1. P.T $c_0+\frac{c_1}{2}x+\frac{c_2}{3}x^2+\cdots.+\frac{c_n}{n+1}x^n=\frac{(1+x)^{n+1}-1}{(n+1)x}$

Sol: $(1+x)^n=c_0+c_1x+c_2x^2+\cdots.+c_nx^n$

$w.k.t\ c_0=1, c_1=n, c_2=\frac{n(n-1)}{2}$

$(1+x)^n=1+nx+\frac{n(n-1)}{2}x^2+\cdots.+x^n$

Now 'n' replacing with 'n + 1'

$(1+x)^{n+1}=1+(n+1)x+\frac{(n+1)(n+1-1)}{2}x^2+\cdots.$

$(1+x)^{n+1}-1=(n+1)x+\frac{(n+1)x.nx}{2}\quad+\cdots$

Taking $(n+1)$ common from each term

$(1+x)^{n+1}-1=(n+1)x\{1+\frac{nx}{2}+\frac{n(n-1)}{3}x^2\ \ldots\ldots\}$

$\frac{(1+x)^{n+1}-1}{(n+1)x}=\{1+\frac{nx}{2}+\frac{n(n-1)}{3}x^2\ \ldots\ldots\}$

$c_0+\frac{c_1}{2}x+\frac{c_2}{3}x^2+\cdots.+\frac{c_n}{n+1}x^n=\frac{(1+x)^{n+1}-1}{(n+1)x}$.....(1)

Put $x=1$ in (1)

$c_0+\frac{c_1}{2}+\frac{c_2}{3}+\cdots.+\frac{c_n}{n+1}=\frac{(2)^{n+1}-1}{(n+1)}$.....(2)

Put $x=-1$ in (1)

$c_0-\frac{c_1}{2}+\frac{c_2}{3}+\cdots.+(-1)^n\frac{c_n}{n+1}=\frac{1}{(n+1)}$.....(3)

(2)-(3)

$\Rightarrow 2\left[\frac{c_1}{2}+\frac{c_3}{4}+\frac{c_5}{6}\ldots.\right]=\frac{[2^{n+1}-1-1]}{(n+1)}$

$\Rightarrow 2\left[\frac{c_1}{2}+\frac{c_3}{4}+\frac{c_5}{6}\ldots.\right]=\frac{2[2^n-1]}{(n+1)}$

$\therefore \left[\frac{c_1}{2}+\frac{c_3}{4}+\frac{c_5}{6}\ldots.\right]=\frac{[2^n-1]}{(n+1)}$

2. P.T

$c_0c_r+c_1c_{r+1}+c_2c_{r+2}+\cdots+c_{n-r}c_n=2n_{C_{n+r}}$

Deduce

(i). $c_0c_1+c_1c_2+c_2c_3+\cdots+c_{n-1}c_n=2n_{C_{n+r}}$

(ii). $c_0{}^2+c_1{}^2+c_2{}^2+\cdots+c_r=2n_{C_n}$

we know that

$(1+x)^n=c_0+c_1x+c_2x^2+\cdots.+c_nx^n$ .....(1)

$(x+1)^n=c_0x^n+c_1x^{n-1}+c_2+x^{n-2}\ldots.+c_n$ ..... (2)

multiplying (1)& (2)

$\Rightarrow (1+x)^n(x+1)^n$

$=[c_0+c_1x+c_2x^2+\cdots.+c_nx^n][c_0x^n+c_1x^{n-1}+c_2+x^{n-2}\ldots.+c_n]$

$\Rightarrow (1+x)^{2n}$

$=[c_0+c_1x+c_2x^2+\cdots.+c_nx^n][c_0x^n+c_1x^{n-1}+c_2+x^{n-2}\ldots.+c_n]$

comparing both sides coefficient of x^{n-r}

$c_0c_r+c_1c_{r+1}+c_2c_{r+2}+\cdots+c_{n-r}c_n=2n_{C_{n-r}}$

$\{\because n_{C_r}=n_{C_{n-r}}\}$

$c_0c_r+c_1c_{r+1}+c_2c_{r+2}+\cdots+c_{n-r}c_n=2n_{C_{[2n-(n-r)]}}$

$\therefore c_0c_r+c_1c_{r+1}+c_2c_{r+2}+\cdots+c_{n-r}c_n=2n_{C_{n+r}}$

put $r=1$

$c_0c_1+c_1c_2+c_2c_3+\cdots+c_{n-1}c_n=2n_{C_{n+r}}$

put $r=0$

$c_0{}^2+c_1{}^2+c_2{}^2+\cdots+c_r=2n_{C_n}$

Hence proved.

3. Prove that

$(C_0-C_1)(C_1-C_2)(C_2-C_3)\ldots(C_{n-1}-C_n)$

$=\frac{(n+1)^n}{n!}C_0C_1C_2\ldots\ldots C_n$

Sol: L.H.S

$(C_0+C_1)(C_1+C_2)(C_2+C_3)\ldots(C_{n-1}+C_n)$

$=C_0\left(1+\frac{C_1}{C_0}\right)C_1\left(1+\frac{C_2}{C_1}\right)\ldots C_{n-1}\left(1+\frac{C_n}{C_{n-1}}\right)$

$=C_0C_1C_2\ldots C_{n-1}\left(1+\frac{n}{1}\right)\left(1+\frac{n(n-1)}{2.n}\right)\ldots\left(1+\frac{1}{n}\right)$

$=C_0C_1C_2\ldots C_{n-1}\left(\frac{1+n}{1}\right)\left(\frac{2+n-1}{2}\right)\ldots\left(\frac{n+1}{n}\right)$

$=C_0C_1C_2\ldots C_{n-1}\left(\frac{1+n}{1}\right)\left(\frac{1+n}{2}\right)\ldots\left(\frac{1+n}{n}\right)$

$=C_0C_1C_2\ldots C_{n-1}\frac{(n+1)^n}{1.2.3\ldots\ldots n}$

$=\frac{(n+1)^n}{n!}C_0C_1C_2\ldots\ldots C_n$ R.H.S

4. **If the 2^{nd}, 3^{rd}, and 4^{th} terms in the expansion of $(a+x)^n$ are respectively 240, 720 and 1080, then find the value of a, x and n.**

Sol:

Given expansion $(a+x)^n$

Genral term $T_{r+1} = n_{C_r} a^{n-r} a^r$

$T_2 = T_{1+1} = n_{C_1} a^{n-1} x^1 = 240 \ldots\ldots(1)$

$T_3 = T_{2+1} = n_{C_2} a^{n-2} x^2 = 720 \ldots\ldots(2)$

$T_4 = T_{3+1} = n_{C_3} a^{n-3} x^3 = 1080 \ldots\ldots(3)$

$\frac{(2)}{(1)} \Rightarrow \frac{n_{C_2} a^{n-2} x^2}{n_{C_1} a^{n-1} x^1} = \frac{720}{240}$

$\Rightarrow \left(\frac{n-1}{2}\right)\frac{x}{a} = 3$

$\Rightarrow \mathbf{(n-1)x = 6a \ldots\ldots (4)}$

$\frac{(3)}{(2)} \Rightarrow \frac{n_{C_3} a^{n-3} x^3}{n_{C_2} a^{n-2} x^2} = \frac{1080}{720}$

$\Rightarrow \left(\frac{n-2}{3}\right)\frac{x}{a} = \frac{3}{2}$

$\Rightarrow \mathbf{(n-2)x = \frac{9a}{2} \ldots\ldots (5)}$

$\frac{(5)}{(4)} \Rightarrow \frac{(n-2)x}{(n-1)x} = \frac{9a}{2(6a)}$

$\Rightarrow \frac{(n-2)}{(n-1)} = \frac{3}{4}$

$\Rightarrow 4n - 8 = 3n - 3$

$\Rightarrow 4n - 3n = -3 + 8$

$\therefore \mathbf{n = 5}$

$sub\ n = 5\ in\ (4) \Rightarrow 4x = 6a$

$\mathbf{x = \frac{3a}{2}} \ldots\ldots (6)$

Sub n and x in $(1) \Rightarrow (5)a^4\left(\frac{3a}{2}\right) = 240$

$\Rightarrow a^5 = \frac{240\times 2}{15} = 32$

$\Rightarrow a^5 = 2^5 \Rightarrow \mathbf{a = 2}$

Sub n=5 and a=2 in (6) $\Rightarrow x = \frac{3(2)}{2} = 3$

$\therefore \mathbf{a = 2, x = 3, n = 5}$

5. **If the coefficients of r^{th}, $(r+1)^{th}$, $(r+2)^{th}$ terms in the expansion of $(1+x)^n$ are in A.P then S.T $n^2 - (4r+1)n + 4r^2 - 2 = 0$.**

Sol:

Given that the coefficients of r^{th}, $(r+1)^{th}$, $(r+2)^{th}$ terms in the expansion of $(1+x)^n$

Are $n_{C_{r-1}}(a), n_{C_r}(b), n_{C_{r+1}}(c)$ in A.P

We know that if a, b, c are in A.P $2b = a + c$

Or $2 = \frac{a}{b} + \frac{c}{b}$

$$\frac{nc_{\ r+1}}{nc_{\ r}} = \frac{n-r}{r+1}$$

$\Rightarrow 2 = \frac{n_{C_{r-1}}}{n_{C_r}} + \frac{n_{C_{r+1}}}{n_{C_r}}$

$\Rightarrow 2 = \frac{r}{n-(r-1)} + \frac{n-r}{r+1}$

$\Rightarrow 2(n-r+1)(r+1) = (r)(r+1) + (n-r+1)(n-r)$

$\Rightarrow 2(nr + n - r^2 - r + r + 1) = r^2 + r + n^2 - nr - nr + r^2 + n - r$

$\Rightarrow (2nr + 2n - 2r^2 + 2) = n^2 - 2nr + 2r^2 + n$

$\Rightarrow n^2 - 4nr - n + 4r^2 - 2 = 0$

$\therefore n^2 - (4r+1)n + 4r^2 - 2 = 0$

If $\left(7+4\sqrt{3}\right)^n = I + f$ *where I and n are* $+ve$ Integers and $0 < f < 1$, then S.T (i) I is an odd integer. (ii) $(I+f)(1-f) = 1$

6. If the coefficients of x^{10} *in the expansion of* $\left(ax^2+\frac{1}{bx}\right)^{11}$, is equal to the coefficients of x^{-10} *in the expansion of* $\left(ax-\frac{1}{bx^2}\right)^{11}$ find the relation between a and b where a and b are real numbers.

Sol: Genral term $T_{r+1}=(-)^r\, n_{c_r}a^{n-r}\,a^r$

in the expansion of $\left(ax^2+\frac{1}{bx}\right)^{11}$

$T_{r+1}=11_{c_r}(ax^2)^{11-r}\left(\frac{1}{bx}\right)^r$

$=(11_{c_r})\left(\frac{a^{11-r}}{b^r}\right)\frac{(x^{22-2r})}{x^r}$

$=(11_{c_r})\left(\frac{a^{11-r}}{b^r}\right)(x^{22-3r})$

To get coefficient of x^{10} put $22-3r=10$

$3r=12 \Rightarrow \boldsymbol{r=4}$

The coefficients of x^{10} in the expansion of $\left(ax^2+\frac{1}{bx}\right)^{11}$ is $=(11_{c_4})\left(\frac{a^7}{b^4}\right)$............(1)

in the expansion of $\left(ax-\frac{1}{bx^2}\right)^{11}$

$T_{r+1}=(-1)^r 11_{c_r}(ax)^{11-r}\left(\frac{1}{bx^2}\right)^r$

$=(11_{c_r})\left(\frac{a^{11-r}}{b^r}\right)\frac{(x^{11-r})}{x^{2r}}$

$=(11_{c_r})\left(\frac{a^{11-r}}{b^r}\right)(x^{11-3r})$

To get coefficient of x^{-10} put $11-3r=-10$

$3r=21 \Rightarrow \boldsymbol{r=7}$

the coefficients of $-x^{10}$ in the expansion of $\left(ax-\frac{1}{bx^2}\right)^{11}$ is $=-(11_{c_7})\left(\frac{a^4}{b^7}\right)$.........(7)

Given that (1) = (2)

$(11_{c_4})\left(\frac{a^7}{b^4}\right)=-(11_{c_7})\left(\frac{a^4}{b^7}\right)\ [\because 11_{c_4}=11_{c_7}]$

$a^3=-\frac{1}{b^3}$

$(ab)^3=-1 \Rightarrow ab=-1$

7. *If the coefficients of 4 consecutive terms in the expansion of* $(1+x)^n$ *are* a_1, a_2, a_3, a_4 *Respectively then show that* $\frac{a_1}{a_1+a_2}+\frac{a_3}{a_3+a_4}=\frac{2a_2}{a_2+a_3}$.

SoL: Let $T_r, T_{r+1}, T_{r+2}, T_{r+3}$ are 4 ConSecutive terms in the expansion of $(1+x)^n$

$a_1=nc_1,\ a_2=nc_2,\ a_3=nc_3,\ a_4=nc_4$

L.H.S ⇨ $\frac{a_1}{a_1+a_2}+\frac{a_3}{a_3+a_4}$

$=\frac{nc_1}{nc_1+nc_2}+\frac{nc_3}{nc_3+nc_4}$

$=\frac{nc_1}{nc_1\left(1+\frac{nc_2}{nc_1}\right)}+\frac{nc_3}{nc_3\left(1+\frac{nc_4}{nc_3}\right)}$

$=\frac{1}{\left(1+\frac{nc_2}{nc_1}\right)}+\frac{1}{\left(1+\frac{nc_4}{nc_3}\right)}$

$$\frac{nc_{r+1}}{nc_r}=\frac{n-r}{r+1}$$

$=\frac{1}{\left(1+\frac{n-1}{2}\right)}+\frac{1}{\left(1+\frac{n-3}{4}\right)}$

$=\frac{1}{\left(\frac{2+n-1}{2}\right)}+\frac{1}{\left(\frac{4+n-3}{4}\right)}$

$=\frac{2}{(n+1)}+\frac{4}{(n+1)}=\frac{6}{(n+1)}$

R.H.S

$\frac{2a_2}{a_2+a_3}=\frac{2nc_2}{nc_2+nc_3}$

$=\frac{2nc_2}{nc_2\left(1+\frac{nc_3}{nc_2}\right)}$

$=\frac{2}{\left(1+\frac{nc_3}{nc_2}\right)}$

$$\frac{nc_{r+1}}{nc_r}=\frac{n-r}{r+1}$$

$=\frac{2}{\left(1+\frac{n-2}{3}\right)}=\frac{2}{\left(\frac{3+n-2}{3}\right)}$

$=\frac{2\times3}{(n+1)}=\frac{6}{(n+1)}$

$\therefore$ ***L.H.S = R.H.S***

1. S.T $1+\frac{1}{3}+\frac{1.3}{3.6}+\frac{1.3.5}{3.6.9}+\quad \ldots=\sqrt{3}.$

Sol: $=1+\frac{1}{3}+\frac{1.3}{3.6}+\frac{1.3.5}{3.6.9}+\quad \ldots$

$=1+\frac{1}{1!}\left(\frac{1}{3}\right)+\frac{1.3}{2!}\left(\frac{1}{3}\right)^2+\quad$ *... Comparing with the* series

$$(1-x)^{-\frac{p}{q}}=1+\frac{p}{1!}\left(\frac{x}{q}\right)+\frac{p(p+q)}{2!}\left(\frac{x}{q}\right)^2+\cdots$$

$p=1$	$p+q=3$ $\Rightarrow q=3-p$ $\Rightarrow q=3-1$ $\therefore q=2$	$\frac{x}{q}=\frac{1}{3}$ $\Rightarrow x=\frac{q}{3}=\frac{2}{3}$

$\therefore(1-x)^{-\frac{p}{q}}=\left(1-\frac{2}{3}\right)^{-\frac{1}{2}}$

$=\left(\frac{1}{3}\right)^{-\frac{1}{2}}$

$=\left(\frac{3}{1}\right)^{\frac{1}{2}}$

$\therefore 1+\frac{1}{3}+\frac{1.3}{3.6}+\frac{1.3.5}{3.6.9}+\quad \ldots=\sqrt{3}$

2. S.T $1+\frac{2}{3}.\frac{1}{2}+\frac{2.5}{3.6}\left(\frac{1}{2}\right)^2+\frac{2.5.8}{3.6.9}\left(\frac{1}{2}\right)^3+\ldots=\sqrt[3]{4}$

Sol:

$$=1+\frac{2}{3}.\frac{1}{2}+\frac{2.5}{3.6}\left(\frac{1}{2}\right)^2+\frac{2.5.8}{3.6.9}\left(\frac{1}{2}\right)^3+\ldots$$

$=1+\frac{2}{6}+\frac{2.5}{6.12}+\cdots$

$=1+\frac{2}{1!}\left(\frac{1}{6}\right)+\frac{2.5}{2!}\left(\frac{1}{6}\right)^2+\quad\ldots$

Comparing with the series

$$(1-x)^{-\frac{p}{q}}=1+\frac{p}{1!}\left(\frac{x}{q}\right)+\frac{p(p+q)}{2!}\left(\frac{x}{q}\right)^2+\cdots$$

$p=2$	$p+q=5$ $\Rightarrow q=5-p$ $\Rightarrow q=5-2$ $\therefore q=3$	$\frac{x}{q}=\frac{1}{6}$ $\Rightarrow x=\frac{q}{6}=\frac{3}{6}$

$\therefore(1-x)^{-\frac{p}{q}}=\left(1-\frac{1}{2}\right)^{-\frac{2}{3}}$

$=\left(\frac{1}{2}\right)^{-\frac{2}{3}}=\left(\frac{2}{1}\right)^{\frac{2}{3}}(4)^{\frac{1}{3}}$

$\therefore 1+\frac{2}{3}.\frac{1}{2}+\frac{2.5}{3.6}\left(\frac{1}{2}\right)^2+\frac{2.5.8}{3.6.9}\left(\frac{1}{2}\right)^3+\ldots=\sqrt[3]{4}$

3. Show that $1+\frac{4}{5}+\frac{4.7}{5.10}+\frac{4.7.10}{5.10.15}+\quad\ldots=\sqrt[3]{\frac{625}{16}}$

Sol:

$=1+\frac{4}{5}+\frac{4.7}{5.10}+\frac{4.7.10}{5.10.15}+\quad\ldots$

$=1+\frac{4}{1!}\left(\frac{1}{5}\right)+\frac{4.7}{2!}\left(\frac{1}{5}\right)^2+\quad\ldots$

Comparing with the series

$$(1-x)^{-\frac{p}{q}}=1+\frac{p}{1!}\left(\frac{x}{q}\right)+\frac{p(p+q)}{2!}\left(\frac{x}{q}\right)^2+\cdots$$

$p=4$	$p+q=7$ $\Rightarrow q=7-p$ $\Rightarrow q=7-4$ $\therefore q=3$	$\frac{x}{q}=\frac{1}{5}$ $\Rightarrow x=\frac{q}{5}=\frac{3}{5}$

$\therefore(1-x)^{-\frac{p}{q}}=\left(1-\frac{3}{5}\right)^{-\frac{4}{3}}$

$=\left(\frac{2}{5}\right)^{-\frac{4}{3}}=\left(\frac{5}{2}\right)^{\frac{4}{3}}$

$\therefore 1-\frac{4}{5}+\frac{4.7}{5.10}-\frac{4.7.10}{5.10.15}+\quad\ldots=\sqrt[3]{\frac{625}{16}}$

4. Show that $\frac{7}{5}\left[1+\frac{1}{10^2}+\frac{1.3}{1.2}\frac{1}{10^4}+\frac{1.3.5}{1.2.3}\frac{1}{10^6}\quad\ldots\right]=\sqrt{2}.$

Sol: Consider $1+\frac{1}{10^2}+\frac{1.3}{1.2}\frac{1}{10^4}+\frac{1.3.5}{1.2.3}\frac{1}{10^6}\quad\ldots$

$1+\left(\frac{1}{100}\right)+\frac{1.3}{1.2}\left(\frac{1}{100}\right)^2+\frac{1.3.5}{1.2.3}\left(\frac{1}{100}\right)^3\ldots$

$1+\frac{1}{1!}\left(\frac{1}{100}\right)+\frac{1.3}{2!}\left(\frac{1}{100}\right)^2+\frac{1.3.5}{3!}\left(\frac{1}{100}\right)^3\ldots$

Comparing with the series

$$(1-x)^{-\frac{p}{q}}=1+\frac{p}{1!}\left(\frac{x}{q}\right)+\frac{p(p+q)}{2!}\left(\frac{x}{q}\right)^2+\cdots$$

$p=1$	$p+q=3$ $\Rightarrow q=3-p\Rightarrow$ $q=3-1$ $\therefore q=2$	$\frac{x}{q}=\frac{1}{100}$ $\Rightarrow x=\frac{q}{100}=\frac{2}{100}$

$\therefore(1-x)^{-\frac{p}{q}}=\left(1-\frac{1}{50}\right)^{-\frac{1}{2}}$

$=\left(\frac{49}{50}\right)^{-\frac{1}{2}}=\left(\frac{50}{49}\right)^{\frac{1}{2}}=\sqrt{\frac{25\times2}{49}}=\frac{5}{7}\sqrt{2}$

$\frac{7}{5}\left[1+\frac{1}{10^2}+\frac{1.3}{1.2}\frac{1}{10^4}+\frac{1.3.5}{1.2.3}\frac{1}{10^6}\quad\ldots\right]=\frac{7}{5}\left(\frac{5}{7}\sqrt{2}\right)=\sqrt{2}.$

5. Show that $\frac{3}{4}+\frac{3.5}{4.8}+\frac{3.5.7}{4.8.12}+\quad ...=\sqrt{8}$ **-1.**

Sol: $let\ S=\frac{3}{4}+\frac{3.5}{4.8}+\frac{3.5.7}{4.8.12}+\quad ...$

Adding '1' an both the sides

$S+1=1+\frac{3}{4}+\frac{3.5}{4.8}+\frac{3.5.7}{4.8.12}+\quad ...$

$=1+\frac{3}{1!}\left(\frac{1}{4}\right)+\frac{3.5}{2!}\left(\frac{1}{4}\right)^2+\quad ...$ Comparing with the series $(1-x)^{-\frac{p}{q}}=1+\frac{p}{1!}\left(\frac{x}{q}\right)+\frac{p(p+q)}{2!}\left(\frac{x}{q}\right)^2+\cdots$

$p=3$	$p+q=5$ $\Rightarrow q=5-p$ $\Rightarrow q=5-3$ $\therefore q=2$	$\frac{x}{q}=\frac{1}{4}$ $\Rightarrow x=\frac{q}{4}=\frac{2}{4}$

$\therefore S+1=(1-x)^{-\frac{p}{q}}=\left(1-\frac{1}{2}\right)^{-\frac{3}{2}}=\left(\frac{1}{2}\right)^{-\frac{3}{2}}=\left(\frac{2}{1}\right)^{\frac{3}{2}}$

$=\sqrt{8}$

$\therefore S=\frac{3}{4}+\frac{3.5}{4.8}+\frac{3.5.7}{4.8.12}+\quad ...=\sqrt{8}$ -1.

6. Show that $\frac{3.5}{5.10}+\frac{3.5.7}{5.10.15}+\quad ...=\frac{5\sqrt{5}}{3\sqrt{3}}$ - $\frac{8}{5}$.

Sol: let S= $\frac{3.5}{5.10}+\frac{3.5.7}{5.10.15}+$

Adding '$1+\frac{3}{5}$' *an both the sides*

$S+1+\frac{3}{5}=1+\frac{3}{5}+\frac{3.5}{5.10}+\frac{3.5.7}{5.10.15}+$

$=1+\frac{3}{1!}\left(\frac{1}{5}\right)+\frac{3.5}{2!}\left(\frac{1}{5}\right)^2+\quad ...$

Comparing with the series

$(1-x)^{-\frac{p}{q}}=1+\frac{p}{1!}\left(\frac{x}{q}\right)+\frac{p(p+q)}{2!}\left(\frac{x}{q}\right)^2+\cdots$

$p=3$	$p+q=5$ $\Rightarrow q=5-p\Rightarrow$ $q=5-3$ $\therefore q=2$	$\frac{x}{q}=\frac{1}{5}$ $\Rightarrow x=\frac{q}{5}=\frac{2}{5}$

$\therefore S+1+\frac{3}{5}=(1-x)^{-\frac{p}{q}}=\left(1-\frac{2}{5}\right)^{-\frac{3}{2}}$

$=\left(\frac{3}{5}\right)^{-\frac{3}{2}}=\left(\frac{5}{3}\right)^{\frac{3}{2}}=\sqrt{\frac{5\times5\times5}{3\times3\times3}}=\frac{5\sqrt{5}}{3\sqrt{3}}$

$\therefore \frac{3.5}{5.10}+\frac{3.5.7}{5.10.15}+\quad ...=\frac{5\sqrt{5}}{3\sqrt{3}}$ - $\frac{8}{5}$.

7. Show that $\frac{3}{4.8}-\frac{3.5}{4.8.12}+\frac{3.5.7}{4.8.12.16}+\quad ...=\sqrt{\frac{2}{3}}$ - $\frac{3}{4}$.

Sol: $let\ S=\frac{1.3}{4.8}-\frac{1.3.5}{4.8.12}+\frac{1.3.5.7}{4.8.12.16}+\quad ...$

Adding '$1-\frac{1}{4}$' *an both the sides*

$S+1-\frac{1}{4}=1-\frac{1}{4}+\frac{1.3}{4.8}-\frac{1.3.5}{4.8.12}+..$

$=1-\frac{1}{1!}\left(\frac{1}{4}\right)+\frac{1.3}{2!}\left(\frac{1}{4}\right)^2-\quad ...$

Comparing with the series

$(1+x)^{-\frac{p}{q}}=1-\frac{p}{1!}\left(\frac{x}{q}\right)+\frac{p(p+q)}{2!}\left(\frac{x}{q}\right)^2+\cdots$

$p=1$	$p+q=3$ $\Rightarrow q=3-p\Rightarrow$ $q=3-1$ $\therefore q=2$	$\frac{x}{q}=\frac{1}{4}$ $\Rightarrow x=\frac{q}{4}=\frac{2}{4}$

$\therefore S+1-\frac{1}{4}=(1+x)^{-\frac{p}{q}}=\left(1+\frac{1}{2}\right)^{-\frac{1}{2}}=\left(\frac{3}{2}\right)^{-\frac{1}{2}}$

$=\left(\frac{2}{3}\right)^{\frac{1}{2}}$

Aims

$\frac{3}{4.8}-\frac{3.5}{4.8.12}+\frac{3.5.7}{4.8.12.16}+\quad ...=\sqrt{\frac{2}{3}}$ - $\frac{3}{4}$.

8. If $x=\frac{1}{5}+\frac{1.3}{5.10}-\frac{1.3.5}{5.10.15}+\quad ...\infty,$ ***then S.T*** $3x^2+6x=2.$

Sol: $x=\frac{1}{5}+\frac{1.3}{5.10}-\frac{1.3.5}{5.10.15}+\quad ...\infty$

Adding '1' an both the sides

$x+1=1+\frac{1}{5}+\frac{1.3}{5,10}+\quad ...$

$=1+\frac{1}{1!}\left(\frac{1}{5}\right)+\frac{1.3}{2!}\left(\frac{1}{5}\right)^2+\quad ...$

Comparing with the series

$(1-x)^{-\frac{p}{q}}=1+\frac{p}{1!}\left(\frac{x}{q}\right)+\frac{p(p+q)}{2!}\left(\frac{x}{q}\right)^2+\cdots$

$p=1$	$p+q=3$ $\Rightarrow q=3-p$ $\Rightarrow q=3-1$ $\therefore q=2$	$\frac{x}{q}=\frac{1}{5}$ $\Rightarrow x=\frac{q}{5}=\frac{2}{5}$

$\therefore x+1=\left(1-\frac{2}{5}\right)^{-\frac{p}{q}}=\left(\frac{3}{5}\right)^{-\frac{3}{2}}=\left(\frac{5}{3}\right)^{\frac{1}{2}}=\sqrt{\frac{5}{3}}$

$x+1=\sqrt{\frac{5}{3}}$ S.O.B

$(x+1)^2=\left(\sqrt{\frac{5}{3}}\right)^2$

⇨ $x^2+2x+1=\frac{5}{3}$

⇨ $3x^2+6x+3=5$

⇨ $3x^2+6x=5-3=2$

9. If $x=\frac{1.3}{3.6}+\frac{1.3.5}{3.6.9}+\ \dots\infty$, then S.T $9x^2+24x=11$.

Sol: $x=\frac{1.3}{3.6}+\frac{1.3.5}{3.6.9}+\ \dots$

Adding '$1+\frac{1}{3}$' an both the sides

$$x+1+\frac{1}{3}=1+\frac{1}{3}+\frac{1.3}{3.6}+\frac{1.3.5}{3.6.9}+\ \dots$$

$=1+\frac{1}{1!}\left(\frac{1}{3}\right)+\frac{1.3}{2!}\left(\frac{1}{3}\right)^2+\ \dots$

Comparing with the series

$$(1-x)^{-\frac{p}{q}}=1+\frac{p}{1!}\left(\frac{x}{q}\right)+\frac{p(p+q)}{2!}\left(\frac{x}{q}\right)^2+\cdots$$

$p=1$	$p+q=3$ ⇨ $q=3-p$ ⇨ $q=3-1$ $\therefore q=2$	$\frac{x}{q}=\frac{1}{3}$ ⇨ $x=\frac{q}{3}=\frac{2}{3}$

$\therefore x+1+\frac{1}{3}=(1-x)^{-\frac{p}{q}}=\left(1-\frac{2}{3}\right)^{-\frac{1}{2}}$

$x+\frac{4}{3}=\left(\frac{1}{3}\right)^{-\frac{1}{2}}=\left(\frac{3}{1}\right)^{\frac{1}{2}}$

$\frac{3x+4}{3}=\sqrt{3}$ S.O.B

⇨ $\left(\frac{3x+4}{3}\right)^2=(\sqrt{3})^2$

⇨ $\frac{9x^2+24x+16}{9}=3$

⇨ $9x^2+24x+16=3\times 9$

⇨ $9x^2+24x=27-16$

$\therefore \mathbf{9x^2+24x=11.}$

10. If $x=\frac{5}{2!3}+\frac{5.7}{3!3^2}+\frac{5.7.9}{4!3^3}\ \dots$, then S.T $x^2+4x=23$.

Sol: $x=\frac{5}{2!3}+\frac{5.7}{3!3^2}+\frac{5.7.9}{4!3^3}\ \dots$

multiplying & dividing by 3

$\frac{3x}{3}=\frac{3.5}{2!3^2}+\frac{3.5.7}{3!3^3}+\frac{3.5.7.9}{4!3^4}\ \dots$

$x=\frac{3.5}{2!}\left(\frac{1}{3}\right)^2+\frac{3.5.7}{3!}\left(\frac{1}{3}\right)^3+$

Adding '$1+\frac{3}{3}$' an both the sides

$x+1+\frac{3}{3}=1+\frac{3}{1!}\left(\frac{1}{3}\right)+\frac{3.5}{2!}\left(\frac{1}{3}\right)^2+\ \dots$

Comparing with the series

$$(1-x)^{-\frac{p}{q}}=1+\frac{p}{1!}\left(\frac{x}{q}\right)+\frac{p(p+q)}{2!}\left(\frac{x}{q}\right)^2+\cdots$$

$p=3$	$p+q=5$ ⇨ $q=5-p$ ⇨ $q=5-3$ $\therefore q=2$	$\frac{x}{q}=\frac{1}{3}$ ⇨ $x=\frac{q}{3}=\frac{2}{3}$

$\therefore x+1+1=(1-x)^{-\frac{p}{q}}=\left(1-\frac{2}{3}\right)^{-\frac{3}{2}}$

$x+2=\left(\frac{1}{3}\right)^{-\frac{3}{2}}=\left(\frac{3}{1}\right)^{\frac{3}{2}}$

$x+2=\sqrt{27}$

S.O.B

⇨ $(x+2)^2=(\sqrt{27})^2$

⇨ $x^2+4x+4=27$

⇨ $x^2+4x=27-4$

$\mathbf{x^2+4x=23}$

11. S.T $1+\frac{x}{2}+\frac{x(x-1)}{2.4}+\frac{x(x-1)(x-2)}{2.4.6}+\cdots$
$=1+\frac{x}{3}+\frac{x(x+1)}{3.6}+\frac{x(x+1)(x+2)}{3.6.9}+\cdots$

Sol:

L.H.S= $1+\frac{x}{2}+\frac{x(x-1)}{2.4}+\frac{x(x-1)(x-2)}{2.4.6}+\cdots$

$=1+\frac{x}{1!}\left(\frac{1}{2}\right)+\frac{x(x-1)}{2!}\left(\frac{1}{2}\right)^2+\cdots$

Comparing with the series

$$(1-x)^{-\frac{p}{q}}=1+\frac{p}{1!}\left(\frac{x}{q}\right)+\frac{p(p+q)}{2!}\left(\frac{x}{q}\right)^2+\cdots$$

$p=x$	$p+q=x-1$ $\Rightarrow q=x-1-p$ $\Rightarrow q=x-1-x$ $\therefore q=-1$	$\frac{x}{q}=\frac{1}{2}$ $\Rightarrow x=\frac{q}{2}=-\frac{1}{2}$

$(1-x)^{-\frac{x}{-1}}=\left(1+\frac{1}{2}\right)^x=\left(\frac{3}{2}\right)^x$ ....(1)

R.H.S

$1+\frac{x}{3}+\frac{x(x+1)}{3.6}+\frac{x(x+1)(x+2)}{3.6.9}+\cdots$

$=1+\frac{x}{1!}\left(\frac{1}{3}\right)+\frac{x(x+1)}{2!}\left(\frac{1}{3}\right)^2+\cdots$

Comparing with the series

$$(1-x)^{-\frac{p}{q}}=1+\frac{p}{1!}\left(\frac{x}{q}\right)+\frac{p(p+q)}{2!}\left(\frac{x}{q}\right)^2+\cdots$$

$p=x$	$p+q=x+1$ $\Rightarrow q=x+1-p$ $\Rightarrow q=x+1-x$ $\therefore q=1$	$\frac{x}{q}=\frac{1}{3}$ $\Rightarrow x=\frac{q}{3}=\frac{1}{3}$

$(1-x)^{-\frac{p}{q}}=\left(1-\frac{1}{3}\right)^{-\frac{x}{1}}=\left(\frac{2}{3}\right)^{-\frac{x}{1}}=\left(\frac{3}{2}\right)^x$ .....(2)

From (1) & (2) L.H.S=R.H.S