+\sqrt{4x}}{3(a-x)})$

$=\lim_{x\to a}\left(\frac{\sqrt{3a+x}+\sqrt{4x}}{3(\sqrt{a+2x}+\sqrt{3x})}\right)$

$=\frac{4\sqrt{a}}{3(2\sqrt{3a})}$

$=\frac{2}{3\sqrt{3}}$

6. $\lim_{x\to 2}\frac{(2x^2-7x-4)}{(2x-1)(\sqrt{x}-2)}$

8. **Compute** $\lim_{x\to 0}\frac{\sin(a+bx)-\sin(a-bx)}{x}$

Sol: $\lim_{x\to 0}\frac{\sin(a+bx)-\sin(a-bx)}{x}$

$=\lim_{x\to 0}\frac{2\cos(\frac{a+bx+a-bx}{2})\sin(\frac{a+bx-a+bx}{2})}{x}$

$=2\cos a.\lim_{x\to 0}\frac{\sin(bx)}{x}$

$=2\cos a.1\ .b$

$=2b\cos a$

7. Show that $\lim_{x\to a}\frac{x\sin a - a\sin x}{x-a} = \sin a - a\cos a$

$\lim_{x\to 0}[\frac{x\sin a - a\sin x}{x-a}]$

Sol:

$=\lim_{x\to 0}[\frac{x\sin a - a\sin a - a\sin x + a\sin a}{x-a}]$

$=\lim_{x\to 0}[\frac{(x\sin a - a\sin a)-(a\sin x - a\sin a)}{x-a}]$

$=\lim_{x\to 0}\frac{(x-a)\sin a}{(x-a)} - a\lim_{x\to 0}\frac{\sin x - \sin a}{x-a}$

$=\sin a - a\lim_{x\to 0}\frac{2\cos(\frac{x+a}{2})\sin(\frac{x-a}{2})}{x-a}$

$=\sin a - a\lim_{x\to 0}\cos\frac{(x+a)}{2}.\lim_{x\to 0}\frac{\sin\frac{(x-a)}{2}}{\frac{x-a}{2}}.$

$=\sin a - a\cos a.$

8. **Compute** $\lim_{x\to 0}\frac{(1+x)^{\frac{1}{8}}-(1-x)^{\frac{1}{8}}}{x}$

Sol: H/W

9. Is 'f' defined by $f(x) =$

$\begin{cases} k^2x - k, & if\ x \geq 0 \\ 2, & if\ \ x < 0 \end{cases}$, *is a continuous function on R, then find the values of k.*

10. Is 'f' defined by $f(x) =$

$\begin{cases} \frac{\sin ax}{x}, & if\ x \neq 0 \\ 0, & if\ \ x = 0 \end{cases}$, *is a continuous at $x = 0$.,*

Q.No.15

1. Find the derivative from First principle of function ,cos ax, cos^2x, sec $3x$, tan $2x$, xsin x, $\sqrt{x+1}$. Logx.

(i)Sol: f(x) =cos ax and f(x + h) =cos$(ax+ah)$.

$\therefore f^1(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$\Rightarrow f^1(x) = \lim_{h\to 0}\frac{\cos(ax+ah).-\cos ax}{h}$

$=\lim_{h\to 0}\frac{-2\sin(\frac{ax+ah+ax}{2})\sin(\frac{ax+ah-ax}{2})}{h}$

$=\lim_{h\to 0}\frac{-2\sin(ax+\frac{ah}{2})\sin(ah/2)}{h}$

$=\lim_{h\to 0} -2\sin(ax+\frac{ah}{2})\lim_{ah/2\to 0}\frac{\sin(ah/2)}{\frac{(ah)}{2}}.\frac{(a)}{2}$

=-2sin (ax). (1). (a/2)

= - a sin (ax).

(ii) Sol: f(x) =cos²x and f(x + h) = cos²(x + h).

$\therefore f^1(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$\Rightarrow f^1(x) = \lim_{h\to 0}\frac{\cos^2(x+h)-cos^2x}{h}$

[Cos²B – cos²A=cos (A+B) cos (A-B)]

$=\lim_{x\to h}\frac{-\sin(x+h+x)\sin(x+h-x)}{h}$

$=\lim_{x\to h} -sin2x\lim_{x\to h}\frac{sinh}{h}$

=-sin2x. (1)

=-sin2x

(iii) Sol: f(x) =sec3x and f(x + h) = sec (3x+3h).

$\therefore f^1(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$\Rightarrow f^1(x) = \lim_{h\to 0}\frac{\sec(3x+3h)-sec3x}{h}$

$=\lim_{h\to 0}[\frac{1}{\cos(3x+3h)}+\frac{1}{\cos(3x)}]\frac{1}{h}$

$=\lim_{h\to 0}[\frac{\cos(3x)-cos(3x+3h)}{cos3x.\cos(3x+3h)}]\frac{1}{h}$

$=\lim_{h\to 0}[\frac{-2\sin\left(\frac{3x+3x+3h}{2}\right)\sin(\frac{3x-3x-3h}{2})}{cos3x.\cos(3x+3h)}]\frac{1}{h}$

$=\lim_{h\to 0}\left[\frac{-2\sin\left(3x+\frac{3h}{2}\right)}{cos3x.\cos(3x+3h)}\right]\lim_{h\to 0}\frac{\sin\left(-\frac{3h}{2}\right)}{\left(-\frac{3h}{2}\right)}.(-3/2)$

$=\frac{-2sin3x}{cos3x.\quad cos3x}(-3/2)$

=3tan3x.sec3x.

(iv) Sol: l: f(x) =tan2x and f(x + h) = tan (2x + 2h).

$\therefore f^1(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$\Rightarrow f^1(x) = \lim_{h\to 0}\frac{\tan(2x+3h)-tan2x}{h}$

$=\lim_{h\to 0}[\frac{\sin(2x+2h)}{\cos(2x+2h)}+\frac{\sin(2x)}{\cos(2x)}]\frac{1}{h}$

$=\lim_{h\to 0}[\frac{\sin(2x+2h)\cos(2x)-cos(2x+2h)sin(2x))}{cos2x.\cos(2x+2h)}]\frac{1}{h}$

$=\lim_{h\to 0}[\frac{\sin(2x+2h-2x)}{cos2x.\cos(2x+2h)}]\frac{1}{h}$

$=\lim_{h\to 0}\frac{sin2h}{2h}(2)\lim_{h\to 0}\frac{1}{cos2x.\cos(2x+2h)}$

$=\frac{2}{cos2x.cos2x}=\frac{2}{cos^2 2x}$

=2sec²2x.

(v). **f(x) =xsinx** [h/w]

2. **If $x^y = e^{x-y}$, then show that $\frac{dy}{dx} = \frac{logx}{(1+logx)^2}$.**

Sol: $x^y = e^{x-y}$

Applying log on both sides

$\Rightarrow \log x^y = \log e^{x-y}$

$\Rightarrow y\log x = (x-y)\log e$

$\Rightarrow y\log x = (x-y)$

$\Rightarrow y+y\log x = x$

$\Rightarrow y(1+\log x) = x$

$\Rightarrow y=\frac{x}{(1+\log x)}$

Diff w. r.to 'x'

$\Rightarrow \frac{dy}{dx} = \frac{(1+\log x).1-x(1+\frac{1}{x})}{(1+\log x)^2}$

$\Rightarrow \frac{dy}{dx} = \frac{1+\log x-1)}{(1+\log x)^2}$

$\Rightarrow \frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$.

3. **If $\sin y = x\sin(a+y)$, then S.T $\frac{dy}{dx} = \frac{sin^2(a+y)}{sina}$.**

Sol: given $\sin y = x\sin(a+y)$

$\Rightarrow x=\frac{siny}{\sin(a+y)}$ $[(uv)'=\frac{vu'-uv'}{v^2}]$

Diff w. r.to 'y'

$\frac{dx}{dy}=\frac{\sin(a+y)cosy-\cos(a+y)siny}{sin^2(a+y)}$

$\{\sin(A-B) = \sin A\cos B - \cos A\sin B\}$

$\Rightarrow \frac{dx}{dy}=\frac{\sin(a+y-y)}{sin^2(a+y)}$

$\Rightarrow \frac{dx}{dy}=\frac{\sin(a)}{sin^2(a+y)}$

$\therefore \frac{dy}{dx} = \frac{sin^2(a+y)}{sina}$.

4. **If f(x) $=\tan^{-1}\left[\frac{\sqrt{1+x^2}-1}{x}\right]$,g(x)$=\tan^{-1} x$ then differentiate f(x) with respect to g(x).**

Sol: f(x) $=\tan^{-1}\left[\frac{\sqrt{1+x^2}-1}{x}\right]$

Put x=$\tan\theta$,

$\Rightarrow$ f(x) $=\tan^{-1}\left[\frac{\sqrt{1+tan^2\theta}-1}{tan\theta}\right]$

$=\tan^{-1}\left[\frac{sec\theta-1}{tan\theta}\right]$

$=\tan^{-1}\left[\frac{\frac{1}{cos\theta}-1}{\frac{sin\theta}{cos\theta}}\right]$ $=\tan^{-1}\left[\frac{1-cos\theta}{sin\theta}\right]$

$=\tan^{-1}\left[\frac{2sin^2\frac{\theta}{2}}{2sin\frac{\theta}{2}cos\frac{\theta}{2}}\right]$ $=\tan^{-1}\left[\frac{sin\frac{\theta}{2}}{cos\frac{\theta}{2}}\right]$

$=\tan^{-1}\left[tan\frac{\theta}{2}\right]$ $=\frac{\theta}{2}=\frac{\tan^{-1}x}{2}$

$\Rightarrow$ f'(x)$=\frac{1}{2(1+x^2)}$

And g(x) $=\tan^{-1} x$

g'(x)$=\frac{1}{2(1+x^2)}$

$\therefore\frac{f'(x)}{g'(x)}=\frac{1}{2}$

5. If $ax^2+2hxy+by^2=0$. then P.T$\frac{d^2x}{dy^2}=\frac{(h^2-ab)}{(hx+by)^3}$.

6. If=x^y $then\ show\ that\ \frac{dy}{dx}=\frac{y^2}{x(1-logy)}=\frac{y^2}{x(1-ylogx)}$.

If$x^y=y^x, then\ S.T\ \frac{dy}{dx}=\frac{x(xlogy-y)}{y(ylogx-x)}$. H

7. If y=$\tan^{-1}\left[\frac{2x}{1-x^2}\right]+\tan^{-1}\left[\frac{3x-x^3}{1-3x^2}\right]-\tan^{-1}\left[\frac{4x-4x^3}{1-6x^2+x^4}\right]$ then show that$\frac{dy}{dx}=\frac{1}{1+x^2}$. (Hint put x= $tan\theta$).

$\Rightarrow$(y-a) $=\frac{-4}{3}$(x-a)

$\Rightarrow$3y-3a=-4x+4a

$\therefore$4x+3y-7a=0.

(i) xy=10 at the point (2, 5).

(ii) Y=x^3+4x^2 at the point (-1, 3).

(iii) Curve y=5x^4 at the point (1, 5).

2. **Show that length of subnormal at any point on the curve $y^2=4ax$ is a constant.{(ii) $y=a^x$}**

Sol: Given curve is $y^2=4ax$

Differentiating w.r.t 'x'

2y$\frac{dy}{dx}=4a \Rightarrow \frac{dy}{dx}=\frac{2a}{y}$ =m

Length of sub normal |y. m|=|y. $\frac{2a}{y}$|=2a (constant)

3. **Find the tangent and normal to the curve y=2$e^{\frac{-x}{3}}$ at the point where the curve meets the Y-axes.**

Sol: given equation of the curve y=2$e^{\frac{-x}{3}}$

Equation of the Y-axis is x=0

$\Rightarrow$y=2. $e^{\frac{-0}{3}}$=2(1) =2

Required point is (0, 2)

y=2$e^{\frac{-x}{3}}$, Differentiating w.r.t 'x'

$\frac{dy}{dx}=2(\frac{-1}{3})\,e^{\frac{-x}{3}}$

$where\ x=0\ and\ y=2$

$\frac{dy}{dx}=-\frac{2}{3}e^0=-\frac{2}{3}$

$Equation\ of\ the\ tangent\ (y-y_1)=m(x-x_1)$

$(y-2)=-\frac{2}{3}(x-0)$

$\Rightarrow$3y-6=-2x

$\therefore$2x+3y-6=0.

Equation of normal at (0, 2) is (y-y$_1$) $=\frac{1}{m}$(x-x$_1$)

$\Rightarrow$y-2=$\frac{3}{2}$(x-0)

$\Rightarrow$2y-4=3x

$\therefore$3x- 2y + 4=0.

Q.No.16

1. **Find the equation of tangent and normal to the Curve y^4 =ax^3 at the point (a, a).**

Sol: Given curve is y^4 =ax^3

Differentiating w.r.t 'x'

4y$^3\frac{dy}{dx}=3ax^2 \Rightarrow \frac{dy}{dx}=\frac{3ax^2}{4y^3}$

Slope(m) of the tangent at (a, a) $=\frac{dy}{dx}_{(a,a)}=\frac{3a(a)^2}{4(a)^3}=\frac{3}{4}$

Slope of the normal at (a, a) $=\frac{-1}{m}=\frac{-4}{3}$

Equation of tangent at (a, a) is (y-y$_1$) =m(x-x$_1$)

$\Rightarrow$y-a=$\frac{3}{4}$ (x-a)

$\Rightarrow$4y-4a=3x-3a

$\therefore$3x-4y+a=0.

Equation of normal at (a, a) is (y-y$_1$) $=\frac{1}{m}$(x-x$_1$)

4. **Show that the tangent at p(x$_1$, y$_1$) on the curve $\sqrt{x}+\sqrt{y}=\sqrt{a}$ is $yy_1^{\frac{-1}{2}}+xx_1^{\frac{-1}{2}}=a^{\frac{1}{2}}$.**

Sol: given equation of the curve $\sqrt{x}+\sqrt{y}=\sqrt{a}$

Differentiating w.r.t 'x'

$\frac{1}{2\sqrt{x}}+\frac{1}{2\sqrt{y}}\frac{dy}{dx}=0$

$\Rightarrow \frac{1}{2\sqrt{y}}\frac{dy}{dx}=-\frac{1}{2\sqrt{x}}$

$\frac{dy}{dx}=-\frac{\sqrt{y}}{\sqrt{x}}=-\frac{y^{\frac{1}{2}}}{x^{\frac{1}{2}}}$

Slope(m) of the tangent at (x_1,y_1) $=\frac{dy}{dx_{(x_1,\ y_1)}}=-\frac{y_1^{\frac{1}{2}}}{x_1^{\frac{1}{2}}}$

Equation of the tangent is $(y-y_1)=m(x-x_1)$

$(y-y_1)=-\frac{y_1^{\frac{1}{2}}}{x_1^{\frac{1}{2}}}(x-x_1)$

$\Rightarrow\frac{(y-y_1)}{y_1^{\frac{1}{2}}}=\frac{(x-x_1)}{x_1^{\frac{1}{2}}}$

$\Rightarrow\frac{y}{y_1^{\frac{1}{2}}}-\frac{y_1}{y_1^{\frac{1}{2}}}=\frac{x}{x_1^{\frac{1}{2}}}-\frac{x_1}{x_1^{\frac{1}{2}}}$

$\Rightarrow yy_1^{\frac{-1}{2}}+xx_1^{\frac{-1}{2}}=x_1^{\frac{1}{2}}+y_1^{\frac{1}{2}}$

$\therefore yy_1^{\frac{-1}{2}}+xx_1^{\frac{-1}{2}}=a^{\frac{1}{2}}.$

5. **Find the lengths of normal and subnormal at a point on the curve $y=\frac{a}{2}(e^{\frac{x}{a}}+e^{\frac{-x}{a}})$.**

Sol: given curve $y=\frac{a}{2}(e^{\frac{x}{a}}+e^{\frac{-x}{a}})$.

$Y=a\cosh\frac{x}{e}$

Differentiating w.r.t 'x'

$\frac{dy}{dx}=a.\sinh\left(\frac{x}{a}\right)(\frac{1}{a})=\sinh\frac{x}{a}=m$

$length\ of\ the\ normal\ |y_1|\sqrt{(1+m^2)}$

$=a\cosh\frac{x}{a}\sqrt{1+sinh^2(\frac{x}{a})}$

$=a\cosh\frac{x}{a}\sinh\frac{x}{a}$

$=acosh^2(\frac{x}{a})$

$length\ of\ the\ subnormal=y_1|m|$

$=a\cosh\frac{x}{a}\sinh\frac{x}{a}$

$=\frac{1}{2}\sinh\frac{2x}{e}$

6. **Find the value of k so that the length of the sub-normal at any point on the curve $x.y^k=a^{k+1}$ is a constants.**

Sol: given curve is curve $x.y^k=a^{k+1}$

Or $x=\frac{a^{k+1}}{y^k}$

Differentiating w.r.t 'x'

$\Rightarrow x.\ k.y^{k-1}\frac{dy}{dx}+y^k.1=0$

$\frac{dy}{dx}=-\frac{y^k}{x.k.y^{k-1}}$

$(m)\frac{dy}{dx}=\frac{-y_1}{x.k.}$

Length of the sub normal$=[y_1.m|$

$=|y_1.\frac{-y_1}{x_1.k.}|$

$=\frac{y_1^2}{kx_1}$

$=\frac{y_1^2}{k\frac{a^{k+1}}{y^k}}=\frac{y_1^{k+2}}{ka^{k+1}}$

Length of the sub-normal at any point on the curve Is independent of x_1,y_1

$\Rightarrow k+2=0$

$\therefore k=02.$

Q.No 17

1. **the distance time formula for the motion of particle along straight line is $s=t^3-9t^2+24t-18$ find when and where velocity becomes zero.**

Sol: given $s=t^3-9t^2+24t-18$

Differentiating w.r.t't'

$\frac{ds}{dt}=3t^2-18t+24$

$\{\frac{ds}{dt}=v\}$

If Velocity becomes zero $\frac{ds}{dt}=0$

$\Rightarrow 3t^2-18t+24=0$

$3(t^2-6t+8)=0$

$\Rightarrow(t-2)(t-4)=0$

$\therefore t=2$ or $t=4$

The velocity is zero after 2 and 4 seconds.

Case (I):	Case (II):
If t=2	If t=4
$S=t^3-9t^2+24t-18$	$S=t^3-9t^2+24t-18$
$=2^3-9(2^2)+24(2)-18$	$S=4^3-9(4)^2+24(4)-18$
$=8-36+48-18$	$=64-144+96-18$
$=56-54=2$	$=160-162=-2$

The particle is at a distance of 2 units from the starting point 'O' on either side.

2. **A container in the shape of an inverted cone has height 12cm and radius 6cm at the top. If it is filled with water at the rate of $12cm^3$/sec, what is the rate of change in the height of water level when the tank is filled 8cm?**

Sol: Here h=OC =8cm

r=AB=6cm

$\frac{dv}{dt}=2m^3/min$

$\Delta OAB\ and\ \Delta OCD\ are\ simillar\ angle$

$\frac{CD}{AB}=\frac{OC}{OA}\Rightarrow\frac{r}{6}=\frac{h}{12}$

$\Rightarrow r=\frac{h}{2}$

Volume of the cone $v=\frac{1}{3}\pi r^2h$

$V=\frac{1}{3}\pi(\frac{h}{2})^2h$

$v=\frac{1}{12}\pi h^3$

Differentiating w.r.t "t"

$\frac{dv}{dt}=\frac{1}{12}\pi 3h^2.\frac{dh}{dt}$

$\Rightarrow\frac{dh}{dt}=\frac{4}{\pi h^2}\frac{dv}{dt}$ [h=8]

$=\frac{4}{\pi 8^2}.12$

$=\frac{3}{4\pi}$cm/sec.

Hence, the rate of change of water level is $\frac{3}{4\pi}$cm/sec. when the water level of the tank is 8cm.

3. **A stone is dropped into a quiet lake and ripples move in circles at the speed of 5cm/sec. at the instant when the radius of circular ripple is 8cm. how fast is the enclosed area increases.**

Sol: suppose r is the value of the outer ripple and A be its area.

Area of the circle A=πr^2 (given r=8, $\frac{dr}{dt}$=5cm/sec)

Differentiating w.r.t "t"

$\frac{dA}{dt} = 2\pi r\frac{dr}{dt}$

$\frac{dA}{dt} = 2\pi(8)(5)$

$\frac{dA}{dt} = 80\pi cm^2/sec$.

4. **A balloon, which always remains spherical on inflation, is being inflated by pumping in 900c.c of gas per second. Find the rate at which the radius of balloon increases when the radius is 15cm.**

Sol: Given r=15cm and $\frac{dV}{dt}$ =900c.c/sec

Volume of the sphere v=$\frac{4}{3}\pi r^3$

Differentiating w.r.t "t"

$\frac{dV}{dt} = \frac{4}{3}\pi.3r^2\frac{dr}{dt}$

$\frac{dr}{dt} = \frac{900}{4\pi r^2}$ Where r=15

$\frac{dr}{dt} = \frac{900}{4\pi 15^2}$

$= \frac{900}{4\pi 225} = \frac{900}{\pi 900} = \frac{1}{\pi}$cm/sec.

5. **The radius of a circle is increasing at the rate of 0.7cm/sec. what is the rate of increases of it circumference.**

Sol: given $\frac{dV}{dt} = 0.7cm/sec.$

Circumference of a circle=$2\pi r$

Differentiating w.r.t "t"

$\frac{dC}{dt} = 2\pi\frac{dr}{dt}$

$=2\pi(0.7)$

$=1.4\ \pi cm/sec.$

6. **The radius of an air bubble is increasing at the rate of $\frac{1}{2}cm/sec$ .at what rate is the volume of the bubble increasing when the radius is 1cm?**

Sol: given$\frac{dr}{dt} = \frac{1}{2}cm/sec$, radius(r) =1 cm

Volume of the sphere v=$\frac{4}{3}\pi r^3$

Differentiating w.r.t "t"

$\frac{dV}{dt} = \frac{4}{3}\pi.3r^2\frac{dr}{dt}$

$\frac{dV}{dt} = 2\ \pi cm^3/sec.$

7. **Suppose we have a rectangular aquarium with dimensions of length 8m, width 4m and height 3m. Suppose we are filling the tank with water at the rate of 0.4m³/sec. how fast is the height of water changing when the water level is 2.5m?**

Sol: length of aquarium l=8m

Width of aquarium b=4m

Height of aquarium h=3m

$\frac{dV}{dt} = 0.4m^3/sec,$

$V = lbh = (8)\ (4)\ (3) = 96$

$V = lbh$

$\Rightarrow \log V = \log l + \log b + \log h$

Differentiating w.r.t "t"

$\frac{1}{v}\frac{dV}{dt} = 0+0+\frac{1}{h}\frac{dh}{dt}$

$\frac{dh}{dt} = \frac{1}{96}$ At h=2.5.

8. **The volume of a cube is increasing a rate of 9 c.c per second. How fast is the surface area increasing when the length of the edge is 10c.ms?**

Sol: suppose 'a' is the edge of the cube and v be the volume of the cube.

V=a³

Given$\frac{dV}{dt} = 8cm^3/sec$ and a=12cm

Surface area of cube(S) =6a²

Differentiating w.r.t "t"

$\frac{dS}{dt} = 12a\frac{da}{dt}$ ..... (1)

V=a³

Differentiating w.r.t "t"

$\frac{dV}{dt} = 3a^2\frac{da}{dt}$

$8=3(144)\frac{da}{dt}$

$\Rightarrow\frac{da}{dt} = \frac{8}{3\times144}cm/sec$... (2)

$\frac{dS}{dt} = 12a\frac{da}{dt}$

$\frac{dS}{dt} = 12(12)\frac{8}{3\times144} = \frac{8}{3}cm^2/sec.$

9. **Let a kind of bacteria grow in such a way that at time t sec. there are t^3/2 bacteria. Find the rate of growth at time t=4 hours.**

Sol: let g be the amount of growth of bacteria at t then g (t) $=t^{3/2}$

The growth rate at time is given by g' (t) $=\frac{3}{2}t^{\frac{1}{2}}$

Given t=4hr

g' (t) $=\frac{3}{2}(4\times 60\times 60)^{\frac{1}{2}}$

$=\frac{3}{2}(2\times 60)=180.$

THE STRAIGHT LINES

1. **If area of triangle formed by lines x = 0: y = 0 and 3x + 4y = a is 6 sq units. Find 'a'.**

   Sol: given equation of the line is 3x + 4y –a=0.

   the area of the triangle formed by the straight line ax+by+c=0 with the coordinate axes $=\frac{1}{2}\left|\frac{c^2}{a.b}\right|$

   $\Rightarrow \frac{1}{2}\left|\frac{(-a)^2}{3.4}\right|=6$

   $\Rightarrow a^2=6.2.3.4$

   $\Rightarrow a^2=12.12$ $\therefore$a=12.

2. **Find the area of triangle formed by lines 3x-4y+12 = 0: with the coordinate axes.**

   $\boldsymbol{Sol}$: *the area of the triangle formed by the straight line* $ax+by+c=0$ *with the coordinate axes* $=\frac{1}{2}\left|\frac{c^2}{a.b}\right|$

   $\Rightarrow \frac{1}{2}\left|\frac{(12)^2}{3.(-4)}\right|=\frac{144}{12}=12$

3. **Find the area of the triangle formed by the line xcosα + $y sin\alpha = p$.with the coordinate axes.**

   Sol:

   the area of the triangle formed by the straight line $ax+by+c=0$ *with the coordinate axes* $=\frac{1}{2}\left|\frac{c^2}{a.b}\right|$

   $\Rightarrow \frac{1}{2}\left|\frac{p^2}{cos\alpha sin\alpha}\right|=\frac{p^2}{sin2\alpha}=p2\ cosec2\alpha\ sq.units.$

4. **If the product of intercepts made by a straight line xtan $\alpha + y\sec\alpha = 1$, on the axes is equal to sin α. Find α.**

   Sol:

   equation of the straight line in the intercept form is

   $$\frac{x}{cot\ \alpha}+\frac{y}{cos\ \alpha}=1$$

Given the product of intercepts made by a straight line is $\sin\alpha$.

$\Rightarrow \cot\alpha\cos\alpha = \sin\alpha$

$\Rightarrow \frac{\cos\alpha}{\sin\alpha}.\cos\alpha = \sin\alpha$

$\Rightarrow \cos^2\alpha = \sin^2\alpha$

$\therefore \alpha = 45.$

5. **Find the value of y if the line joining (3,y) and (2,7) is parallel to the line joining the points(-1, 4) and (0, 6).**

Sol: given points A (3, y), B (2, 7), C (-1, 4) and D (0, 6)

Given AB and CD are parallel

Slope of AB =slope of CD

$\Rightarrow \frac{7-y}{2-3} = \frac{6-4}{0-1}$

$\Rightarrow 7 - y = -2$

$\Rightarrow y = 7 + 2$

$\therefore y = 9$

6. **Find the value of "x" if the slope of the line passing through (2, 5) and (x, 3) is 2.**

Sol: given points A (2, 5), B (x, 3)

And slope of AB =2

$\frac{3-5}{x-2} = 2$

$\Rightarrow \frac{-2}{2} = x - 2$

$\Rightarrow$x=-1+2 $\therefore$x=1.

7. **Find the condition for the points (a, 0), (h, k) and (0, b) when ab≠0 to be collinear.**

Sol: given A (a, 0), B (h, k), C (0, b) are collinear

Slope of AB =slope of BC

$\Rightarrow \frac{k-0}{h-a} = \frac{b-k}{0-h}$

$\Rightarrow$K (-h) = (b-k) (h-a)

$\Rightarrow -hk = bh - ab - hk + ka$

$\Rightarrow$bh +ak=ab or $\frac{h}{a} + \frac{k}{b} = 1.$

8. **Find distance between parallel lines given by 5x-3y-4=0 and 10x-6y-9=0.**

Sol: given 5x-3y-4=0 and 10x-6y-9=0 are parallel to each other

10x-6y-9=0

5x-3y-4=0 $\Rightarrow$ 10x-6y-8=0

Here a=10, b=6, c_1=-9 and c_2=-8

Distance between || lines $=\frac{|c_2-c_1|}{\sqrt{a^2+b^2}}$.

$=\frac{|-8+9|}{\sqrt{10^2+6^2}}=\frac{|1|}{136}$.

9. **Find the equation of the straight line perpendicular to the line 5x-3y+1=0 and passing through (4, -3).**

Sol: given line 5x-3y+1=0 and point (4, -3).

Eq'n of a line $\perp$ax + by +c=0 and passing through (x_1, y_1)

b $(x - x_1)$-a$(y - y_1)$=0

$\Rightarrow$-3(x-4)-5(y+3) =0

$\Rightarrow$-3x+12-5y-15=0 $\therefore$3x+5y-3=0.

10. **Find the equation of line passing through A(-1,3) and perpendicular to the straight line passing through B(2,-5) and C(4,6)**

Sol: the equation of line passing through A(-1,3) and perpendicular to the straight line passing through B(2,-5) and C(4,6)

$(y - y_1) =-\frac{1}{m}(x - x_1)$ where m=slope of BC $=\frac{6+5}{4-2} = \frac{11}{2}$

$\Rightarrow$(y-3) $=-\frac{2}{11}(x - 1)$

$\Rightarrow$11y-33=-2x+2

$\Rightarrow$2x+11y-33=0.

11. **Find the equation of the straight line passing through (-4, 5) and cutting off equal intercepts on the coordinate axis.**

Sol: equation of the straight line making equal intercepts is$\frac{x}{a} + \frac{y}{a} = 1$ or x+y=a…….. (1)

Given eq'n (1) is passing through (-4, 5)

-4+5=a $\Rightarrow$ a=1

$\therefore$ the required eq'n is x+y=1.

12. **Find the equation of the straight line passing through (2, 3) and making non-zero intercepts on the coordinate axis whose sum is zero.**

Sol: equation of the straight line in the intercept form is$\frac{x}{a} + \frac{y}{b} = 1$ and a+b=0$\Rightarrow$ b= -a

Is$\frac{x}{a} + \frac{y}{-a} = 1$ or x-y=a…….. (1)

Given eq'n (1) is passing through (2, 3)

2-3=a $\Rightarrow$a=-1

$\therefore$ the required eq'n is x-y=-1or x-y+1=0.

13. **Find the value of "k" if straight lines 6x-10y+3=0 and kx-5y+8=0 are parallel.**

Sol: if a_1x+b_1y+c_1=0 and a_2x+b_2y+c_2=0 are parallel to each other $\frac{a_1}{a_2} = \frac{b_1}{b_2}$

$\Rightarrow\frac{6}{k} = \frac{-10}{-5}$ $\Rightarrow$k=3.

14. **Find the equation of the straight line making an angle 150⁰ with the positive direction of X-axes and passing through the point (-2, -1).**

Sol: required eq''n of the straight line is (y-y_1) =m(x-x_1)

Where m= tanθ (θ = 150), P (-2, -1)

m= tan150=tan (90+60)

=-cot60=-$\frac{1}{\sqrt{3}}$

Y+1=-$\frac{1}{\sqrt{3}}(x + 2)$ $\Rightarrow$x+$\sqrt{3}y + 3 = 0.$

15. **If the straight line x+p=0, y-2=0 and 3x+2y+5=0are concurrent, find the value of p?**

Sol: given x+p=0…… (1)
Y-2=0 … (2)
3 x+2y+5=0….. (3) are concurrent.
From eq''n (2) y=2
sub y=2 in (3)
⇒3x+2(2) +5=0
⇒3x=-9 or x=-3
Sub x=-3 in eq''n (1)
-3+p=0 ∴p=3.

16. **Find the foot of the ⊥drawn from (4, 1) upon the straight line 3x-4y+12=0.**

Sol: foot of the ⊥$=\frac{h-x_1}{a}=\frac{k-y_1}{b}=\frac{-(ax_1+by_1+c)}{a^2+b^2}$

$=\frac{h-4}{3}=\frac{k-1}{-4}=\frac{-(3(4)-4(1)+12)}{3^2+4^2}$

$\frac{h-4}{3}=\frac{k-1}{-4}=\frac{-(12-4+12)}{25}$

$\frac{h-4}{3}=\frac{k-1}{-4}=\frac{-(20)}{25}$=-4/5

$\frac{h-4}{3}=-4/5$⇒h-4 =- $\frac{12}{5}$ $\frac{k-1}{-4}=-\frac{4}{5}\Rightarrow k-1=$ 16/5

h=$-\frac{12}{5}+4$ k=$\frac{16}{5}+1$

= 8/5. = 21/5.

∴foot of the perpendicular$(\frac{8}{5},\frac{21}{5})$

17. **Find length of the ⊥distance from the point (3, 4) to the line 3x-4y+10=0.**

Sol: length of the perpendicular

$=\frac{|ax_1+by_1+c|}{\sqrt{(a^2+b^2)}}$

$=\frac{|3(3)-4(4)+10|}{\sqrt{(3^2+(-4)^2)}}$

$=\frac{|9-16+10|}{\sqrt{(9+16)}}=\frac{3}{\sqrt{25}}=\frac{3}{5.}$

18. **If θ is angle between lines x/a +y/b=1&x/b+y/a=1 find the value of sin θ.**

19. **If a, b, c are in A.P, then show that eq''n ax+by+c=0 represents a family of concurrent lines and find the point of concurrency?**

Sol: a, b, c are in A.P
2b=a + c
a- 2b + c=0
a.1+b(-2) + c=0each number of family of straight lines ax + by + c=0 passes through the fixed point (1, -2).
∴ Set of lines ax + by +c =0 for parametric values of a, b and c is a family of concurrent lines.
∴point of concurrency is (1, -2).

20. **Find the angle between the lines 2x+y+4=0 and y-3x=7.**

Sol: $\cos\theta=[\frac{|a_1a_2+b_1b_2|}{\sqrt{a_1{}^2}+b_1{}^2\sqrt{b_1{}^2+b_2{}^2}}]$

Here $a_1=2, a_2=-3,\ b_1=1, b_2=1$

$\text{Cos}\theta\ \frac{2(-3)+1.1}{\sqrt{(2^2+1^2)}\sqrt{(1^2+1^2)}}=\sqrt{\frac{5}{2}}.$

21. **Find k, if the lines y-3xk+4=0 and (2k-1) x-(8k-1) y-6=0 are perpendicular.**

Sol: given lines are lines
Y-3xk+4=0
(2k-1) x-(8k-1) y-6=0
$a_1a_2+b_1b_2=0$
⇒ (-3k) (2k-1) + (1) (-8k+1) =0
⇒-6k²+3k-8k+1=0
⇒6k²+5k-1=0 ⇒ (k+1) (6k-1) =0
s ⇒ (k+1) =0 or 6k-1 =0 ∴K=-1 or k=1/6.

22. **Transformed the eq''n x+y+1=0 into normal form.**

23. **Transformed eq''n √3x+y=4 into intercept form and normal form**

24. **Find the equation of the straight line passing through**
$(at_1^2, 2at_1)$, $(at_2^2, 2at_2)$.
Sol: given points $(at_1^2, 2at_1)$, $(at_2^2, 2at2)$.
The equation of the straight line passing through A (x_1, y_1), B (x_2, y_2) is (y-y₁) =$\frac{(y_2-y_1)}{(x_2-x_1)}(x-y_1)$

Q.No .3

1. **If (3, 2, -1), (4, 1, 1), (6, 2, 5) are three vertices and (4, 2, 2) is the centroid of a tetrahedron, find the fourth vertex.**

Sol: let A (3, 2, -1), B (4, 1, 1), C (6, 2, 5) and D (x, y, z) are the vertices of a tetrahedron (4, 2, 2) = (α,β,γ)
D (x, y, z) =
$(4\ \alpha-x_1-x_2-x_3, 4\beta-y_1-y_2-y_3, 4\gamma-z_1-z_2-z_3\ )$
= (16-3-4-6, 8-2-1-2, 8+1-1-5)
= (3, 3, 3) is the co-ordinates of D.

2. **Find the fourth vertex of the parallelogram whose consecutive vertices are (2, 4, -1), (3, 6, -1), (4, 5, 1).**

3. Sol: suppose A(2, 4, -1), B(3, 6, -1),C(4, 5, 1) and D(x, y, z)
ABCD is a parallelogram
Mid point of AC=Mid point of BD
$(\frac{2+4}{2},\frac{4+5}{2},\frac{-1+1}{2})=(\frac{3+x}{2},\frac{6+y}{2},\frac{-1+z}{2})$
$\frac{3+x}{2}=\frac{6}{2}||\ \frac{6+y}{2}=\frac{9}{2}||\frac{z-1}{2}=\frac{0}{2}$

3+x=6 | 6+y=9 | z-1=0
X=3, y=3 and z=1
∴ Coordinates of the fourth vertex are (3, 3, 1)

4. **Find the ratio in which YZ-plane divides the line joining A (2, 4, 5) and B (3, 5, -4). Also find the point of intersection.**
Sol: ratio in which the XZ-plane divides the line joining A (2, 4, 5) and B (3, 5, -4) =-y_1:y_2
=-4:5.
Point of intersection $(\frac{mx_2-nx_1}{m-n},\frac{my_2-ny}{m-n},\frac{mz_2-nz_1}{m-n})$
$=(\frac{4(3)-5(2)}{4-5},\frac{4(5)-5(4)}{4-5},\frac{4(-4)-5(5)}{4-5})$
=(-2, 0, 41)

5. **Find the x if the distance between (5, -1, 7) and (x, 5, 1) is 9 units.**
Sol: A (5, -1, 7) and B (x, 5, 1) are the given points and AB=9
AB=$\sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}$
$\Rightarrow\sqrt{(5-x)^2+(-1-5)^2+(7-1)^2}$=9
S.O.B
$\Rightarrow(5-x)^2+36+36=81$
$\Rightarrow(5-x)^2=81-72$
$\Rightarrow(5-x)^2=9$
⇒5-x=±3
x=5-3=2 or x=5+3=8.

6. **Show that the point A (-4, 9, 6), B (-1, 6, 6) and C (0, 7, 10) form a right angled isosceles triangle.**
Sol: A (-4, 9, 6), B (-1, 6, 6) and C (0, 7, 10) are the given points
AB^2= $(2+1)^2$+ $(3-5)^2$+ $(5+1)^2$
=9+4+36
=49
BC^2= $(-1-4)^2$+ $(5+3)^2$+ $(-1-2)^2$
=25 + 64+9
=98
AC^2= $(4-2)^2$ + $(-3-3)^2$ + $(2-5)^2$
=4+36+9
=49

AB^2+ BC^2=49+49=98=AC^2
ABC is a right angled isosceles triangle.

7. **Find the coordinates of the vertex 'C' of triangle ABC if its centroid is the origin and the vertices A, B are (1, 1, 1) and (-2, 4, 1) respectively.**
Sol: let A (1, 1, 1), B (-2, 4, 1), and C (x, y, z) are the vertices of a tringle (0, 0, 0) = (α,β,γ)
C (x, y, z) = $(3\alpha-x_1-x_2, 4\beta-y_1-y_2, 4\gamma-z_1-z_2)$
= (0-1+2, 0-1-4, 0-1-1)
= (1, -5, -2)

8. **Show that the points (3, -2, 4), (1, 1, 1) and C (-1, 4, -2) are collinear.**
Sol: AB=$\sqrt{(3-1)^2+(-2-1)^2+(4-1)^2}$
=$\sqrt{4+9+9}$
=$\sqrt{22}$
BC=$\sqrt{(1+1)^2+(1-4)^2+(1+2)^2}$
=$\sqrt{4+9+9}$
AC=$\sqrt{(3+1)^2+(-2-4)^2+(4+2)^2}$
=$\sqrt{16+36+36}$
=$\sqrt{88}$=$2\sqrt{22}$
AB+BC=$\sqrt{22}+\sqrt{22}$=$2\sqrt{22}=AC$
∴A, B, C are collinear.

Q.No .4

1. **Find intercepts of plane 4x +3y-2z+2=0 on the co-ordinate axes.**
Sol: Equation of the plane is 4x +3y-2z+2=0
⇒ - 4x - 3y + 2z=2
$\Rightarrow\frac{-4x}{2}-\frac{3y}{2}+\frac{2z}{2}=1$
$\Rightarrow\frac{x}{(\frac{-2}{4})}+\frac{3y}{(\frac{-2}{3})}+\frac{2z}{(\frac{2}{2})}=1$
$\Rightarrow\frac{x}{(\frac{-1}{2})}+\frac{3y}{(\frac{-2}{3})}+\frac{2z}{(1)}=1$
∴ X, Y and Z-intercepts are -1/2, -2/3, 1.

2. **Transform the equation 4x-4y+2z-5=0 into intercept form.**
Similar as (1)

3. **Find the angle between planes x+2y +2z-5=0, 3x+3y+2z-8=0. (ii) 2x-y+z=6, x+y+2z=7.**
Sol: Sol: Equation of the planes are x+2y +2z-5=0, 3x+3y+2z-8=0.
$\cos\theta=\frac{|a_1a_2+b_1b_2+b_1b_2|}{\sqrt{a_1{}^2+b_1{}^2+c_1{}^2}.\sqrt{a_2{}^2+b_2{}^2+c_2{}^2}}$
$=\frac{|1.(-2)+2.(1)+(-1).1|}{\sqrt{1^2+2^2+1^2}.\sqrt{2^2+1^2+1^2}}$
$=\frac{|-2+2-1|}{\sqrt{1+4+1}.\sqrt{4+1+1}}$
$=\frac{|-1|}{\sqrt{6}.\sqrt{6}}=\frac{1}{6}$
$\therefore\theta=\cos^{-1}(\frac{1}{6})$.

4. **Reduce the equation x+2y-3z-6=0 of the plane into the normal form.**
Sol: Equation of the plane is x + 2y -3z=6
Dividing both sides by $\sqrt{a^2+b^2+c^2}$
$\sqrt{(1)^2+(2)^2+(-3)^2}$=$\sqrt{14}$
the equation of the plane in the normal form is

$$\frac{1x}{\sqrt{14}}+\frac{2y}{\sqrt{14}}+\frac{-3z}{\sqrt{14}}=\frac{6}{\sqrt{14}}$$

5. **Find the equation of the plane whose intercepts on X, Y, Z-axes are 1, 2, 4 respectively.**

Sol: Equation of the plane in the intercept form is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

Here given a=1, b=2, c=3

Equation of the required plane in the intercept form is

$\frac{x}{1}+\frac{y}{2}+\frac{z}{4}=1$

4x+2y+y=4.

6. **Find equation of plane passing through point (1,1,1) and parallel to the plane x+2y+3z-7=0.**

Sol: given Equation of the plane is x+2y+3z-7=0.

Required parallel equation of the plane is x+2y+3z=k

This plane is passing through (1, 1, 1)

⇒ x+2y+3z=k ⇒ 1+2+3=k

⇒k=6.

Equation of the required plane is x+2y+3z=6.

7. **Find equation of plane passing through point (-2, 1, 3) and having (3, -5, 4). As direction ratios of its normal.**

Sol: Equation of the plane passing through (x_1, y_1, z_1) and having (a, b, c) as d.r.'s is $a(x-x_1)+b(y-y_1)+c(z-z_1)=0$

3(x+2) -5(y-1) +4(z-3) =0

⇒3x – 5y +4z +6 +5 -12=0

3x – 5y +4z =1.

Q.No .5 and 6

1. Compute $\lim_{x\to a}\left(\frac{x\sin a-a\sin x}{x-a}\right)$

Sol: $\lim_{x\to a}\left(\frac{x\sin a-a\sin x}{x-a}\right)$

$=\lim_{x\to a}\left(\frac{x\sin a-asia)-(a\sin x-a\sin a)}{x-a}\right)$

$=\lim_{x\to a}\left(\frac{(x-a)\sin a-a(siax-\sin a)}{x-a}\right)$

$=\lim_{x\to a}\frac{(x-a)sina}{(x-a)}-a\lim_{x\to a}\frac{(sinx-sina)}{(x-a)}$

=sina –a $\lim_{x\to a}\frac{(2\cos(\frac{x+a}{2})\sin(\frac{x-a}{2}))}{(x-a)}$

=sina – a $\lim_{x\to a}2\cos\left(\frac{x+a}{2}\right)\lim_{x\to a}\frac{(\sin(\frac{x-a}{2}))}{\frac{(x-a)}{2}}.\frac{1}{2}$

=sina – acosa.

2. **Compute** $\lim_{x\to 0}\left(\frac{3^x-1}{\sqrt{1+x}-1}\right)$

Sol: $\lim_{x\to 0}\left(\frac{3^x-1}{\sqrt{1+x}-1}\right)$

$=\lim_{x\to 0}\left(\frac{3^x-1}{x}\right)\lim_{x\to 0}\left(\frac{1}{\sqrt{1+x}-1}\right)$

=

$=2\frac{m^2}{n^2}$

4. **Compute** $\lim_{x\to 0}\frac{\sin(a+bx)-\sin(a-bx)}{x}$

Sol: $\lim_{x\to 0}\frac{\sin(a+bx)-\sin(a-bx)}{x}$

$=\lim_{x\to 0}\frac{2\cos(\frac{a+bx+a-bx}{2})\sin(\frac{a+bx-a+bx}{2})}{x}$

$=2cosa\lim_{x\to 0}\frac{sinbx}{bx}\times b$

$=2cosa(1)b=2bcosa.$

5. **Compute** $\lim_{x\to\infty}\frac{x^2-x+3}{x^2-2x+5}$

Sol: $\lim_{x\to\infty}\frac{x^2-x+3}{x^2-2x+5}$

$\lim_{x\to 0}(3^x-1)\lim_{x\to 0}\frac{1}{(\sqrt{1+x}-1)}\frac{(\sqrt{1+x}+1)}{(\sqrt{1+x}+1)}$

$=\lim_{x\to 0}\frac{(3^x-1)}{x}\lim_{x\to 0}(\sqrt{1+x}+1)$

=(log 3). (1+1)

=2(log 3).

3. **Compute** $\lim_{x\to 0}\left(\frac{1-\cos 2mx}{\sin^2 nx}\right)$

Sol: $\lim_{x\to 0}\frac{1-cos2mx}{sin^2nx}$

$=\lim_{x\to 0}\frac{2sin^2mx}{sin^2nx}$

$=2.\frac{(\lim_{x\to 0}\frac{sinmx}{mx})^2}{\lim_{x\to 0}(\frac{sinnx}{nx})^2}\times\frac{m^2}{n^2}$

Aims tutorial

$=\lim_{x\to\infty}\frac{x^2(1-\frac{1}{x}+\frac{3}{x^2})}{x^2(1-2/x+5/x^2)}$

$=\lim_{x\to\infty}\frac{(1-\frac{1}{x}+\frac{3}{x^2})}{(1-2/x+5/x^2)}$

$=\lim_{\substack{x\to\infty \\ \frac{1}{x}\to 0}}\frac{(1-0+0)}{(1-0+0)}$

$=1$

6. Compute $\lim_{x\to 0}\left(\frac{e^x-1}{\sqrt{1+x}-1}\right)$

Sol: $\lim_{x\to 0}(e^x-1)\lim_{x\to 0}\frac{1}{(\sqrt{1+x}-1)}\frac{(\sqrt{1+x}+1)}{(\sqrt{1+x}+1)}$

$=\lim_{x\to 0}\frac{(e^x-1)}{x}\lim_{x\to 0}(\sqrt{1+x}+1)$

$=1.(1+1)$

$=2$

7. Compute $\lim_{x\to\infty}\frac{8|x|+3x}{3|x|-2x}$

$=\lim_{x\to\infty}\frac{8|x|+3x}{3|x|-2x}$

Sol:

$=x\to\infty \Rightarrow x>0 \therefore |x|=x$

$=\lim_{x\to\infty}\frac{8|x|+3x}{3|x|-2x}$

$=\lim_{x\to\infty}\frac{8x+3x}{3x-2x}$

$=\lim_{x\to\infty}\frac{11x}{1x}=11.$

8. If the function f is defined by

$$f(x)=\begin{cases}k^2x-k & \text{if } x\geq 1\\ 2 & 1<x\end{cases}$$

it is continuous on R find k

$\lim_{x\to 1-}f(x)=\lim_{x\to 1-}2=2$

Sol:

$\lim_{x\to 1+}f(x)=\lim_{x\to 1-}(k^2x-k)=k^2-k$

Given f(x) is continuous at x=0

$\lim_{x\to 1-}f(x)=\lim_{x\to 1+}f(x)$

$2=k^2-k$

$\Rightarrow k^2-k-2=0$

$\Rightarrow$ (k-2) (k-1) =0

K=2 or k=-1

9. Is f defined by f(x)=$\begin{cases}\frac{\sin 2x}{x} & \text{if } x\neq 0\\ 1 & if\ x=0\end{cases}$

Continuous at x=0.

Sol: $\lim_{x\to 1-}f(x)=\lim_{x\to 0}\frac{\sin 2x}{x}=2$

But f (0) =1

$\lim_{x\to 1-}f(x)\neq f(0)$

$\therefore$f is not continuous at x=o

10. Show that $\lim_{x\to\infty}\left(\sqrt{x^2+x}-x\right)$**=1/2**

Sol: $\lim_{x\to\infty}(\sqrt{x^2+x}-x)$

$=\lim_{x\to\infty}\frac{(\sqrt{x^2+x}-x)(\sqrt{x^2+x}+x)}{(\sqrt{x^2+x}+x)}$

$=\lim_{x\to\infty}\frac{x^2+x-x^2}{(\sqrt{x^2+x}+x)}$

$=\lim_{x\to\infty}\frac{x}{(\sqrt{x^2+x}+x)}$

$=\lim_{x\to\infty}\frac{x}{(x\sqrt{1+1/x}+1)}$

$=\lim_{\substack{x\to\infty \\ \frac{1}{x}\to 0}}\frac{1}{(\sqrt{1+\frac{1}{x}}+1)}$

$=\frac{1}{1+1}$

$=\frac{1}{2}$

11. ***compute*** $\lim_{x\to 0}\frac{a^x-1}{b^x-1}$.

sol: $\lim_{x\to 0}\frac{a^x-1}{b^x-1}$.

$=\frac{\lim_{x\to 0}\frac{a^x-1}{x}}{\lim_{x\to 0}\frac{b^x-1}{x}}$

$=\frac{\log a}{\log b}$

$=\log_b a$

12. Compute $\lim_{x\to\frac{\pi}{2}} \frac{cosx}{(x-\frac{\pi}{2})}$.

Sol: $\lim_{x\to\frac{\pi}{2}} \frac{cosx}{(x-\frac{\pi}{2})}$.

Let x - $\frac{\pi}{2}=\theta \Rightarrow x = \frac{\pi}{2}+\theta$

$\lim_{x-\frac{\pi}{2}\to 0} \frac{\sin(\frac{\pi}{2}+\theta)}{\theta}$

$=\lim_{\theta\to 0} -\frac{\sin(\theta)}{\theta}$

=-1.

13. Evaluate $\lim_{x\to 1} \frac{\sin(x-1)}{(x^2-1)}$.

sol: $\lim_{x\to 1} \frac{\sin(x-1)}{(x^2-1)}$ s

$=\lim_{x\to 1} \frac{\sin(x-1)}{(x-1)(x+1)}$.

$= \lim_{x-1\to 0} \frac{\sin(x-1)}{(x-1)}. \lim_{x\to 1} \frac{1}{x+1}$

$=1.\frac{1}{1+1}$

$=\frac{1}{2}$

14. Evaluate $\lim_{x\to 1} \frac{e^{3+x}-e^3}{(x)}$.

Sol:$\lim_{x\to 1} \frac{e^{3+x}-e^3}{(x)}$

$= \lim_{x\to 0} \frac{e^{3+x}-e^3}{x}$

$= \lim_{x\to 0} \frac{e^3(e^x-1)}{x}$

$= e^3.\lim_{x\to 0} \frac{(e^x-1)}{x}$

$=e^3(1)$

$=e^3$.

15. $\lim_{x\to 0} \frac{\log_e(1+5x)}{x}$

Sol: As $x \to 0 \Rightarrow 5x \to 0$

$= \lim_{5x\to 0} \frac{\log_e(1+5x)}{5x} \times 5$

$= (1)5$ $\{\therefore \lim_{x\to 0} \frac{\log_e(1+x)}{x} = 1\}$

=5

Q.NO. 7 & 8

1. **If f(x) =log $(\sec x + \tan x)$, find f'(x).**

   Sol: f(x) =log $(\sec x + \tan x)$

   Differentiating w.r.t 'x'

   $\Rightarrow$ f'(x)=$\frac{1}{(\sec x+\tan x)}\frac{d}{dx}(\sec x + \tan x)$

   $=\frac{1}{(\sec x+\tan x)} \times (\sec x \tan x + sec^2 x)$

   $=\frac{secx(secx+tanx)}{(\sec x+\tan x)}$

   $=\sec x$.

2. **If f(x) =1+x+x²+x³+x⁴..... +x¹⁰⁰, then f' (1).**

   Sol: f(x) =1+x+x²+x³+x⁴..... +x¹⁰⁰

   Differentiating w.r.t 'x'

   $\Rightarrow$f'(x) =1+2x+3x²+4x³+.....+ 100x⁹⁹

   $\Rightarrow$ f' (1) =1+2+3+4.....+100

   $=\frac{(100)(101)}{2}$

   =5050 $(\therefore \sum n = \frac{n(n+1)}{2})$.

3. **Find the derivates of the following functions.**

   $\boldsymbol{y = \log(tan5x)}$

   sol: $y = \log(tan5x)$

   Differentiating w.r.t 'x'

   $\frac{dy}{dx} = \frac{1}{tan5x} \times \frac{d}{dx}(tan5x)$

   $= \frac{1}{tan5x} sec^2 5x \times (5)$

   $=\frac{5}{sin5x.cos5x} = \frac{2\times 5}{2sin5xcos5x}$

   $=\frac{10}{sin10x}$

   $=10cosec10x$.

4. $\boldsymbol{y = \sinh^{-1}\left(\frac{3x}{4}\right)}$

   $sol: y = \sinh^{-1}\left(\frac{3x}{4}\right)$

   $differentiating\ w.r.t\ 'x'$

   $\frac{dy}{dx} = \frac{1}{\sqrt{1+(\frac{3x}{4})^2}} \times \frac{d}{dx}(\frac{3x}{4})$

   $= \frac{1}{\sqrt{1+\frac{9x^2}{16}}} \times \frac{3}{4}$

   $= \frac{3}{\sqrt{16+9x^2}}$.

5. $\boldsymbol{x = tanh^2 y}$

   Sol: $x = tanh^2 y \Rightarrow \sqrt{x} = \pm tanhy$

   $y = \tanh^{-1}\sqrt{x}$

   $differentiating\ w.r.t\ 'x'$

   $\frac{dy}{dx} = \frac{1}{\pm(1-x)}\frac{d}{dx}(\sqrt{x})$

$= \pm \frac{1}{(1-x)} \frac{1}{2\sqrt{x}}$

6. $\boldsymbol{x = \tan(e^{-y})}$

$sol: x = \tan(e^{-y}) \Rightarrow \tan^{-1} x = e^{-y}$

$differentiating\ w.r.t\ 'x'$

$\frac{1}{1+x^2} = e^{-y} \times (-\frac{dy}{dx})$

$\frac{dy}{dx} = \frac{-e^{-y}}{1+x^2}$

7. $\boldsymbol{y = \cos[\log(cotx)]}$

8. Sol: $y = \cos[\log(cotx)]$

$\frac{dy}{dx} = -\sin[\log(cotx)] \times \frac{d}{dx}([\log(cotx)])$

$= -\sin[\log(cotx)] \times \frac{1}{\text{cotx}} \times \frac{d}{dx}(cotx)$

$= \frac{-\sin[\log(cotx)]}{cotx} \times (-cosec^2 x)$

$= -\frac{\sin[\log(cotx)](cosecx)}{cosx}.$

9. $\boldsymbol{y = \sin^{-1}(3x - 4x^3)}$

$sol: y = \sin^{-1}(3x - 4x^3)$

$put\ x = sin\theta \Rightarrow \theta = \sin^{-1} x$

$y = \sin^{-1}(3sin\theta - 4sin^3\theta)$

$y = \sin^{-1}(sin3\theta)$

$y = 3\theta = 3\sin^{-1} x$

$differentiating\ w.r.t\ 'x'$

$\frac{dy}{dx} = 3\frac{1}{\sqrt{1-x^2}}$

$\frac{dy}{dx} = \frac{3}{\sqrt{1-x^2}}.$

10. $\boldsymbol{f(x) = a^x e^{x^2}}$

$sol:\ f(x) = a^x e^{x^2}$

$\frac{d}{dx}(uv)' = uv' + vu'.$

$differentiating\ w.r.t\ 'x'$

$\Rightarrow f'(x) = a^x \frac{d(e^{x^2})}{dx} + e^{x^2} \frac{d}{dx}(a^x)$

$= a^x e^{x^2}.2x + e^{x^2}\ a^x$

$= a^x e^{x^2}(2x + 1).$

11. $\boldsymbol{y = \log[sin(logx)]}$

$sol: \boldsymbol{y = \log[sin(logx)]}$

$differentiating\ w.r.t\ 'x'$

$\frac{dy}{dx} = \frac{1}{\sin(logx)} \times \frac{d}{dx}(\sin(logx))$

$= \frac{1}{\sin(logx)} \times \cos(logx) \frac{d}{dx}(logx)$

$= \frac{\frac{1}{\sin(logx)} \times (\cos(logx))}{x}.$

$= \frac{1}{\sin(logx)} \frac{(cos(logx))}{x}$

12. $\boldsymbol{y = \sin[\tan^{-1}(e^{-x})]}$

$sol: y = \sin[\tan^{-1}(e^{-x})]$

$differentiating\ w.r.t\ 'x'$

$\boldsymbol{\frac{dy}{dx} = cos[\tan^{-1}(e^{-x})] \frac{d}{dx}[\tan^{-1}(e^{-x})]}$

$\boldsymbol{= cos[\tan^{-1}(e^{-x})] \frac{1}{1+(e^{-x})^2} \frac{d}{dx}(e^{-x})}$

$\boldsymbol{= cos[\tan^{-1}(e^{-x})] \frac{1}{1+e^{-2x}}(-e^{-x})}$

$\boldsymbol{= cos[\tan^{-1}(e^{-x})] \frac{(e^x)}{1+(e)^{2x}}}$

13. $\boldsymbol{y = \tan^{-1}(\frac{2x}{1+x^2})}$

$\boldsymbol{sol}$: Put x=$\tan\theta \Rightarrow \theta = \tan^{-1} x$

$\boldsymbol{y = \tan^{-1}(\frac{2x}{1+x^2})}$

$\boldsymbol{y = \tan^{-1}(\frac{2tan\theta}{1+tan^2\theta})}$

$\boldsymbol{y = \tan^{-1}(tan2\theta)}$

$\boldsymbol{y = 2\theta = 2}\tan^{-1} x$

$differentiating\ w.r.t\ 'x'$

$\boldsymbol{\frac{dy}{dx} = \frac{2}{1+x^2}}$

14. $\boldsymbol{if\ f(x) = 7^{x^2+3x}}$

sol: $f(x) = 7^{x^2+3x}$

$differentiating\ w.r.t\ 'x'$

$\Rightarrow f'(x) = 7^{x^2+3x} \log 7 \times \frac{d}{dx}(x^2 + 3x)$

$= 7^{x^2+3x} \log 7(2x + 3)$

15. $\boldsymbol{y = x^x}$

$\boldsymbol{sol}$: $\boldsymbol{applying\ log\ on\ both\ sides}$

$logy = logx^x$

$\Rightarrow logy = xlogx$

$differentiating\ w.r.t\ 'x'$

$\frac{1}{y}\frac{dy}{dx} = x\frac{d}{dx}logx + logx.\frac{dx}{dx}$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = x(\frac{1}{x}) + logx.1$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = 1 + logx$

$\frac{dy}{dx} = y(1 + logx)$

16. $\boldsymbol{if\ x = acos^3t,\ y = asin^3t, find\ \frac{dy}{dx}}.$

Sol:$x = acos^3t\ \ and\ y = asin^3t$

$differentiating\ w.r.t\ 't'$

$\frac{dx}{dt} = a.3.cos^2t(-sint) = -3acos^2t.sint$

$and\ \frac{dy}{dt} = a.3.sin^2t(cost)$

$\frac{dy}{dx} = \frac{-3acos^2t.sint}{3asin^2t.cost} = -\frac{cost}{sint} = -cott.$

17. $\boldsymbol{y = \{\cot^{-1} x^3\}^2}$

Sol: $differentiating\ w.r.t\ 'x'$

$$\frac{dy}{dx} = 2\{\cot^{-1} x^3\}\frac{-1}{1+(x^3)^2}.3x^2$$

$=\frac{\{\cot^{-1} x^3\}3x^2}{1+x^6}$.

Q.No.9 & 10

1. Find the approximate value of $\sqrt{82}, \sqrt[3]{65}$, sin60°1', $\sqrt[3]{7.8}$.
   Sol (I). 82=81+1=81(1+1/81)
   Here let x=81, $\Delta x = 1, f(x) = \sqrt{x}$
   $dy = f'(x).\Delta x$
   $dy = \frac{1}{2\sqrt{x}}.\Delta x = \frac{1}{2\sqrt{81}}.1$
   $=\frac{1}{18} = 0.0555$
   $f(x+\delta x) - f(x) = dy$
   $\Rightarrow f(x+\delta x) = f(x) + dy$
   $=\sqrt{81}+0.0555$
   $=9+0.0555$
   $=9.0555$

2. Find dy and Δy if y=x²+3x+6, when x=10, $\Delta x = 0.01$.
   Sol: $here\ x = 10, \Delta x = 0.01 and\ \Delta y = f(x+\Delta x) - f(x)$
   $\Delta y = f(10.01) - f(10)$
   $=([10.01]^2 + 3[10.10] + 6) - (10^2 + 3(10) + 6)$
   = (100.2001+30.03+6)-(100+30+6)
   =136.2031-136
   =0.2301.
   y=x²+3x+6
   $dy = f'(x).\Delta x$
   = (2x+3) (0.01) {x=10}
   =23(0.01)
   =0.23

3. Find dy and Δy if y=$e^x + x$ when x=5, $\Delta x = 0.02$.
   Sol: $here\ x = 5, \Delta x = 0.02\ and\ \Delta y = f(x+\Delta x) - f(x)$
   $\Delta y = f(5.02) - f(5)$
   $=e^{5.02} + 5.02 - (e^5 + 5)$
   $=e^{5.02} + 0.02 - e^5$
   $=e^5(e^{0.02} - 1)+0.02.$
   y=$e^x + x$
   $dy = f'(x).\Delta x$
   =9.056.
   $(ii).\ let\ x = 64, \Delta x = 1, f(x) = \sqrt[3]{x} = x^{1/3}$
   $\Rightarrow$f'(x) $=\frac{1}{3}x^{\frac{-2}{3}}$
   $f(x+\delta x) = f(x) + f'(x).\Delta x$
   $= \sqrt[3]{x} + \frac{1}{3}x^{\frac{-2}{3}}.1$
   $= \sqrt[3]{64} + \frac{1}{3}(64)^{\frac{-2}{3}}$
   $= 4 + \frac{1}{3}(4^3)^{\frac{-2}{3}}$
   $= 4 + \frac{1}{3}(\frac{1}{16})$
   $=4 + \frac{1}{48}=\frac{192+1}{48} = 4.020$
   $=(e^x + 1).\Delta x$
   $=(e^5 + 1).(0.02)$

4. Find dy and Δy if y=x²+x, when x=10, $\Delta x = 0.1$.(h/w)
5. If the side of a square is increased from 3 to 3.01cm. find the approximate increase in the area of the square.
6. The diameter of a sphere is measured to be 40cm, if an error of 0.02cm. Occurs in this, find the error in volume and surface area of the sphere?

Q.No .10

1. Verify Rolle's Theorem for the function f(x) =x² +4 on [-3, 3].
   Sol: f(x) =x² +4. f is continuous on [-3, 3].
   f is differentiable on[-3, 3].
   And f (3) =9+4=13
   f (-3)=9+4=13
   f(3)=f(-3)
   thus f satisfies all the conditions of Rolle's theorem.
   ∴by roole's theorem ∃ c∈ (3, −3) such that f'(c)=0
   f '(x)=2x =0 ⇒ x=0
   The point c=0 ∈ (3, −3). $thus\ rolle'stheorem\ is\ verified.$
2. Verify Rolle's Theorem for the function f(x) =x(x+3) $e^{-x/2}$ on [-3, 0].

3. Verify Rolle's Theorem for the function f(x) =sinx-sin2x on $[0,\pi]$.
4. Verify the conditions of the Lagrange's mean value theorem for the following functions in each case find a point 'c' in the interval as stated by the theorem.(i)x^2-1 on[2, 3], (ii) sinx-sin2x on $[0,\pi]$.

Aims tutorial

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