LOCUS

Definition: The set of all points (and only those points) which satisfy the given geometrical condition(s) (or properties) is called a locus.

Eg. The set of points in a plane which are at a constant distance r from a given point C is a locus. Here the locus a circle.

2. The set of points in a plane which are equidistant from two given points A and B is a locus. Here the locus is a straight line and it is the perpendicular bisector of the line segment joining A and B.

EQUATION OF A LOCUS

An equation $f(x, y) = 0$ is said to be the equation of a locus S if every point of S satisfies

$f(x, y) = 0$ and every point that satisfies $f(x, y) = 0$ belongs to S.

An equation of a locus is an algebraic description of the locus. This can be obtained in the following way

(i) Consider a point $P(x, y)$ on the locus

(ii) Write the geometric condition(s) to be satisfied by P in terms of an equation or in equation in symbols.

(iii) Apply the proper formula of coordinate geometry and translate the geometric condition(s) into an algebraic equation.

(iv) Simplify the equation so that it is free from radicals.

The equation thus obtained is the required equation of locus.

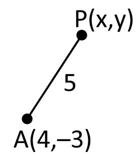
EXERCISE - 1A

I.

1. Find the equation of locus of a point which is at a distance 5 from A(4, –3).

Sol. Let P(x, y) be a point in the locus.

Given A(4, –3)



Given that CP = 5

$\Rightarrow CP^2 = 25$

$\Rightarrow (x - 4)^2 + (y + 3)^2 = 25$

$\Rightarrow x^2 - 8x + 16 + y^2 + 6y + 9 - 25 = 0$

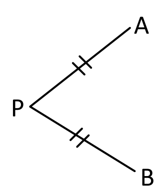
$\therefore$ Equation of the locus of P is:

$x^2 + y^2 - 8x + 6y = 0$

2. Find the equation of locus of a point which is equidistant from the points A(–3, 2) and B(0, 4).

Sol. Given points are A(–3, 2), B(0, 4)

Let P(x, y) be any point in the locus



Given that PA = PB

$\Rightarrow PA^2 = PB^2$

$\Rightarrow (x + 3)^2 + (y - 2)^2 = (x - 0)^2 + (y - 4)^2$

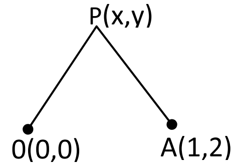
$\Rightarrow x^2 + 6x + 9 + y^2 - 4y + 4 = x^2 + y^2 - 8y + 16$

$\Rightarrow$ 6x + 4y = 3 is the equation of the locus.

3. Find the equation of locus of a point P such that the distance of P from the origin is twice the distance of P from A (1, 2).

Sol. Given points are O (0, 0), A (1, 2)

Let P(x, y) be any point in the locus



Given that OP = 2AP

$\Rightarrow OP^2 = 4AP^2$

$\Rightarrow x^2 + y^2 = 4[(x-1)^2 + (y-2)^2]$

$= 4(x^2 - 2x + 1 + y^2 - 4y + 4)$

$\Rightarrow x^2 + y^2 = 4x^2 + 4y^2 - 8x - 16y + 20$

$\therefore$ Equation to the locus of P is

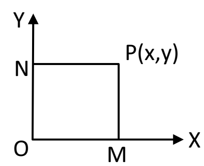
$3x^2 + 3y^2 - 8x - 16y + 20 = 0$

4. Find the equation of locus of a point which is equidistant from the coordinate axes.

Sol. Let P(x, y) be any point in the locus.

Let PM = perpendicular distance of P from X-axis. = $|x|$

Let PN = perpendicular distance of P from Y-axis. = $|y|$



Given PM = PN $\Rightarrow |x| = |y|$

Squaring on both sides, $x^2 = y^2$

Therefore, Locus of P is $x^2 - y^2 = 0$

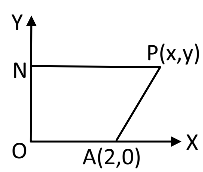
5. Find the equation of locus of a point equidistant from A(2, 0) and the Y-axis.

Sol. Given point is A (2, 0)

Let P(x, y) be any point in the locus.

Draw PN perpendicular to Y-axis.= $|x|$

Given that is PA = PN



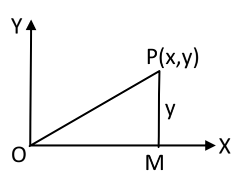
$\Rightarrow PA^2 = PN^2$

$\Rightarrow (x - 2)^2 + (y - 0)^2 = x^2$

$\Rightarrow x^2 - 4x + 4 + y^2 = x^2$

$\therefore$ Locus of P is $y^2 - 4x + 4 = 0$

6. Find the equation of locus of a point P, the square of whose distance from the origin is 4 times its y coordinates.



Sol. Let P(x, y) be any point in the locus.

Now $OP^2 = x^2 + y^2$

Given condition is $OP^2 = 4y \Rightarrow x^2 + y^2 = 4y$

Equation of the locus of P is $x^2 + y^2 - 4y = 0$

7. Find the equation of locus of a point P such that $PA^2 + PB^2 = 2c^2$, where A= (a, 0), B(–a, 0) and 0 < |a| < |c|.

Sol. Let P(x, y) be a point in locus.

Given A = (a, 0), B = (–a, 0)

Given that $PA^2 + PB^2 = 2c^2$

$\Rightarrow (x - a)^2 + (y - 0)^2 + (x + a)^2 + (y - 0)^2 = 2c^2$

$\Rightarrow x^2 - 2ax + a^2 + y^2 + x^2 + 2ax + a^2 + y^2 = 2c^2$

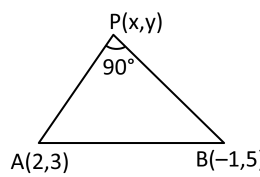
$\Rightarrow 2x^2 + 2y^2 = 2c^2 - 2a^2$

$\therefore x^2 + y^2 = c^2 - a^2$ is the locus of P.

II.

1. Find the equation of locus of P, if the line segment joining (2, 3) and (–1, 5) subtends a right angle at P.

Sol. Given points A (2, 3), B (–1, 5).



Let P(x, y) be any point in the locus.

Given condition is: $\angle APB = 90°$

$\Rightarrow$ (Slope of $\overline{AP}$) (Slope of $\overline{BP}$) = –1

$\Rightarrow \dfrac{y-3}{x-2} \cdot \dfrac{y-5}{x+1} = -1$

$(y-3)(y-5) + (x-2)(x+1) = 0$

$x^2 + y^2 - x - 8y + 13 = 0$

$\therefore$ Locus of P is $x^2 + y^2 - x - 8y + 13 = 0$

2. The ends of the hypotenuse of a right angled triangle are (0, 6) and (6, 0). Find the equation of locus of its third vertex.

Sol. Same as above.

3. Find the equation of locus of a point, the difference of whose distances from (–5, 0) and (5, 0) is 8 units.

Sol. Given points are A (5, 0), B (–5, 0)

Let P(x, y) be any point in the locus

Given |PA – PB| = 8

$\Rightarrow PA - PB = \pm 8$

$\Rightarrow PA = \pm 8 + PB$

Squaring on both sides

$PA^2 = 64 + PB^2 \pm 16PB$

$PA^2 - 64 - PB^2 = \pm 16PB$

$\Rightarrow (x-5)^2 + y^2 - (x+5)^2 - y^2 - 64 = \pm 16PB$

$-4 \cdot 5 \cdot x - 64 = \pm 16PB$

$-5x - 16 = \pm 4PB$

Squaring on both sides

$25x^2 + 256 + 160x = 16(PB)^2$

$= 16[(x+5)^2 + y^2]$

$= 16x^2 + 400 + 160x + 16y^2$

$\Rightarrow 9x^2 - 16y^2 = 144$

$\Rightarrow \dfrac{9x^2}{144} - \dfrac{16y^2}{144} = 1$

$\Rightarrow$ locus of P is $\dfrac{x^2}{16} - \dfrac{y^2}{9} = 1$

4. **Find the equation of locus of P, if A(4, 0), B(–4, 0) and |PA – PB| = 4.**

Sol. Same as above.

5. **Find the equation of locus of a point, the sum of whose distances from (0, 2) and (0, –2) is 6.**

Sol. Given points are A (0, 2) and B (0, –2)

Let P(x, y) be any point in the locus.

Given PA + PB = 6

$\Rightarrow PA = 6 - PB$

Squaring on both sides

$PA^2 = 36 + PB^2 - 12PB$

$12PB = PB^2 - PA^2 + 36$

$= x^2 + (y+2)^2 - [x^2 + (y-2)^2] + 36$

$\Rightarrow 12PB = 4 \cdot 2 \cdot y + 36$

$\Rightarrow 3PB = 2y + 9$

squaring on both sides

$9PB^2 = 4y^2 + 36y + 81$

$\Rightarrow 9[x^2 + (y+2)^2] = 4y^2 + 36y + 81$

$\Rightarrow 9x^2 + 9y^2 + 36 + 36y = 4y^2 + 36y + 81$

$\Rightarrow 9x^2 + 5y^2 = 45$

$\Rightarrow \dfrac{9x^2}{45} + \dfrac{5y^2}{45} = 1 \Rightarrow$ Locus of P is $\dfrac{x^2}{5} + \dfrac{y^2}{9} = 1$.

6. **Find the equation of locus of P, if A(2, 3), B(2, –3) and PA + PB = 8**.

Sol. Same as above.

7. A(5, 3) and B(3, –2) are two fixed points. Find the equation of locus of P, so that the area of triangle PAB is 9.

Sol. Given points are A(5, 3), B(3, –2)

Let P(x, y) be a point in the locus.

Given, area of ΔAPB = 9.

$$\Rightarrow \frac{1}{2}\begin{vmatrix} x-5 & y-3 \\ 3-5 & -2-3 \end{vmatrix} = 9$$

$$\Rightarrow \begin{vmatrix} x-5 & y-3 \\ -2 & -5 \end{vmatrix} = 18$$

$$\Rightarrow |-5x+25+2y-6| = 18$$

$$\Rightarrow |-5x+2y+19| = 18$$

$$\Rightarrow -5x+2y+19 = \pm 18$$

$$\Rightarrow -5x+2y+19 = 18 \text{ or } -5x+2y+19 = 18$$

$$\Rightarrow 5x-2y-1 = 0 \text{ or } 5x-2y-37 = 0$$

$\therefore$ Locus of P is :

$(5x – 2y – 1)(5x – 2y – 37) = 0.$

8. Find the equation of locus of a point which forms a triangle of area 2 with the point A(1, 1) and B(–2, 3).

Sol. Same as above.

Ans . $(2x + 3y – 1)(2x + 3y – 9) = 0$

9. If the distance from P to the points (2, 3) and (2, –3) are in the ratio 2 : 3, then find the equation of locus of P.

Sol. Let P(x, y) be a point in locus.

Given points are A(2, 3), B(2, –3)

Given that PA : PB = 2 : 3

$\Rightarrow$ 3PA = 2PB

$\Rightarrow 9PA^2 = 4PB^2$

$\Rightarrow 9[(x–2)^2 + (y–3)^2] = 4[(x–2)^2 + (y+3)^2]$

$\Rightarrow 9[x^2 – 4x + 4 + y^2 – 6y + 9] = 4[x^2 – 4x + 4 + y^2 + 6y + 9]$

$\Rightarrow 5x^2 + 5y^2 – 20x – 78y + 65 = 0$ which is the equation of locus.

10. A(1, 2), B(2, –3) and C(–2, 3) are three points. A point P moves such that $PA^2 + PB^2 = 2PC^2$. Show that the equation to the locus of P is 7x – 7y + 4 = 0.

Sol. Let P(x, y) be a point in locus.

Given points are A (1, 2), B (2, –3) and C(–2, 3)

Given that $PA^2 + PB^2 = 2PC^2$

$\Rightarrow (x-1)^2 + (y-2)^2 + (x-2)^2 + (y+3)^2 = 2[(x+2)^2 + (y-3)^2]$

$\Rightarrow 2x^2 + 2y^2 - 6x + 2y + 18 = \quad 2x^2 + 2y^2 + 8x - 12y + 26$

$\Rightarrow 14x - 14y + 8 = 0$

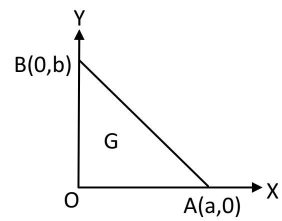
$\Rightarrow 7x - 7y + 4 = 0$

Therefore, equation of locus is $7x - 7y + 4 = 0$

11. A straight rod of length 9 slides with its ends A, B always on the X and Y-axes respectively. Then find the locus of the centroid of ΔOAB.

Sol. The given rod AB meets X-axis at A and Y-axis at B.

Let OA = a and OB = b and AB =9.



Coordinates of A are (a, 0) and B are (0, b).

Let G (x,y) be the centroid of ΔOAB

But Coordinates of G of **ΔOAB** are $\left(\frac{a}{3}, \frac{b}{3}\right)$

Therefore, $\left(\frac{a}{3}, \frac{b}{3}\right) = (x, y)$

$\Rightarrow \frac{a}{3} = x, \frac{b}{3} = y \Rightarrow a = 3x, b = 3y$

But $OA^2 + OB^2 = AB^2$ and given AB = 9

$\Rightarrow a^2 + b^2 = 81$

$\Rightarrow 9(x^2 + y^2) = 81$

$\therefore$ Equation to the locus of P is $x^2 + y^2 = 9$.

Problems for practice

1. Find the equation of the locus of a point which is at a distance 5 from (–2, 3) in a plane.

Ans. $x^2 + y^2 + 4x - 6y - 12 = 0$.

2. Find the equation of locus of a point P, if the distance of P from A(3, 0) is twice the distance of P from B(–3, 0).

Ans. $x^2 + y^2 + 10x + 9 = 0$.

3. Find the locus of the third vertex of a right angled triangle, the ends of whose hypotenuse are (4, 0) and (0, 4).

Ans. $x^2 + y^2 - 4x - 4y = 0$

4. Find the equation of locus of P, if the ratio of the distances from P to (5, –4) and (7, 6) is 2 : 3.

Ans. $5(x^2 + y^2) - 34x + 120y + 29 = 0$.

5. A(2, 3) and B(–3, 4) are two given points. Find the equation of locus of P so that the area of the triangle PAB is 8.5.

Ans. $x^2 + 10xy + 25y^2 - 34x - 170y = 0$