1. ***Prove that*** $n_{P_r} = \frac{(n)!}{[n-r]!}$

Sol: clearly n_{P_r} is equal to the number of ways of filling r blank places which are arranged in a row by n dissimilar things.

The first blank place can be filled by any one of n things and hence it can be done in n ways.

Then second blank place can be filled in (n-1) ways. Similarly third blank place can be filled in (n-2) ways and so on. Proceeding in this way, the rth blank place can be filled in n-(r-1)=n-r+1 ways

By counting principle, the r blank places can be filled in $n(n-1)(n-2)\ldots.(n-r+1)$

$$= \frac{(n)(n-1)\ldots(n-r+1)(n-r)\ldots.4.3.2.1}{1.2.3\ldots.(n-r)}$$

$$= \frac{n!}{(n-r)!}$$

2. ***Prove that*** $n_{P_r} = r.(n-1)_{P_{(r-1)}} + (n-1)_{P_r}$.

Sol:

R.H.S

$\Rightarrow (n-1)_{P_r} + r.(n-1)_{P_{(r-1)}}$

$\Rightarrow \frac{(n-1)!}{[n-1-r]!} + r.\frac{(n-1)!}{[n-1-r+1]!}$

$\Rightarrow (n-1)!\left[\frac{1}{[n-1-r]!} + \frac{r}{[n-r]!}\right]$

$\Rightarrow (n-1)!\left[\frac{(n-r)}{(n-r)[n-1-r]!} + \frac{r}{[n-r]!}\right]$

$\Rightarrow (n-1)!\left[\frac{(n-r)}{(n-r)!} + \frac{r}{[n-r]!}\right]$

$\Rightarrow (n-1)!\left[\frac{(n-r+r)}{(n-r)!}\right]$

$\Rightarrow \left[\frac{(n)(n-1)!}{(n-r)!}\right]$

$\Rightarrow \left[\frac{(n)!}{(n-r)!}\right] = n_{P_r}$.L.H.S

$\therefore n_{P_r} = r.(n-1)_{P_{(r-1)}} + (n-1)_{P_r}$.

3. ***Find the number of 4 letter words that can be formed using the letters of the word 'MIXTURE' which i) contains x ii) do not contain the letter X.***

Sol: we have to fill 4 blanks using 7 letters of the word 'MIXTURE'. Take 4 blanks _ _ _ _

- First we put X in one of the 4 blanks. This can be done in 4 ways

Now we can fill the remaining 3 places with the remaining 6 letters in 6_{P_3}ways. Thus the number of 4 letter words containing the letter X =4. 6_{P_3}

$$= 4.6 \times 5 \times 4 = 480.$$

- Leaving the letter X, we have to fill 4 blanks with the remaining 6 letters in 6_{P_4} *ways*

Thus the number of 4 letter words do not containing the letter X

$=6_{P_4} = 6 \times 5 \times 4 \times 3 = 360$

4. **Find the number of ways of arranging 6 boys and 6 girls in a row so that (i) all the girls sit together. (ii) No two girls sit together. (iii) Boys and girls sit alternately.**

Sol: given

No. of boys = 6

No. of girls = 6

<u>(i)All the girls sit together.</u>

Treat 6 girls as 1 unit then we have 6units of boys + I unit of girls these 7 units can be arrange in 7! ways

Now the 6 girls can be arrange among themselves in 6! ways

$\therefore$ Total no of arrangements = 7! 6!

<u>(ii)No two girls sit together.</u>

First we can arrange 6 boys in a row in 6! ways

There are 7 gaps (including the beginning gap and the ending gap) they can be arrange with 6 girls in 7_{P_6}ways

$\therefore$ Total no of arrangements = 6! 7_{P_6}.

<u>(iii) Boys and girls sit alternately.</u>

First we can arrange with either a boy or a girl it can be done in 2 ways.

Start with a boy: 6 boys can be arrange in 6! ways, after boys there are 6 gaps they can be arrange with 6 girls in 6! ways .

Total no of arrangements = 2. 6! 6!

5. Find the number of ways of permuting the letters of the word 'PICTURE' so that (i) all vowels come together. (ii) No two vowels come together.

Sol:

The word picture has 3 vowels {E, I, U}

and 4 consonants {P, C, R, T}

(i) **all vowels come together.**

Treat 3 vowels as one unit, then we can arrange 4 consonants + 1unit of vowels in 5! Ways.

Now, 3 vowels among themselves can be arranged in3! Ways

Total number of arrangements

5! 3! =720.3

(ii) No two vowels come together.

First we can arrange the 4 consonants in 4! Ways then in b/w the vowels, in the beginning and in the ending, there are 5 gaps

These 5 gaps can be filled with 3 vowels in 5_{P_3}ways

Total number of arrangements

$= 4!\times 5_{P_3}=24\times 5\times 4\times 3 = 1440 ways.$

6. Find the number of ways of arranging 5 different mathematics books, 4 different physics books and 3 different chemistry books such that the books of the same subject are together.

Sol: number of mathematic books=5

Number of physics books=4

Number of chemistry books=3

Given condition: Books of same subject are together.

Treat 5 mathematics books as 1st unit,

4 mathematics books as 2nd unit,

3 mathematics books as 3rd unit

Now these 3 units can be arranged in a row in 3!ways.

And 5 mathematics books can be arranged among them self in 5!,

4 physics books can be arranged among them self in 4!

and 3 chemistry books can be arranged among them self in 3!

Required total number of arrangements=3! 5! 4! 3!=1, 03,680.

7. Find the rank of the word (i)"MASTER."

Sol: Given word is MASTER

The alphabetical order of the letter of the word MASTER is A, E, M, R, S, T

The number of words starting with

$\underline{A}$ _ _ _ _ _ $= 5! = 120$

$\underline{E}$ _ _ _ _ _ $= 5! = 120$

M A E _ _ _ $= 3! = 6$

M A R _ _ _ $= 3! = 6$

M A S E _ _ $= 2! = 2$

M A S R _ _ $= 2! = 2$

M A S T E R $=0! = 1$

$\therefore$ *Rank of the word MASTER is*

$= 120 + 120 + 6 + 6 + 2 + 2 + 1 = 257$

Find the rank of the word (ii)"REMAST."

Sol: Given word is REMAST

The alphabetical order of the letter of the word REMAST is

A, E, M, R, S, T

The number of words starting with

$\underline{A}$ _ _ _ _ _ $= 5! = 120$

$\underline{E}$ _ _ _ _ _ $= 5! = 120$

$\underline{M}$ _ _ _ _ _ $= 5! = 120$

R A _ _ _ _ $= 4! = 24$

R E A _ _ _ $= 3! = 6$

R E M A S T $=0! = 1$

$\therefore$ *Rank of the word REMAST is*

$= 120 + 120 + 120 + 24 + 6 + 1 = 391$

(iii)Find the rank of the word "PRISON."

Sol: Given word is PRISON

The alphabetical order of the letter of the word PRISON is I, N, O, P, R, S

The number of words starting with

I _ _ _ _ _ = 5! = 120

N _ _ _ _ _ = 5! = 120

O _ _ _ _ _ = 5! = 120

P I _ _ _ _ = 4! = 24

P N _ _ _ _ = 4! = 24

P O _ _ _ _ = 4! = 24

P R I N _ _ = 2! = 2

P R I O _ _ = 2! = 2

P R I S N _ = 1! = 1

P R I S O N = 0! = 1

$\therefore$ *Rank of the word PRISON is*

$= 120 + 120 + 120 + 24 + 24 + 24 + 24 + 2 + 2 + 1 + 1 = 438$

(iv)Find the rank of the word "EAMCET."

Sol: Given word is EAMCET

The alphabetical order of the letter of the word EAMCET is A, C, E, E, M, T

The number of words starting with

A _ _ _ _ _ $= \frac{5!}{2!} = 60$

C _ _ _ _ _ $= \frac{5!}{2!} = 60$

E A C _ _ _ = 3! = 6

E A E _ _ _ = 3! = 6

E A M C E T = 3! =1

$\therefore$ *Rank of the word EAMCET is*

$= 60 + 60 + 6 + 6 + 1 = 133$

(v)Find the rank of the word "JANATA."

Sol: Given word is JANATA."

The alphabetical order of the letter of the word JANATA." is

A, A, A, J, N, T

The number of words starting with

A _ _ _ _ _ $= \frac{5!}{2!} = 60$

J A A _ _ _ = 3! = 6

J A N A A _ =1! = 1

J A N A T A =1! = 1

$\therefore$ *Rank of the word* JANATA *is*

$= 60 + 6 + 1 + 1 = 68$

(vi)Find the rank of the word "AJANTA."

Sol: Given word is AJANTA."

The alphabetical order of the letter of the word AJANTA." is

A, A, A, J, N, T

The number of words starting with

A A _ _ _ _ = 4! =24

A J A A _ _ = 2! = 2

A J A N A _ = 1! = 1

A J A N T A =1! = 1

$\therefore$ *Rank of the word* AJANTA *is* $= 24 + 1 + 1 = 28$

8. **Find the number of numbers that are greater than 4000 which can be formed using the digits 0, 2, 4, 6, 8 without repetition.**

Sol:

Given digits 0, 2, 4, 6, 8

Case (i)

The number of 4 digit numbers which are greater than 4000. Create 4 blanks

4 or 6 or 8			

a 4 digit number is greater than 4000 only if it's first blanks can be filled with either 4 or 6 or 8, it can be done in 3 ways.

Now the remaining 3 blanks can be arrange with leftover 4 digits in4_{P_3}.

Total number of arrangements $=3.\, 4_{P_3}. = 3\times 4\times 3\times 2 = 72$

Case (ii) every 5 digit number is greater than 4000.

Create 5 blanks

A 5 digit number is greater than 4000 if its first blank can be filled with either 2 or 4 or 6 or 8 it can be done in 4 ways.

Now the remaining 4 blanks can be arrange with leftover 4 digits in4_{P_4}.

Total number of arrangements

$=4.\, 4_{P_4}. = 4\times 4\times 3\times 2\times 1 = 96.$

∴ Total number of numbers which greater than 4000 is 72 + 96 =168.

9. **Find the sum of all 4 digited numbers that can be formed using the digits 1, 2, 4, 5, 6 without repetition.**

Sol: Given numbers 1, 2, 4, 5, 6 (n=5)

And r = 4

The sum of the r-digited numbers that can be formed using given n digits is

$(n-1)_{p_{(r-1)}}[(summ\ of\ all\ ndigits)(11\dots 1(r)]$

$=(5-1)_{p_{(4-1)}}[(1+2+4+5+6)(1111)]$

$=4_{p_3}[(18)(1111)]$

$=(4\times 3\times 2)(18).[1111]$

$=24[18]\ (1111)$

$=(432)\ (1111)$

$=4, 79, 952$

10. **Find the sum of all 4 digited numbers that can be formed using the digits 1, 3, 5, 7, 9 without repetition.**

Sol: Given numbers 1, 3, 5, 7, 9 (n=5)

And r = 4

The sum of the r-digited numbers that can be formed using given n digits is

$(n-1)_{p_{(r-1)}}[(summ\ of\ all\ ndigits)(11\dots 1(r)]$

$=(5-1)_{p_{(4-1)}}[(1+3+5+7+9)(1111)]$

$=4_{p_{3)}}[(25)(1111)]$

$=(4\times 3\times 2)[25]\ (1111)$

$=24.[25]\ (1111)$

=600(1111)

=6, 66, 600

11. **Find the sum of all 4 digited numbers that can be formed using the digits 0, 2, 4, 7, 8 without repetition.**

Sol: Given numbers 0, 2, 4, 7, 8 (n=5)

And r = 4

The sum of the r-digited numbers that can be formed using given n digits is

$(n-1)_{p_{(r-1)}}[(summ\ of\ all\ ndigits)(11\dots 1(r)]$
$-(n-2)_{p_{(r-2)}}[(summ\ of\ all\ ndigits)(11\dots 1(r-1)]$

$=(5-1)_{p_{(4-1)}}[(0+2+4+7+8)(1111)]$
$-(5-2)_{p_{(4-2)}}[(0+2+4+7+8)(1111)]$

$=4_{p_3}[(21)(1111)]$ -$3_{p_2}[(21)(111)]$

$=(4\times 3\times 2)(21)[1111]$ - $(3\times 2)(21)[111]$

= (24)(21)[1111] - (6)(21)[111]

= (504)[1111] - (126)[111]

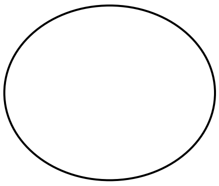
=559944-13986

=5, 45, 958.

12. Find the number of ways of arranging 7 gents and 4 ladies around a circular table if no two ladies wish to sit together.

Sol:

Given 7 gents & 4 ladies



First we arrange 7 gents around the Circular table in (7-1)! Ways

In b/w 7 gents there are 7 gaps are there, now we can arrange 4 ladies in these 7 gaps in 7_{P_4} *ways*

Total number of arrangements is $6!\ 7_{P_4}$.

13. Find the number of different ways of preparing a garland using 7 distinct red roses and 4 distinct yellow roses such that no two yellow roses come together.

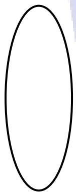
Sol:

Given red roses =7

Yellow roses=4

Condition:

No two yellow roses come together



First we can arrange 7 roses in a garland in

$$\frac{(7-1)!}{2} = \frac{6!}{2}\ ways \left[\frac{(n-1)!}{2}\right]$$

In b/w we have 7 gaps they can be arrange with 4 roses in 7_{P_4} *ways*.

Total number of arrangements is $\frac{6!}{2} \times 7_{P_4}$.

14. Find the number of ways of arranging the letters of the word SINGING so that (i) they begin and end with I. (ii) the two G's come together.

Sol: given word 'SINGING'

I						I

S----1

I----2

N---2

G---2

Total=7

First we can arrange first & last places with I's in 1 way as shown above

Now, we can arrange remaining 5 places with the remaining 5 letters S, N, G, N, G in which 2 G's and 2 N's in

$$= \frac{5!}{2!2!} = \frac{5\times4\times3\times2\times1}{2\times2} = 30 \text{ ways.}$$

(ii) The two G's come together.

Treat the 2 G's as one unit.

Then we have 6 letters in which there are 2 I's and 2 N's, they can be arrange in

$$= \frac{6!}{2!2!} = \frac{6\times5\times4\times3\times2\times1}{2\times2} = 180 \text{ ways}$$

COMBINATIONS

1. $n_{c_r} = \frac{n!}{(n-r)!r!}$.

sol:

Proof: consider one of the n_{c_r} combinations. this combination consists of r dissimilar things. If we permute the r things we get r! Permutations. Thus each combination gives rise to r! Permutations and hence n_{c_r} combinations give rise to $n_{c_r}\, r!\, permutations$. But the number of permutations of n dissimilar things taken r at a time is n_{P_r}.

$\therefore n_{c_r}\, r! = n_{P_r} \Rightarrow \frac{n_{P_r}}{r!} = \frac{n!}{r!(n-r)!}$.

2. If $n_{C_r} = n_{C_s}$, ***then prove that*** $r = s$ ***or*** $n = r + s$.

Sol:

given $n_{C_r} = n_{C_s}$

Suppose that r > s

$\Rightarrow n - r < n - s$

Now $n_{C_r} = n_{C_s}$

$\Rightarrow \frac{n!}{r!(n-r)!} = \frac{n!}{s!(n-s)!}$

$\Rightarrow (n-r)!\, r! = (n-s)!\, s!$

$\Rightarrow s!\,(s+1)(s+2)\ldots r(n-r)!$
$= s!\,(n-r)!\,(n-r+1)(n-r+2)\ldots\ldots(n-s)$

$\Rightarrow (s+1)(s+2)\ldots r = (n-r+1)(n-r+2)\ldots\ldots(n-s)$

Since each side of the above relation is a product $r - s$ consecutive positive integer, we get

$r = n - s$

$\Rightarrow n = r + s$

similarly if $r < s$, Then also we can prove that $n = r + s$.

$\therefore$ If $n_{C_r} = n_{C_s}$, ***then*** $r = s$ ***or*** $n = r + s$.

3. ***Show that*** $\frac{4n_{c_{2n}}}{2n_{c_n}} = \frac{1.3.5\ldots\ldots(4n-1)}{\{1.3.5\ldots\ldots(2n-1)\}^2}$

Sol: $\frac{4n_{c_{2n}}}{2n_{c_n}} = \frac{\left[\frac{(4n)!}{(4n-2n)!2n!}\right]}{\left[\frac{(2n)!}{(2n-n)!n!}\right]} = \frac{\left[\frac{(4n)!}{2n!2n!}\right]}{\left[\frac{(2n)!}{n!n!}\right]} = \frac{(4n)!}{\{2n!\}^2} \times \frac{n!^2}{2n!}$

$= \frac{[(4n)(4n-1)(4n-2)\ldots\ldots5.4.3.2.1]}{\{(2n)(2n-1)(2n-2)\ldots\ldots5.4.3.2.1\}^2} \times \frac{n!^2}{2n!}$

$= \frac{[(4n)(4n-2)\ldots\ldots4.2.][(4n-1)\ldots.5.3.1]}{\{[(2n)(2n-2)\ldots\ldots4.2][(2n-1)\ldots.5.3.1]\}^2} \times \frac{n!^2}{2n!}$v

Taking 2 common from Nr and Dr

$= \frac{2^{2n}[(2n)(2n-1)\ldots\ldots3.2.1.][(4n-1)\ldots.5.3.1]}{\{2^n[(n)(n-1)\ldots\ldots3.2.1][(2n-1)\ldots.5.3.1]\}^2} \times \frac{n!^2}{2n!}$

$= \frac{2^{2n}[(2n)(2n-1)\ldots\ldots3.1.][(4n-1)\ldots.5.3.1]}{2^{2n}\{[(n)(n-1)\ldots\ldots3.2.1][(2n-1)\ldots.5.3.1]\}^2} \times \frac{n!^2}{2n!}$

$= \frac{[2n!][(4n-1)\ldots.5.3.1]}{\{[n!][(2n-1)\ldots.5.3.1]\}^2} \times \frac{n!^2}{2n!}$

$= \frac{[2n!][(4n-1)\ldots.5.3.1]}{n!^2[(2n-1)\ldots.5.3.1]^2} \times \frac{n!^2}{2n!} = \frac{1.3.5\ldots\ldots(4n-1)}{\{1.3.5\ldots\ldots(2n-1)\}^2}$.

4. ***Prove that*** $(n+1)_{C_r} = n_{C_r} + n_{C_{r-1}}$.

Sol: R.H.S $\Rightarrow n_{C_r} + n_{C_{r-1}}$.

$= \frac{(n)!}{(n-r)!r!} + \frac{(n)!}{(n-r+1)!(r-1)!}$

$= (n)!\left[\frac{1}{(n-r)!r!} + \frac{1}{(n-r+1)!(r-1)!}\right]$

$= (n)!\left[\frac{n+1-r}{(n+1-r)(n-r)!r!} + \frac{r}{(n-r+1)!r(r-1)!}\right]$

$= (n)!\left[\frac{n+1-r}{(n+1-r)!r!} + \frac{r}{(n-r+1)!r!}\right]$

$= (n)!\left[\frac{n+1-r+r}{(n+1-r)!r!}\right]$

$= \left[\frac{(n+1)n!}{(n+1-r)!r!}\right]$

$= \left[\frac{(n+1)!}{(n+1-r)!r!}\right] = (n+1)_{C_r}$ R.H.S

5. Simplify

$\mathbf{34_{C_5} + \sum_{r=0}^{4}(38-r)_{C_4}}$

Sol:

$= 34_{C_5} + \sum_{r=0}^{4}(38-r)_{C_4}$

$= 34_{C_5} + (38-0)_{C_4} + (38-1)_{C_4}$
$+(38-2)_{C_4} + (38-3)_{C_4} + (38-4)_{C_4}$

$n_{C_r} + n_{C_{r+1}}$
$= (n+1)_{C_{(r+1)}}$

$= (34_{C_5} + 34_{C_4}) + 35_{C_4} + 36_{C_4} + 37_{C_4} + 38_{C_4}$

$= (35_{C_5} + 35_{C_4}) + 36_{C_4} + 37_{C_4} + 38_{C_4}$

$= (36_{C_5} + 36_{C_4}) + 37_{C_4} + 38_{C_4}$

$= (37_{C_5} + 37_{C_4}) + 38_{C_4}$

$= (38_{C_5} + 38_{C_4}) = 39_{C_5}$

$\therefore 34_{C_5} + \sum_{r=0}^{4}(38-r)_{C_4} = 39_{C_5}$

6. Prove that $3 \le r \le n$,

$\boldsymbol{n-3_{C_r} + 3n-3_{C_{r-1}} + 3n_{C_{r-2}} + n-3_{C_{r-3}} = n_{C_r}}$.

Sol:

$= (n-3)_{C_r} + 3(n-3)_{C_{r-1}} + 3(n-3)_{C_{r-2}}$
$+ (n-3)_{C_{r-3}}$

$= \{(n-3)_{C_r} + 1(n-3)_{C_{r-1}}\}$
$+2\{(n-3)_{C_{r-1}} + (n-3)_{C_{r-2}}\}$
$+(n-3)_{C_{r-2}} + (n-3)_{C_{r-3}}$

$= (n-3+1)_{C_r} + 2(n-3+1)_{C_{r-1}} + (n-3+1)_{C_{r-2}}$

$= \{(n-2)_{C_r} + (n-2)_{C_{r-1}}\}$
$+\{(n-2)_{C_{r-1}} + (n-2)_{C_{r-2}}\}$

$= (n-2+1)_{C_r} + (n-2+1)_{C_{r-1}}$

$= (n-1)_{C_r} + (n-1)_{C_{r-1}}$

$= (n-1+1)_{C_r}$

$= n_{C_r}$

Aims

7. Find the number of subsets of A having (i)at least 3 elements.(ii) at most 3 elements, if a set has 12 elements.

Sol:

Number of elements in A =12

(i) Number of subsets of A having at least 3 elements

$= 12_{C_3} + 12_{C_4} + 12_{C_5} \dots \dots + 12_{C_{12}}$

$= \{12_{C_0} + 12_{C_1} + 12_{C_2} + 12_{C_3} + \cdots + 12_{C_{12}}\}$
$-\{12_{C_0} + 12_{C_1} + 12_{C_2}\}$

$[\because n_{C_0} + n_{C_1} + n_{C_2} + n_{C_3} + \cdots + n_{C_n} = 2^n]$

$= 2^{12} - [1 + 12 + 66]$

$= 4096 - 79 = 4017$.

(ii) Number of subsets of A having at most 3 elements

$= 12_{C_0} + 12_{C_1} + 12_{C_2} + 12_{C_3}$

=1+12+66+220

=299.

8. Find the number of ways of selecting 11 member cricket team from 7 batsmen, 6 bowlers such that there will be at least 5 bowlers in the team.

Sol: given condition; team contains at least 5 bowlers [5or 6]

Batsmen(7)	Bowlers(6)
7_{C_6}	6_{C_5}
7_{C_5}	6_{C_6}

$= 7_{C_6} \times 6_{C_5} + 7_{C_5} \times 6_{C_6}$

$= 7_{C_1} \times 6_{C_1} + 7_{C_2} \times 1$

$= 7 \times 6 + \frac{7\times 6}{2\times 1} \times 1 = 42 + 21 = 63$

9. A question paper is divided into 3 sections A, B, C containing 3, 4, 5 questions respectively. Find the number of ways of attempting 6 questions choosing at least one from each section.

Sol: Total number of questions from 3 sections (3+4+5) =12

Number of ways of attempting 6 questions from 12 is 12_{C_6} ways (total)

The number of ways of selecting 6 questions from sections B and C only is 9_{C_6} ways

The number of ways of selecting 6 questions from sections A and C only is 8_{C_6} ways

The number of ways of selecting 6 questions from sections A and B only is 7_{C_6} ways

Number of ways of selecting 6 questions choosing at least one from each section= total – (Non)

$=12_{C_6} - 9_{C_6} - 8_{C_6} - 7_{C_6}$.

10. Find the number of ways of selecting 11 member cricket team from 7 batsmen, 6 bowlers and 2 wicket keepers so that the team contains 2 wicket keepers and at least 4 bowlers.

Sol: given condition; team contains 2kt and at least 4 bowlers [4 or 5or 6]

Batsmen(7)	Bowlers(6)	Wicket keepers
7_{C_5}	6_{C_4}	2_{C_2}
7_{C_4}	6_{C_5}	2_{C_2}
7_{C_3}	6_{C_6}	2_{C_2}

$= 7_{C_5} \times 6_{C_4} \times 2_{C_2} + 7_{C_4} \times 6_{C_5} \times 2_{C_2} + 7_{C_3} \times 6_{C_6} \times 2_{C_2}$

$= 7_{C_2} \times 6_{C_2} \times 1 + 7_{C_3} \times 6_{C_1} \times 1 + 7_{C_3} \times 1 \times 1$

$=\frac{7\times6}{2\times1} \times \frac{6\times5}{2\times1} \times 1 + \frac{7\times6\times5}{3\times2\times1} \times 6 \times 1 + \frac{7\times6\times5}{3\times2\times1} \times 1 \times 1$

$=7\times 3 \times 15 + 7 \times 6 \times 5 + 35$

=315+210+35

=560

11. Find the number of ways of forming a committee of 5 members out of 6 Indians and 5 Americans so that always Indians will be in majority in committee.

Sol: given condition; committee contains 5 members [I>A]

Indians(6)	Americans(5)
6_{C_5}	5_{C_0}
6_{C_4}	5_{C_1}
6_{C_3}	5_{C_2}

$= 6_{C_5} \times 5_{C_0} + 6_{C_4} \times 5_{C_1} + 6_{C_3} \times 5_{C_2}$

$= 6_{C_1} \times 1 + 6_{C_2} \times 5 + 6_{C_3} \times 5_{C_2}$

$=6 \times 1 + \frac{6\times5}{2\times1} \times 5 + \frac{6\times5\times4}{3\times2\times1} \times \frac{5\times4}{2\times1}$

=6 +75+200

=281

The required no of ways of selecting a committee of 5 members is 281.

12. If 5 vowels and 6 consonants are given, then how many 6 letter words can be formed with 3 vowels and 3 consonants?

Sol:

Given vowels=5

Consonants=6

Number of ways of selecting 3 vowels=5_{C_3}

Number of ways of selecting 3 consonants=6_{C_3}

Total number of selections=$5_{C_3} \times 6_{C_3}$

Number of arrangements of 5 letter words with 3 vowels and 3 consonants=$5_{C_3} \times 6_{C_3} \times 6!$