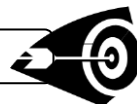
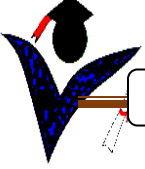




<p>1. If $Z = 3 - 5i$, then show that $z^3 - 10z^2 + 58z + 136 = 0$.</p> <p>Sol: $Z = 3 - 5i = z - 3 = 5i$ S.O.B $\Rightarrow (z - 3)^2 = (-5i)^2$ $\Rightarrow z^2 + 9 - 6z = 25i^2$ $\Rightarrow z^2 + 9 - 6z = -25$ $\Rightarrow z^2 - 6z + 34 = 0 \dots (1)$</p> <p>Now $z^3 - 6z^2 - 4z^2 + 58z + 136$ $\Rightarrow z(z^2 - 6z + 34) - 4z^2 + 24z - 136$ $\Rightarrow z(0) - 4(z^2 - 6z + 34)$ $\Rightarrow 0 - 4(0)$ $= 0$.</p>	<p>2. If $Z = 2 - i\sqrt{7}$, then show that $3z^3 - 4z^2 + z + 88 = 0$.</p> <p>Sol: $Z = 2 - i\sqrt{7} \Rightarrow z - 2 = -i\sqrt{7}$ S.O.B $\Rightarrow (z - 2)^2 = (-i\sqrt{7})^2$ $\Rightarrow z^2 + 4 - 4z = 7i^2$ $\Rightarrow z^2 + 4 - 4z = -7$ $\Rightarrow z^2 - 4z + 15 = 0 \dots (1)$</p> <p>Now $3z^3 - 4z^2 + z + 88 = 0$. $\Rightarrow 3z(z^2 - 4z + 15) + 8z^2 - 32z - 88$ $\Rightarrow 3z(0) - 8(z^2 - 4z + 15)$ $\Rightarrow 0 - 8(0)$ $= 0$.</p>
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<p>1. Show that the four points in the argand diagram represented by the complex numbers $2 + i, 4 + 3i, 2 + 5i, 3i$ are the vertices of square.</p> <p>Sol: let $A = -2 + 7i = (-2, 7)$, $B = -\frac{3}{2} + \frac{1}{2}i = (-\frac{3}{2}, \frac{1}{2})$, $C = 4 - 3i = (4, -3)$, $D = \frac{7}{2}(1 + i) = (\frac{7}{2}, \frac{7}{2})$</p> <p>$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$</p> <p>$A(-2, 7)$, $B(-\frac{3}{2}, \frac{1}{2})$</p> <p>$AB = \sqrt{(-\frac{3}{2} + 2)^2 + (\frac{1}{2} - 7)^2}$</p> <p>$= \sqrt{(\frac{-3+4}{2})^2 + (\frac{1-14}{2})^2} = \sqrt{\frac{1}{4} + \frac{169}{4}} = \frac{\sqrt{170}}{2}$</p> <p>$B(-\frac{3}{2}, \frac{1}{2})$, $C(4, -3)$</p> <p>$BC = \sqrt{(4 + \frac{3}{2})^2 + (-3 - \frac{1}{2})^2} = \sqrt{(\frac{11}{2})^2 + (-\frac{7}{2})^2} = \sqrt{\frac{121+49}{4}} = \frac{\sqrt{170}}{2}$</p> <p>$C(4, -3)$ $D(\frac{7}{2}, \frac{7}{2})$</p> <p>$CD = \sqrt{(\frac{7}{2} - 4)^2 + (\frac{7}{2} + 3)^2}$</p> <p>$= \sqrt{(\frac{7-8}{2})^2 + (\frac{7+6}{2})^2} = \sqrt{\frac{1+169}{4}} = \frac{\sqrt{170}}{2}$</p> <p>$D(\frac{7}{2}, \frac{7}{2})$, $A(-2, 7)$</p> <p>$DA = \sqrt{(-2 - \frac{7}{2})^2 + (7 - \frac{7}{2})^2} = \sqrt{(\frac{-4-7}{2})^2 + (\frac{14-7}{2})^2}$</p> <p>$= \sqrt{(\frac{121}{4} + \frac{49}{4})} = \frac{\sqrt{170}}{2}$</p> <p>$A(-2, 7)$, $C(4, -3)$</p> <p>$AC = \sqrt{(4 + 2)^2 + (-3 - 7)^2} = \sqrt{(6)^2 + (-10)^2} = \sqrt{36 + 100} = \sqrt{136}$</p> <p>$B(-\frac{3}{2}, \frac{1}{2})$ $D(\frac{7}{2}, \frac{7}{2})$, $BD = \sqrt{(\frac{7}{2} + \frac{3}{2})^2 + (\frac{7}{2} - \frac{1}{2})^2}$</p> <p>$= \sqrt{(\frac{10}{2})^2 + (\frac{6}{2})^2} = \sqrt{\frac{100}{4} + \frac{36}{4}} = \frac{\sqrt{136}}{2}$ $AB=BC=CD=DA$ and $AC \neq BD$</p> <p>∴ Given complex number are the vertices of a rhombus.</p>	<p>2. Show that the four points in the argand diagram represented by the complex numbers $2 + i, 4 + 3i, 2 + 5i, 3i$ are the vertices of square.</p> <p>Sol: let $A = 2 + i = (2, 1)$, $B = 4 + 3i = (4, 3)$, $C = 2 + 5i = (2, 5)$, $D = 3i = (0, 3)$</p> <p>$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$</p> <p>$A(2, 1)$, $B(4, 3)$</p> <p>$AB = \sqrt{(4 - 2)^2 + (3 - 1)^2}$</p> <p>$= \sqrt{(2)^2 + (2)^2} = \sqrt{4 + 4} = \sqrt{8}$</p> <p>$B(4, 3)$, $C(2, 5)$</p> <p>$BC = \sqrt{(2 - 4)^2 + (5 - 3)^2} = \sqrt{(-2)^2 + (2)^2} = \sqrt{4 + 4} = \sqrt{8}$</p> <p>$C(2, 5)$ $D(0, 3)$,</p> <p>$CD = \sqrt{(0 - 2)^2 + (3 - 5)^2}$</p> <p>$= \sqrt{(-2)^2 + (-2)^2} = \sqrt{4 + 4} = \sqrt{8}$</p> <p>$D(0, 3)$ $A(2, 1)$,</p> <p>$DA = \sqrt{(2 - 0)^2 + (1 - 3)^2}$</p> <p>$= \sqrt{(2)^2 + (-2)^2} = \sqrt{4 + 4} = \sqrt{8}$</p> <p>$A(2, 1)$, $C(2, 5)$</p> <p>$AC = \sqrt{(2 - 2)^2 + (5 - 1)^2} = \sqrt{(0)^2 + (4)^2} = \sqrt{16} = 4$</p> <p>$B(4, 3)$ $D(0, 3)$, $BD = \sqrt{(0 - 4)^2 + (3 - 3)^2}$</p> <p>$= \sqrt{(-4)^2 + (0)^2} = \sqrt{16} = 4$</p> <p>$AB=BC=CD=DA$ and $AC=BD$ ∴ Given complex number are the vertices of a square.</p>
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1. Show that the points in the argand diagram represented by the complex numbers $2 + 2i, -2 - 2i, -2\sqrt{3} + 2\sqrt{3}i$ are the vertices of an equilateral triangle.

Sol: let
 $A = 2 + 2i = (2, 2),$
 $B = -2 - 2i = (-2, -2),$
 $C = -2\sqrt{3} + 2\sqrt{3}i = (-2\sqrt{3}, 2\sqrt{3})$

$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$A(2, 2), \quad B(-2, -2)$

$AB = \sqrt{(-2 - 2)^2 + (-2 - 2)^2}$

$= \sqrt{(-4)^2 + (-4)^2} = \sqrt{16 + 16} = \sqrt{32}$

$B(-2, -2), \quad C(-2\sqrt{3}, 2\sqrt{3})$

$BC = \sqrt{(-2\sqrt{3} + 2)^2 + (2\sqrt{3} + 2)^2}$

$= \sqrt{12 + 4 - 8\sqrt{3} + 12 + 4 + 8\sqrt{3}}$

$= \sqrt{16 + 16} = \sqrt{32}$

$C(-2\sqrt{3}, 2\sqrt{3}), \quad A(2, 2),$

$CA = \sqrt{(2 + 2\sqrt{3})^2 + (2 - 2\sqrt{3})^2}$

$= \sqrt{12 + 4 + 8\sqrt{3} + 12 + 4 - 8\sqrt{3}}$

$= \sqrt{16 + 16} = \sqrt{32}$

$AB = BC = CA$

3. if $(x - iy)^{1/3} = a - ib$, then show that $\frac{x}{a} + \frac{y}{b} = 4(a^2 - b^2)$.

Sol: Given $(x - iy)^{1/3} = a - ib$

$\Rightarrow (x - iy) = (a - ib)^3$

$\Rightarrow (x - iy) = a^3 - (ib)^3 - 3a^2(ib) + 3a(ib)^2$

$\Rightarrow (x - iy) = a^3 - b^3(-i) - i3a^2b + 3a(b^2(-1))$

$\Rightarrow (x - iy) = a^3 + 3ab^2 + ib^3 - i3a^2b$

$\Rightarrow (x - iy) = (a^3 - 3ab^2) + i(b^3 - 3a^2b)$

equating the real & imaginary parts on both sides we get,

$x = (a^3 - 3ab^2), y = (b^3 - 3a^2b)$

$\Rightarrow x = a(a^2 - 3b^2), \quad y = -b(b^2 - 3a^2)$

$\Rightarrow \frac{x}{a} = (a^2 - 3b^2), \quad \frac{y}{b} = -(b^2 - 3a^2)$

Now $\frac{x}{a} + \frac{y}{b} = a^2 - 3b^2 - b^2 + 3a^2$

$\Rightarrow \frac{x}{a} + \frac{y}{b} = 4a^2 - 4b^2 \quad \therefore \frac{x}{a} + \frac{y}{b} = 4(a^2 - b^2)$

1. If $Z = x + iy$ and if the point p in the argand plane represented by Z, find the locus of Z satisfying the equation $|z - 2 - 3i| = 5$.

Sol:

Given $|z - 2 - 3i| = 5$, and $Z = x + iy$

$\Rightarrow |x + iy - 2 - 3i| = 5$

$\Rightarrow |(x - 2) + i(y - 3)| = 5$

$\Rightarrow \sqrt{(x - 2)^2 + (y - 3)^2} = 5 \quad \text{S.O.B}$

$\Rightarrow (x - 2)^2 + (y - 3)^2 = 25$

$\Rightarrow x^2 + 4 - 4x + y^2 + 9 - 6y - 25 = 0$

$\therefore x^2 + y^2 - 4x - 6y - 12 = 0$

If $x + iy = \frac{1}{1 + \cos\theta + i\sin\theta}$ then S.T $4x^2 - 1 = 0$.

Sol: $x + iy = \frac{1}{1 + \cos\theta + i\sin\theta} = \frac{1}{(1 + \cos\theta) + i\sin\theta}$

$= \frac{1}{2\cos(\frac{\theta}{2}) + i(2\sin(\frac{\theta}{2})\cos(\frac{\theta}{2}))}$

$= \frac{1}{2\cos(\frac{\theta}{2})[\cos(\frac{\theta}{2}) + i\sin(\frac{\theta}{2})]}$

$= \frac{1}{2\cos(\frac{\theta}{2})[\cos(\frac{\theta}{2}) + i\sin(\frac{\theta}{2})][\cos(\frac{\theta}{2}) - i\sin(\frac{\theta}{2})]}$

$= \frac{\cos(\frac{\theta}{2}) - i\sin(\frac{\theta}{2})}{2\cos(\frac{\theta}{2})[\cos^2(\frac{\theta}{2}) - i^2\sin^2(\frac{\theta}{2})]}$

$= \frac{\cos(\frac{\theta}{2}) - i\sin(\frac{\theta}{2})}{2\cos(\frac{\theta}{2})[\cos^2(\frac{\theta}{2}) + \sin^2(\frac{\theta}{2})]}$

$= \frac{\cos(\frac{\theta}{2}) - i\sin(\frac{\theta}{2})}{2\cos(\frac{\theta}{2})}$

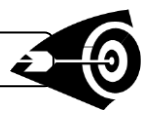
$x + iy = \frac{\cos(\frac{\theta}{2})}{2\cos(\frac{\theta}{2})} - \frac{i\sin(\frac{\theta}{2})}{2\cos(\frac{\theta}{2})}$

$x + iy = \frac{1}{2} - \frac{i}{2}\tan\left(\frac{\theta}{2}\right)$

equating the real & imaginary parts on both sides we get,

$x = \frac{1}{2} \Rightarrow 2x = 1 \quad \text{S.O.B}$

$4x^2 = 1 \text{ or } 4x^2 - 1 = 0.$





If the point p denotes the complex number $Z = x + iy$ in the argand plane and if $\frac{Z-i}{z-1}$ is a purely imaginary no. find the locus of p.

Sol: Given $Z = x + iy$

$$\Rightarrow \frac{Z-i}{z-1} = \frac{x+iy-i}{x+iy-1} = \frac{x+i(y-1)}{(x-1)+iy}$$

$$\Rightarrow \frac{[x+i(y-1)] \times [(x-1)-iy]}{[(x-1)+iy] \times [(x-1)-iy]}$$

$$\Rightarrow \frac{x(x-1) - xyi + i(y-1)(x-1) - i^2(y-1)y}{(x-1)^2 - (iy)^2}$$

$$\Rightarrow \frac{x^2 - x - xyi + i(xy - y - x + 1) + y^2 - y}{(x-1)^2 - y^2 i^2}$$

$$\Rightarrow \frac{[x^2 + y^2 - x - y] + i[xy - y - x + 1]}{(x-1)^2 + y^2}$$

$\therefore \frac{Z-i}{z-1}$ is a purely imaginary number \Rightarrow real part = 0

$$\frac{x^2 + y^2 - x - y}{(x-1)^2 + y^2} = 0 \Rightarrow x^2 + y^2 - x - y = 0.$$

1. If the amplitude of $\left(\frac{Z-2}{z-6i}\right) = \frac{\pi}{2}$, find its locus.

Sol: let $Z = x + iy$

$$\Rightarrow \frac{Z-2}{z-6i} = \frac{x+iy-2}{x+iy-6i} = \frac{(x-2)+iy}{x+i(y-6)}$$

$$\Rightarrow \frac{[(x-2)+iy] \times [x-i(y-6)]}{[x+i(y-6)] \times [x-i(y-6)]}$$

$$\Rightarrow \frac{(x-2)x - (x-2)(y-6)i + xyi - i^2y(y-6)}{(x)^2 - (iy)^2(y-6)^2}$$

$$\Rightarrow \frac{x^2 - 2x - (xy - 6x - 2y + 12)i + i(xy) + y^2 - 6y}{x^2 + (y-6)^2}$$

$$\Rightarrow \frac{[x^2 + y^2 - 2x - 6y] - i[6x - 2y + 12]}{x^2 + (y-6)^2}$$

$$\text{Amplitude of } \left(\frac{Z-2}{z-6i}\right) = \frac{\pi}{2} \left\{ \tan^{-1} \left| \frac{b}{a} \right| = \frac{\pi}{2} \right\}$$

\Rightarrow real part(a)=0, $b > 0$

$$\Rightarrow \frac{x^2 + y^2 - 2x - 6y}{x^2 + (y-6)^2} = 0$$

$$\therefore x^2 + y^2 - 2x - 6y = 0 \text{ and}$$

$$3x + y - 6 > 0$$

2. Determine the locus of z, $z \neq 2i$, such that

$$\left(\frac{Z-4}{z-2i}\right) = 0.$$

Sol: let $Z = x + iy$

$$\Rightarrow \frac{Z-4}{z-2i} = \frac{x+iy-4}{x+iy-2i} = \frac{(x-4)+iy}{x+i(y-2)}$$

$$\Rightarrow \frac{[(x-4)+iy] \times [x-i(y-2)]}{[x+i(y-2)] \times [x-i(y-2)]}$$

$$\Rightarrow \frac{(x-4)x - (x-4)(y-2)i + xyi - i^2y(y-2)}{(x)^2 - (iy)^2(y-2)^2}$$

$$\Rightarrow \frac{x^2 - 4x - (xy - 2x - 4y + 8)i + i(xy) + y^2 - 2y}{x^2 + (y-2)^2}$$

$$\Rightarrow \frac{[x^2 + y^2 - 4x - 2y] + i[2x + 4y - 8]}{x^2 + (y-2)^2}$$

$$\text{Real part of } \left(\frac{Z-4}{z-2i}\right) = 0$$

\Rightarrow real part(a)=0,

$$\Rightarrow \frac{x^2 + y^2 - 4x - 2y}{x^2 + (y-2)^2} = 0 \therefore x^2 + y^2 - 4x - 2y = 0$$

9. Find the real value of θ in order that $\left(\frac{3+2i\sin\theta}{1-2i\sin\theta}\right)$ is a

(a) Purely imaginary number

(b) Real number.

Sol:

$$\left(\frac{3+2i\sin\theta}{1-2i\sin\theta}\right) = \frac{[3+2i\sin\theta][1+2i\sin\theta]}{[1-2i\sin\theta][1+2i\sin\theta]}$$

$$\Rightarrow \frac{[3+6i\sin\theta+2i\sin\theta+4i^2\sin^2\theta]}{(1)^2 - (2i)^2(\sin\theta)^2}$$

$$\Rightarrow \frac{[3-4\sin^2\theta] + i(8\sin\theta)}{1+4\sin^2\theta}$$

$$\Rightarrow \left(\frac{3-4\sin^2\theta}{1+4\sin^2\theta}\right) + i\left(\frac{8\sin\theta}{1+4\sin^2\theta}\right)$$

(a) Purely imaginary number \Rightarrow real = 0

$$\Rightarrow \frac{3-4\sin^2\theta}{1+4\sin^2\theta} = 0 \Rightarrow 3 - 4\sin^2\theta = 0$$

$$\Rightarrow \sin^2\theta = \frac{3}{4} \Rightarrow \sin\theta = \frac{\sqrt{3}}{2}$$

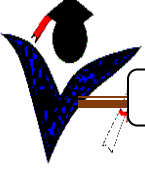
$$\Rightarrow \theta = n\pi \pm \frac{\pi}{3}, n \in \mathbb{Z}$$

(b) Real number \Rightarrow imaginary part = 0

$$\Rightarrow \frac{8\sin\theta}{1+4\sin^2\theta} = 0 \Rightarrow \sin\theta = 0 \Rightarrow \theta = 0$$

$$\Rightarrow \theta = n\pi, n \in \mathbb{Z}$$





10. The points, P, Q denote the complex numbers z_1, z_2 in the argand diagram. O is the origin. If $z_1 \overline{z_2} + \overline{z_1} z_2 = 0$, then show that $\angle POQ = 90^\circ$.

Sol:

let $p(x_1, y_1) \Rightarrow z_1 = x_1 + iy_1; \Rightarrow \overline{z_1} = x_1 - iy_1$
 $Q(x_2, y_2) \Rightarrow z_2 = x_2 + iy_2; \Rightarrow \overline{z_2} = x_2 - iy_2$
 and $O(0, 0)$

given $z_1 \overline{z_2} + \overline{z_1} z_2 = 0$

$$\Rightarrow (x_1 + iy_1)(x_2 - iy_2) + (x_1 - iy_1)(x_2 + iy_2) = 0$$

$$\Rightarrow x_1x_2 - ix_1y_2 + iy_1x_2 - y_1y_2i^2 + x_1x_2 + ix_1y_2 - iy_1x_2 - y_1y_2i^2 = 0$$

$$\Rightarrow 2x_1x_2 + 2y_1y_2 = 0$$

$$\Rightarrow y_1y_2 = -x_1x_2 \Rightarrow \left(\frac{y_1}{x_1}\right) = -\left(\frac{x_2}{y_2}\right)$$

$$\Rightarrow \left(\frac{y_1}{x_1}\right) \left(\frac{y_2}{x_2}\right) = -1$$

$$\Rightarrow (\text{slope of } \overline{OP})(\text{slope of } \overline{OQ}) = -1$$

$$\Rightarrow \angle POQ = 90^\circ.$$

9. Show that the points in the argand diagram represented by the complex numbers z_1, z_2, z_3 are collinear if and only if there exist three real numbers p, q, r not all zero, satisfying $pz_1 + qz_2 + rz_3 = 0$ and $p + q + r = 0$

Sol: $p + q + r = 0 \dots\dots(1)$
 $pz_1 + qz_2 + rz_3 = 0 \dots\dots(2)$

$$\Rightarrow z_1 = -\frac{(qz_2 + rz_3)}{p}$$

$$\Rightarrow z_1 = -\frac{(qz_2 + rz_3)}{-(q+r)}$$

$$\therefore z_1 = \frac{(qz_2 + rz_3)}{(q+r)}$$

$\therefore z_1$ divides the line segment joining z_2, z_3
 In the ratio r:q.
 z_1, z_2, z_3 are collinear

