1. ***State and explain the axioms that define 'probability function'. Prove addition theorem on probability. i.e., $P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$.***

A: **<u>Axiomatic Approach to Probability:</u>** Let S be the sample space of a random experiment then a function P: P(S)→R
Satisfying the following axioms is called a probability function.

(1) $P(A) \geq 0, \ \ \forall A \in p(S)$, *axiom of non – negativity*

(2) $P(S) = 1$, *axiom of certainity*

(3) $P(A \cup B) = P(A) + P(B)$, if $A \cap B = \emptyset$ and $A, B \in S$ axioms of union

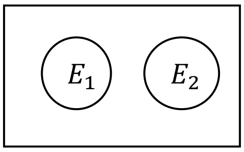
.

<u>Theorem:</u> Addition Theorem on Probability. If E_1, E_2 are two events in a sample space S, Then

$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$.

<u>Proof:</u>

Case 1:



Suppose that

$(E_1 \cap E_2) = \emptyset$

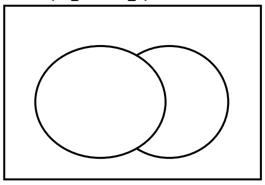
$P(E_1 \cup E_2) = P(E_1) + P(E_2)$

by axioms of union

$= P(E_1) + P(E_2) - 0$

$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

Case 2:



Suppose that

$(E_1 \cap E_2) \neq \emptyset$

From Venn diagram

$P(E_1 \cup E_2) = P[E_1 \cup (E_2 - E_1)]$

$= P(E_1) + P(E_2 - E_1)$

$\therefore (E_2 - E_1) = [E_2 - (E_1 \cap E_2)]$

$P(E_1 \cup E_2) = P(E_1) + [E_2 - (E_1 \cap E_2)]$

$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$.

2. ***A, B, C are three horse in a race. The probability of A to win the race is twice that of B and probability of B is twice that of C. what are the probabilities of A, B and C to win the race?***

<u>Sol.</u> Let A, B, C be the events that the horses A, B, C wins the race respectively.

Given $P(A) = 2P(B)$, $P(B) = 2P(C)$

$\therefore P(A) = 2P(B) = 2[2P(C)] = 4P(C)$

Since the horses A, B and C run the race,

$A \cup B \cup C = S$ and A, B, C are mutually disjoint

$P(A \cup B \cup C) = P(A) + P(B) + P(C)$

$\Rightarrow P(S) = 4p(C) + 2p(C) + P(C) = 1$

$\Rightarrow 7P(C) = 1$

$P(C) = \frac{1}{7}$

$P(B) = 2P(C)$

$= 2[\frac{1}{7}]$

$P(B) = \frac{2}{7}$

$P(A) = 4P(C)$

$= 4[\frac{1}{7}]$

$P(A) = \frac{4}{7}$

$P(A) = \frac{4}{7}$, $P(B) = \frac{4}{7}$, $P(C) = \frac{4}{7}$

Probability that horse A loses in the race

$= P(A)$

$= 1 - P(A) = 1 - \frac{4}{7}$

$= \frac{3}{7}$.

3. ***In a committee of 25 members, each member is proficient either in mathematics or in statistics or in both. if 19 of these proficient in mathematics, 16 in statistics, find the probability that a person selected from the committee is proficient in both.***

Sol:

Let A, the events of a member proficient in Mathematics. $P(A)=\frac{19}{25}$

Let B, the events of a member proficient in Statistics.

$P(B)=\frac{16}{25}$,

$P(A\cup B)=\frac{25}{25}$

By addition theorem

$P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$\Rightarrow P(A\cap B)=P(A)+P(B)-P(A\cup B)$

$=\frac{19}{25}+\frac{16}{25}-\frac{25}{25}$

$=\frac{10}{25}=\frac{2}{5}$

4. ***(i)A single die is rolled twice in succession. What is the probability that the number on the second rolling is greater than that on the first rolling?***
(ii) if one ticket is randomly selected from tickets numbered 1 to 30, then find the probability that the number on the ticket is multiple of 3 or 5.

Sol:

(i)Let A be the event that the number on the second toss is greater than that on the first rolling.

$$A=\left\{\begin{matrix}(1,2),(1,3),(1,4)(1,5),(1,6),\\(2,3),(2,4),(2,5),(2,6),(3,4),\\(3,5),(3,6),(4,5),(4,6),(5,6),\end{matrix}\right\}$$

$n(A)=15$

Let S be the sample space then $n(S)=6^2=36$

$Required\ probabillity\ P(A)=\frac{n(A)}{n(S)}$

$=\frac{15}{36}=\frac{5}{12}$

(ii) Let A be the event of a ticket having number is multiple of 5.

Let B be the event of a ticket having number is multiple of 7.

$A=\{5,10,15,20,25,30\}\Rightarrow n(A)=6$

$B=\{7,14,21,28\}\Rightarrow n(B)=4,\quad A\cap B=\emptyset$

By addition theorem

$P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$=\frac{6}{30}+\frac{4}{30}-0$

$=\frac{10}{30}=\frac{1}{3}$

5. ***A, B, C are three news papers from a city, 20% of the population read A, 16% read B, 14% read C, 8% read both A and B, 5% read both A and C, 4% read both B and C, 2% read all three. Find the percentage of the population who read at least one news paper and find the percentage of the population who read paper A only.***

Sol:

Let A, B, C be the events that a person selected from the city reads newspapers A, B, C respectively.

Given that $P(A)=\frac{20}{100}, P(B)=\frac{16}{100}, P(c)=\frac{14}{100}$

$P(A\cap B)=\frac{8}{100},\quad P(A\cap C)=\frac{5}{100},$

$P(B\cap C)=\frac{4}{100},\quad P(A\cap B\cap C)=\frac{2}{100},$

$P(A\cup B\cup C)=?$

Probability that a person selected from the city reads at least one news paper

$P(A\cup B\cup C)=P(A)+P(B)+P(c)-P(A\cap B)-P(A\cap C)-P(B\cap C)+P(A\cap B\cap C)$

$\Rightarrow P(A\cup B\cup C)=\frac{20}{100}+\frac{16}{100}+\frac{14}{100}-\frac{8}{100}-\frac{4}{100}-\frac{5}{100}+\frac{2}{100}$

$\Rightarrow P(A\cup B\cup C)=\frac{20+16+14-8-4-5+2}{100}$

$=\frac{35}{100}$

$\therefore Required\ percentage\ of\ population$ who read at least one newspaper

$P(A\cup B\cup C)\times 100$

$=\frac{35}{100}\times 100=35\%$

Probability that the selected person read the news paper A only$=P(A)-P(A\cap B)-P(A\cap C)+P(A\cap B\cap C)$

$=\frac{20}{100}-\frac{8}{100}-\frac{5}{100}+\frac{2}{100}$

$=\frac{9}{100}$

$=9\%$

6. ***The probabilities of three events A,B,C are such that P (A) = $o.3$, $P(B) = 0.4$, $P(c) = 0.8$, $P(A\cap B) = 0.08$, $P(A\cap C) = 0.28$, $P(A\cap B\cap C) = 0.09$, and $P(A\cup B\cup C) \geq 0.75$. S.T $P(B\cap C)$ lies in the interval [0.23, 0.48].***

Sol: P (A) = $o.3$, P (B) = 0.4, P (c) = 0.8,
$P(A\cap B) = 0.08$,
$P(A\cap C) = 0.28$,
$P(A\cap B\cap C) = 0.09$,
and $P(A\cup B\cup C) \geq 0.75$.
We know that
$P(A\cup B\cup C) = P(A) + P(B) + P(c)$
$-P(A\cap B) - P(B\cap C)$
$-P(A\cap C) + P(A\cap B\cap C)$

$\therefore\ P(s) = P(A\cup B\cup C) \leq 0$

$\Rightarrow 0.75 \leq o.3 + 0.4 +, 0.8 - 0.08 - 0.28 - P(B\cap C) + 0.09 \leq 1$

$\Rightarrow 0.75 \leq 1.23 - P(B\cap C) \leq 1$
$\Rightarrow 0.75 - 1.23 \leq 1.23 - 1.23 - P(B\cap C) \leq 1 - 1.23$

$\Rightarrow -0.48 \leq -P(B\cap C) \leq -0.23$

$\Rightarrow 0.48 \geq P(B\cap C) \geq 0.23$
$\therefore P(B\cap C)$ lies in the interval [0.23, 0.48].

7. ***The probability of three mutually exclusive events are respectively, given as $\frac{1+3p}{3}, \frac{1-p}{4}, \frac{1-2p}{2}$ prove that $\frac{1}{3} \leq p \leq \frac{1}{2}$.***

Sol: let A, B, C be the given 3 mutually exclusive events.
$P(A) = \frac{1+3p}{3}, P(B) = \frac{1-p}{4}, P(C) = \frac{1-2p}{2}$

$0 \leq P(A) \leq 1$
$\Rightarrow 0 \leq \frac{1+3p}{3} \leq 1$
$\Rightarrow 0 \leq 1 + 3p \leq 3$
$\Rightarrow -1 \leq 3p \leq 2$
$\Rightarrow -\frac{1}{3} \leq p \leq \frac{2}{3}$ ... (1)

$0 \leq P(B) \leq 1$
$\Rightarrow 0 \leq \frac{1-p}{4} \leq 1$
$\Rightarrow 0 \leq 1 - p \leq 4$
$\Rightarrow -1 \leq -p \leq 4 - 1$
$\Rightarrow -1 \leq -p \leq 3$
$\Rightarrow -3 \leq p \leq 1$ …….. (2)

$0 \leq P(C) \leq 1$
$\Rightarrow 0 \leq \frac{1-2p}{2} \leq 1$
$\Rightarrow 0 \leq 1 - 2p \leq 2$
$\Rightarrow -1 \leq -2p \leq 2 - 1$
$\Rightarrow -\frac{1}{2} \leq -p \leq \frac{1}{2}$
$\Rightarrow -\frac{1}{2} \leq p \leq \frac{1}{2}$ … …. (3)

Also $0 = P(A\cup B\cup C) \leq 1$
$\Rightarrow 0 \leq \frac{1+3p}{3} + \frac{1-p}{4} + \frac{1-2p}{2} \leq 1$
$\Rightarrow 0 \leq 4 + 12p + 3 - 3p + 6 - 12p \leq 12$
$\Rightarrow 0 \leq 13 - 3p \leq 12$
$\Rightarrow 0 \leq 3p - 13 \leq -12$
$\Rightarrow 13 \leq 3p \leq 1$
$\Rightarrow \frac{13}{3} \leq p \leq \frac{1}{3}$ …..(4)
From (1), (2), (3) & (4) we get $\frac{1}{3} \leq p \leq \frac{1}{2}$.

8. ***(i). for any two events A and B, show that $P(\bar{A}\cap\bar{B}) = 1 + P(A\cap B) - P(A) - P(B)$.***
Sol:
Given $P(\bar{A}\cap\bar{B})$

$\Rightarrow P(\bar{A}\cap\bar{B}) = P(\overline{A\cup B})$

$= 1 - P(A\cup B)$

$= 1 - [P(A) + P(B) - P(A\cap B)]$

$= 1 - P(A) - P(B) + P(A\cap B)$

$\therefore P(\bar{A}\cap\bar{B}) = 1 + P(A\cap B) - P(A) - P(B)$.

(ii) If A and B are two events with $P(A\cup B) = 0.65$, $P(A\cap B) = 0.15$, then Find $P(\bar{A}) + P(\bar{B})$.
Sol:

$P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$P(A\cup B) = 1 - P(\bar{A}) + 1 - P(\bar{B}) - P(A\cap B)$

$\Rightarrow P(\bar{A}) + P(\bar{B}) = 2 - P(A\cup B) - P(A\cap B)$

$\Rightarrow P(\bar{A}) + P(\bar{B}) = 2 - (0.65) - (0.15)$

$\Rightarrow P(\bar{A}) + P(\bar{B}) = 2 - (0.80)$

$= 1.2$

9. A, B, C are aiming to shoot a balloon. A will succeed 4 times out of 5 attempts. The chance of B to shoot the balloon is 3 out of 4 and that of C is 2 out of 3. If three aim the balloon Simultaneously, then find the probability that at least two of them hit the balloon.

Sol: Let A, B, C be the events that the shooters A, B, C succeed in shooting the balloon.

Given that

$P(A) = \frac{4}{5} \Rightarrow P(\overline{A}) = 1 - P(A) = \frac{1}{5}$

$P(B) = \frac{3}{4} \Rightarrow P(\overline{B}) = 1 - P(B) = \frac{1}{4}$

$P(C) = \frac{2}{3} \Rightarrow P(\overline{C}) = 1 - P(C) = \frac{1}{3}$

Clearly A, B, C are independent events.

Probability that at least two of them hit the balloon

$= P(\mathbf{A} \cap \mathbf{B} \cap \overline{c}) + P(\mathbf{A} \cap \overline{B} \cap \boldsymbol{C})$

$+ P(\overline{A} \cap \mathbf{B} \cap \boldsymbol{C}) + P(\mathbf{A} \cap \mathbf{B} \cap \boldsymbol{C})$

$= P(A)\,P(B)\,P(\overline{C}) + P(A)\,P(\overline{B})\,P(C)$

$+ P(\overline{A})\,P(B)\,P(C) + P(A)\,P(B)\,P(C)$

$= \frac{4}{5} \cdot \frac{3}{4} \cdot \left(\frac{1}{3}\right) + \frac{4}{5} \cdot \left(\frac{1}{4}\right) \cdot \frac{2}{3} + \left(\frac{1}{5}\right) \cdot \frac{3}{4} \cdot \frac{2}{3} + \frac{4}{5} \cdot \frac{3}{4} \cdot \frac{2}{3}$

$= \frac{12}{60} + \frac{8}{60} + \frac{6}{60} + \frac{24}{60} = \frac{50}{60} = \frac{5}{6}$

10. In a shooting test the probability of A, B, C hitting the targets are $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}$ respectively. It all of them fire at the same target. Find the probability that
i) Only one of them hits the target.
ii) At least one of them hits the target.

Sol. The probabilities that A, B, C hitting the targets are denoted by

$P(A) = \frac{1}{2} \Rightarrow P(\overline{A}) = 1 - P(A) = \frac{1}{2}$

$P(B) = \frac{2}{3} \Rightarrow P(\overline{B}) = 1 - P(B) = \frac{1}{3}$

$P(C) = \frac{3}{4} \Rightarrow P(\overline{C}) = 1 - P(C) = \frac{1}{4}$

Clearly A, B, C are independent events.

Probability that only one of them hit the target

$= P(A \cap \overline{B} \cap \overline{C}) + P(\overline{A} \cap B \cap \overline{C})$

$+ P(\overline{A} \cap \overline{B} \cap C)$

$= P(A)\,P(\overline{B})\,P(\overline{C}) + P(\overline{A})\,P(B)\,P(\overline{C})$
$+ P(\overline{A})\,P(\overline{B})\,P(C)$

$= \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{4} + \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{1}{4} + \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{3}{4}$

$= \frac{1}{24} + \frac{2}{24} + \frac{3}{24}$

$= \frac{1+2+3}{24}$

$= \frac{6}{24}$

$= \frac{1}{4}.$

ii) **Probability that at least one of them hits the target**

= 1 – Probability that none of them hits the target. $= 1 - P(\overline{A} \cap \overline{B} \cap \overline{C})$

$= 1 - P(\overline{A})\,P(\overline{B})\,P(\overline{C})$

$= 1 - \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{4}$

$= 1 - \frac{1}{24}$

$= \frac{23}{24}.$

11. A, B, C are three independent events of an experiment such that P $(A \cap \bar{B} \cap \bar{C}) = \frac{1}{4}$,

P $(\bar{A} \cap B \cap \bar{C}) = \frac{1}{8}$, P $(\bar{A} \cap \bar{B} \cap \bar{C})$=1/4 then find P (A), P (B) and P (C).

Sol: given that A, B, C are independent events and

$P(A \cap \bar{B} \cap \bar{C}) = \frac{1}{4} \Rightarrow P(A).P(\bar{B}).P(\bar{C}) = \frac{1}{4}$ ...... (1)

$P(\bar{A} \cap B \cap \bar{C}) = \frac{1}{8} \Rightarrow P(\bar{A}).P(B).P(\bar{C}) = \frac{1}{8}$......... (2)

$P(\bar{A} \cap \bar{B} \cap \bar{C})=1/4 \Rightarrow P(\bar{A}).P(\bar{B}).P(\bar{C}) = \frac{1}{4}$......... (3)

$= \frac{P(A)P(\bar{B})P(\bar{C})}{P(\bar{A})P(\bar{B})P(\bar{C})} = \frac{\frac{1}{4}}{\frac{1}{4}}$

$P(A) = 1 - P(\bar{A})$

$\Rightarrow \frac{P(A)}{1-P(A)} = 1$

$\Rightarrow P(A) = 1 - P(A)$

$\Rightarrow 2P(A) = 1$

$\Rightarrow P(A) = \frac{1}{2} \Rightarrow P(\bar{A}) = \frac{1}{2}$ .... (4)

$= \frac{P(\bar{A})P(B)P(\bar{C})}{P(\bar{A})P(\bar{B})P(C)} = \frac{\frac{1}{4}}{\frac{1}{8}}$

$\Rightarrow \frac{P(B)}{1-P(B)} = \frac{1}{2}$.

$\Rightarrow 2P(B) = 1 - P(B)$

$\Rightarrow 3P(B) = 1$

$\Rightarrow P(B) = \frac{1}{3}. \Rightarrow P(\bar{B}) = \frac{2}{3}$....... (5)

From (4) & (5), $P(A)P(\bar{B})P(\bar{C}) = \frac{1}{4}$

$\frac{1}{2}.\frac{2}{3}.P(\bar{C}) = \frac{1}{4} \Rightarrow P(\bar{C}) = \frac{1}{4}.\frac{3}{1} = \frac{3}{4}$..... (6)

$P(C) = 1 - P(\bar{C}) = 1 - \frac{3}{4} = \frac{1}{4}$

$P(A) = \frac{1}{2}, P(B) = \frac{1}{3}, P(C) = \frac{1}{4}$

12. Define conditional probability. There are 3 black and 4 white balls in one bag; 4 black and 3 white balls in the second bag. A die is rolled and the first bag is selected if it is 1 or 3, and second bag for the rest. Find the probability of drawing a black ball from the selected bag.

<u>Sol:</u> conditional probability: if A, B are two events in a sample space S and P (A) ≠0,then the probability of B after the event A has occurred is called conditional probability of B given by A. it is denoted by $P(\frac{B}{A})$

$P(\frac{B}{A}) = \frac{n(A \cap B)}{n(A)} = \frac{P(A \cap B)}{P(A)}$

Let E_1, E_2 be the events of selecting first and second bags respectively

Let B be the event of drawing a black ball from the first bag now $P(E_1) = \frac{2}{6} = \frac{1}{3}, P(E_2) = \frac{4}{6} = \frac{2}{3}$

Probability of drawing a black ball from the first bag is $P(\frac{B}{E_1}) = \frac{3}{7}$

Probability of drawing a black ball from the 2nd bag is

$P(\frac{B}{E_2}) = \frac{4}{7}$

By total probability theorem,

$P(B) = P(E_1).P(\frac{B}{E_1}) + P(E_2).P(\frac{B}{E_2})$

$= \frac{1}{3}.\frac{3}{7} + \frac{2}{3}.\frac{4}{7}$

$= \frac{3+8}{21}$

$= \frac{11}{21}$.

13. ***State and prove Baye's theorem.***

Sol: if $E_1, E_2, E_3 \ldots .. E_n$ are mutually exclusive and exhaustive events in a sample space S such that

P (E_1) >0 for i=1, 2, 3 …n and A is any event with

P (A)>0, then $P\left(\frac{E_k}{A}\right) = \frac{P(E_k)P\left(\frac{A}{E_k}\right)}{\sum_{i=1}^{n} P(E_i)P\left(\frac{A}{E_i}\right)}$

Proof: given that $E_1, E_2, E_3 \ldots .. E_n$ are mutually exclusive and exhaustive events in a sample space S.

$\Rightarrow, E_1 \cup E_2 \cup E_3 \cup \ldots . . \cup E_n = S$

$\Rightarrow \coprod_{i=1}^{n} E_i = S$

Given 'A" is any event of a random experiment, then

Since $A = A \cap S$

$A = A \cap \coprod_{i=1}^{n} E_i \Rightarrow A = \coprod_{i=1}^{n} E_i \cap A$

Applying p on both sides we get

$P(A) = P\sum_{i=1}^{n}(E_i \cap A)$

[By multiplication theorem]

$P(A) = \sum_{i=1}^{n} P(E_i)\, P\left(\frac{A}{E_i}\right)$ …. (1).

for k=1, 2, 3…….

And also by using conditional

Probability: $P\left(\frac{E_k}{A}\right) = \frac{P(E_k \cap A)}{P(A)}$……. (2)

And w. k. t $P(E_k \cap A) = P(E_k)\, P\left(\frac{A}{E_k}\right)$…. (3)

[By multiplication theorem]

Sub (1) & (3) in (2)

$P\left(\frac{E_k}{A}\right) = \frac{P(E_k)P\left(\frac{A}{E_k}\right)}{\sum_{i=1}^{n} P(E_i)P\left(\frac{A}{E_i}\right)}$ for i=1, 2, 3……

14. ***Three boxes numbered I,II,III contain 1 white ,2 black and 3 red balls; 2 white, 1 black and 1 red ball;4 white , 5 black and 3 red balls. one box is randomly selected and a ball is drawn from it. If the ball is red then find the probability that it is from box II.***

Sol: given

Box	white	black	red	total
I	1	2	3	6
II	2	1	1	4
III	4	5	3	12

Let B_1, B_2, B_3 be the events of selecting the boxes I, II, III respectively.

$P(B_1) = P(B_2) = P(B_3) = \frac{1}{3}$

Let R be the event of drawing a red ball.

$P\left(\frac{R}{B_1}\right) = \frac{3}{6} = \frac{1}{2}$, $P\left(\frac{R}{B_2}\right) = \frac{1}{4}$, $P\left(\frac{R}{B_3}\right) = \frac{3}{12} = \frac{1}{4}$.

$\therefore$ *the probability that the*

red ball from box – II

$P\left(\frac{B_2}{R}\right) = \frac{P(B_2)P\left(\frac{R}{B_2}\right)}{P(B_1)P\left(\frac{R}{B_1}\right) + P(B_2)P\left(\frac{R}{B_2}\right) + P(B_3)P\left(\frac{R}{B_3}\right)}$

$= \frac{\frac{1}{3}\cdot\frac{1}{4}}{\frac{1}{3}\left[\frac{1}{2}+\frac{1}{4}+\frac{1}{4}\right]}$

$= \frac{\frac{1}{4}}{\frac{2+1+1}{4}}$

$= \frac{\frac{1}{4}}{\frac{4}{4}}$

$= \frac{1}{4}$

15. Three boxes contains balls
With different colours as follows:

	w	B	R
B_1	2	1	2
B_2	3	2	4
B_3	4	3	2

A die is thrown. If 1 or 2 turns up on the dice, Box B_1 is selected: if 3 or 4 turn up Box B_2 is selected; if 5 or 6 turn up Box B_3 is selected;if a box is selected like this, a ball is drawn from that box . if the ball is red, then find the probability that it was drawn from B_2.

: given

Box	white	black	red	total
B_1	2	1	2	5
B_2	3	2	4	9
B_3	4	3	2	9

Let B_1, B_2, B_3 be the events of selecting the boxes B_1, B_2, B_3 respectively.

$P(B_1) = P(B_2) = P(B_3) = \frac{1}{3}$

Let R be the event of drawing a red ball.

$P\left(\frac{R}{B_1}\right) = \frac{2}{9}, P\left(\frac{R}{B_2}\right) = \frac{4}{9}, P\left(\frac{R}{B_3}\right) = \frac{2}{9}$

$\therefore$ *the probability that the*

red ball from box $- B_2$

$$P\left(\frac{B_2}{R}\right) = \frac{P(B_2)P\left(\frac{R}{B_2}\right)}{P(B_1)P\left(\frac{R}{B_1}\right)+P(B_2)P\left(\frac{R}{B_2}\right)+P(B_3)P\left(\frac{R}{B_3}\right)}$$

$$= \frac{\frac{1}{3}\times\frac{4}{9}}{\frac{1}{3}\left[\frac{2}{5}+\frac{4}{9}+\frac{2}{9}\right]}$$

$$= \frac{\frac{4}{9}}{\frac{18+20+10}{45}} \quad = \frac{4}{9}\times\frac{45}{48} = \frac{5}{12}$$

16. **In a certain college, 25% of the boys and 10% of the girls are studying mathematics. The girls constitute 60% of the student strength. If a student selected at random is found studying mathematics, find the probability that the student is a girl.**

Sol:

Let G, B denotes the events of selecting student is a girl, a boy respectively and M denote the selected student is studying mathematics.

Then $P(G) = \frac{60}{100}; P(B) = \frac{40}{100}$

$P\left(\frac{M}{G}\right) = \frac{10}{100}, P\left(\frac{M}{B}\right) = \frac{25}{100}$

By Baye'stheorem

$$P\left(\frac{G}{M}\right) = \frac{P(G)P\left(\frac{M}{G}\right)}{P(G)P\left(\frac{M}{G}\right)+P(B)P\left(\frac{M}{B}\right)}$$

$$= \frac{\frac{60}{100}\cdot\frac{10}{100}}{\frac{60}{100}\cdot\frac{10}{100}+\frac{40}{100}\cdot\frac{25}{100}}$$

$$= \frac{60.10}{60.10+40.25}$$

$$= \frac{600}{1600}$$

$$= \frac{6}{16}$$

$$= \frac{3}{8}$$