1. If $Z = 3 - 5i$, then show that $z^3 - 10z^2 + 58z + 136 = 0$.

Sol: $Z = 3 - 5i = z - 3 = 5i$

S.O.B

$\Rightarrow (z-3)^2 = (-5i)^2$

$\Rightarrow z^2 + 9 - 6z = 25i^2$

$\Rightarrow z^2 + 9 - 6z = -25$

$\Rightarrow \mathbf{z^2 - 6z + 34 = 0}$ .....(1)

Now $z^3 - 6z^2 - 4z^2 + 58z + 136$

$\Rightarrow z(\mathbf{z^2 - 6z + 34}) - 4z^2 + 24z - 136$

$\Rightarrow z(0) - 4(\mathbf{z^2 - 6z + 34})$

$\Rightarrow 0 - 4(0)$

$=0.$

2. If $Z = 2 - i\sqrt{7}$, then show that $3z^3 - 4z^2 + z + 88 = 0$.

Sol: $Z = 2 - i\sqrt{7} \Rightarrow z - 2 = -i\sqrt{7}$

S.O.B

$\Rightarrow (z-2)^2 = \left(-i\sqrt{7}\right)^2$

$\Rightarrow z^2 + 4 - 4z = 7i^2$

$\Rightarrow z^2 + 4 - 4z = -7$

$\Rightarrow \mathbf{z^2 - 4z + 15 = 0}$ .....(1)

Now $3z^3 - 4z^2 + z + 88 = 0$.

$\Rightarrow 3z(\mathbf{z^2 - 4z + 15}) + 8z^2 - 32z - 88$

$\Rightarrow 3z(0) - 8(\mathbf{z^2 - 4z + 15})$

$\Rightarrow 0 - 8(0)$

$=0.$

1. **Show that the four points in the argand diagram represented by the complex numbers $2+i, 4+3i, 2+5i, 3i$ are the vertices of square.**

Sol: **let** $A = -2 + 7i = (-2, 7)$, B$= -\frac{3}{2} + \frac{1}{2}i = \left(-\frac{3}{2}, \frac{1}{2}\right)$,

C$= 4 - 3i = (4, -3)$, D$= \frac{7}{2}(1+i) = \left(\frac{7}{2}, \frac{7}{2}\right)$

AB$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

A (-2, 7), B $\left(-\frac{3}{2}, \frac{1}{2}\right)$

AB$= \sqrt{\left(-\frac{3}{2} + 2\right)^2 + \left(\frac{1}{2} - 7\right)^2}$

$= \sqrt{\left(\frac{-3+4}{2}\right)^2 + \left(\frac{1-14}{2}\right)^2} = \sqrt{\frac{1}{4} + \frac{169}{4}} = \frac{\sqrt{170}}{2}$

B $\left(-\frac{3}{2}, \frac{1}{2}\right)$, C (4, -3)

BC$= \sqrt{\left(4 + \frac{3}{2}\right)^2 + \left(-3 - \frac{1}{2}\right)^2} = \sqrt{\left(\frac{11}{2}\right)^2 + \left(-\frac{7}{2}\right)^2} = \sqrt{\frac{121+49}{4}} = \frac{\sqrt{170}}{2}$

C (4, -3) D $\left(\frac{7}{2}, \frac{7}{2}\right)$

CD$= \sqrt{\left(\frac{7}{2} - 4\right)^2 + \left(\frac{7}{2} + 3\right)^2}$

$= \sqrt{\left(\frac{7-8}{2}\right)^2 + \left(\frac{7+6}{2}\right)^2} = \sqrt{\frac{1+169}{4}} = \frac{\sqrt{170}}{2}$

D $\left(\frac{7}{2}, \frac{7}{2}\right)$, A (-2, 7)

DA$= \sqrt{\left(-2 - \frac{7}{2}\right)^2 + \left(7 - \frac{7}{2}\right)^2} = \sqrt{\left(\frac{-4-7}{2}\right)^2 + \left(\frac{14-7}{2}\right)^2}$

$= \sqrt{\left(\frac{121}{4} + \frac{49}{4}\right)} = \frac{\sqrt{170}}{2}$

A (-2, 7), C (4, -3)

AC$= \sqrt{(4+2)^2 + (-3-7)^2} = \sqrt{(6)^2 + (-10)^2} =$
$\sqrt{36 + 100} = \sqrt{136}$

B $\left(-\frac{3}{2}, \frac{1}{2}\right)$ D $\left(\frac{7}{2}, \frac{7}{2}\right)$, BD$= \sqrt{\left(\frac{7}{2} + \frac{3}{2}\right)^2 + \left(\frac{7}{2} - \frac{1}{2}\right)^2}$

$= \sqrt{\left(\frac{10}{2}\right)^2 + \left(\frac{6}{2}\right)^2} = \sqrt{\frac{100}{4} + \frac{36}{4}} = \frac{\sqrt{136}}{2}$

AB=BC=CD=DA and AC≠BD

∴ Given complex number are the vertices of a rhombus.

2. **Show that the four points in the argand diagram represented by the complex numbers $2+i, 4+3i, 2+5i, 3i$ are the vertices of square.**

Sol: **let**

$A = 2 + i = (2, 1)$,

B$= 4 + 3i = (4, 3)$,

C$= 2 + 5i = (2, 5)$,

D$=3i=(0, 3)$

AB$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

A (2, 1), B (4, 3)

AB$= \sqrt{(4-2)^2 + (3-1)^2}$

$= \sqrt{(2)^2 + (2)^2} = \sqrt{4+4} = \sqrt{8}$

B (4, 3), C (2, 5)

BC$= \sqrt{(2-4)^2 + (5-3)^2} = \sqrt{(-2)^2 + (2)^2} = \sqrt{4+4} = \sqrt{8}$

C (2, 5) D (0, 3),

CD$= \sqrt{(0-2)^2 + (3-5)^2}$

$= \sqrt{(-2)^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8}$

D (0, 3) A (2, 1),

DA$= \sqrt{(2-0)^2 + (1-3)^2}$

$= \sqrt{(2)^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8}$

A (2, 1), C (2, 5)

AC$= \sqrt{(2-2)^2 + (5-1)^2} = \sqrt{(0)^2 + (4)^2} = \sqrt{16} = 4$

B (4, 3) D (0, 3), BD$= \sqrt{(0-4)^2 + (3-3)^2}$

$= \sqrt{(-4)^2 + (0)^2} = \sqrt{16} = 4$

AB=BC=CD=DA and AC=BD

∴ Given complex number are the vertices of a square.

1. Show that the points in the argand diagram represented by the complex numbers $2+2i, -2-2i, -2\sqrt{3}+2\sqrt{3}i$ are the vertices of an equilateral triangle.

Sol: ***let***

$A = 2+2i = (2,2)$,

B$= -2-2i = (-2,-2)$,

C$= -2\sqrt{3}+2\sqrt{3}i = (-2\sqrt{3},\ 2\sqrt{3})$

AB$=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

A (2, 2), B (-2, -2)

AB$=\sqrt{(-2-2)^2+(-2-2)^2}$

$=\sqrt{(-4)^2+(-4)^2}=\sqrt{16+16}=\sqrt{32}$

B (-2, -2), C ($-2\sqrt{3}, 2\sqrt{3}$)

BC$=\sqrt{\left(-2\sqrt{3}+2\right)^2+\left(2\sqrt{3}+2\right)^2}$

$=\sqrt{12+4-8\sqrt{3}+12+4+8\sqrt{3}}$

$=\sqrt{16+16}=\sqrt{32}$

C ($-2\sqrt{3}, 2\sqrt{3}$) A (2, 2),

CA$=\sqrt{\left(2+2\sqrt{3}\right)^2+\left(2-2\sqrt{3}\right)^2}$

$=\sqrt{12+4+8\sqrt{3}+12+4-8\sqrt{3}}$

$=\sqrt{16+16}=\sqrt{32}$

$$AB = BC = CA$$

1. If $Z = x+iy$ and if the point p in the argand plane represented by Z, find the locus of Z satisfying the equation $|z-2-3i| = 5$.

Sol:

Given $|z-2-3i| = 5$, *and* $Z = x+iy$

⇨ $|x+iy-2-3i| = 5$

⇨ $|(x-2)+i(y-3)| = 5$

⇨$\sqrt{(x-2)^2+(y-3)^2}=5$ S.O.B

⇨$(x-2)^2+(y-3)^2 = 25$

⇨$x^2+4-4x+y^2+9-6y-25=0$

$\therefore x^2+y^2-4x-6y-12=0$

3. *if* $(x-iy)^{1/3} = a-ib$, *then show that* $\frac{x}{a}+\frac{y}{b} = 4(a^2-b^2)$.

Sol: Given $(x-iy)^{1/3} = a-ib$

⇨ $(x-iy) = (a-ib)^3$

⇨ $(x-iy) = a^3 - (ib)^3 - 3a^2(ib) + 3a(ib)^2$

⇨ $(x-iy) = a^3 - b^3(-i) - i3a^2b + 3a(b^2(-1))$

⇨ $(x-iy) = a^3 + 3ab^2 + ib^3 - i3a^2b$

⇨ $(x-iy) = (a^3-3ab^2) + i(b^3-3a^2b)$

equating the real & imaginary parts on both sides we get,

$x = (a^3-3ab^2)$, $y = (b^3-3a^2b)$

⇨ $x = a(a^2-3b^2)$, $y = -b(b^2-3a^2)$

⇨ $\frac{x}{a} = (a^2-3b^2)$, $\frac{y}{b} = -(b^2-3a^2)$

Now $\frac{x}{a}+\frac{y}{b} = a^2-3b^2-b^2+3a^2$

⇨ $\frac{x}{a}+\frac{y}{b} = 4a^2-4b^2$ $\therefore \frac{x}{a}+\frac{y}{b} = 4(a^2-b^2)$

If $x+iy = \frac{1}{1+cos\theta+isin\theta}$ then S.T $4x^2-1=0$.

Sol: $x+iy = \frac{1}{1+cos\theta+isin\theta} = \frac{1}{(1+cos\theta)+isin\theta}$

$= \frac{1}{\left(2cos^2\left(\frac{\theta}{2}\right)\right)+i\left(2sin\left(\frac{\theta}{2}\right)cos\left(\frac{\theta}{2}\right)\right)}$

$= \frac{1}{2cos\left(\frac{\theta}{2}\right)\left[cos\left(\frac{\theta}{2}\right)+isin\left(\frac{\theta}{2}\right)\right]}$

$= \frac{1}{2cos\left(\frac{\theta}{2}\right)\left[cos\left(\frac{\theta}{2}\right)+isin\left(\frac{\theta}{2}\right)\right]}\frac{\left[cos\left(\frac{\theta}{2}\right)-isin\left(\frac{\theta}{2}\right)\right]}{\left[cos\left(\frac{\theta}{2}\right)-isin\left(\frac{\theta}{2}\right)\right]}$

$= \frac{cos\left(\frac{\theta}{2}\right)-isin\left(\frac{\theta}{2}\right)}{2cos\left(\frac{\theta}{2}\right)\left[cos^2\left(\frac{\theta}{2}\right)-i^2sin^2\left(\frac{\theta}{2}\right)\right]}$

$= \frac{cos\left(\frac{\theta}{2}\right)-isin\left(\frac{\theta}{2}\right)}{2cos\left(\frac{\theta}{2}\right)\left[cos^2\left(\frac{\theta}{2}\right)+sin^2\left(\frac{\theta}{2}\right)\right]}$

$= \frac{cos\left(\frac{\theta}{2}\right)-isin\left(\frac{\theta}{2}\right)}{2cos\left(\frac{\theta}{2}\right)}$

$x+iy = \frac{cos\left(\frac{\theta}{2}\right)}{2cos\left(\frac{\theta}{2}\right)} - \frac{isin\left(\frac{\theta}{2}\right)}{2cos\left(\frac{\theta}{2}\right)}$

$x+iy = \frac{1}{2} - \frac{i}{2}tan\left(\frac{\theta}{2}\right)$

equating the real & imaginary parts on both sides we get,

$x = \frac{1}{2}$ ⇨ $2x = 1$ *S.O.B*

$4x^2 = 1$ *or* $4x^2-1=0$.

If the point p denotes the complex number Z= $x+iy$ in the argand plane and if $\frac{Z-i}{z-1}$ is a purely imaginary no. find the locus of p.

Sol: Given Z= $x+iy$

$\Rightarrow \frac{Z-i}{z-1} = \frac{x+iy-i}{x+iy-1} = \frac{x+i(y-1)}{(x-1)+iy}$

$\Rightarrow \frac{[x+i(y-1)]\times[(x-1)-iy]}{[(x-1)+iy]\times[(x-1)-iy]}$

$\Rightarrow \frac{x(x-1)-xyi+i(y-1)(x-1)-i^2(y-1)y}{(x-1)^2-(iy)^2}$

$\Rightarrow \frac{x^2-x-xyi+i(xy-y-x+1)+y^2-y}{(x-1)^2-y^2i^2}$

$\Rightarrow \frac{[x^2+y^2-x-y]i[xy-y-x+1]}{(x-1)^2+y^2}$

$\therefore \frac{Z-i}{z-1}$ is a purely imaginary number$\Rightarrow$ real part=0

$\frac{x^2+y^2-x-y}{(x-1)^2+y^2} = 0 \Rightarrow\ x^2+y^2-x-y=0.$

1. If the amplitude of $\left(\frac{Z-2}{z-6i}\right) = \frac{\pi}{2}$, *find its locus*.

Sol: *let* $Z = x+iy$

$\Rightarrow \frac{Z-2}{z-6i} = \frac{x+iy-2}{x+iy-6i} = \frac{(x-2)+iy}{x+i(y-6)}$

$\Rightarrow \frac{[(x-2)+iy]\times[x-i(y-6)]}{[x+i(y-6)]\times[x-i(y-6)]}$

$\Rightarrow \frac{(x-2)x-(x-2)(y-6)i+xyi-i^2y(y-6)}{(x)^2-(i)^2(y-6)^2}$

$\Rightarrow \frac{x^2-2x-(xy-6x-2y+12)i+i(xy)+y^2-6y}{x^2+(y-6)^2}$

$\Rightarrow \frac{[x^2+y^2-2x-6y]-i[6x-2y+12]}{x^2+(y-6)^2}$

Amplitude of $\left(\frac{Z-2}{z-6i}\right) = \frac{\pi}{2}\left\{\tan^{-1}\left|\frac{b}{a}\right| = \frac{\pi}{2}\right\}$

$\Rightarrow$ real part(a)=0, b>0

$\Rightarrow \frac{x^2+y^2-2x-6y}{x^2+(y-6)^2} = 0$

$\therefore x^2+y^2-2x-6y=0$ and

$3x+y-6>0$

2. **Determine the locus of z, z$\neq$ $2i$, *such that* $\left(\frac{Z-4}{z-2i}\right) = 0$.**

Sol: *let* $Z = x+iy$

$\Rightarrow \frac{Z-4}{z-2i} = \frac{x+iy-4}{x+iy-2i} = \frac{(x-4)+iy}{x+i(y-2)}$

$\Rightarrow \frac{[(x-4)+iy]\times[x-i(y-2)]}{[x+i(y-2)]\times[x-i(y-2)]}$

$\Rightarrow \frac{(x-4)x-(x-4)(y-2)i+xyi-i^2y(y-2)}{(x)^2-(i)^2(y-2)^2}$

$\Rightarrow \frac{x^2-4x-(xy-2x-4y+8)i+i(xy)+y^2-2y}{x^2+(y-2)^2}$

$\Rightarrow \frac{[x^2+y^2-4x-2y]+i[2x+4y-8]}{x^2+(y-2)^2}$

Real part of $\left(\frac{Z-4}{z-2i}\right) = 0$

$\Rightarrow$ real part(a)=0,

$\Rightarrow \frac{x^2+y^2-4x-2y}{x^2+(y-2)^2} = 0 \therefore x^2+y^2-4x-2y=0$

9. **Find the real value of θ *in order that* $\left(\frac{3+2isin\theta}{1-2isin\theta}\right)$ *is a***

(a) Purely imaginary number

(b) Real number.

Sol:

$\left(\frac{3+2isin\theta}{1-2isin\theta}\right) = \frac{[3+2isin\theta][1+2isin\theta]}{[1-2isin\theta][1+2isin\theta]}$

$\Rightarrow \frac{[3+6isin\theta+2isin\theta+4i^2sin^2\theta]}{(1)^2-(2i)^2(sin\theta)^2}$

$\Rightarrow \frac{[(3-4sin^2\theta)+i(8sin\theta)]}{1+4sin^2\theta}$

$\Rightarrow \left(\frac{(3-4sin^2\theta)}{1+4sin^2\theta}\right) + i\left(\frac{8sin\theta}{1+4sin^2\theta}\right)$

(a) Purely imaginary number $\Rightarrow$real =0

$\Rightarrow \frac{3-4sin^2\theta}{1+4sin^2\theta} = 0 \Rightarrow 3-4sin^2\theta = 0$

$\Rightarrow sin^2\theta = \frac{3}{4} \Rightarrow sin\theta = \frac{\sqrt{3}}{2}$

$\Rightarrow \theta = n\pi \pm \frac{\pi}{3}, n \in z$

(b) Real number $\Rightarrow$imaginary part =0

$\Rightarrow \frac{8sin\theta}{1+4sin^2\theta} = 0 \Rightarrow sin\theta = 0 \Rightarrow \theta = 0$

$\Rightarrow \theta = n\pi, n \in z$

10. **The points, P, Q denote the complex numbers z_1, z_2 in the argand diagram. O is the origin. If $z_1\overline{z_2}+\overline{z_1}z_2=0$, then show that $\lfloor POQ=90°$.**

Sol:

$let\ p(x_1,y_1)$⇨$z_1=x_1+iy_1$; ⇨$\overline{z_1}=x_1-iy_1$

$Q(x_2,y_2)$⇨$z_2=x_2+iy_2$; ⇨$\overline{z_2}=x_2-iy_2$

$and\ O(0,0)$

$given\ z_1\overline{z_2}+\overline{z_1}z_2=0$

⇨$(x_1+iy_1)(x_2-iy_2)+(x_1-iy_1)(x_2+iy_2)=0$

⇨$x_1x_2-ix_1y_2+iy_1x_2-y_1y_2i^2$

$+x_1x_2+ix_1y_2-iy_1x_2-y_1y_2i^2=0$

⇨$2x_1x_2+2y_1y_2=0$

⇨$y_1y_2=-x_1x_2$ ⇨$\left(\frac{y_1}{x_1}\right)=-\left(\frac{x_2}{y_2}\right)$

⇨$\left(\frac{y_1}{x_1}\right)\left(\frac{y_2}{x_2}\right)=-1$

⇨$(slope\ of\ \overline{OP})(slope\ of\ \overline{OQ})=-1$

⇨$\lfloor POQ=90°$.

9. **Show that the points in the argand diagram represented by the complex numbers z_1, z_2, z_3 are collinear if and only if there exist three real numbers p, q, r not all zero, satisfying $pz_1+qz_1+r\,z_3=0$ *and***

$$p+q+r=0$$

Sol: $p+q+r=0$.......(1)

$pz_1+qz_1+r\,z_3=0$ ... . (2)

⇨$z_1=-\frac{(qz_1+r\,z_3)}{p}$

⇨$z_1=-\frac{(qz_1+r\,z_3)}{-(q+r)}$

$\therefore z_1=\frac{(qz_1+r\,z_3)}{(q+r)}$

$\therefore z_1\ divides\ the\ line\ segment\ joining\ z_2, z_3$

In the ratio r:q.

z_1, z_2, z_3 are collinear