MATHS 1A & 1B CHAPTER WISE WEIGHTAGE

MATHS - 1A

S.NO	NAME OF THE CHAPTER	LAQ(7M)	SAQ(4M)	VSAQ(2M)	TOTAL
1	FUNCTIONS	1		2	11M
2	MATHEMATICAL INDUCTION	1			7M
3	ADDITION OF VECTORS		1	2	8M
4	MULTIPLICATION OF VECTORS	1	1	1	13M
5	TRIGONOMETRY UPTO TRASFORMATIONS	1	1	2	15M
6	TRIGONOMETRIC EQUATIONS		1		4M
7	INVERSE TRIGONOMETRIC FUNCTIONS		1		4M
8	HYPERBOLIC FUNCTIONS			1	2M
9	PROPERTIES OF TRIANGLES	1	1		11M
10	MATRICES	2	1	2	22M
TOTAL		**7**	**7**	**10**	**97M**

MATHS - 1B

S.NO	NAME OF THE CHAPTER	LAQ(7M)	SAQ(4M)	VSAQ(2M)	TOTAL
1	LOCUS		1		4M
2	CHANGE OF AXES		1		4M
3	STRAIGHT LINES	1	1	2	15M
4	PAIR OF STRAIGHT LINES	2			14M
5	3D-GEOMETRY			1	2M
6	D.C's & D.R's	1			7M
7	PLANES			1	2M
8	LIMITS & CONTINUITY		1	2	8M
9	DERIVATIVES	1	1	2	15M
10	APPLICATIONS OF DIFFERENTIATION	2	2	2	26M
TOTAL		**7**	**7**	**10**	**97M**

BOARD OF INTERMEDIATE EDUCATIONS A.P : HYDERABAD

MODEL QUESTION PAPER w.e.f. 2012-13

MATHEMATICS-IA

Time : 3 Hours **Max.Marks : 75**

Note : The Question Paper consists of three A,B and C

Section-A **10 X 2 = 20 Marks**

I. Very Short Answer Questions :
i) Answer All questions
ii) Each Questions carries Two marks

1. If $A=\left\{0,\frac{\pi}{6},\frac{\pi}{4},\frac{\pi}{3},\frac{\pi}{2}\right\}$ and $f: A \rightarrow B$ is a surjection defined by f(x) = cos x then find B.

2. Find the domain of the real-valued function $f(x)=\frac{1}{\log(2-x)}$

3. A certain bookshop has 10 dozen chemistry books, 8 dozen physics books, 10 dozen economics books. Their selling prices are Rs. 80, Rs.60 and Rs.40 each respectively. Find the total amount the bookshop will receive by selling all the books, using matrix algebra.

4. If $A=\begin{bmatrix} 2 & -4 \\ -5 & 3 \end{bmatrix}$, then find A+A' and A A'.

5. Show that the points whose position vectors are $-2\bar{a}+3\bar{b}+5\bar{c}, \bar{a}+2\bar{b}+3\bar{c}, 7\bar{a}-\bar{c}$ are collinear when $\bar{a},\bar{b},\bar{c}$ are non-coplanar vectors.

6. Let $\bar{a}=2\bar{i}+4\bar{j}-5\bar{k}, b=\bar{i}+\bar{j}+\bar{k}$ and $\bar{c}=\bar{j}+2\bar{k}$. Find unit vector in the opposite direction of $\bar{a}+\bar{b}+\bar{c}$.

7. If $\bar{a}=\bar{i}+2\bar{j}-3\bar{k}$ and $\bar{b}=3\bar{i}-2\bar{j}+2\bar{k}$ then show that $\bar{a}+\bar{b}$ and $\bar{a}-\bar{b}$ are perpendicular to each other.

8. Prove that $\frac{\cos 9^0+\sin 9^0}{\cos 9^0-\sin 9^0}=\cot 36^0$.

9. Find the period of the function defind by $f(x)=\tan\left(x+4x+9x+......+n^2x\right)$.

10. If sinh x = 3 then show that $x=\log_e\left(3+\sqrt{10}\right)$.

Section-B **5 X 4 = 20 Marks**

I. Short Answer Questions :
i) Answer any Five questions
ii) Each Questions carries Four marks

11. Show that $\begin{vmatrix} bc & b+c & 1 \\ ca & c+a & 1 \\ ab & a+b & 1 \end{vmatrix}=(a-b)(b-c)(c-a)$.

12. Let ABCDEF be regular hexagon with centre 'O'. Show that
$\overline{AB}+\overline{AC}+\overline{AD}+\overline{AE}+\overline{AF}=3\overline{AD}=6\overline{AO}$.

13. If $\bar{a}=\bar{i}-2\bar{j}-3\bar{k}, \bar{b}=2\bar{i}+\bar{j}-\bar{k}$ and $\bar{c}=\bar{i}+3\bar{j}-2\bar{k}$ find $\bar{a}\times\left(\bar{b}\times\bar{c}\right)$.

14. If A is not an integral multiple of $\frac{\pi}{2}$, prove that
i) tan A + cot A = 2 cosec 2A
ii) cot A - tan A = 2 cot 2 A

15. Slove : $2\cos^2\theta-\sqrt{3}\sin\theta+1=0$.

16. Prove that $\cos\left(2\tan^{-1}\frac{1}{7}\right)=\sin\left(4\tan^{-1}\frac{1}{3}\right)$.

17. In a ΔABC prove that $\tan\left(\frac{B-C}{2}\right)=\frac{b-c}{b+c}\cot\frac{A}{2}$.

Section-C 5 X 7 = 35 Marks

I. Long Answer Questions :
i) Answer any Five questions
ii) Each Questions carries Seven marks

18. Let $f: A\to B, g: B\to C$ be bijections. Then prove that $(gof)^{-1}=f^{-1}og^{-1}$..

19. By using mathematical induction show that $\forall n\in N, \frac{1}{1.4}+\frac{1}{4.7}+\frac{1}{7.10}+....$ upto n terms $=\frac{n}{3n+1}$.

20. If $A=\begin{bmatrix}1 & -2 & 3\\ 0 & -1 & 4\\ -2 & 2 & 1\end{bmatrix}$ then find $\left(A'\right)^{-1}$

21. Solve the following equations by Gauss-Jordan method 3x+4y+5z=18, 2x-y+8z = 13 and 5x-2y+7z=20.

22. If $A=(1,-2,-1), B(4,0,-3), C=(1,2,-1)$ and $D=(2,-4,-5)$, find the distance between $\overline{AB}$ and $\overline{CD}$

23. If A,B,C are angles of a triangle, then prove that
$\sin^2\frac{A}{2}+\sin^2\frac{B}{2}-\sin^2\frac{C}{2}=1-2\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}$.

24. In a ΔABC, if a = 13, b = 14, c = 15, find R, r, r_1, r_2 and r_3.

MATHS-1A

2 MARKS IMP.QUESTIONS

FUNCTIONS

1.* If $f : R \rightarrow R, g : R \rightarrow R$ are defind by $f(x) = 4x - 1$ and $g(x) = x^2 + 2$ then find

i) (gof)(x) ii) $(gof)\left(\frac{a+1}{4}\right)$ iii) fof(x) iv) go(fof)(o)

Ans : i) $16x^2 - 8x + 3$ ii) $a^2 + 2$ iii) $16x - 5$ iv) 27

2.* If f and g are real valued functions defined by $f(x) = 2x–1$ and $g(x) = x^2$ then find

i) (3f –2g) (x) ii) (fg) (x) iii) $\left(\frac{\sqrt{f}}{g}\right)(x)$ iv) (f+g+2) (x)

3.* If $f = \{(4,5),(5,6)(6,-4)\}$ and $g = \{(4,-4),(6,5)(8,5)\}$ then find

i) f + g ii) f – g iii) 2f + 4g iv) f + 4 v) fg

vi) f / g vii) $|f|$ viii) $\sqrt{f}$ ix) f^2 x) f^3

4.* i) If $A = \left\{0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}\right\}$ and $f : A \rightarrow B$ is a surjection defined by $f(x) = \cos x$ then find B.

ii) If $A = \{-2, -1, 0, 1, 2\}$ & $f : A \rightarrow B$ is a surjection defined by $f(x) = x^2 + x + 1$ then find B.

iii) If $A = \{1,2,3,4\}$ and $f : A \rightarrow R$ is a surjection defined by $f(x) = \frac{x^2 + x + 1}{x + 1}$ then find rangect f.

5. If $f(x) = 2, g(x) = x^2, h(x) = 2x$ for all $x \in R$, then find $(fo(goh)(x))$

6. If $f(x) = \frac{x+1}{x-1} (x \neq \pm 1)$ then find i) (fofof) (x) ii) (fofofof) (x)

7. If $f : R \rightarrow R, g : R \rightarrow R$ defined by $f(x) = 3x - 2, g(x) = x^2 + 1$, then find

i) $(gof^{-1})(2)$ ii) $(gof)(x-1)$

8. Define the following functions and write an example for each

i) one – one ii) onto iii) even and odd iv) bijection

9. If $f : N \rightarrow N$ is defined as $f(x) = 2x + 5$, Is ' f ' onto? Explain with reason.

10. Find the inverse of the following functions

i) If $a, b \in R, f : R \rightarrow R$ defined by $f(x) = ax + b (a \neq 0)$

ii) $f : R \rightarrow (0, \infty)$ defined by $f(x) = 5^x$ iii) $f : (0, \infty) \rightarrow R$ defined by $f(x) = \log_2^x$

iv) $f(x) = e^{4x+7}$ v) $f : R \rightarrow R, f(x) = \frac{2x+1}{3}$

11. i) If $f : R \rightarrow R$ is defined by $f(x) = \frac{1-x^2}{1+x^2}$, then show that $f(\tan\theta) = \cos 2\theta$

ii) If $f:R-\{\pm 1\}\rightarrow$ is defined by $f(x)=\log\left|\frac{1+x}{1-x}\right|$ then show that $f\left(\frac{2x}{1+x^2}\right)=2f(x)$

12. If the function $f:R\rightarrow R$ defined by $f(x)\frac{3^x+3^{-x}}{2}$, then show that $f(x+y)+f(x-y)=2f(x)f(y)$

13. If $f(x)=\cos(\log x)$, then show that $f\left(\frac{1}{x}\right)f\left(\frac{1}{y}\right)-\frac{1}{2}\left(f\left(\frac{x}{y}\right)+f(xy)\right)=0$

14.* Find the domain of the following real valued functions

i) $f(x)=\frac{1}{6x-x^2-5}$ ii) $f(x)=\sqrt{x^2-1}+\frac{1}{\sqrt{x^2-3x+2}}$

iii) $f(x)=\frac{1}{\sqrt{|x|-x}}$ iv) $f(x)=\sqrt{x+2}+\frac{1}{\log_{10}(1-x)}$

v) $f(x)=\frac{\sqrt{3+x}+\sqrt{3-x}}{x}$ vi) $f(x)=\sqrt{4x-x^2}$

vii) $f(x)=\log(x^2-4x+3)$ viii) $f(x)=\frac{1}{x+|x|}$

15.* Find the range of the following real valued functions

i) $\log\left|4-x^2\right|$ ii) $\frac{x^2-4}{x-2}$

16.* Find the domain and range of the following real valued functions

i) $f(x)=\frac{x}{1+x^2}$ *ii) $f(x)=\sqrt{9-x^2}$ iii) $f(x)=|x|+|1+x|$ iv) [x]

VECTORADDITION

1. ABCD is a parallelogram if L&M are middle points of BC and CD. Then find
i) $\overline{AL}$ and $\overline{AM}$ interms of $\overline{AB}$ and $\overline{AD}$ ii) l , if $\overline{AM}=l\,\overline{AD}-\overline{LM}$
2. In triangle ABC, P,Q, & R are the mid points of the sides AB, BC, and CA. If D is any point then (i) express $\overline{DA}+\overline{DB}+\overline{DC}$ interms of $\overline{DP},\overline{DQ},\overline{DR}$
ii) If $\overline{PA}+\overline{QB}+\overline{RC}=\overline{a}$ then find $\overline{a}$
3. If G is the centroid of ΔABC, then show that $\overline{OG}=\frac{\overline{a}+\overline{b}+\overline{c}}{3}$ when $\overline{a},\overline{b},\overline{c}$ are pv's. of the vertices of ΔABC.
4. i) $\overline{a}=2\overline{i}+5\overline{j}+\overline{k},\overline{b}=4\overline{i}+m\overline{j}+n\overline{k}$ are collinear then find m and n.
ii) If the vectors $-3\overline{i}+4\overline{j}+l\,\overline{k}$ and $m\overline{i}+8\overline{j}+6\overline{k}$ are colliner then find l and m.
5. If the position vectors of the points A, B and C are $-2\mathbf{i}+\mathbf{j}-\mathbf{k}$, $-4\mathbf{i}+2\mathbf{j}+2\mathbf{k}$ and $6\mathbf{i}-3\mathbf{j}-13\mathbf{k}$ respectively and $\mathbf{AB}=\lambda\mathbf{AC}$, then find the value of λ
6. If $\mathbf{OA}=\mathbf{i}+\mathbf{j}+\mathbf{k}$, $\mathbf{AB}=3\mathbf{i}-2\mathbf{j}+\mathbf{k}$, $\mathbf{BC}=\mathbf{i}+2\mathbf{j}-\mathbf{k}$ and $\mathbf{CD}=2\mathbf{i}+\mathbf{j}+3\mathbf{k}$,then find the vector **OD**
7. i) Let $\mathbf{a}=2\mathbf{i}+4\mathbf{j}-5\mathbf{k}$, $\mathbf{b}=\mathbf{i}+\mathbf{j}+\mathbf{k}$ and $\mathbf{c}=\mathbf{j}+2\mathbf{k}$. Find unit vector in the opposite direction of $\mathbf{a}+\mathbf{b}+\mathbf{c}$.
ii) Let $\mathbf{a}=\mathbf{i}+2\mathbf{j}+3\mathbf{k}$, and $\mathbf{b}=3\mathbf{i}+\mathbf{j}$. Find unit vector in the direction of $\mathbf{a}+\mathbf{b}$.

8. ABCDE is apentagon. If the sum of the vectors $\overline{AB}, \overline{AE}, \overline{BC}, \overline{DC}, \overline{ED}$ and $\overline{AC}$ is $l\,\overline{AC}$ then find the value of l .
9. Using the vector eaquation of the straigth line passing though two points, prove that the points whose position vectors are $\bar{a}, \bar{b}$ and $3\bar{a} - 2\bar{b}$ are collinear
10. If $\bar{a}, \bar{b}, \bar{c}$ are the pv's of the vertices A,B and C respectively of triangle ABC, then find the vector equation of the median throgh the vertex A.
11. OABC is a parallelogram. If **OA** = **a** and **OC** = **c** then find the vector equation of the side **BC**.
12. Is the triangle formed by the vector by the vectors 3**i**+5**j**+2**k**, 2**i** –3**j** –5**k** and – 5**i** – 2**j** + 3**k** equilateral.
13. Find the vector equation of the line passing through the point 2**i**+3**j**+**k** and parallel to the vector 4**i** - 2**j** + 3**k**
14. Find the vector equation and cartesian equation to the line passing through the points 2i+j+3k, –4i+3j–k
15. i) Find the vector equation of the plane passing through the points $\bar{i} - 2\bar{j} + 5\bar{k}, - 5\bar{j} - \bar{k} \,\&\, - 3\bar{i} + 5\bar{j}$
    ii) Find the vector equation of the plane passing through the points (0, 0, 0), (0, 5, 0), and (2, 0, 1).
16. Let A,B,C and D be four points with position vectors $\bar{a} + 2\bar{b}, 2\bar{a} - \bar{b}, a$ and $3\bar{a} + \bar{b}$ respectively. Express the vectors $\overline{AC}, \overline{DA}, \overline{BA}$ and $\overline{BC}$ interms of $\bar{a}$ and $\bar{b}$

MULTIPLICATION OF VECTORS

1. If **a** = **i** + 2**j** – 3**k** and **b** = 3**i** – **j** + 2**k**, then show that **a** + **b** and **a** – **b** are perpendicular to each other
2. If the vectors $\lambda\bar{i} - 3\bar{j} + 5\bar{k}$ and $2\lambda\bar{i} - \lambda\bar{j} - \bar{k}$ are perpendicular to each other, find λ.
3. Find the cartesian equation of the plane through the point A (2, –1, –4) ane parallel to the plane 4x – 12y – 3z – 7 = 0
4. Let **a** = **i** + **j** + **k** and **b** = 2**i** + 3**j** + **k** find
   i) The projection vector of **b** and **a** its magnitude
   ii) The vector components of **b** in the direction of **a** and perpendicular to **a**
5. If **a** = 2**i** + 2**j** – 3**k**, **b** = 3**i** – 2**j** + 2**k**, then find angle between 2**a** + **b** and **a** + 2**b**
6. If α, β and γ be the angles made by the vector 3**i** – **6j** + **2k** with the positive directions of the coordinate axes, the find $\cos\alpha, \cos\beta$ and $\cos\gamma$
7. If $|\mathbf{a}| = 2, |\mathbf{b}| = 3$ and $|\mathbf{c}| = 4$ and each of **a, b, c** is perpendicular to the sum of the other two vectors, then find the magnitude of **a + b + c.**
8. Let **a** = 4**i** + 5**j** – **k, b** = **i** – 4**j** + 5**k** and **c** = 3**i** + **j** – **k**. Find the vector which is perpendicular to both **a** and **b** whose magnitude is twenty one times the magnitude of **c**.
9.* If **a** = 2**i** – 3**j** + 5**k, b** = –**i** + 4**j** + 2**k** then find $\bar{a} \times \bar{b}$ and unit vector perpendicular to both **a** and **b**
10. Let **a** = 2**i** – **j** + **k** and **b** = 3**i** + 4**j** – **k**. if θ is the angle between **a** and **b**, then find $\sin\theta$
11. If $|\bar{p}| = 2, |\bar{q}| = 3$ and $(\bar{p}, \bar{q}) = \frac{\pi}{6}$, then find $|\bar{p} \times \bar{q}|^2$
12. If $4\bar{i} + \frac{2p}{3}\bar{j} + p\bar{k}$ is parallel to the vector $\bar{i} + 2\bar{j} + 3\bar{k}$, find p
13.* Find the area of the parallelgram having $\bar{a} + 2\bar{j} - \bar{k}$ and $\bar{b} = -\bar{i} + \bar{k}$ as adjacent sides
14.* Find the area of the parallelogram whose diagonals are $3\bar{i} + \bar{j} - 2\bar{k}$ and $\bar{i} - 3\bar{j} + 4\bar{k}$
15. If the vectors **a** = 2**i** – **j** + **k**, **b** = **i** + 2**j** – 3**k** and **c** = 3**i** + p**j** + 5**k** are coplanar, then find **p**
16. **a,b,c** are non-zero vectors and **a** is perpendicular to both **b** and **c**. If |**a**| = 2, |**b**| = 3, |**c**| = 4 and (**b,c**) = $\frac{2\pi}{3}$, then find |[**abc**]|.
17. Show that for any vector $\bar{i} \times (\bar{a} \times \bar{i}) + \bar{j} \times (\bar{a} \times \bar{j}) + \bar{k} \times (\bar{a} \times \bar{k}) = 2\bar{a}$

18. Find the equation of the plane passing through the point $\bar{a} = 2\bar{i} + 3\bar{j} - \bar{k}$ and perpendicular to the vector $3\bar{i} - 2\bar{j} - 2\bar{k}$ and the distance of this plane from the origin.

19. Find the angle between the line $\frac{x+1}{2} = \frac{y}{3} = \frac{z-3}{6}$ and the plane $10x + 2y - 11z = 3$.

Formula : If q is the angle between $\frac{x-x1}{l} = \frac{y-y1}{m} = \frac{z-z1}{m}$ and $ax + by + cz + d = 0$ then

$$\sin q = \left| \frac{al+bm+cn}{\sqrt{l^2+m^2+n^2}\sqrt{a^2+b^2+c^2}} \right|$$

20. Let **b = 2i + j – k, c = i + 3k**. If **a** is a unit vector then find the maximum value of **[a b c]**

21. For any three vectors **a, b, c** prove that [**b** × **c** **c** × **a** **a** × **b**] = $\left[\bar{a}\,\bar{b}\,\bar{c}\right]^2$.

22. Determine λ, for which the volume of the parallelopiped having coteminus edges i + j, 3i -j and $3j + \lambda k$ is 16 cubic units

23. Find the volume of the tetrahedron having the edges i + j + k, i - j and i + 2j + k.

24. If the vectors $2i + \lambda j - k$ and $4i - 2j + 2k$ are perpendicular to each other, then find λ

25. $a = 2i - j + k,\ b = i - 3j - 5k$. Find the vector c such that a, b and c form the sides of a triangle

26. Let e_1 and e_2 be unit vectors containing angle θ. If $\frac{1}{2}|e_1 - e_2| = \sin \lambda\theta$, then find λ

27. Find the equaition of the plane through the point (3, -2, 1) and perpendicular to the vector (4, 7, - 4).

28. Find the angles made by the straight line passing through the points (1, -3, 2) and (3, -5, 1) with the coordinate axes

MATRICES (2 MARKS)

1. ** If $A = \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}$ then show that $A^2 = -I$

2. If $A = \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}, B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, C = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$ and I is the unit matrix of order 2 then prove that

   i) $A^2 = B^2 = c^2 = -I$

   ii) $AB = -BA = -C$

3. ** If $A = \begin{bmatrix} 2 & 4 \\ -1 & k \end{bmatrix}$ and $A^2 = 0$ then find k. [Ans :-2]

4. ** Find the trace of A if i) $A = \begin{bmatrix} 1 & 2 & -\frac{1}{2} \\ 0 & -1 & 2 \\ -\frac{1}{2} & 2 & 1 \end{bmatrix}$ [Ans = 1]

** ii) $A=\begin{bmatrix}1 & 3 & -5\\ 2 & -1 & 5\\ 2 & 0 & 1\end{bmatrix}$ [Ans = 1]

5. ** Find the additive inverse of $A=\begin{bmatrix}i & 0 & 1\\ 0 & -i & 2\\ -1 & 1 & 5\end{bmatrix}$ [Hint : additive inverse of A is -A]

6. * If $\begin{bmatrix}x-1 & 2 & y-5\\ z & 0 & 2\\ 1 & -1 & 1+a\end{bmatrix}=\begin{bmatrix}1-x & 2 & -y\\ 2 & 0 & 2\\ 1 & -1 & 1\end{bmatrix}$ then find the values of x,y,z & a

7. ** If $A=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}, B\begin{bmatrix}3 & 8\\ 7 & 2\end{bmatrix}$ and 2 X+A=B then find X.

8. * Construct 3 x 2 matrix whose elements are defined by $a_{ij}=\frac{1}{2}|i-3j|$

9. * If $A=\begin{bmatrix}1 & -2 & 3\\ -4 & 2 & 5\end{bmatrix}$ and $B=\begin{bmatrix}2 & 3\\ 4 & 5\\ 2 & 1\end{bmatrix}$, do AB and BA exist ? If they exist , find them. Do A and B commute with respect to multiplication ?

10. **(i) If $A=\begin{bmatrix}0 & 4 & -2\\ -4 & 0 & 8\\ 2 & -8 & x\end{bmatrix}$ is a skew symmetric matrix, find the value of x [Hint : A^T=-A]

[Ans :0]

**(ii) Define symmetric & skew symmettric matrias

11. **If $A=\begin{bmatrix}-1 & 2 & 3\\ 2 & 5 & 6\\ 3 & x & 7\end{bmatrix}$ is a symmetric matrix, then find x [Hint : A^T=A] [Ans : 6]

12. **If $A=\begin{bmatrix}-2 & 1 & 0\\ 3 & 4 & -5\end{bmatrix}$ and $B=\begin{bmatrix}1 & 2\\ 4 & 3\\ -1 & 5\end{bmatrix}$ then find $A+B^T$

13. If $A=\begin{bmatrix}2 & 0 & 1\\ -1 & 1 & 5\end{bmatrix}$ and $B=\begin{bmatrix}-1 & 1 & 0\\ 0 & 1 & -2\end{bmatrix}$ then find $(AB^1)^1$

14. **If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$ then show that $AA^1 = A^1A = I$

15. *Find the minors of -1 and 3 in the matrix $\begin{bmatrix} 2 & -1 & 4 \\ 0 & -2 & 5 \\ -3 & 1 & 3 \end{bmatrix}$ [Ans:15,-4]

16. *Find the co-factors of the elements 2,-5 in the matrix $\begin{bmatrix} -1 & 0 & 5 \\ 1 & 2 & -2 \\ -4 & -5 & 3 \end{bmatrix}$ [Ans:17,3]

17. If w is complex cuberoot of unity then show that $\begin{vmatrix} 1 & w & w^2 \\ w & w^2 & 1 \\ w^2 & 1 & w \end{vmatrix} = 0$ [Hint : $1+w+w^2=0$]

18. **If $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 4 \\ 5 & -6 & x \end{bmatrix}$ and det A=45 then find x

19. **Find the adjoint and the inverse of the matrix

i) $A = \begin{bmatrix} 1 & 2 \\ 3 & -5 \end{bmatrix}$ ii) $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$ iii) $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$

iv) Find the inverse of $\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} (abc \neq 0)$

20. Define rank of matrix and find the rank of the following matrices

1. i) $\begin{bmatrix} 1 & 2 & 1 \\ -1 & 0 & 2 \\ 0 & 1 & -1 \end{bmatrix}$ [Ans:3] ii) $\begin{bmatrix} -1 & -2 & -3 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix}$ [Ans 2]

iii) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 1 & 2 \end{bmatrix}$ [Ans 3] iv) $\begin{bmatrix} 1 & 0 & -4 \\ 2 & -1 & 3 \end{bmatrix}$ Ans :2

v) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ Ans :1 vi) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ Ans :3

2. *Find the rank of the matrix using elementary transformations>

i) $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 2 & 1 \end{bmatrix}$ [Ans :2] ii) $\begin{bmatrix} 1 & 2 & 0 & -1 \\ 3 & 4 & 1 & 2 \\ -2 & 3 & 2 & 5 \end{bmatrix}$ [Ans :3]

21. ** If $A = \begin{bmatrix} 2 & -4 \\ -5 & 3 \end{bmatrix}$ then find $A + A^1$ and AA^1

23. A certain book has 10 dozen chemistery books, 8 dozen physic books, 10 dozen economics book thier selling prices are Rs.80,Rs 60, Rs,40 each. Find the total amount the book shop will receive by selling all the books,using matrix Algbra.

UPTO TRANSFORMATIONS

1. If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$, prove that $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$
2. If A,B,C are angles of a triangle, then prove that $\cos\left(\frac{A+2B+3C}{2}\right) + \cos\left(\frac{A-C}{2}\right) = 0$
3. Prove that $\cot\frac{\pi}{20}.\cot\frac{3\pi}{20}.\cot\frac{5\pi}{20}.\cot\frac{7\pi}{20}.\cot\frac{9\pi}{20} = 1$
4. Find the period of the following functions

   i) $f(x) = \tan 5x$ ii) $f(x) = \cos\left(\frac{4x+9}{5}\right)$ iii) $f(x) = |\sin x|$ iv) $f(x) = \cos^4 x$

   iv) $f(x) = \sin(x + 2x + ..nx) \forall x \in R, n \in Z^+$ v) $f(x) = \sin^4 x + \cos^4 x . \forall x \in R$
5. Prove that $\cos 12^0 + \cos 84^0 + \cos 132^0 + \cos 156^0 = -1/2$
6. Find the maximum and minimum values of the following functions over R.

   i) $f(x) = 5\sin x + 12\cos x + 13$ ii) $f(x) = \sin 2x - \cos 2x$

   iii) $\cos\left(x + \frac{\pi}{3}\right) + 2\sqrt{2}\sin\left(x + \frac{\pi}{3}\right) - 3$ iv) $5\cos x + 3\cos\left(x + \frac{\pi}{3}\right) + 8$
7. Find the value of

   i) $\sin^2 82^0 \frac{1}{2} - \sin^2 22^0 \frac{1}{2}$ ii) $\cos^2 112\frac{1^0}{2} - \sin^2 52\frac{1^0}{2}$

   iii) $\sin^2 52\frac{1^0}{2} - \sin^2 22\frac{1^0}{2}$ iv) $\sin^2 24^0 - \sin^2 6^0$ v) $\sin^2 42^0 - \cos^2 78^0$
8. Prove that $\frac{1}{\sin 10^0} - \frac{\sqrt{3}}{\cos 10^0} = 4$
9. If $\sec\theta + \tan\theta = \frac{2}{3}$, find the value of $\sin\theta$ and determine the quadrant in which θ lies

10. Show that $\cos^4\alpha+2\cos^2\alpha\left(1-\frac{1}{\sec^2\alpha}\right)=1-\sin^4\alpha$

11. Find the period of the function f defined by f(x) = x-[x] for all $x\in R$, where [x]=integral part of x

12. If $3\sin\theta+4\cos\theta=5$, then find the value of $4\sin\theta-3\cos\theta$

13. Prove that $(\tan\theta+\cot\theta)^2=\sec^2\theta+\operatorname{cosec}^2\theta=\sec^2\theta.\operatorname{cosec}^2\theta$

14. If $\tan\theta=\frac{-4}{3}$ and θ does not lie in 4th quadrant, prove that
$5\sin\theta+10\cos\theta+9\sec\theta+16\operatorname{cosec}\theta+4\cot\theta=0$

15. If $\frac{2\sin\theta}{1+\cos\theta+\sin\theta}=x$, find the value of $\frac{1-\cos\theta+\sin\theta}{1+\sin\theta}$

16. If $\tan 20^0=p$, then prove that $\frac{\tan 610^0+\tan 700^0}{\tan 560^0-\tan 470^0}=\frac{1-p^2}{1+p^2}$

17. If A,B,C,D are angles of a cyclic quadrilateral, then prove that
i) sin A – sinC = sinD – sin B   ii) cosA+ cosB+cosC+cos D=0

18. If $\operatorname{cosec}\theta-\sin\theta=a^3$ and $\sec\theta-\cos\theta=b^3$, then prove that $a^2b^2\left(a^2+b^2\right)=1$

19. i) Draw the graph of $y=\tan x$ in $\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$

ii) Draw the graph of $y=\cos^2x$ in $(0,\pi)$

iii) Draw the graph of $y=\sin 2x$ in $(-\pi,\pi)$

20. Find the expansion of the following if A,B,C are real numbers
i) sin (A+B–C)   ii) cos (A–B–C)

21. If θ is not an integral multiple of $\frac{\pi}{2}$, prove that $\tan\theta+2\tan 2\theta+4\tan 4\theta+8\cot 8\theta=\cot\theta$

22. If $\tan A=\frac{-12}{5}$ and $630^0<A<720^0$, find the values of

i) $\sin\frac{A}{2}$ ii) $\cos\frac{A}{2}$ iii) $\tan\frac{A}{2}$ iv) $\cot\frac{A}{2}$

23. Find the value of $\tan 100^0+\tan 125^0+\tan 100^0.\tan 125^0$

24. If θ is not an ood multiple of $\frac{\pi}{2}$ and if $\tan\theta\neq-1$, then show that $\frac{1+\sin 2\theta-\cos 2\theta}{1+\sin 2\theta+\cos 2\theta}=\tan\theta$

25. Prove that $2(\cos 60^0+\sin 84^0)=\sqrt{3}+\sqrt{15}$

26. Prove that $\cos 20^0\cos 40^0-\sin 50\sin 25^0=\frac{\sqrt{3}+1}{4}$

HYPERBOLIC FUNCTIONS

1. If Cosh x = 3/2, find the value of (i) sinh 2x (ii) cosh 2x

2. If tanh x = 1/4, then prove that $x=\frac{1}{2}+\log_e\left(\frac{5}{3}\right)$

3. If $\cosh x=\frac{5}{2}$, find the values of i) cosh (2x) and ii) sinh (2x)

4. Show that $\tanh^{-1}\left(\frac{1}{2}\right)=\frac{1}{2}\log_e 3$

5. $\sinh x=\frac{3}{4}$, find $\cosh(2x)$ and $\sinh(2x)$

6. If $\sinh x = 3$, then show that $x=\log_e\left(3+\sqrt{10}\right)$

7. If $\sinh x = 5$, show that $x=\log_e(5+\sqrt{26})$

8. If $\sinh x=\frac{1}{2}$, find the value of $\cosh 2x+\sinh 2x$.

9. Prove that, for any $x \in R$, $\tanh 3x=\frac{3\tanh x+\tanh^3 x}{1+3\tanh^2 x}$

10. Prove that, for any $x \in R$, $\sinh(3x)=3\sinh x+4\sinh^3 x$

11. Prove that i) $\tanh(x-y)=\frac{\tanh x-\tanh y}{1-\tanh x\tanh y}$ ii) $\coth(x-y)=\frac{\coth x.\coth y-1}{\coth x-\coth y}$

12. Prove that $\frac{\tanh x}{\operatorname{sech} x-1}+\frac{\tanh x}{\operatorname{sech} x+1}=-2\operatorname{cosech} x$ for $x\neq 0$

13. Prove that $\frac{\cosh x}{1-\tanh x}+\frac{\sinh x}{1-\coth x}=\sinh x+\cosh x$ for $x\neq 0$

14. If $u=\log_e\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)$ and if $\cos\theta>0$, then prove that $\cosh u=\sec\theta$.

15. Prove that i) $(\cosh x-\sinh x)^n=\cosh(nx)-\sinh(nx)$, for any $n\in R$

ii) $(\cosh x+\sinh x)^n=\cosh(nx)+\sinh(nx)$, for any $n\in R$

16. For any $x\in R$, prove that $\cosh^4 x-\sinh^4 x=\cosh(2x)$

17. If $\theta\in\left(-\frac{\pi}{4},\frac{\pi}{4}\right)$ and $x=\log_e\cot\left(\frac{\pi}{4}+\theta\right)$ then prove that

i) $\cosh x=\sec 2\theta$ and ii) $\sinh x=-\tan 2\theta$

18. If $\cosh x=\sec\theta$ then prove that $\tanh^2\left(\frac{x}{2}\right)=\tan^2\frac{\theta}{2}$ [Hint : $\tanh^2\left(\frac{x}{2}\right)=\frac{\cosh x-1}{\cosh x+1}$]

PROPERTIES OF TRIANGLES

1. If the lengths of the sides of a triangle are 3,4,5 find the circumradius of the triangle
2. In ΔABC, show that $\sum(b+c)\cos A=2s$
3. If the sides of a triangle are 13, 14, 15, then find the circum diameter
4. In ΔABC, if (a+b+c) (b+c–a) = 3bc, find A
5. In ΔABC, find $b\cos^2\frac{C}{2}+c\cos^2\frac{B}{2}$.
6. If $\tan\frac{A}{2}=\frac{5}{6}$ and $\tan\frac{C}{2}=\frac{2}{5}$, determine the relation between a,b,c
7. If $\cot\frac{A}{2}=\frac{b+c}{a}$, find angle B
8. Show that $\frac{c-b\cos A}{b-c\cos A}=\frac{\cos B}{\cos C}$
9. If $a=\sqrt{3}+1$cms., $\angle B=30^0$, $\angle C=45^0$, then find c

10. If a = 26 cms., b = 30 cms. and $\cos C = \frac{63}{65}$, then find c.
11. If the angles ae in the ratio 1 : 5 : 6, then find the ratio of its sides
12. Prove that $\frac{a^2+b^2-c^2}{c^2+a^2-b^2} = \frac{\tan B}{\tan C}$
13. Prove that (b–acos C) sin A = a cos A sin C
14. If $\frac{a}{\cos A} = \frac{b}{\cos B} = \frac{c}{\cos C}$, then show that ΔABC is equilateral
15. In ΔABC, prove that $\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}=\frac{1}{r}$
16. Show that $rr_1r_2r_3 = \Delta^2$
17. In an equilateral triangle, find the value of r/R
18. In $\Delta ABC, \Delta = 6$ sq.cm and $s = 1.5$ cm., find r.
19. If $rr_2 = r_1r_3$, then find B
20. If $A = 90^0$, show that 2(r+R) = b + c
21. In ΔABC, express $\Sigma r_1 \cot\frac{A}{2}$ interms of s.
22. Show that $\Sigma \frac{r_1}{(s-b)(s-c)} = \frac{3}{r}$
23. If A=60^0 and I is the incentre of ΔABC, then find 'AI' interms of r
24. Show that $a^2\sin 2C + c^2\sin 2A = 4\Delta$
25. In ΔABC, if a = 3, b = 4, and $\sin A = \frac{3}{4}$, find angle B.
26. if a = 6, b = 5, c = 9, then find angle of A
27. If a = 4, b = 5, c = 7, find $\cos\frac{B}{2}$
28. if $\tan\left(\frac{C-A}{2}\right) = k\cot\frac{B}{2}$, find k
29. In ΔABC, show that $\frac{b^2-c^2}{a^2} = \frac{\sin(B-C)}{\sin(B+C)}$
30. Prove that $a(b\cos C - c\cos B) = b^2 - c^2$
31. Show that $\frac{c-b\cos A}{b-c\cos A} = \frac{\cos B}{\cos C}$
32. Show that $\sum a(\sin B - \sin C) = 0$
33. Prove that $2(bc\cos A + ca\cos B + ab\cos C) = a^2+b^2+c^2$
34. Prove that $(b+c)\cos A + (c+a)\cos B + (a+b)\cos C = a+b+c$
35. If 4 , 5 are two sides of a triangle and the included angle is 60^0, find its area
36. Show that $b\cos^2\frac{C}{2} + c\cos^2\frac{B}{2} = s$

4 MARKS IMP.QUESTIONS

ADDITION OF VECTORS

1.* Let A B C D E F be a regular hexagon with centre 'O'.
Show that $\mathbf{AB} + \mathbf{AC} + \mathbf{AD} + \mathbf{AE} + \mathbf{AF} = 3\,\mathbf{AD} = 6\,\mathbf{AO}$.

2.* In ABC, if 'O' is the circumcentre and H is the orthocentre, then show that
i) **OA + OB + OC = OH** ii) **HA + HB + HC = 2HO.**

3.* If the points whose position vectors are $3\mathbf{i} - 2\mathbf{j} - \mathbf{k}$, $2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$, $-\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ and $4\mathbf{i} + 5\mathbf{j} + \lambda\mathbf{k}$ are coplanar, then show that $\lambda = \frac{-146}{17}$.

4.* **a, b, c** are non-coplanar vectors. Prove that the following four points are coplanar
i) $6\mathbf{a} + 2\mathbf{b} - \mathbf{c}$, $2\mathbf{a} - \mathbf{b} + 3\mathbf{c}$, $-\mathbf{a} + 2\mathbf{b} - 4\mathbf{c}$, $-12\mathbf{a} - \mathbf{b} - 3\mathbf{c}$.
ii) $4\bar{i} + 5\bar{j} + \bar{k}, -\bar{j} - \bar{k}, 3\bar{i} + 9\bar{j} + 4\bar{k}$ & $-4\bar{i} + 4\bar{j} + 4\bar{k}$

5. i) If $\bar{a}, \bar{b}, \bar{c}$ are non coplanar vectors , then test for collinearity of the points with position vectors $3\bar{a} - 4\bar{b} + 3\bar{c}, -4\bar{a} + 5\bar{b} - 6\bar{c}, 4\bar{a} - 7\bar{b} + 6\bar{c}$ (Ans : Non collinear)
ii) $\bar{a} - 2\bar{b} + 3\bar{c}, 2\bar{a} + 3\bar{b} - 4\bar{c}, -7\bar{b} + 10\bar{c}$

6.* In the two dimensional plane, prove by using vector method, the equation of the line whose intercepts on the axes are 'a' and 'b' is $\frac{x}{a} + \frac{y}{b} = 1$.

7.* i) Show that the line joining the pair of points $6\mathbf{a} - 4\mathbf{b} + 4\mathbf{c}$, $-4\mathbf{c}$ and the line joining the pair of points $-\mathbf{a} - 2\mathbf{b} - 3\mathbf{c}$, $\mathbf{a} + 2\mathbf{b} - 5\mathbf{c}$ intersect at the point $-4\mathbf{c}$ when **a, b, c** are non-coplanar vectors.
ii) If $\bar{a}, \bar{b}, \bar{c}$ are non-coplanar find the point of intersection of the line passing through the points $2\bar{a} + 3\bar{b} - \bar{c}, 3\bar{a} + 4\bar{b} - 2\bar{c}$ with the line joining points $\bar{a} - 2\bar{b} + 3\bar{c}, \bar{a} - 6\bar{b} + 6\bar{c}$.

8.* Find tha point of intersection of the line $\bar{r} = 2\bar{a} + \bar{b} + t(\bar{b} - \bar{c})$ and the plane $\bar{r} = \bar{a} + x(\bar{b} + \bar{c}) + y(\bar{a} + 2\bar{b} - \bar{c})$

9.* Find the equation of the line parallel to the vector $2\bar{i} - \bar{j} + 2\bar{k}$, and which passes through the point A whose position vector is $3\bar{i} + \bar{j} - \bar{k}$. If P is a point on this line such that AP = 15, find the position vector of P.

10.* Let $\bar{a}, \bar{b}$ be non-collinear vectors, if $\bar{\alpha} = (x + 4y)\bar{a} + (2x + y + 1)\bar{b}$ and $\bar{\beta} = (y - 2x + 2)\bar{a} + (2x - 3y - 1)\bar{b}$ are such that $3\bar{\alpha} = 2\bar{\beta}$, then find x and y.

11.* Find the vector equation of the plane passing through points $4\mathbf{i} - 3\mathbf{j} - \mathbf{k}$, $3\mathbf{i} + 7\mathbf{j} - 10\mathbf{k}$ and $2\mathbf{i} + 5\mathbf{j} - 7\mathbf{k}$ and show that the point $\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$ lies in the plane.

12. If $\bar{a} + \bar{b} + \bar{c} = \alpha\bar{d}, \bar{b} + \bar{c} + \bar{d} = \beta\bar{a}$, and $\bar{a}, \bar{b}, \bar{c}$ are non coplanar vectors, than show that $\bar{a} + \bar{b} + \bar{c} + \bar{d} = \bar{0}$

MULTIPLICATION OF VECTORS(4marks)

1.* Find the volume of the parallelo piped whose conterminus adges are represented by the vectors $2\mathbf{i} - 3\mathbf{j} + \mathbf{k}$, $\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ and $2\mathbf{i} + \mathbf{j} - \mathbf{k}$ **Formula :** $\left|[\bar{a}\,\bar{b}\,\bar{c}]\right|$

2.* Find λ, for which the volume of the parallelo piped whose conterminus adges are represented by the vectors $\mathbf{i} + \mathbf{j}$, $3\mathbf{i} - \mathbf{j}$ and $3\mathbf{j} + \lambda\mathbf{k}$ is 16 cubic units

3.* Find the volume of the tetrahedron having edges $\mathbf{i} + \mathbf{j} + \mathbf{k}$, $\mathbf{i} - \mathbf{j}$ and $\mathbf{i} + 2\mathbf{j} + \mathbf{k}$.

Formula : $\frac{1}{6}\left|[\bar{a}\,\bar{b}\,\bar{c}]\right|$

4.* Find the volume of the tetrahedron whose vertices are (1, 2, 1), (3,2, 5), (2, –1, 0) and (–1, 0, 1). **Formula :** $\frac{1}{6}\left|\left[\overline{AB}\,\overline{AC}\,\overline{AD}\right]\right|$

5.* Show that angle in a semicircle is a right angle.

6. Show that for any two vectors $\bar{a}$ and $\bar{b}$, $|\bar{a}\times\bar{b}|^2=(\bar{a}.\bar{a})(\bar{b}.\bar{b})-(\bar{a}.\bar{b})^2=a^{-2}b^{-2}-(\bar{a}.\bar{b})^2$.

7. Show that in any triangle, the perpendicular bisectors of the sides are concurrent.

9. Show that in any triangle, the altitudes are concurrent.

10.* Let **a, b** and **c** be unit vectors such that **b** is not parallel to **c** and $\bar{a}\times(\bar{b}\times\bar{c})=\frac{1}{2}\bar{b}$. Find the angles made by **a** with each of **b** and **c**.

11. If **a + b + c = 0**, |**a**| = 3, |**b**| = 5 and |**c**| = 7, then find the angle between **a** and **b**.

12.* Find the area of the triangle whose vertices are A(1, 2, 3), B(2, 3, 1) and C(3, 1, 2).

Formula : $\frac{1}{2}\left|\overline{AB}\times\overline{AC}\right|$

13.* Find a unit vector perpendicular to the plane determined by the points P(1, –1, 2), Q(2, 0, –1) and R(0, 2, 1). Formula : $\pm\frac{(\overline{PQ}\times\overline{PR})}{\left|\overline{PQ}\times\overline{PR}\right|}$

14.* If **a** = 2**i** + 3**j** + 4**k, b = i + j – k** and **c = i – j + k**, then compute $\bar{a}\times(\bar{b}\times\bar{c})$ and vertify that it is perpendicular to $\bar{a}$.

15.* If $\bar{a}=\bar{i}-2\bar{j}+3\bar{k}, \bar{b}=2\bar{i}+\bar{j}+\bar{k}, \bar{c}=\bar{i}+2\bar{j}-\bar{k}$ Then find $(\bar{a}\times\bar{b})\times\bar{c}$ and $\left|\bar{a}\times(\bar{b}\times\bar{c})\right|$

16.* **a** = 2**i + j** - 3**k, b = i** - 2**j + k, c = -i + j –** 4**k** and **d = i + j + k**, then compute the following.
i) $(\bar{a}\times\bar{b})\times(\bar{c}\times\bar{d})$ and ii) $\left|(\bar{a}\times\bar{b})\times(\bar{c}\times\bar{d})\right|$

17.* i) If $\bar{a}+\bar{b}+\bar{c}=\bar{0}$ then prove that $\bar{a}\times\bar{b}=\bar{b}\times\bar{c}=\bar{c}\times\bar{a}$
ii) In ΔABC if $\overline{BC}=\bar{a},\overline{CA}=\bar{b}$ and $\overline{AB}=\bar{c}$ then prove that $\bar{a}\times\bar{b}=\bar{b}\times\bar{c}=\bar{c}\times\bar{a}$

18.* If **a = i –** 2**j –** 3**k, b =** 2**i + j – k** and **c = i +** 3**j –** 2**k**, vertify that $\bar{a}\times(\bar{b}\times\bar{c})\neq(\bar{a}\times\bar{b})\times\bar{c}$.

19.* Let $\bar{a}$ and $\bar{b}$ be vectors, satisfying $|\bar{a}|=|\bar{b}|=5$ and $(\bar{a},\bar{b})=45^0$. Find the area of the triangle having $\bar{a}-2\bar{b}$ and $3\bar{a}+2\bar{b}$ as two of its sides.

20*. If **a, b, c** are unit vectors such that **a** is perpendicular to the plane of **b, c** and the angle between **b** and **c** is $\frac{\pi}{3}$, then find $|\bar{a}+\bar{b}+\bar{c}|$.

21*. For any vector $\bar{a}$, show that $|\bar{a}\times\bar{i}|^2+|\bar{a}\times\bar{j}|^2+|\bar{a}\times\bar{c}|^2=2|\bar{a}|^2$.

22*. If $\bar{a}$ is a non zero vector and $\bar{b},\bar{c}$ are two vector such that $\bar{a}\times\bar{b}=\bar{a}\times\bar{c}$ and $\bar{a}.\bar{b}=\bar{a}.\bar{c}$, then prove that $\bar{b}=\bar{c}$.

MATRICES (4 MARKS)

1. ** i) If $A=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$ then show that . $A^n=\begin{bmatrix}\cos n\theta & \sin n\theta\\ -\sin n\theta & \cos n\theta\end{bmatrix}$ $\forall n\in N$ by using mathematical induction

** ii) If $A=\begin{bmatrix}3 & -4\\ 1 & -1\end{bmatrix}$, then show taht $A^n=\begin{bmatrix}1+2n & -4n\\ n & 1-2n\end{bmatrix}$ $\forall n\in N$ by using mathematical induction.

2. ** If $\theta-\phi=\frac{\pi}{2}$,then show that $\begin{bmatrix}\cos^2\theta & \cos\theta\sin\theta\\ \cos\theta\sin\theta & \sin^2\theta\end{bmatrix}\begin{bmatrix}\cos^2\phi & \cos\phi\sin\phi\\ \cos\phi\sin\phi & \sin^2\phi\end{bmatrix}=0$

3. ** If $I=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}, E=\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}$ then show that $(aI+bE)^3=a^3I+3a^2bE$.

4. ** i) If $A=\begin{bmatrix}1 & 2 & 2\\ 2 & 1 & 2\\ 2 & 2 & 1\end{bmatrix}$ then show that $A^2-4A-5I=0$

** ii) If $A=\begin{bmatrix}1 & -2 & 1\\ 0 & 1 & -1\\ 3 & -1 & 1\end{bmatrix}$ then find A^3-3A^2-A-3I

5. <u>Problems on inverse.</u>

** i) Show that $A=\begin{bmatrix}1 & 2 & 1\\ 3 & 2 & 3\\ 1 & 1 & 2\end{bmatrix}$ is non-singular matrix and find A^{-1}.

***ii) If $3A=\begin{bmatrix}1 & 2 & 2\\ 2 & 1 & -2\\ -2 & 2 & -1\end{bmatrix}$ then show that $A^{-1}=A^T\left(\text{Hint}: AA^T=A^{T}\cdot A=I\right)$

**iii) If $A=\begin{bmatrix}3 & -3 & 4\\ 2 & -3 & 4\\ 0 & -1 & 1\end{bmatrix}$ then show that $A^{-1}=A^3\left(\text{Hint } A.A^3=A^3.A=I\right)$

*iv) If $A=\begin{bmatrix}1 & -2 & 3\\ 0 & -1 & 4\\ -2 & 2 & 1\end{bmatrix}$ find $\left(A^1\right)^{-1}$

<u>Theorems :</u>

1. For any $n\times n$ matrix , show that A can be uniquely expressed as sum of symmetric &

skew-symmetric matrices.

Define inverse matrix

2. If A,B are invertiable matrices Prove that $(AB)^{-1} = B^{-1}A^{-1}$

TRIGNOMETY UPTO TRASFORMATIONS

1*. If $A + B = 45^0$, then prove that

(1 + tan A) (1 + tan B) = 2, and hence deduce that $\text{Tan}\,22\frac{1}{2}^0 = \sqrt{2} - 1$

2*. If $A + B = 225^0$, then prove that $\frac{\cot A}{1+\cot A}.\frac{\cot B}{1+\cot B} = \frac{1}{2}$.

3. Prove that $\tan 70^0 - \tan 20^0 = 2\tan 50^0$.

4.* For $A \in R$, prove that i) $\sin A.\sin(60 + A)\sin(60 - A) = \frac{1}{4}\sin 3A$.and hence deduce that

$\sin 20^0 \sin 40^0 \sin 60^0 \sin 80^0 = \frac{3}{16}$.

ii) $\cos A.\cos(60 + A)\cos(60 - A) = \frac{1}{4}\cos 3A$ and hence deduce that

$\cos\frac{\pi}{9}\cos\frac{2\pi}{9}\cos\frac{3\pi}{9}\cos\frac{4\pi}{9} = \frac{1}{16}$.

5. If 3A is not an odd multiple of $\frac{\pi}{2}$, prove that $\tan A.\tan(60 + A).\tan(60 - A) = \tan 3A$ and hence find the value of $\tan 6^0 \tan 42^0 \tan 66^0 \tan 78^0$.

6.* If A is not an integral multiple of π, prove that $\cos A.\cos 2A.\cos 4A.\cos 8A = \frac{\sin 16A}{16\sin A}$ and

hence deduce that $\cos\frac{2\pi}{15}.\cos\frac{4\pi}{15}.\cos\frac{8\pi}{15}.\cos\frac{16\pi}{15} = \frac{1}{16}$

7. Prove the following

i) $\cos\frac{2\pi}{7}.\cos\frac{4\pi}{7}.\cos\frac{8\pi}{7} = \frac{1}{8}$ ii) $\cos\frac{\pi}{11}.\cos\frac{2\pi}{11}.\cos\frac{3\pi}{11}.\cos\frac{4\pi}{11}.\cos\frac{5\pi}{11} = \frac{1}{32}$.

8.* $\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)\left(1+\cos\frac{7\pi}{8}\right) = \frac{1}{8}$

9.* i) Prove that $\sin^4\frac{\pi}{8} + \sin^4\frac{3\pi}{8} + \sin^4\frac{5\pi}{8} + \sin^4\frac{7\pi}{8} = \frac{3}{2}$.

ii) $\cos^4\frac{\pi}{8} + \cos^4\frac{3\pi}{8} + \cos^4\frac{5\pi}{8} + \cos^4\frac{7\pi}{8} = \frac{3}{2}$

10.* If θ is not an integral multiple of $\pi/2$ then prove that $\tan\theta + 2\tan 2\theta + 4\tan 4\theta + 8\cot 8\theta = \cot\theta$

11. Let ABC be a triangle such that $\cot A + \cot B + \cot C = \sqrt{3}$, then prove that ABC is an equilateral triangle.

12. Prove that $\tan 3A.\tan 2A.\tan A = \tan 3A - \tan 2A - \tan A$

13. Prove that $(\cos\alpha + \cos\beta)^2 + (\sin\alpha + \sin\beta)^2 = 4\cos^2\left(\frac{\alpha - \beta}{2}\right)$

14. If $A + B + C = \frac{\pi}{2}$ and if none of A, B, C is an odd multiple of $\frac{\pi}{2}$, then prove that

$\sum \frac{\cos(B+C)}{\cos B \cos C} = 2.$

15.* Prove that (if none of the denominators is zero)

$$\left(\frac{\cos A+\cos B}{\sin A-\sin B}\right)^n + \left(\frac{\sin A+\sin B}{\cos A-\cos B}\right)^n = \begin{cases} 2.\cos^n\left(\frac{A-B}{2}\right), \text{if n is even} \\ 0, \quad \text{if n is odd} \end{cases}.$$

16. Prove that the roots of the quadratic equation $16x^2 - 12x + 1 = 0$ are $\sin^2 18^0$ and $\cos^2 36^0$.

17.* Prove that $\sin 18^0 = \frac{\sqrt{5}-1}{4}$

18.* If $\tan\theta = \frac{b}{a}$, then prove that $a\cos 2\theta + b\sin 2\theta = a$ or

$\frac{a}{\cos\theta} = \frac{b}{\sin\theta}$, then P.T $a\cos 2\theta + b\sin 2\theta = a$

TRIGONOMETRIC EQUATIONS

1. solve $\cos x + (2+\sqrt{3})\sin x = 1$
2. Solve $2\sin^2\theta + 3\cos\theta = 3$
3.* Solve $2\cos^2\theta + 11\sin\theta = 7$
4.* Solve $\tan\theta + \tan 2\theta + \sqrt{3}\tan\theta\tan 2\theta = \sqrt{3}$
5. Solve sin6x = sin4x –sin2x
6.* Solve the following and write the general solution

i) $2\cos^2\theta - \sqrt{3}\sin\theta + 1 = 0$ ii) $\sqrt{2}(\sin x + \cos x) = \sqrt{3}$

iii) $\sin x + \sqrt{3}\cos x = \sqrt{2}$ iv) $\tan\theta + 3\cot\theta = 5\sec\theta$

v) $\sin 7\theta + \sin 4\theta + \sin\theta = 0$ vi) $7\text{Sin}^2\theta + 3\text{Cos}^2\theta = 4$

7.* If $\tan\left(\frac{\pi}{2}\sin\theta\right) = \cot\left(\frac{\pi}{2}\cos\theta\right)$ then prove that $\left(\theta + \frac{\pi}{4}\right) = \pm\frac{1}{\sqrt{2}}$.

8.* If $\tan(\pi\cos\theta) = \cot(\pi\sin\theta)$, then prove that $\cos\left(\theta - \frac{\pi}{4}\right) = \pm\frac{1}{2\sqrt{2}}$.

9.* If θ_1, θ_2 are solutions of the equation $a\cos 2\theta + b\sin 2\theta = c$, $\tan\theta_1 \neq \tan\theta_2$ and $a + c \neq 0$, then find the values of

i) $\tan\theta_1 + \tan\theta_2$ ii) $\tan\theta_1 . \tan\theta_2$ iii) $\tan(\theta_1 + \theta_2)$

10. Solve (i) sin 2x – cos 2x = sin x – cos x ii) $\sin x + \sqrt{3}\cos x = \sqrt{2}$

iii) $1 + \sin^2\theta = 3\sin\theta\cos\theta$. iv) $\sqrt{2}(\sin x + \cos x) = \sqrt{3}$

11.* If $0 < \theta < \pi$, solve $\cos\theta.\cos 2\theta.\cos 3\theta = 1/4$.

12.* Solve the equation $\cot^2 x - \left(\sqrt{3}+1\right)\cot x + \sqrt{3} = 0\left(0 < x < \frac{\pi}{2}\right)$.

13.* Solve the equation tan x + tan 2x + tan 3x = 0.

14 If $x + y = \frac{2\pi}{3}$ and $\text{Sin}x + \text{Sin}y = \frac{3}{2}$ find x and y

15 Solve Sin x + Sin 2x + Sin 3x = Cos x + Cos 2x + Cos 3x

15.* Find all values of x in $(-\pi, \pi)$ satisfying the equation $8^{1+\cos x+\cos^2 x+....} = 4^3$.

16. Slove 4 sin x sin 2x sin 4x = sin 3x

17.* If α,β are solutions of the equation $a\cos\theta + b\sin\theta = c$ then show that

i) $\sin\alpha + \sin\beta = \dfrac{2bc}{a^2+b^2}$ ii) $\sin\alpha.\sin\beta = \dfrac{c^2-a^2}{a^2+b^2}$

iii) $\cos\alpha + \cos\beta = \dfrac{2ac}{a^2+b^2}$ iv) $\cos\alpha.\cos\beta = \dfrac{c^2-b^2}{a^2+b^2}$

INVERSE TRIGONOMETRIC FUNCTIONS

*1. i) If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \pi$, then prove that $x\sqrt{1-x^2} + y\sqrt{1-y^2} + z\sqrt{1-z^2} = 2xyz$.

ii) If $\cos^{-1}p + \cos^{-1}q + \cos^{-1}r = \pi$, then prove that $p^2 + q^2 + r^2 + 2pqr = 1$.

iii) If $\cos^{-1}\dfrac{p}{a} + \cos^{-1}\dfrac{q}{b} = \alpha$, then prove that $\dfrac{p^2}{a^2} - \dfrac{2pq}{ab}.\cos\alpha + \dfrac{q^2}{b^2} = \sin^2\alpha$.

iv) If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$, then prove that $x + y + z = xyz$.

v) If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi/2$, then prove that $xy + yz + zx = 1$.

vi) If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \pi$, then prove that $x^4 + y^4 + z^4 + 4x^2y^2z^2 = 2(x^2y^2 + y^2z^2 + z^2x^2)$

2. i) Show that $\text{Cos}^{-1}\left(\dfrac{63}{65}\right) + 2\,\text{Tan}^{-1}\left(\dfrac{1}{5}\right) = \text{Sin}^{-1}\left(\dfrac{3}{5}\right)$ *ii) Show that $2\text{Tan}^{-1}\dfrac{1}{3} + \text{Tan}^{-1}\dfrac{1}{7} = \dfrac{\pi}{4}$

iii) Show that $\cos\left(2\tan^{-1}\dfrac{1}{7}\right) = \sin\left(4\tan^{-1}\dfrac{1}{3}\right)$ *iv) $\sin^{-1}\dfrac{4}{5} + 2\tan^{-1}\dfrac{1}{3} = \dfrac{\pi}{2}$

v) $2\text{Sin}^{-1}\dfrac{3}{5} - \cos^{-1}\dfrac{5}{13} = \cos^{-1}\left(\dfrac{323}{325}\right)$.

3. *i) Prove that $\sin^{-1}\dfrac{4}{5} + \sin^{-1}\dfrac{7}{25} = \sin^{-1}\dfrac{117}{125}$. *ii)Show that $\text{Sin}^{-1}\left(\dfrac{3}{5}\right) + \text{Sin}^{-1}\left(\dfrac{8}{17}\right) = \cos^{-1}\left(\dfrac{36}{85}\right)$

*iii) $\sin^{-1}\dfrac{4}{5} + \sin^{-1}\dfrac{5}{13} + \sin^{-1}\dfrac{16}{25} = \dfrac{\pi}{2}$ iv) $\text{Sin}^{-1}\dfrac{3}{5} + \cos^{-1}\dfrac{12}{13} = \cos^{-1}\dfrac{33}{65}$

*v) Prove that $\tan^{-1}\dfrac{1}{2} + \tan^{-1}\dfrac{1}{5} + \tan^{-1}\dfrac{1}{8} = \dfrac{\pi}{4}$.

4. Solve

*i) $\text{arc}\sin\left(\dfrac{5}{x}\right) + \text{arc}\sin\left(\dfrac{12}{x}\right) = \dfrac{\pi}{2}.(x > 0)$. ii) $\sin^{-1}\dfrac{3x}{5} + \sin^{-1}\dfrac{4x}{5} = \sin^{-1}x$

iii) $\sin^{-1}x + \sin^{-1}2x = \dfrac{\pi}{3}$ *iv) $3\sin^{-1}\dfrac{2x}{1+x^2} - 4\cos^{-1}\dfrac{1-x^2}{1+x^2} + 2\tan^{-1}\dfrac{2x}{1-x^2} = \dfrac{\pi}{3}$

*v) $\text{Tan}^{-1}\dfrac{x-1}{x-2} + \tan^{-1}\dfrac{x+1}{x+2} = \dfrac{\pi}{4}$. *vi) $\tan^{-1}\left(\dfrac{x+1}{x-1}\right) + \tan^{-1}\left(\dfrac{x-1}{x}\right) = p + \tan^{-1}(-7)$

*vii) If $\text{Sin}^{-1}\left(\dfrac{2p}{1+p^2}\right) - \text{Cos}^{-1}\left(\dfrac{1-q^2}{1+q^2}\right) = \text{Tan}^{-1}\left(\dfrac{2x}{1-x^2}\right)$ then prove that $x = \dfrac{p-q}{1+pq}$

5. Prove that $\cos\left[\text{Tan}^{-1}\left\{\sin\left(\cos^{-1}x\right)\right\}\right] = \sqrt{\dfrac{x^2+1}{x^2+2}}$.

*6. Prove that $\tan\left(\dfrac{\pi}{4} + \dfrac{1}{2}\cos^{-1}\dfrac{a}{b}\right) + \tan\left(\dfrac{\pi}{4} - \dfrac{1}{2}\cos^{-1}\dfrac{a}{b}\right) = \dfrac{2b}{a}$.

7.* If $\alpha = \tan^{-1}\left(\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right)$ prove that $x^2 = \sin 2\alpha$

PROPERTIES OF TRIANGLES

1.* i) Show that $a^2 \cot A + b^2 \cot B + c^2 \cot C = \frac{abc}{R}$.

ii) Prove that $a \cos A + b \cos B + c \cos C = 4R \sin A \sin B \sin C$

iii) Show that $a\cos^2\frac{A}{2} + b\cos^2\frac{B}{2} + c\cos^2\frac{C}{2} = s + \frac{\Delta}{R}$.

2.* i) Prove that $\frac{a}{bc} + \frac{\cos A}{a} = \frac{b}{ca} + \frac{\cos B}{b} = \frac{c}{ab} + \frac{\cos C}{c}$

ii) Prove that $\frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{c} = \frac{a^2+b^2+c^2}{2abc}$

3.* Show that $(b-c)^2\cos^2\frac{A}{2} + (b+c)^2\sin^2\frac{A}{2} = a^2$.

4.* i) In ΔABC, if $\frac{1}{a+c} + \frac{1}{b+c} = \frac{3}{a+b+c}$, show that $C = 60^0$.

ii) If $C = 60^0$, then show that (1) $\frac{a}{b+c} + \frac{b}{c+a} = 1$ (2) $\frac{b}{c^2-a^2} + \frac{a}{c^2-b^2} = 0$.

5.* Show that in ΔABC, $a = b \cos c + c \cos B$.

6. Show that in ΔABC, $\tan\left(\frac{B-C}{2}\right) = \frac{b-c}{b+c}\cot\frac{A}{2}$.

7. If p_1, p_2, p_3 are the altitudes of the vertices A, B, C of a triangle respectively, show that $\frac{1}{p_1^2} + \frac{1}{p_2^2} + \frac{1}{p_3^2} = \frac{\cot A + \cot B + \cot C}{\Delta}$.

8.* If a : b : c = 7 : 8 : 9, find cos A : cos B : cos C.

9.* If $\cot\frac{A}{2}, \cot\frac{B}{2}, \cot\frac{C}{2}$ are in A.P., then prove that a, b, c are in A.P.

10.* P.T $\text{Cot } A + \text{Cot } B + \text{Cot } C = \frac{a^2+b^2+c^2}{4\Delta}$.

11.* Show that $b^2 \sin 2C + c^2 \sin 2B = 2bc \sin A$

12.* i) Prove that $\tan\frac{A}{2} + \tan\frac{B}{2} + \tan\frac{C}{2} = \frac{bc+ca+ab-s^2}{\Delta}$.

ii) Prove that $\cot\frac{A}{2} + \cot\frac{B}{2} + \cot\frac{C}{2} = \frac{s^2}{\Delta}$. iii) $\frac{\cot A/2 + \cot B/2 + \cot C/2}{\cot A + \cot B + \cot C} = \frac{(a+b+c)^2}{a^2+b^2+c^2}$

13.* If $\cot\frac{A}{2} : \cot\frac{B}{2} : \cot\frac{C}{2} = 3 : 5 : 7$, show that a : b : c = 6 : 5 : 4.

14. If $\sin^2\frac{A}{2}, \sin^2\frac{B}{2}, \sin^2\frac{C}{2}$ are in H.P., then show that a, b, c are in H.P.

15. In triangle ABC if a cos A = b cos B, then prove that the triangle is either isosceles (or) right angled.

16.* If $a^2 + b^2 + c^2 = 8R^2$, then prove that the triangle is right angled.

7 MARKS IMP.QUESTIONS
FUNCTIONS

1.* Let $f: A \to B$, $g: B \to C$ be bijections. Then show that $gof: A \to C$ is a bijection.

2.* Let $f: A \to B$, $g: B \to C$ be bijections. Then show that $(gof)^{-1} = f^{-1}og^{-1}$.

3.* Let $f: A \to B$ be a bijection. Then show that $fof^{-1} = I_B$ and $f^{-1}of = I_A$.

4.* Let $f: A \to B$, I_A and I_B be identity functions on A and B respectively. Then show that $foI_A = f = I_Bof$.

5.* Let $f: A \to B$ be a bijection. Then show that 'f' is a bijection if and only if there exists a functions $g: B \to A$ such that $fog = I_B$ and $gof = I_A$ and in this case, $g = f^{-1}$.

6.* Let $f: A \to B$, $g: B \to C$ and $h: C \to D$. Then show that ho(gof) = (hog)of, that is composition of functions is associative.

7. If $f: A \to B, g: B \to A$ & $f = \{(1,a),(2,c),(4,d),(3,b)\}, g^{-1} = \{(2,a),(4,b),(1,c),(3,d)\}$ verify $(gof)^{-1} = f^{-1}og^{-1}$

8. Let A = { 1, 2, 3 } B = { a, b , c } C = { p, q, r }. If $f: A \to B, g: B \to C$ are defined by $f = \{(1, a), (2, c), (3, b)\}$, $g = \{(a, q), (b, r), (c, p)\}$ then show that $(gof)^{-1} = f^{-1}og^{-1}$.

9. Show that $f: Q \to Q$ defined by f(x) = 5x + 4 is a bijection and find f^{-1}.

MATHEMATICAL INDUCTION

1.** Show that $1^2 + (1^2 + 2^2) + (1^2 + 2^2 + 3^2) + \ldots$ upto n terms $= \frac{n(n+1)^2(n+2)}{12}, \forall n \in N$.

2.** i) Show that $\frac{1^3}{1} + \frac{1^3+2^3}{1+3} + \frac{1^3+2^3+3^3}{1+3+5} + \ldots\ldots$ upto n terms $= \frac{n}{24}\left[2n^2 + 9n + 13\right]$.

ii) Show that $1^3 + 2^3 + 3^3 + \ldots\ldots + n^3 = \frac{n^2(n+1)^2}{4} \forall n \in N$

3. *i) Show that 2.3 + 3.4 + 4.5 + ... upto n terms $= \frac{n\left(n^2 + 6n + 11\right)}{3} \forall n \in N$.

*ii) Show that 1.2.3 + 2.3.4 + 3.4.5 + ... upto n terms $\frac{n(n+1)(n+2)(n+3)}{4}, \forall n \in N$.

4. *i) Show that $\forall n \in N, \frac{1}{1.4} + \frac{1}{4.7} + \frac{1}{7.10} + \ldots$ upto n terms $= \frac{n}{3n+1}$.

ii) Show that $\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \ldots\ldots +$ upto n terms $= \frac{n}{2n+1}, \forall n \in N$.

5.* i) $a + (a+d) + (a+2d) + \ldots\ldots$ upto n terms $= \frac{n}{2}[2a + (n-1)d]$. $\boxed{t_n = a + (n-1)d}$

*ii) $a + ar + ar^2 + \ldots\ldots$ upto n terms $= \frac{a(r^n - 1)}{r-1}, r \neq 1$ $\boxed{t_n = a.r^{n-1}}$

*iii) $2 + 3.2 + 4.2^2 + \ldots\ldots +$ upto n terms $= n.2^n, \forall n \in N$.

iv) $4^3+8^3+12^3+.......\text{upto } n \text{ terms}=16n^2(n+1)^2, \forall n\in N$.

*v) Using M.I, P.T $\sum_{k=1}^{n}(2k-1)^2=\frac{n(2n-1)(2n+1)}{3}$

6. Show that *i) $49^n+16n-1$ divisible by 64 for all positive intergers n.

*ii) $3.5^{2n+1}+2^{3n+1}$ is divisible by 17, $\forall n\in N$

*iii) $2.4^{2n+1}+3^{3n+1}$ is divisible by 11, $\forall n\in N$

iv) 4^n-3n-1 is divisible by 9 using mathematical induction

7. i) Using mathematical induction, show that x^m+y^m is divisibleby x + y. If 'm' is an odd natural number and x, y are natural numbers.

*ii) If x & y are natural numbers and $x\neq y$. Using mathematical induction. Show that x^n-y^n is divisible by $x-y, \forall n\in N$.

8. Using mathematical induction, show that $2+7+12+.....+(5n-3)=\frac{n(5n-1)}{2}$

9. i) Use mathematical induction prove that $(2n-3)\le 2^{n-2}\forall n\ge 5$

ii) Use mathematical induction prove that $(1+x)^n>1+nx, \forall n\ge 2$

iii) Use mathematical induction prove that $1^2+2^2+3^2+.....+n^2>\frac{n^3}{3}$

TRANSFORMATIONS

1.* If A, B, C are angles in a triangle, then prove that

i) $\sin\frac{A}{2}+\sin\frac{B}{2}+\sin\frac{C}{2}=1+4\sin\frac{\pi-A}{4}.\sin\frac{\pi-B}{4}.\sin\frac{\pi-C}{4}$.

ii) $\cos\frac{A}{2}+\cos\frac{B}{2}+\cos\frac{C}{2}=4\cos\frac{\pi-A}{4}\cos\frac{\pi-B}{4}\cos\frac{\pi-C}{4}$.

iii) $\cos\frac{A}{2}+\cos\frac{B}{2}-\cos\frac{C}{2}=4\cos\frac{\pi+A}{4}\cos\frac{\pi+B}{4}\cos\frac{\pi-C}{4}$.

iv) $\sin\frac{A}{2}+\sin\frac{B}{2}-\sin\frac{C}{2}=-1+4\cos\frac{\pi-A}{4}\cos\frac{\pi-B}{4}\sin\frac{\pi-C}{4}$.

2.* If $A+B+C=180^0$, then prove that

i) $\cos^2\frac{A}{2}+\cos^2\frac{B}{2}+\cos^2\frac{C}{2}=2\left(1+\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}\right)$.

ii) $\cos^2\frac{A}{2}+\cos^2\frac{B}{2}-\cos^2\frac{C}{2}=2\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}$

iii) $\sin^2\frac{A}{2}+\sin^2\frac{B}{2}-\sin^2\frac{C}{2}=1-2\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}$.

iv) $\cos A+\cos B+\cos C=1+4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$.

v) $\sin 2A+\sin 2B+\sin 2C=4\sin A\sin B\sin C$.

vi) $\text{Cos } 2A+\text{Cos } 2B+\text{Cos } 2C=-1-4\cos A\cos B\cos C$

3.* If $A+B+C=2S$, then prove that $\cos(S-A)+\cos(S-B)+\cos(S-C)+\cos S=4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$.

4. $A+B+C=0^0$ then prove that $\cos^2 A+\cos^2 B+\cos^2 C=1+2\cos A\cos B\cos C$

5. $A+B+C=270^0$ then prove that $\cos 2A+\cos 2B+\cos 2C=1-4\sin A\sin B\sin C$

MATRICES (7 MARKS)

1. ** Without expansion – prove that $\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

2. ***i) Show that $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$

**ii) Show that $\begin{vmatrix} b+c & c+a & c+b \\ a+b & b+c & c+a \\ a & b & c \end{vmatrix} = a^3+b^3+c^3-3abc$

***iii) Show that $\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$

**iv) Show that $\begin{vmatrix} bc & b+c & 1 \\ ca & c+a & 1 \\ ab & a+b & 1 \end{vmatrix} = (a-b)(b-c)(c-a)$

3. *** Show that $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2 = \begin{vmatrix} 2bc-a^2 & c^2 & b^2 \\ c^2 & 2ac-b^2 & a^2 \\ b^2 & a^2 & 2ab-c^2 \end{vmatrix} = \left(a^3+b^3+c^3-3abc\right)^2$

4. *a) Show that $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$

**b) Show that $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^b & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)\,abc.$

***ii) Show that $\begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(ab+bc+ca)$

***iii) If $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0$ and $\begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} \neq 0$, then Show that abc = -1

5. * i) Find the value of 'x' if $\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0$

ii) Show that $\begin{vmatrix} y+z & x & x \\ y & z+x & y \\ z & z & x+y \end{vmatrix} = 4xyz$ iii) Show that $\begin{vmatrix} -2a & a+b & c+a \\ a+b & -2b & b+c \\ c+a & c+b & -2c \end{vmatrix} = 4(a+b)(b+c)(c+a)$

Solve the following by using (i) crammer (ii) matrix inversion (iii) Gauss-jordan-methods.

***i) $3x+4y+5z=18, 2x-y+8z=13, 5x-2y+7z=20$ $[Ans: x=3, y=1, z=1]$

** ii) $x+y+z=9, 2x+5y+7z=52, 2x+y-z=0$ $[Ans: x=1, y=3, z=5]$

** iii) $2x-y+3z=9, x+y+z=6, x-y+z=2$ [Ans : x = 1,y = 2,z = 3]

Consistancy and In consistancy :

Examine whether the following system of equations are consitanat (or) in consistant and If consistant find the complete solution.

I unique solution

** i) $x+y+z=6, x-y+z=2, 2x-y+3z=9$ Ans : consistent : x=1,y=2,z=3

II In finite solutions

** i) $x+y+z=3, 2x+2y-z=3, x+y-z=1$ Ans : consistennt : x=k,y=2-k,z=1

***ii) $x+y+z=1, 2x+y+z=2, x+2y+2z=1$ Ans : consistennt : x=1,y=-k,z=k

III Find the non-trivial solutions for the equations

i) 2x + 5y + 6z = 0, x - 3y + 8z = 0, 3x + y - 4z = 0 [Ans : x = 2k, y = -2k, z = k]

ii) By using gauss jordan method, show that system of equations 2x + 4y - z = 0, x + 2y + 2z = 5, 3x + 6y - 7z = 2 has no solution.

iii) Solve x + y + z = 3, 2x + 2y - z = 3, x + y - z = 1 by gauss fordan method

[Ans : x = k, y = 2 - k, z = 1, when KI R]

Inverse theorem

** Statement : If $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$ is a non-singular matrix then prove that $A^{-1} = \frac{\text{Adj}\,A}{\det A}$ (or) $\frac{1}{|A|}\text{Adj}(A)$

HEIGHTS & DISTANCES

1. The angle of elevation of the top point P of the vertical tower PQ of height h from a point A is 45^0 and from a point B is 60^0, where B is a point at a distance 30 meters from the point A measured along the line AB which makes an angle 30^0 with AQ. Find the height of the tower.
2. Two trees A and B are on the same side of a river. From a point C in the river the distances of the trees A and B are 250 m and 300 m respectively. If the angle C is 45^0, find the distance between the tree $\left(use\sqrt{2}=1.414\right)$.
3. A lamp post is situated at the middle point M of the side AC of a triangular plot ABC with BC=7m, CA=8 m and AB =9m. Lamp post subtends an angle 15^0 at the point B. Find the height of the lamp post.

PROPERTIES OF TRIANGLES

Theorems

*i) SINE RULE : $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}=2R$ where R is the circumradius.

ii) COSINE RULE : $a^2 = b^2 + c^2 - 2bc\cos A$.

iii) Prove that $r = 4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

$$r = 4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$$

$$r_1 = 4R\sin\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$$

$$r_2 = 4R\cos\frac{A}{2}\sin\frac{B}{2}\cos\frac{C}{2}$$

$$r_3 = 4R\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}$$

1.* In Δ ABC, prove that

i) $r_1 + r_2 + r_3 - r = 4R$.

ii) $r + r_3 + r_1 - r_2 = 4R\cos B$.

iii) $r + r_1 + r_2 - r_3 = 4R\cos C$

iv) If $r : R : r_1 = 2 : 5 : 12$, then prove that the triangle is right angled at A.

2.* i) $\cos A+\cos B+\cos C=1+\frac{r}{R}$

ii) $\cos^2\frac{A}{2}+\cos^2\frac{B}{2}\cos^2\frac{C}{2}=2+\frac{r}{2R}$. iii) $\sin^2\frac{A}{2}+\sin^2\frac{B}{2}+\sin^2\frac{C}{2}=1-\frac{r}{2R}$

3.* Show that $\frac{r_1}{bc}+\frac{r_2}{ca}+\frac{r_3}{ab}=\frac{1}{r}-\frac{1}{2R}$.

4.* Show that i) $\frac{ab-r_1r_2}{r_3}=\frac{bc-r_2r_3}{r_1}=\frac{ca-r_3r_1}{r_2}$.

ii) $(r_1 + r_2)\sec^2\frac{C}{2} = (r_2 + r_3)\sec^2\frac{A}{2} = (r_3 + r_1)\sec^2\frac{B}{2}$.

iii) Show that $\frac{1}{r^2}+\frac{1}{r_1^2}+\frac{1}{r_2^2}+\frac{1}{r_3^2}=\frac{a^2+b^2+c^2}{\Delta^2}$

5.* i) If $a = 13, b = 14, c = 15$, show that $R = \frac{65}{8}, r = 4, r_1 = \frac{21}{2}$, $r_2 = 12$ and $r_3 = 14$.

ii) If $a = 26, b = 30 \cos C = \frac{63}{65}$, then prove that $R = \frac{65}{4}$, $r = 3, r_1 = 16, r_2 = 48$ and $r_3 = 4$.

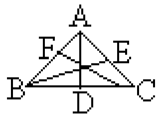
6.* i) If $r_1 = 2, r_2 = 3, r_3 = 6$ and $r = 1$, prove that $a = 3, b = 4$ and $c = 5$

ii) In ΔABC, if $r_1 = 8, r_2 = 12, r_3 = 24$, find a, b, c.

7. Prove that (i) $\frac{r_1(r_2+r_3)}{\sqrt{r_1r_2+r_2r_3+r_3r_1}} = a$. (ii) $a = (r_2+r_3)\sqrt{\frac{rr_1}{r_2r_3}}$ (iii) $\Delta = r_1r_2\sqrt{\frac{4R-r_1-r_2}{r_1+r_2}}$.

8.* In ΔABC, if AD, BE, CF are the perpendiculars drawn from the vertices A,B,C to the opposite sides, show that

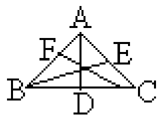
i) $\frac{1}{AD}+\frac{1}{BE}+\frac{1}{CF}=\frac{1}{r}$ ii) $AD.BE.CF = \frac{(abc)^2}{8R^3}$



$AD = \frac{2\Delta}{a}, BE = \frac{2\Delta}{b}, CF = \frac{2\Delta}{c}$

9.* If p_1, p_2, p_3 are altitudes drawn from vertices A,B,C to the opposite sides of a triangle respectively, then show that

i) $\frac{1}{p_1}+\frac{1}{p_2}+\frac{1}{p_3}=\frac{1}{r}$ ii) $\frac{1}{p_1}+\frac{1}{p_2}-\frac{1}{p_3}=\frac{1}{r_3}$



$P_1 = \frac{2\Delta}{a}, P_2 = \frac{2\Delta}{b}, P_3 = \frac{2\Delta}{c}$

iii) $p_1p_2p_3 = \frac{(abc)^2}{8R^3} = \frac{8\Delta^3}{abc}$

10.* Prove that $a^3\cos(B-C) + b^3\cos(C-A) + c^3\cos(A-B) = 3abc$.

11.* i) If $a = (b-c)\sec\theta$, prove that $\tan\theta = \frac{2\sqrt{bc}}{b-c}\sin\frac{A}{2}$. (March-2011)

ii) If $\sin\theta = \frac{a}{b+c}$, then show that $\cos\theta = \frac{2\sqrt{bc}}{b+c}\cos\frac{A}{2}$.

MULTIPLICATION OF VECTORS

1.* Let **a,b,c** be three vectors, Then show that

(i) $(\bar{a}\times\bar{b})\times\bar{c} = (\bar{a}.\bar{c})\bar{b} - (\bar{b}.\bar{c})\bar{a}$ (ii) $\bar{a}\times(\bar{b}\times\bar{c}) = (\bar{a}.\bar{c})\bar{b} - (\bar{a}.\bar{b})\bar{c}$

2.* Find the **shortest distance** between the skew lines $r = (6\bar{i}+2\bar{j}+2\bar{k}) + t(\bar{i}-2\bar{j}+2\bar{k})$ and $r = (-4\bar{i}-\bar{k}) + s(3\bar{i}-2\bar{j}-2\bar{k})$ where s, t are scalars

3.* If A=(1,–2,–1), B=(4,0,–3),C=(1,2,–1) and D=(2,–4,–5), find the **distance** between the **AB** and **CD**

4.* If q is the smaller angle between any two diagonals of a cube then prove that $\cos q = 1/3$

5.* A line makes angles $\theta_1, \theta_2, \theta_3$ and θ_4 with the diagonals of a cube. Show that

$$\cos^2\theta_1 + \cos^2\theta_2 + \cos^2\theta_3 + \cos^2\theta_4 = \frac{4}{3}$$

6. If **[b c d]** + **[c a d]** + **[a b d]** = **[a b c]** then show that the points with position vectors **a,b,c** and **d** are coplanar.

7.* Find the equation of the plane passing through the points A = (2,3,-1), B = (4,5,2) and C = (3,6,5). Hint : $[\overline{AP}\ \overline{AB}\ \overline{AC}] = 0$, Here $\overline{OP} = x\bar{i} + y\bar{j} + z\bar{k}$

8.* Find the equation of the plane passing through the point A = (3,-2,-1) and parallell to the vectors $\bar{b} = \bar{i} - 2\bar{j} + 4\bar{k}$ and $\bar{c} = 3\bar{i} + 2\bar{j} - 5\bar{k}$ Hint : $[\overline{AP}\ \bar{b}\ \bar{c}] = 0$

9*. For any four vectors $\bar{a}, \bar{b}, \bar{c}$ & $\bar{d}$, show that $(\bar{a} \times \bar{b}) \times (\bar{c} \times \bar{d}) = [\bar{a}\,\bar{c}\,\bar{d}]\bar{b} - [\bar{b}\,\bar{c}\,\bar{d}]\bar{a}$ and $(\bar{a} \times \bar{b}) \times (\bar{c} \times \bar{d}) = [\bar{a}\,\bar{b}\,\bar{d}]\bar{c} - [\bar{a}\,\bar{b}\,\bar{c}]\bar{d}$

10*. Find the vector equation of the plane passing through the intersection of the planes $\bar{r}.(\bar{i} + \bar{j} + \bar{k}) = 6$ & $\bar{r}.(2\bar{i} + 3\bar{j} + 4\bar{k}) = -5$ and the point (1,1,1).