FRICTION

POINTS TO REMEMBER

1) The force that always opposes the relative motion between two surfaces in contact and parallel to the surfaces, opposite to the direction of motion is called frictional force.

2) The frictional force is of three types. They are

1. Static friction 2.Dynamic friction 3.Rolling friction

Laws of friction: i) The frictional force is independent of the area of contact.

ii) The frictional force is directly proportional to the normal reaction.

3) Normal reaction is the resultant contact force acting on a body placed on a rigid surface perpendicular to the plane of contact. It is equal to mg on a horizontal surface and $mg\ \cos\theta$ on an inclined plane Coefficient of static friction $\mu_s = \tan\phi$

4)Angle of repose is defined as the angle of inclination of a plane with respect to horizontal for which the body will be in limiting equilibrium on the inclined plane .If α is the angle of repose $\mu_s = \tan\alpha$ • When a body slides down an inclined plane of angle of inclination (θ)greater than the angle of response (α) i.e., $\theta > \alpha$,the acceleration of the body $a = g(\sin\theta - \mu_k \cos\theta)$,

5) **The coefficient of friction** is, $\mu = \frac{f_s}{N} = \tan\theta$ where θ is the angle of repose.

6) **Acceleration of a body on a rough horizontal surface**, $a = \frac{P - \mu_k mg}{m}$

7) $a = g\sin\theta$ for a smooth inclined plane.

8) $a = g\left(\sin\theta - \mu_k \cos\theta\right)$ for a body sliding down rough inclined plane.

9) For a body moving up a rough inclined plane under the action of a force F,

$$a = \frac{F - mg(\sin\theta + \mu_k \cos\theta)}{m}$$

10) When a body is to be moved up a rough inclined plane with uniform velocity the force to be applied F=mg(sinθ+μ_k cosθ)

11) Pulling force $F = \frac{W\sin\phi}{\cos(\theta - \phi)}$ and pushing force $F = \frac{W\sin\phi}{\cos(\theta + \phi)}$ where W is the weight of body, ϕ is the angle of friction and θ is the angle made by F with the horizontal

LONG ANSWER QUESTIONS:

****1. Define coefficient of static and dynamic friction. Mention the laws of static and dynamic friction. Derive an expression for the accelerations of the body sliding on a horizontal plane having coefficients of friction μ_k .**

A. The ratio of limiting friction F_l to normal reaction N is called Coefficient of static friction i.e. $F_s \ \alpha \ N$

$$\mu_s = \frac{f_s}{N}$$

The ratio of dynamic frictional froce to the normal reaction. Coefficient of dynamic friction

$f_k \alpha N,\ f_k = \mu_k N$

$$\mu_k = \frac{f_k}{N}$$

Laws of static friction

1. The Limiting friction depends on nature of surface in contact and on their surface roughness. It is independent of area of contact
2. The Limiting friction is tangential to the surface in contact and opposite to the motion of the body.
3. Magnitude of limiting friction is directly proportional to normal reaction,
$f_s \alpha N \ \ or \ \ f_s = \mu_s N$

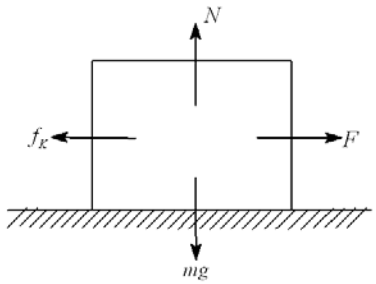
Laws of dynamic or kinetic friction :

1. The kinetic friction depends on nature of surface and independent of area of contact for smaller relative speeds.
2. The kinetic friction is tangential and opposite to motion of the bodies.
3. Dynamic friction is directly proportional to normal reaction

$F_k \propto N$

$\Rightarrow F_k = \mu_k N$

Motion of a body on rough horizontal surface:



Consider a horizontal force F is applied on a block of mass ‘m’, which is kept on a rough horizontal surface. The coefficient of kinetic friction between the block and the surface is μ_k .

The kinetic friction acting on the body is opposite to applied force and tangential at contact surface.

From laws of friction:

Kinetic frictional force $f_k = \mu_k N$ and $N = mg$

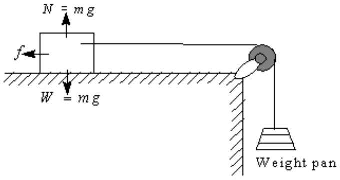
Net force acting on the body

$F_{net} = F - f_k = ma$

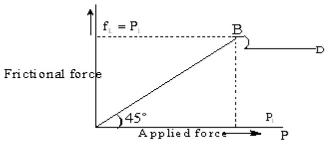
$a = \frac{F - \mu_k mg}{m}$

2. Describe with an experiment how static friction is explained. Distinguish static friction and dynamic friction.

A.



Consider a body be placed on a rough horizontal surface of a table. As shown the body is connected to a weight pan by a thread through smooth pulley. When weight in the pan is small, the block does not slide, since the tension in the thread is balanced by frictional force on block, this frictional force is called static friction. Static friction is self adjusting; as the weight on the pan is gradually increased, static friction also increases to a maximum limit. After the limit, the body slides with an acceleration. The maximum value of static friction is called limiting friction.



Variation of frictional force with applied force

If a graph is drawn with applied force (weight) on X-axis and frictional force on Y-axis, the graph is as shown.

Starting from zero applied force, static friction increases up to the maximum value. After the limit, body starts sliding suddenly reducing frictional force by small amount (BC). After motion has started, the kinetic frictional force remains constant. This is represented by line PQ.

It is clear for diagram that, BE > PQ. Limiting friction > Kinetic friction

I.e. $f_s > f_k$ hence $\mu_s > \mu_k$

3. Derive an equation for the acceleration of a body sliding down an inclined plane without friction. Find also the time taken by the body to slide down the smooth inclined plane through a length '*l*' starting from rest.

A. Consider a body of mass 'm' is placed on a smooth inclined plane of inclination θ is as shown in figure.

The forces acting on the body are

(i) The weight of the body (mg) acts vertically downwards

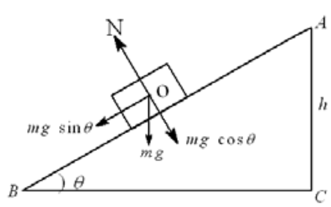
(ii) The normal reaction (N), acts perpendicular to the inclined plane

The weight of the body 'mg' can be resolved in to two components. They are mg $\cos\theta$,and mg $\sin\theta$. The component mg $\cos\theta$ acts perpendicular to the inclined plane and is balanced by normal reaction (N=mg $\cos\theta$) and mg $\sin\theta$ acts parallel to the inclined plane and it is un balanced force. The net down ward force on the body along the plane is given by

$$F_{net} = mg\ \sin\theta$$

$$ma = mg\ \sin\theta \quad (\because F_{net} = ma)$$

$$a = g\sin\theta$$



Consider the body starts from rest from the top of the plane and acquires a velocity V after travelling through displacement 'l' along the plane, then

From the equation of motion,

$v^2 - u^2 = 2as$, as u = 0, $s = l$ and $a = g\sin\theta$

$v^2 = 2\left(g\sin\theta\right)l$

$V = \sqrt{2gl\sin\theta} \qquad \left(\because l\sin\theta = h\right)$

$V = \sqrt{2gh}$

$\rightarrow$Hence the velocity acquired by the body when it reaches the foot of the plane is independent of the inclination of the plane and depends only on the vertical height (h) through which the body is falling.

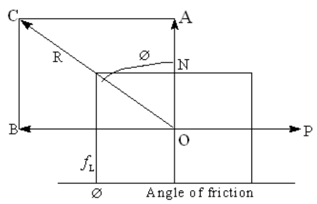
→ If ‘t’ time taken to travel the distance ‘l’ with initial velocity u = 0, at the top of the plane, from $V = u + at$,
We get

$$t = \frac{V}{g\sin\theta} \qquad t = \frac{\sqrt{2lg\sin\theta}}{g\sin\theta} \qquad t = \sqrt{\frac{2l}{g\sin\theta}}$$

4. **Define angle of friction and angle of repose.**

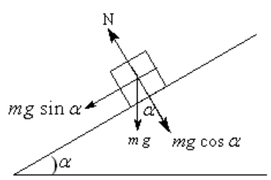
Show that angle of friction is equal to angle of repose for a rough inclined plane.

A. The angle made by the resultant of the normal reaction and the limiting frictional force With normal reaction is called angle of friction (ϕ).

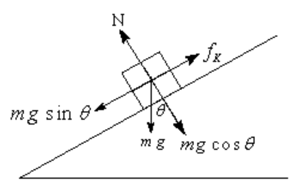


From the figure $\tan\phi = \frac{f_s}{N}$ and $\mu_s = \frac{f_s}{N}$. Hence $\mu_s = \tan\phi$.

The angle of repose (α) is defined as the angle of inclination of a rough inclined plane with horizontal for which the body will be just ready to slide down.



<u>To show that the angle of repose is equal to the angle of friction :</u>



When the body tends to move down the rough inclined plane, the friction on the block is called limiting or static friction f_s .Hence

$f_s = mg\sin\alpha$ and $N = mg\cos\alpha$

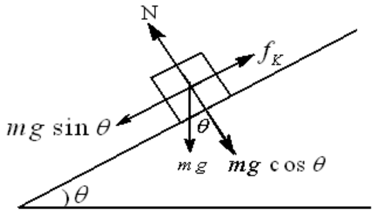
$\therefore$ Coefficient of static friction $\mu_s = \frac{mg\sin\alpha}{mg\cos\alpha} = \tan\alpha$

. $\mu_s = \tan\alpha$ Here α is the angle of repose.

Hence the angle of repose of a rough inclined plane is equal to the angle of friction of horizontal plane

5. **Obtain an expression for the acceleration of a body down a rough inclined plane. Find the time taken by the body to slide down the inclined plane through a length ‘l’ starting from rest. (June 2003, March 2002, 2003, and 2007)**

A. Consider a body of mass ‘m’ is placed on a rough inclined plane of inclination θ as shown in figure.



The forces acting on the body are

(i) The weight of the body (mg) acts vertically downwards

(ii) The normal reaction (N), acts perpendicular to the inclined plane

(iii) The kinetic friction (f_k) acting up the inclined plane

The weight of the body 'mg' can be resolved into two components.They are mg cosθ and mg sin θThe component mg cosθ acts perpendicular to the inclined plane and is balanced by normal reaction (N=mg cosθ) and the component mgsinθ acts parallel to the inclined plane and it is unbalanced force.

The net downward force acting on the body is

$F_{net} = mg\sin\theta - f_k$

$F_{net} = mg\sin\theta - \mu_k N$ $\quad (\because f_k = \mu_k N)$

$F_{net} = mg\sin\theta - \mu_k mg\cos\theta$ $\quad (\because$ N=mg cos$\theta)$

$ma = mg(\sin\theta - \mu_k\cos\theta)$ $\quad (\because F_{net} = ma)$

$$\boxed{a = g(\sin\theta - \mu_k\cos\theta)}$$

Consider body starts from rest from the top of the plane and acquires a velocity V after travelling through displacement 'l' along the plane, then

From the equation of motion, $v^2 - u^2 = 2as$,

$$v^2 - o^2 = 2g\left(\sin\theta - \mu_k\cos\theta\right)s$$

$$\boxed{V = \sqrt{2gl\left(\sin\theta - \mu_k\cos\theta\right)}}\left(\because s = l\right)$$

The time taken by the body to slide down is given by $S = ut + \frac{1}{2}at^2$

Here, $S = l,\ u = 0$

$$\therefore l = 0(t) + \frac{1}{2}g\left(\sin\theta - \mu_k\cos\theta\right)t^2$$

$$\boxed{\therefore t = \sqrt{\frac{2l}{g\left(\sin\theta - \mu_k\cos\theta\right)}}}$$

Friction: It is the force which always opposes the relative motion between two surfaces in contact. It acts parallel to the surfaces and opposite to the direction of motion.

Laws of friction: i) The frictional force is independent of the area of contact.
ii) The frictional force is directly proportional to the normal reaction.

Static friction : Coefficient of static friction is defined as the ratio of limiting friction to normal reaction N.

$i.e., F_l \propto N$

$$\mu_s = \frac{f_s}{N}$$

Dynamic friction: Coefficient of dynamic friction is defined as ratio of dynamic frictional force to the normal reaction $f_k \alpha N$. $\mu_k = \dfrac{f_k}{N}$.

Rolling friction: The rolling friction is directly proportional to the normal reaction.

Coefficient of rolling friction, $\mu_r = \dfrac{f_r}{N}$

Angle of repose: It is defined as the angle of inclination of a plane with the horizontal for which the body will be in limiting equilibrium on the inclined plane. If α is the angle of repose $\mu_s = \tan\alpha$

Angle of friction: The angle made by the resultant of the normal reaction and the limiting friction with normal reaction is called angle of friction.

SHORT ANSWERS QUESTIONS

1. Explain the terms dynamic limiting frictions and rolling friction. (June 2002, 2003)

A. **Limiting friction: The** maximum frictional force developed between the bodies at rest is called limiting friction.

Dynamic or kinetic friction: When applied force is equals to or greater than limiting friction then the body will move. Frictional force between moving bodies is called dynamic or kinetic friction. Kinetic friction is always less than limiting friction.

Rolling friction : When one body rolls on the surface of the other body, (like a ball, a drum or a cylinder) then the existed friction in between that body and the surface of the other body is called rolling friction.

2. Write a note on the causes of friction.

A 1) Frictional force is caused due to the roughness of two surfaces in contact and interlocking of the irregularities between the surfaces.
2) Nature of surfaces.
3) Impurities between surfaces.
4) Shape of surfaces.

3. Explain advantages and disadvantages of friction. (March 2005, 2006)

A. **Advantages of friction:**
1) Safe walking on the floor is possible due to friction.
2) Nails, screws etc. are driven into walls; wooden surfaces etc. due to friction.
3) Writing with pens, pencils, holding objects with hands etc is possible due to friction.
4) A match stick is lightened due to friction.

Disadvantages of friction:
1) Due to friction wear and tear of engine occurs.
2) Due to friction some energy gets converted into heat which goes as waste
3) The large amount of power loss in engines due to friction.

4. Mention the methods used to decrease friction? (July 2001 March 2001, 2003, 2009)

A. 1. By polishing the surface in contact, friction can be reduced.
2 Wheels of vehicles are provided with ball bearings to reduce the friction here ball bearing convert sliding friction to rolling friction.
3. The lubricants form a thin layer between surfaces in contact. It reduces the friction. In light vehicles or machines, oils like "three in one" are used as lubricants. In heavy machines grease is used. In addition to this they guard the mechanical parts from over heating.
4. Automobiles and aero planes are streamlined to reduce the friction due to air.

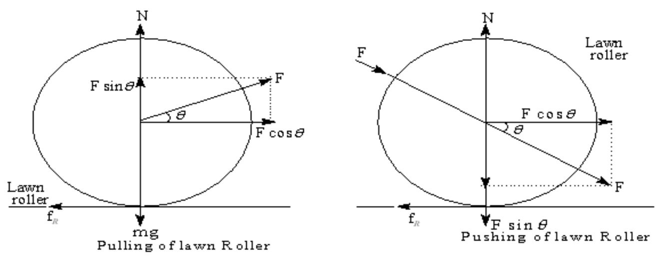
5. Sate the laws of rolling friction?

A. 1) The smaller the area of contact, a lesser will be the rolling friction.
2) The larger the radius of the rolling body, the lesser will be the rolling friction.
3) The rolling friction is directly proportional to the normal reaction.

i.e., $F_r \alpha N \quad or \quad \mu_r = \frac{F_r}{N}$

6. Why pulling the lawn roller is preferred than pushing the lawn roller? (March 2006)

A. **Pulling of lawn roller** :In the case of Pulling a lawn roller the vertical component of the applied force, $F \sin\theta$ is acting in the upward direction and reduces the normal reaction. The normal reactions becomes $N = mg - F\sin\theta$



The frictional force while pulling

$$f_R = \mu_R N$$
$$f_R = \mu_R (mg - F\sin\theta) \quad \text{---------------------} \ (1)$$

Hence less frictional force develops and hence less force is required to pull the lawn roller. In the case of Pushing the lawn roller, the vertical component of the applied force, $F\sin\theta$, is acting in the direction of weight, and increases the normal reaction. Then normal reaction becomes $N = mg + F\sin\theta$

The frictional force while pushing

$$f_R^1 = \mu_R N$$
$$f_R^1 = \mu_R (mg + F\sin\theta) \quad \text{---------------------} \ (2)$$

As $f_R < f_R^1$, less force is required to pull the lawn roller than pushing it.

Hence Pulling is easier than pushing

7. Show that velocity at the bottom of the smooth inclined plane is independent of slope of the inclined

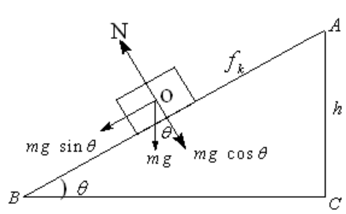
A. Consider a smooth inclined smooth plane of inclination $'\theta'$, height ‘h’ and length ‘L’. A block is released from the top of the inclined plane

Weight of the block is resolved into components. They are

1) Component along the incline $mg\sin\theta$, and

2) Component normal to the incline $mg\cos\theta$,

Only $mg\sin\theta$ creates acceleration of the block along the incline.



From the equation of motion

$$v^2 - u^2 = 2as$$

Acceleration of the block along the inclined plane

$$a = \frac{mg\sin\theta}{m} = g\sin\theta$$

Distance covered by the block on reaching the bottom s = L

$\therefore$ From $v^2 - u^2 = 2as$