

DE MOIVRE 'S THEOREM- Q.NO. 18(14-1-19 & 15-1-19)

1. (A). If n is an integer then show that $(1 + i)^{2n} + (1 - i)^{2n} = 2^{n+1} \cos \frac{n\pi}{2}$.
 - (B). If n is a positive integer, show that $(1 + i)^n + (1 - i)^n = 2^{\frac{n+2}{2}} \cos\left(\frac{n\pi}{4}\right)$
 - (C). If α, β are the roots of the equation $x^2 - 2x + 4 = 0$ then for any $n \in \mathbb{N}$ show that $\alpha^n + \beta^n = 2^{n+1} \cos\left(\frac{n\pi}{3}\right)$
 - (D). If n is an integer then show that $(1 + \cos \theta + i \sin \theta)^n + (1 + \cos \theta - i \sin \theta)^n = 2^{n+1} \cos^n\left(\frac{\theta}{2}\right) \cos\left(\frac{n\theta}{2}\right)$
 - (E). If n is a positive integer, show that $(p + iQ)^{\frac{1}{n}} + (P - iQ)^{\frac{1}{n}} = 2(P^2 + Q^2)^{\frac{1}{2n}} \cdot \cos\left[\frac{1}{n} \tan^{-1} \frac{Q}{P}\right]$
2. Show that one value of $\left[\frac{1 + \sin \frac{\pi}{8} + i \cos \frac{\pi}{8}}{1 + \sin \frac{\pi}{8} - i \cos \frac{\pi}{8}} \right]^{\frac{8}{3}}$ is - 1.
 3. a) If $\cos \alpha + \cos \beta + \cos \gamma = 0 = \sin \alpha + \sin \beta + \sin \gamma$ then show that
 i) $\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3\cos(\alpha + \beta + \gamma)$
 ii) $\sin 3\alpha + \sin 3\beta + \sin 3\gamma = 3\sin(\alpha + \beta + \gamma)$
 b). If $\cos \alpha + \cos \beta + \cos \gamma = 0 = \sin \alpha + \sin \beta + \sin \gamma$, Prove that $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = \frac{3}{2} = \sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma$
 4. (A). If n is an integer and $z = \text{cis } \theta$, $\left(\theta \neq (2n + 1)\frac{\pi}{2}\right)$, then show that $\frac{z^{2n} - 1}{z^{2n} + 1} = i \tan n\theta$.
 - (B). Find all the roots of the equation i) $x^{11} - x^7 + x^4 - 1 = 0$ ii) $x^9 - x^5 + x^4 - 1 = 0$.
 - (C). If $(1+x)^n = a_0 + a_1x + a_2x^2 + \dots + a_nx^n$, then show that
 i) $a_0 - a_2 + a_4 - \dots = 2^{\frac{n}{2}} \cos\left(\frac{n\pi}{4}\right)$. ii) $a_1 - a_3 + a_5 - \dots = 2^{\frac{n}{2}} \sin\left(\frac{n\pi}{4}\right)\sqrt{3}$.

THEORY OF EQUATIONS 18(12-1-19 & 13-1-19)

1. (A). Solve : $(8x^3 - 36x^2 - 18x + 81 = 0)$, $(4x^3 - 24x^2 + 23x + 18 = 0)$ given that the roots are in A.P.
 (B). Solve the equation $x^3 - 7x^2 + 14x - 8 = 0$, given that the roots are in geometric progression.
 (C). Solve the equation $15x^3 - 23x^2 + 9x - 1 = 0$, given that the roots are in H.P.
 (D). Solve $18x^3 + 81x^2 + 121x + 60 = 0$ given that a root is equal to half the sum of the remaining roots.
2. (A). Solve : $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$, $(2x^4 + x^3 - x^2 + x + 2 = 0)$, $(6x^4 - 35x^3 + 62x^2 - 35x + 6 = 0)$
 (B). Solve : $2x^5 + x^4 - 12x^3 - 12x^2 + x + 2 = 0$, $(x^5 - 5x^4 + 9x^3 - 9x^2 + 5x - 1 = 0)$
 (C). Solve the equation : $6x^6 - 25x^5 + 31x^4 - 31x^2 + 25x - 6 = 0.11$.
3. (A). Solve the equation $x^4 + 2x^3 - 5x^2 + 6x + 2 = 0$, given that one root of it is $1 + i$.
 (B). Solve the equation $x^4 - 4x^2 + 8x + 35 = 0$, given that one root of it is $2 + i\sqrt{3}$
 (C). Find the algebraic equation of degree 5 whose roots are the translates of the roots of $x^5 + 4x^3 - x^2 + 11 = 0$ by -3.
4. (A). Solve the equation $x^4 + 4x^3 - 2x^2 - 12x + 9 = 0$, if it has a pair of equal roots.
 (B). Find the repeated roots of the equation $x^5 - 3x^4 - 5x^3 + 27x^2 - 32x + 12 = 0$.
 (C). Solve $x^3 - 9x^2 + 14x + 24 = 0$, given that two of the roots are in the ratio 3:2.
 (D). Solve $x^4 + x^3 - 16x^2 - 4x + 48 = 0$, given that the product of two of the roots is 6.
 (E). Solve $x^4 - 2x^3 + 4x^2 + 6x - 21 = 0$, given that sum of the two roots is zero.
5. Find the roots of $x^4 - 16x^3 + 86x^2 - 176x + 105 = 0$.

BINOMIAL THEOREM (QUESTION NO : 20) (10-1-19 & 11-1-19)

- 1.(A). Prove that $C_0 + \frac{C_1}{2}x + \frac{C_2}{3}x^2 + \dots + \frac{C_n}{n+1}x^n = \frac{(1+x)^{n+1} - 1}{(n+1)x}$.
- Deduce that $\frac{C_1}{2} + \frac{C_3}{4} + \frac{C_5}{6} + \dots = \frac{2^n - 1}{n+1}$.
- (B). Prove that $C_0C_r + C_1C_{r+1} + C_2C_{r+2} + \dots + C_{n-r}C_n = {}^{2n}C_{n+r}$.
- Deduce that i) $C_0^2 + C_1^2 + C_2^2 + \dots + C_n^2 = {}^{2n}C_n$. ii) $C_0C_1 + C_1C_2 + C_2C_3 + \dots + C_{n-1}C_n = {}^{2n}C_{n+1}$.
- (C). If n is a positive integer, prove that $\sum_{r=1}^n r^3 \left(\frac{{}^nC_r}{{}^nC_{r-1}} \right)^2 = \frac{n(n+1)^2(n+2)}{12}$.
- (D). Prove that $(C_0 + C_1)(C_1 + C_2)(C_2 + C_3)\dots(C_{n-1} + C_n) = \frac{(n+1)^n}{n!} C_0 C_1 \dots C_n$.
- 2.(A.) If the 2nd, 3rd and 4th terms in the expansion of $(a + x)^n$ are respectively 240, 720, 1080, find a, x, n.
- (B). If the coefficient of x^{10} in the expansion of $(ax^2 + \frac{1}{bx})^{11}$ is equal to the coefficient of x^{-10} in the expansion of $(ax - \frac{1}{bx^2})^{11}$, find the relation between a and b, where a and b are real numbers.
- (C). If $(7 + 4\sqrt{3})^n = I + f$ where I and n are positive integers and $0 < f < 1$ then show that (i) I is an odd positive integer (ii) $(I + f)(1 - f) = 1$.
- 3.(A). If the coefficients of rth, (r + 1)th, (r + 2)nd terms in the expansion of $(1 + x)^n$ are in A.P, then show that $n^2 - (4r + 1)n + 4r^2 - 2 = 0$.
- (B). If the coefficients of 4 consecutive terms in the expansion of $(1 + x)^n$ are a_1, a_2, a_3, a_4 respectively, then show that $\frac{a_1}{a_1+a_2} + \frac{a_3}{a_3+a_4} = \frac{2a_2}{a_2+a_3}$.
- (C). If P and Q are the sum of odd terms and the sum of even terms respectively in the expansion of $(x + a)^n$, then prove that i) $P^2 - Q^2 = (x^2 - a^2)^n$ ii) $4PQ = (x + a)^{2n} - (x - a)^{2n}$.

BINOMIAL THEOREM (QUESTION NO : 21) (8-1-19 & 9-1-19)

1. (A). Find the sum of the infinite series $1 + \frac{2}{3} \cdot \frac{1}{2} + \frac{2.5}{3.6} \left(\frac{1}{2}\right)^2 + \frac{2.5.8}{3.6.9} \left(\frac{1}{2}\right)^3 + \dots \dots \dots \infty$.
- (B). Find the sum of infinite series $\frac{3}{4} + \frac{3.5}{4.8} + \frac{3.5.7}{4.8.12} + \dots \dots \dots \infty$.
- (C). Find the sum of infinite series $\frac{3.5}{5.10} + \frac{3.5.7}{5.10.15} + \dots \dots \dots$
- (D). Find the sum of infinite series $\frac{3}{4.8} - \frac{3.5}{4.8.12} + \frac{3.5.7}{4.8.12.16} + \dots \dots$
- (E). If $t = \frac{4}{5} + \frac{4.6}{5.10} + \frac{4.6.8}{5.10.15} + \dots \dots$. Then prove that $9t = 16$.

RANDOM VARIABLE AND DISTRIBUTIONS (2-1-19 & 3-1-19)

1 (A). The probability distribution of a random variable X is given below:

$X = x_i$	1	2	3	4	5
$P(X=x_i)$	k	2k	3k	4k	5k

(B). Find the value of k and the mean, variance of X.

$X = x$	-2	-1	0	1	2	3
$P(X = x)$	0.1	k	0.2	2k	0.3	k

is the probability distribution of a random variable X. Find the value of K and the variance of X.

(C). A random variable X has the following probability distribution.

$X = x$	0	1	2	3	4	5	6	7
$P(X = x)$	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

Find (i) k (ii) The mean (iii) $P(0 < X < 5)$

(D). A cubical die is thrown. Find the mean and variance of X, giving the number on the face that shows up.

2.(A). The range of a random variable X is {0, 1, 2}. Given that $P(X = 0) = 3C^3$, $P(X = 1) = 4C - 10C^2$, $P(X = 2) = 5C - 1$. Find (i) the value of C (ii) $P(X < 1)$ (iii) $P(1 < X \leq 2)$ (iv) $P(0 < X \leq 3)$.

(B). If X is a random variable with the probability distribution $P(X = K) = \frac{(K+1)C}{2^k}$ ($K = 0, 1, 2, \dots$), then find C.

(C). The range of a random variable X is {1, 2, 3,.....} and $P(X = k) = \frac{c^k}{k!}$; $k = 1, 2, 3, \dots$. Find the value of c and $P(0 < x < 3)$.

3. (A). In the experiment of tossing a coin n times, if the variable X denotes the number of heads and $P(X = 4)$, $P(X = 5)$, $P(X = 6)$ are in A.P, then find n.

(B). One in nine ships is likely to be wrecked when they set on sail. When 6 ships are set on sail, find the probability for : i) atleast one will arrive safely ii) exactly three will arrive safely

(C). If the difference between the mean and variance of binomial variate is $\frac{5}{9}$ then, find the probability for the event of 2 successes when the experiment is conducted 5 times.

(D). If the mean and variance of a binomial variate X are 2.4 and 1.44 respectively, find $P(1 < X \leq 4)$.

4. If $X : S \rightarrow R$ is a discrete random variable with range $\{x_1, x_2, x_3, \dots\}$; μ is mean and σ^2 is variance of X then prove that $\sigma^2 + \mu^2 = \sum X_r^2 P(X=X_r)$.

SHORT ANSWER QUESTIONS (4 MARKS)

COMPLEX NUMBERS (16-1-19)

1. If $x + iy = \frac{3}{2 + \cos \theta + i \sin \theta}$ then show that $x^2 + y^2 = 4x - 3$.
2. If the point P denotes the complex number $z = x + iy$ in the argand plane and if $\frac{z-i}{z-1}$ is a purely imaginary number, find the locus of P.
3. If $x + iy = \frac{1}{1 + \cos \theta + i \sin \theta}$, then show that $4x^2 - 1 = 0$.
- 4.a) Show that the points in the Argand plane represented by the complex numbers $-2 + 7i$, $\frac{-3}{2} + \frac{1}{2}i$, $4 - 3i$, $\frac{7}{2}(1+i)$ are the vertices of rhombus.
- b) Show that the points in the Argand diagram represented by the complex numbers $2 + 2i$, $-2 - 2i$, $-2\sqrt{3} + 2\sqrt{3}i$ are the vertices of an equilateral triangle.
- c) Show that the four points in the Argand plane represented by the complex numbers $2 + i$, $4 + 3i$, $2 + 5i$, $3i$ are the vertices of a square.
- 5.a) If $z = 3 - 5i$, then show that $z^3 - 10z^2 + 58z + 136 = 0$.
- b) If $z = 2 - i, \sqrt{7}$ then show that $3z^3 - 4z^2 + z + 88 = 0$.
- c) If $(x - iy)^{1/3} = a - ib$, then show that $\frac{x}{a} + \frac{y}{b} = 4(a^2 - b^2)$.
- 6.a) If $z = x+iy$ and if the point P in the Argand plane represents z, find the locus of z satisfying the equation $|z - 2 - 3i| = 5$.
- b) If $z = x+iy$ and if the point P in the Argand plane represents z, find the locus of z satisfying the equation $|z - 3 + i| = 4$.
- 7.a) Find the real value of x and y if $\frac{x-1}{3+i} + \frac{y-1}{3-i} = i$.
- b) If the amplitude of $\left(\frac{z-2}{z-6i}\right) = \frac{\pi}{2}$, find its locus.
- c) Determine the locus of z, $z \neq 2i$, such that $\text{Re}\left(\frac{z-4}{z-2i}\right) = 0$.
- d) Find the real values of θ in order that $\frac{3 + 2i \sin \theta}{1 - 2i \sin \theta}$ is a
 - a) real number
 - b) purely imaginary number

QUADRATIC EXPRESSIONS (17-1-19)

1. If x is a real number, find the range of i) $\frac{x+2}{2x^2+3x+6}$ ii) $\frac{x^2+x+1}{x^2-x+1}$, iii) $\frac{2x^2-6x+5}{x^2-3x+2}$
2. Show that $\frac{x}{x^2-5x+9}$ lies between $\frac{-1}{11}$, 1.
3. If x is real, find the maximum value of the expression $\frac{x^2+14x+9}{x^2+2x+3}$.
4. Prove that $\frac{1}{3x+1} + \frac{1}{x+1} - \frac{1}{(3x+1)(x+1)}$ does not lie between 1 and 4, if x is real.
5. If the expression $\frac{x-p}{x^2-3x+2}$ takes all real values for $x \in \mathbb{R}$, then find the bounds for p.
6. If x_1, x_2 are the roots of the Q.E. $ax^2 + bx + c = 0$ then find the value of $(ax_1 + b)^{-2} + (ax_2 + b)^{-2}$ in terms of ab.

PERMUTATIONS (18-1-19 & 19-1-19)

- Find the number of 4 letter words that can be formed using the letters of the word 'M I X T U R E' which
i) contain the letter X ii) do not contain the letter X
- Find the number of ways of arranging 6 boys and 6 girls in a row so that
i) all the girls sit together ii) no two girls sit together iii) boys and girls sit alternately.
- Find the number of ways of permuting the letters of the word 'PICTURE' so that
I) All vowels come together II) No two vowels come together
- Find the number of ways of arranging 5 different mathematics books, 4 different physics books and 3 different chemistry books such that the books of the same subject are together.
- Find the rank of the words i) "M A S T E R" ii) "R E M A S T" iii) "P R I S O N" iv) "EAMCET"
- Find the number of numbers that are greater than 4000 which can be formed using the digits 0, 2, 4, 6, 8 without repetition.
- Find the sum of all 4 digit numbers that can be formed using the digits 1, 3, 5, 7 and 9 (without repetition)
- Find the sum of all 4 digit number that can be formed using the digits 0, 2, 4, 7, 8 without repetition.

COMBINATIONS (20-1-19 & 21-1-19)

- Show that
$$\frac{{}^{4n}C_{2n}}{{}^{2n}C_n} = \frac{1.3.5.....(4n-1)}{\{1.3.....(2n-1)\}^2}$$
- Simplify: ${}^{34}C_5 + \sum_{r=0}^4 {}^{38-r}C_4$.
- Prove that for $3 \leq r \leq n$, ${}^{n-3}C_r + 3 {}^{n-3}C_{r-1} + 3 {}^{n-3}C_{r-2} + {}^{n-3}C_{r-3} = {}^nC_r$
- Find the number of ways of selecting a cricket team of 11 players from 7 batsmen and 6 bowlers such that there will be atleast 5 bowlers in the team.
- Find the number of ways of forming a committee of 5 members out of 6 Indians and 5 Americans so that always Indians will be in majority in the committee.
- A question paper is divided into 3 sections A, B, C containing 3, 4, 5 questions respectively. Find the number of ways of attempting 6 questions choosing atleast one from each section.
- Find the number of ways of selecting 11 member cricket team from 7 batsmen, 6 bowlers and 2 wicket keepers so that the team contains 2 wicket keepers and atleast 4 bowlers.

PARTIAL FRACTIONS (22-1-19 & 23 -1-19)

- a) Resolve $\frac{x+4}{(x^2-4)(x+1)}$ b) $\frac{3x+7}{x^2-3x+2}$
- a) $\frac{x^2-x+1}{(x+1)(x-1)^2}$ b) $\frac{1}{(x-1)^2(x-2)}$
- a) Resolve $\frac{2x^2+3x+4}{(x-1)(x^2+2)}$ b) Resolve $\frac{x^2-3}{(x+2)(x^2+1)}$ c) Resolve $\frac{2x^2+2x+1}{(x^3+x^2)}$
- a) Resolve $\frac{x^3}{(x-a)(x-b)(x-c)}$ b) Find the partial fractions of $\frac{x^3}{(2x-1)(x+2)(x-3)}$
- a) Resolve $\frac{3x-18}{x^3(x+3)}$ into partial fractions.
- b) Resolve $\frac{3x^3-8x^2+10}{(x-1)^4}$ into partial fractions.
- Find the coefficient of x^n in the power series expansion of $\frac{x-4}{x^2-5x+6}$ specifying the region in which the expansion is valid.

PROBABILITY (24-1-19 & 25-1-19)

1. In a committee of 25 members each member is proficient either in mathematics or in statistics or in both if 19 of these are proficient in mathematics and 16 are in statistics then find the probability that a person selected from the committee is proficient in both.
2. Find the probability of drawing an ace or a spade from a well shuffled pack of 52 playing cards.
3. If one ticket is randomly selected from tickets numbered 1 to 30 then find the probability that the number on the ticket is (i) a multiple of 5 or 7 (ii) multiple of 3 or 5.
4. In a class of 60 boys and 20 girls, half of the boys and half of the girls know cricket. Find the probability of a person selected from the class is either a boy or a girl who know cricket
5. If A, B, C are three events in a sample space s, then S.T $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$
6. If two numbers are selected randomly from 20 consecutive natural numbers, find the probability that the sum of the two numbers is (i) an even number (ii) an odd number.
7. A bag contains 12 two rupee coins, 7 one rupee coins, and 4 half rupee coins. If three coins are selected at random, then find the probability that (a) The sum of three coins is maximum (b) the sum of three coins is minimum
8. A speaks truth in 75% of the cases and B in 80% of the cases. What is the probability that their statements about an incident do not match.
9. Two people A and B are rolling a die on the condition that the person who gets 3 will win the game. if A starts the game, then find the probability of A and B respectively to win the game.
10. Find the probability that a non leap year contains (i) 53 Sundays (ii) 52 Sundays.
11. In a box containing 15 bulbs, 5 are defective if 5 are selected at random from the box, find the probability that (i) none of them is defective (ii) only one is defective (iii) at least one is defective.
12. State and prove multiplication theorem
13. If one card is drawn from a pack of cards, then show that the events of an ace and getting a heart card are independent events.

Question no 17 - PROBABILITY (26-1-19)

1. The probability that Australia wins a match against India in a cricket game is given to be $\frac{1}{3}$. If India and Australia play 3 matches, what is the probability that (i) Australia will lose all the three matches? (ii) Australia will win at least one match?
2. Three screws are drawn at random from a lot of 50 screws, 5 of which are defective. Find the probability of the event that all the 3 screws are non-defective, assuming that the drawing is (a) with replacement (b) without replacement
3. A problem in calculus is given to two students A and B whose chances of solving it are $\frac{1}{3}$ and $\frac{1}{4}$ respectively. Find the probability of the problem being solved if both of them by independently
4. If A and B are independent events with $P(A)=0.6, P(B)=0.7$ then compute (i) $P(A \cap B)$ (ii) $P(A \cup B)$ (iii) $P(\bar{A} \cap \bar{B})$ (iv) $P(\bar{A} \cup \bar{B})$.
5. If A and B are independent events with $P(A)=0.2, P(B)=0.5$ then compute (i) $P(A \cap B)$ (ii) $P(\bar{A} \cap \bar{B})$ (iii) $P(\bar{A} \cup \bar{B})$ (iv) $P(A \cup B)$.
6. A, B are two independent events such that the probability of both the events to occur is $\frac{1}{6}$ and the probability of both the events do not occur is $\frac{1}{3}$. Find $P(A)$.
7. A bag contains 4 white and 2 black balls; another bag contains 3 white and 4 black balls, a bag is drawn at random and a ball is chosen at random from it. Then what is the probability that the ball is white.
8. If A, B are two events with $P(A \cap B) = 0.65, P(A \cup B) = 0.15$ then find .
9. For any two events A and B show that
10. If A and B are two events with $P(A)=0.5, P(B)=0.4, P(A \cap B) = 0.3$ find the probability that (i) A does not occur (ii) neither A or B occurs.