1. Solve $4^{x-1} - 3.2^{x-1} + 2 = 0$

Sol: $4^{x-1} - 3.2^{x-1} + 2 = 0$

$let\ 2^{x-1} = a$

$\Rightarrow 4^{x-1} = (2^2)^{x-1} = (2^{x-1})^2 = a^2$

$4^{x-1} - 3.2^{x-1} + 2 = 0$

$\Rightarrow a^2 - 3a + 2 = 0$

$\Rightarrow a^2 - 2a - 1a + 2 = 0$

$\Rightarrow a(a-2) - 1(a-2) = 0$

$\Rightarrow (a-2)(a-1) = 0$

$\Rightarrow (a-2) = 0, (a-1) = 0$

$\Rightarrow a = 2,\ a = 1$

Case (i)	Case (ii)
$a = 2$ $\Rightarrow 2^{x-1} = 2^1$ $\Rightarrow x - 1 = 1$ $\therefore x=2$	$a = 1$ $\Rightarrow 2^{x-1} = 2^0$ $\Rightarrow x - 1 = 0$ $\therefore x=1$

2. Solve $2x^4 + x^3 - 11x^2 + x + 2 = 0$

Sol: $2x^4 + x^3 - 11x^2 + x + 2 = 0$

$[\div\ by\ x^2]$

$\Rightarrow \frac{2x^4}{x^2} + \frac{x^3}{x^2} - \frac{11x^2}{x^2} + \frac{x}{x^2} + \frac{2}{x^2} = 0$

$\Rightarrow 2x^2 + x - 11 + \frac{1}{x} + \frac{2}{x^2} = 0$

$\Rightarrow 2(x^2 + \frac{1}{x^2}) + (x + \frac{1}{x}) - 11 = 0$

$let\ \left(x + \frac{1}{x}\right) = a \qquad S.O.B$

$\Rightarrow x^2 + \frac{1}{x^2} + 2 = a^2 \Rightarrow \left(x^2 + \frac{1}{x^2}\right) = a^2 - 2$

$\Rightarrow 2(a^2 - 2) + (a) - 11 = 0$

$\Rightarrow 2a^2 + a - 4 - 11 = 0$

$\Rightarrow 2a^2 + a - 15 = 0$

$\Rightarrow 2a^2 + 6a - 5a - 15 = 0$

$\Rightarrow 2a(a+3) - 5(a+3) = 0$

$\Rightarrow (a+3)(2a-5) = 0 \Rightarrow a + 3 = 0, (2a-5) = 0$

$x + \frac{1}{x} + 3 = 0$ $\Rightarrow x^2 + 3x + 1 = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{-3 \pm \sqrt{9 - 4.1.1}}{2.1}$ $x = \frac{-3 \pm \sqrt{5}}{2}$	$2\left(x + \frac{1}{x}\right) - 5 = 0$ $\Rightarrow 2x^2 - 5x + 2 = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{5 \pm \sqrt{25 - 4.2.2}}{2.1}$ $x = \frac{5 \pm \sqrt{9}}{4} = \frac{5 \pm 3}{4}$ $x = 2, \frac{1}{2}$

3. Determine the range of the expression $\frac{x^2+x+1}{x^2-x+1}$.

Sol: $let\ y = \frac{x^2+x+1}{x^2-x+1}$

$\Rightarrow y(x^2 - x + 1) = x^2 + x + 1$

$\Rightarrow yx^2 - xy + y = x^2 + x + 1$

$\Rightarrow yx^2 - x^2 - xy - x + y - 1 = 0$

$\Rightarrow x^2(y-1) - x(y+1) + (y-1) = 0$

$comparing\ with\ ax^2 + bx + c = 0$

$a = (y-1), b = -(y+1), c = (y-1)$

Given $x\ is\ real \Rightarrow \Delta \equiv b^2 - 4ac \geq 0$

$\Rightarrow [-(y+1)]^2 - 4(y-1)(y-1) \geq 0$

$\Rightarrow (y+1)^2 - 4(y-1)^2 \geq 0$

$\Rightarrow y^2 + 1 + 2y - 4(y^2 + 1 - 2y) \geq 0$

$\Rightarrow y^2 + 1 + 2y - 4y^2 - 4 + 8y \geq 0$

$\Rightarrow -3y^2 + 10y - 3 \geq 0\ [\div\ by\ -]$

$\Rightarrow 3y^2 - 10y + 3 \leq 0$

$\Rightarrow 3y^2 - 9y - 1y + 3 \leq 0$

$\Rightarrow 3y(y-3) - 1(y-3) \leq 0$

$\Rightarrow (3y-1)(y-3) \leq 0$

$\Rightarrow y \in \left[\frac{1}{3}, 3\right]$

4. **If x is a real number, find the range of** $\frac{x+2}{2x^2+3x+6}$

Sol: $let\ y = \frac{x+2}{2x^2+3x+6}$

$\Rightarrow y(2x^2+3x+6) = x+2$

$\Rightarrow (2x^2y+3xy+6y) = x+2$

$\Rightarrow 2x^2y+3xy-x+6y-2=0$

$\Rightarrow x^2 2y + x(3y-1) + (6y-2) = 0$

$comparing\ with\ ax^2+bx+c=0$

$a=(2y), b=(3y-1), c=(6y-2)$

Given $x\ is\ real \Rightarrow \Delta \equiv b^2-4ac \geq 0$

$\Rightarrow [3y-1]^2 - 4(2y)(6y-2) \geq 0$

$\Rightarrow 9y^2+1-6y-(48y^2-16y) \geq 0$

$\Rightarrow 9y^2+1-6y-48y^2+16y \geq 0$

$\Rightarrow -39y^2+10y+1 \geq 0$ $[\div by -]$

$\Rightarrow 39y^2-10y-1 \leq 0$

$\Rightarrow 39y^2-13y+3y-1 \leq 0$

$\Rightarrow 13y(3y-1)+1(3y-1) \leq 0$

$\Rightarrow (3y-1)(13y+1) \leq 0$

$\Rightarrow y \in \left[-\frac{1}{13}, \frac{1}{3}\right]$

5. **S.T** $\frac{x}{x^2-5x+9}$ **lies between** $-\frac{1}{11}, 1$

Sol: $let\ y = \frac{x}{x^2-5x+9}$

$\Rightarrow y(x^2-5x+9) = x$

$\Rightarrow x^2y-5xy+9y-x=0$

$\Rightarrow x^2y - x(5y+1) + 9y = 0$

$comparing\ with\ ax^2+bx+c=0$

$a=(y), b=(5y+1), c=(9y)$

Given $x\ is\ real \Rightarrow \Delta \equiv b^2-4ac \geq 0$

$\Rightarrow [5y+1]^2 - 4(y)(9y) \geq 0$

$\Rightarrow 25y^2+1+10y-36y^2 \geq 0$

$\Rightarrow -11y^2+10y+1 \geq 0$

$[\div by -]$

$\Rightarrow 11y^2-10y-1 \leq 0$

$\Rightarrow 11y^2-11y+1y-1 \leq 0$

$\Rightarrow 11y(y-1)+1(y-1) \leq 0$

$\Rightarrow (y-1)(11y+1) \leq 0$

$\Rightarrow y \in \left[-\frac{1}{11}, 1\right]$

6. **If x is a real number, find the value of expression** $\frac{x^2+34x-71}{x^2+2x-7}$. **Do not lie between 5 and 9.**

Sol: $let\ y = \frac{x^2+34x-71}{x^2+2x-7}$

$\Rightarrow y(x^2+2x-7) = x^2+34x-71$

$\Rightarrow x^2y+2xy-7y-x^2-34x+71=0$

$\Rightarrow x^2(y-1)+2x(y-17)+(71-7y)=0$

$comparing\ with\ ax^2+bx+c=0$

$a=(y-1), b=2(y-17), c=(71-7y)$

Given $x\ is\ real \Rightarrow \Delta \equiv b^2-4ac \geq 0$

$\Rightarrow 4[y-17]^2 - 4(y-1)(71-7y) \geq 0$

$\Rightarrow y^2+289-34y-(71y-7y^2-71+7y) \geq 0$

$\Rightarrow 8y^2-112y+360 \geq 0$

$[\div by 8]$

$\Rightarrow y^2-14y+45 \geq 0$

$\Rightarrow y^2-9y-5y+45 \geq 0$

$\Rightarrow y(y-9)-5(y-9) \geq 0$

$\Rightarrow (y-9)(y-5) \geq 0$

$\Rightarrow y \in (-\infty, 5] \cup [9, \infty)$

Hence the expression $\frac{x^2+34x-71}{x^2+2x-7}$ do not lie between 5 and 9.

7. **If x is a real number, find the maximum value of expression $\frac{x^2+14x+9}{x^2+2x+3}$.**

Sol: $let\ y = \frac{x^2+14x+9}{x^2+2x+3}$

$\Rightarrow y(x^2+2x+3) = x^2+14x+9$

$\Rightarrow x^2y+2xy+3y-x^2-14x-9=0$

$\Rightarrow x^2(y-1)+2x(y-7)+(3y-9)=0$

$comparing\ with\ ax^2+bx+c=0$

$a=(y-1), b=2(y-7), c=(3y-9)$

Given $x\ is\ real \Rightarrow \Delta \equiv b^2-4ac \geq 0$

$\Rightarrow 4[y-7]^2-4(y-1)(3y-9) \geq 0$

$\Rightarrow y^2+49-14y-(3y^2-9y-3y+9) \geq 0$

$\Rightarrow y^2+49-14y-3y^2+12y-9 \geq 0$

$\Rightarrow -2y^2-y+40 \geq 0$
$[\div\ by -2]$

$\Rightarrow y^2+y-20 \leq 0$

$\Rightarrow y^2+5y-4y+20 \leq 0$

$\Rightarrow y(y+5)-4(y+5) \leq 0$

$\Rightarrow (y+5)(y-4) \leq 0$

$\Rightarrow y \in [-5,4]$ max value is 4

8. **Prove that $\frac{1}{3x+1}+\frac{1}{x+1}-\frac{1}{(3x+1)(x+1)}$ does not lie between 1 and 4 if x is real.**

Sol: $\frac{1}{3x+1}+\frac{1}{x+1}-\frac{1}{(3x+1)(x+1)} = \frac{x+1+3x+1-1}{(3x+1)(x+1)}$

$=\frac{4x+1}{(3x+1)(x+1)}$

$let\ y = \frac{4x+1}{(3x+1)(x+1)}$

$\Rightarrow y = \frac{4x+1}{3x^2+4x+1}$

$\Rightarrow 3x^2y+4xy+y=4x+1$

$\Rightarrow 3x^2y+4xy-4x+y-1=0$

$\Rightarrow x^2(3y)+2x(2y-2)+(y-1)=0$

$comparing\ with\ ax^2+bx+c=0$

$a=(3y), b=2(2y-2), c=(y-1)$

Given $x\ is\ real \Rightarrow \Delta \equiv b^2-4ac \geq 0$

$\Rightarrow 4[2y-2]^2-4(3y)(y-1) \geq 0$

$\Rightarrow 4y^2+4-8y-(3y^2-3y) \geq 0$

$\Rightarrow 4y^2+4-8y-3y^2+3y \geq 0$

$\Rightarrow y^2-5y+4 \geq 0$

$\Rightarrow y^2-y-4y+4 \geq 0$

$\Rightarrow y(y-1)-4(y-1) \geq 0$

$\Rightarrow (y-1)(y-4) \geq 0$

$\Rightarrow y \in (-\infty,1] \cup [4,\infty)$

Hence the expression do not lie between 1 and 4.

9. If the expression $\frac{x-p}{x^2-3x+2}$ takes all real values for $x \in R$, *then find the bounds for p.*

Sol:

$let\ y = \frac{x-p}{x^2-3x+2}$

$\Rightarrow x^2y - 3xy + 2y = x - p$

$\Rightarrow x^2y - 3xy - x + 2y + p = 0$

$\Rightarrow x^2y - x(3y+1) + (2y+p) = 0$

$comparing\ with\ ax^2 + bx + c = 0$

$a = (y), b = -(3y+1), c = (2y+p)$

Given $x\ is\ real \Rightarrow \Delta \equiv b^2 - 4ac \geq 0$

$\Rightarrow [3y+1]^2 - 4(y)(2y+P) \geq 0$

$\Rightarrow 9y^2 + 1 + 6y - (8y^2 + 4py) \geq 0$

$\Rightarrow 9y^2 + 1 + 6y - 8y^2 - 4py \geq 0$

$\Rightarrow y^2 - 2y(2p-3) + 1 \geq 0$

Here coefficient of y^2>0

So, the roots of above eq"n are imaginary or real equal

$\Rightarrow \Delta \equiv b^2 - 4ac \leq 0$

$comparing\ with\ ay^2 + by + c = 0$

$a = (1), b = -2(2p-3), c = (1)$

$\Delta \equiv b^2 - 4ac \leq 0$

$\Rightarrow 4(2p-3)^2 - 4(1)(1) \leq 0$

$\Rightarrow 4(4p^2 + 9 - 12p) - 4 \leq 0 \quad \{\div by\ 4\}$

$\Rightarrow 4p^2 + 9 - 12p - 1 \leq 0$

$\Rightarrow 4p^2 - 12p + 8 \leq 0 \quad \{\div by\ 4\}$

$\Rightarrow p^2 - 3p + 2 \leq 0$

$\Rightarrow p^2 - 2p - 1p + 2 \leq 0$

$\Rightarrow p(p-2) - 1(p-2) \leq 0$

$\Rightarrow (p-2)(p-1) \leq 0$

$\Rightarrow p \in [1,2]$

But $\frac{x-p}{x^2-3x+2}$ is not define for p=1, 2

$p \in (1,2)$

10. **Find set of values of x for which the equalities q $x^2 - 3x - 10 < 0$, $10x - x^2 - 16 > 0$ hold simultaneously.**

Sol: consider $x^2 - 3x - 10 < 0$

$\Rightarrow x^2 - 5x + 2x - 10 < 0$

$\Rightarrow x(x-5) + 2(x-5) < 0$

$\Rightarrow (x-5)(x+2) < 0$

$\Rightarrow x \in (-2,5)$

Now $10x - x^2 - 16 > 0$

$\Rightarrow x^2 - 10x + 16 < 0$

$\Rightarrow x^2 - 8x - 2x + 16 < 0$

$\Rightarrow x(x-8) - 2(x-8) < 0$

$\Rightarrow (x-8)(x-2) < 0$

$\Rightarrow x \in (2,8)$

Required solution set is $(-\mathbf{2}, \mathbf{5}) \cap (\mathbf{2}, \mathbf{8}) = (\mathbf{2}, \mathbf{5})$

11. **If the roots of $ax^2 + bx + c = 0$ are imaginary,show that all $x \in R$, $ax^2 + bx + c$ and a have the same sign.**

Sol: Given that the roots of $ax^2 + bx + c = 0$ are imaginary,

$\Rightarrow \Delta \equiv b^2 - 4ac < 0$

$\Rightarrow 4ac - b^2 > 0$.....(1)

Consider $\frac{ax^2+bx+c}{a}$

$= x^2 + \frac{b}{a}x + \frac{c}{a}$

$= x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2 + \frac{c}{a}$

$= (x + \frac{b}{2a})^2 + \frac{c}{a} - \frac{b^2}{4a^2}$

$= (x + \frac{b}{2a})^2 + \frac{4ac-b^2}{4a^2}$ from (1)

$\geq$ $>$

>0 $\therefore$ **all $x \in R$, $ax^2 + bx + c$ and a have the same sign.**

12. ***Let α, β be the real roots of $ax^2 + bx + c = 0$ where***

$\alpha < \beta$, then prove that
(i) for $\alpha < x < \beta$,
$ax^2 + bx + c$ *and a have opposite signs.*

(i) for $\alpha < x$ or $x > \beta$ $ax^2 + bx + c$ *and a have same sign.*

Sol: *Given* α, β *be the real roots of* $ax^2 + bx + c = 0$ where $\alpha < \beta$

$\Rightarrow ax^2 + bx + c = a(x - \alpha)(x - \beta)$

$$\Rightarrow \frac{ax^2+bx+c}{a} = (x - \alpha)(x - \beta) \ldots.(1)$$

(i) suppose $x \in R$, $\alpha < x < \beta$,