

PREREQUISITES

(2-D GEOMETRY)

DISTANCE BETWEEN TWO POINTS

(i) The distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$

$$AB(\text{or } BA) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

(ii) The distance from origin O to the point $A(x_1, y_1)$ is $OA = \sqrt{x_1^2 + y_1^2}$

(iii) The distance between two points $A(x_1, 0)$ and $B(x_2, 0)$ lying on the X -axis

$$\text{is } AB = \sqrt{(x_1 - x_2)^2 + (0 - 0)^2} = \sqrt{(x_1 - x_2)^2} = |x_1 - x_2|$$

(iv) The distance between two points $C(0, y_1)$ and $D(0, y_2)$ lying on the Y -axis is $CD = |y_1 - y_2|$

SECTION FORMULA

(i) The point P which divides the line segment joining the points $A(x_1, y_1)$,

$B(x_2, y_2)$ in the ratio $m : n$ internally is given by

$$P = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n} \right) \quad (m+n \neq 0)$$

(ii) If P divides in the ratio $m:n$ externally then $P = \left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n} \right)$ $(m \neq n)$

Note: If the ratio $m : n$ is positive then P divides internally and if the ratio is negative P divides externally.

MID POINT

The mid point of the line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$ is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$

POINTS OF TRISECTION

The points which divide the line segment \overline{AB} in the ratio $1 : 2$ and $2 : 1$ (internally) are called the points of trisection of \overline{AB} .

AREA OF A TRIANGLE

The area of the triangle formed by the points $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ is

$$\text{Area} = \frac{1}{2} |(x_1 y_2 - x_2 y_1) + (x_2 y_3 - x_3 y_2) + (x_3 y_1 - x_1 y_3)|$$

$$\text{i.e., Area of ABC} = \frac{1}{2} \left| \sum (x_1 y_2 - x_2 y_1) \right|$$

Note:1. The area of the triangle formed by the points

(x_1, y_1) , (x_2, y_2) , (x_3, y_3) is the positive value of the determinant $\frac{1}{2} \begin{vmatrix} x_1 - x_2 & y_1 - y_2 \\ x_2 - x_3 & y_2 - y_3 \end{vmatrix}$.

2. The area of the triangle formed by the points (x_1, y_1) , (x_2, y_2) and the origin is $\frac{1}{2} |x_1 y_2 - x_2 y_1|$

AREA OF A QUADRILATERAL

The area of the quadrilateral formed by the points (x_1, y_1) , (x_2, y_2) , (x_3, y_3) , (x_4, y_4) taken in that order is

$$\frac{1}{2} |x_1 y_2 - x_2 y_1 + x_2 y_3 - x_3 y_2 + x_3 y_4 - x_4 y_3 + x_4 y_1 - x_1 y_4|$$

Note 1: The area of the quadrilateral formed by the points (x_1, y_1) , (x_2, y_2) , (x_3, y_3) , (x_4, y_4) taken in order is

$$\text{Area} = \frac{1}{2} \begin{vmatrix} x_1 - x_3 & y_1 - y_3 \\ x_2 - x_4 & y_2 - y_4 \end{vmatrix}$$

CENTRES OF A TRIANGLE

Median : In a triangle, the line segment joining a vertex and the mid point of its opposite side is called a median of the triangle. The medians of a triangle are concurrent.

The point of concurrence of the medians of a triangle is called the centroid (or) centre of gravity of the triangle. It is denoted by G.

IN CENTRE OF A TRIANGLE

Internal bisector : The line which bisects the internal angle of a triangle is called an internal angle bisector of the triangle.

The point of concurrence of internal bisectors of the angles of a triangle is called the incentre of the triangle. It is denoted by I.

$$I = \left(\frac{ax_1 + bx_2 + cx_3}{a + b + c}, \frac{ay_1 + by_2 + cy_3}{a + b + c} \right)$$

EXCENTRES OF A TRIANGLE:

The point of concurrence of internal bisector of angle A and external bisectors of angles B, C of ABC is called the ex-centre opposite to vertex A. It is denoted by I_1 . The excentres of ABC opposite to the vertices B, C are respectively denoted by I_2, I_3 .

$$I_1 = \text{Excentre opposite to A} = \left(\frac{-ax_1 + bx_2 + cx_3}{-a + b + c}, \frac{-ay_1 + by_2 + cy_3}{-a + b + c} \right)$$

$$I_2 = \text{Excentre opposite to B} = \left(\frac{ax_1 - bx_2 + cx_3}{a - b + c}, \frac{ay_1 - by_2 + cy_3}{a - b + c} \right)$$

$$I_3 = \text{Excentre opposite to C} = \left(\frac{ax_1 + bx_2 - cx_3}{a + b - c}, \frac{ay_1 + by_2 - cy_3}{a + b - c} \right)$$

ORTHO CENTRE OF A TRIANGLE

Altitude: The line passing through vertex and perpendicular to opposite side of a triangle is called an altitude of the triangle. Altitudes of a triangle are concurrent. The point of concurrence is called the ortho centre of the triangle. It is denoted by "O" or 'H'.

Circum centre of a Triangle:

Perpendicular bisector: The line passing through mid point of a side and perpendicular to the side is called the perpendicular bisector of the side.

The perpendicular bisectors of the sides of a triangle are concurrent. The point of concurrence is called **the** circum centre or the triangle. It is denoted by S.

EXERCISE

I. Find the distance between the following pairs of points. i) (4, 5), (5, 4) ii) (-3, 1), (3, 2) iii) $(a \cos a, a \sin a)$, $(a \cos b, a \sin b)$

Sol.

i) Let A = (4, 5), B = (5, 4) $\Rightarrow AB = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$$\Rightarrow AB = \sqrt{(4-5)^2 + (5-4)^2} = \sqrt{1+1} = \sqrt{2}$$

ii) Let A = (-3, 1), B = (3, 2) $\Rightarrow AB = \sqrt{(3+3)^2 + (2-1)^2} = \sqrt{36+1} = \sqrt{37}$

iii) Let $A=(a \cos a, a \sin a)$, $B=(a \cos b, a \sin b)$

$$AB = \sqrt{(a \cos \alpha - a \cos \beta)^2 + (a \sin \alpha - a \sin \beta)^2} = \sqrt{a^2 [2 - 2 \cos(\alpha - \beta)]} = \sqrt{2a^2 [1 - \cos(\alpha - \beta)]}$$

$$= \sqrt{4a^2 \sin^2 \left(\frac{\alpha - \beta}{2} \right)} = 2 \left| a \sin \left(\frac{\alpha - \beta}{2} \right) \right|$$

2. Find the value of 'a' if the distance between the points (a, 2), (3, 4) is $2\sqrt{2}$.

Sol. P (a, 2), Q (3, 4) are the given points.

$$PQ = 2\sqrt{2} \Rightarrow PQ^2 = 8$$

$$\Rightarrow (a - 3)^2 + (2 - 4)^2 = 8$$

$$\Rightarrow (a - 3)^2 = 8 - 4 = 4$$

$$\Rightarrow a - 3 = \pm 2 \Rightarrow a = 3 \pm 2 = 5 \text{ or } 1.$$

3. Find the point on the x-axis, which is equidistant from (7, 6) and (-3, 4).

Sol. Let A(7, 6), B(-3, 4)

Let P(x, 0) be the point on x-axis which is equidistant from A and B.

$$\text{Then } PA = PB \Rightarrow PA^2 = PB^2$$

$$\Rightarrow (x - 7)^2 + (0 - 6)^2 = (x + 3)^2 + (0 - 4)^2$$

$$\Rightarrow x^2 - 14x + 49 + 36 = x^2 + 6x + 9 + 16$$

$$\Rightarrow -14x + 85 = 6x + 25 \Rightarrow 20x = 60 \Rightarrow x = 3$$

The required point is P (3, 0).

4. Find the relation between x and y, if the point (x,y) is to be equidistant from (6,-1) and (2,3).

Sol. P(x, y), A (6, -1), B (2, 3) are the given points.

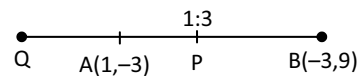
$$PA = PB \Rightarrow PA^2 = PB^2$$

$$\Rightarrow (x - 6)^2 + (y + 1)^2 = (x - 2)^2 + (y - 3)^2$$

$$\Rightarrow x^2 - 12x + 36 + y^2 + 2y + 1 = x^2 - 4x + 4 + y^2 - 6y + 9$$

$$\Rightarrow 8y = 8x - 24 \Rightarrow y = x - 3$$

5. Find the points which divide the line segment joining A (1, -3) and B (-3, 9) in the ratio 1: 3 (i) internally and (ii) externally.



Sol. (i) Given points are A(1, -3) and B(-3, 9)

Let P be the point dividing AB internally in the ratio 1: 3.

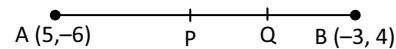
Then coordinates of P are

$$\left(\frac{1(-3) + 3 \cdot 1}{1+3}, \frac{1 \cdot 9 + 3(-3)}{1+3} \right) = \left(\frac{3-3}{4}, \frac{9-9}{4} \right) = (0, 0)$$

Let Q be the point dividing AB externally in the ratio 1 : 3. Then coordinates of Q are

$$\left(\frac{(1)(-3) - 3 \cdot 1}{1-3}, \frac{1 \cdot 9 - 3(-3)}{1-3} \right) = \left(\frac{-3-3}{-2}, \frac{9+9}{-2} \right) = (3, -9)$$

6. Find the points of trisection of the line segment joining (5, -6) and (-3, 4).



Sol. Given points are A (5, -6), B (-3, 4)

Let P and Q be the points of trisection of AB.

Let P divides AB in the ratio 1: 2 then Coordinates of P are $\left(\frac{1(-3) + 2(5)}{1+2}, \frac{1(4) + 2(-6)}{1+2} \right)$

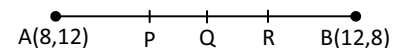
$$= \left(\frac{-3+10}{3}, \frac{4-12}{3} \right) = \left(\frac{7}{3}, \frac{-8}{3} \right)$$

Let Q divides AB in the ratio 2 : 1, then Coordinates of Q are $\left(\frac{2(-3) + 1(5)}{2+1}, \frac{2(4) + 1(-6)}{2+1} \right)$

$$= \left(\frac{-6+5}{3}, \frac{8-6}{3} \right) = \left(\frac{-1}{3}, \frac{2}{3} \right)$$

7. Find the points which divide the line segment joining (8, 12) and (12, 8) into four equal parts.

Sol. Given points A(8, 12), B(12, 8).



Let P, Q, R be the points dividing AB into four equal parts.

Let P divides AB in the ratio 1: 3, then

Coordinates of P are $\left(\frac{1(12) + 3(8)}{1+3}, \frac{1(8) + 3(12)}{1+3} \right) = (9, 11)$

Q divides AB in the ratio 1: 1

Coordinates of Q are $\left(\frac{8+12}{1+1}, \frac{12+8}{2} \right) = \left(\frac{20}{2}, \frac{20}{2} \right) = (10, 10)$

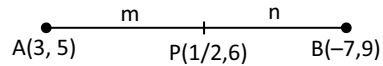
R divides AB in the ratio 3: 1, then Coordinates of R are:

$$\left(\frac{3(12) + 1(8)}{3+1}, \frac{3(8) + 1(12)}{3+1} \right) = (11, 9)$$

P (9, 11), Q (10, 10), R (11, 9) are the required points.

8. Find the ratio in which the point (1/2, 6) divides the line segment joining (3, 5) and (-7, 9).

Sol. Let A (3, 5), B (-7, 9), P (1/2, 6). P divides AB in the ratio m: n.

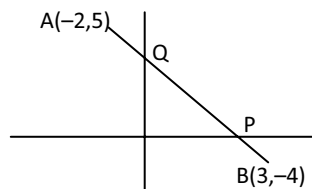


$$\frac{AP}{PB} = \frac{m}{n} = \frac{x - x_1}{x_2 - x} = \frac{\frac{1}{2} - 3}{-7 - \frac{1}{2}} = \frac{-\frac{5}{2}}{-\frac{15}{2}} = \frac{1}{3}$$

∴ P divides AB in the ratio 1 : 3.

9. In what ratio do the coordinate axes divide the line segment joining (-2, 5) and (3, -4).

Sol. A(-2, 5), B(3, -4) are the given points. AB meets the X-axis in P and Y-axis in Q.



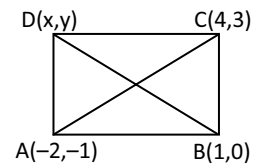
Ratio in which X-axis divides AB = $-y_1 : y_2$ i.e., $= -5 : -4 = 5 : 4$

Ratio in which Y-axis divides AB = $-x_1 : x_2$ i.e., $= 2 : 3$

10. If (-2, -1), (1, 0) and (4, 3) are three successive vertices of a parallelogram, find the fourth vertex.

Sol. Vertices of a parallelogram are A (-2, -1), B (1, 0), C (4, 3)

Let 4th vertex be D(x, y)



Diagonals AC and BD bisect each other.

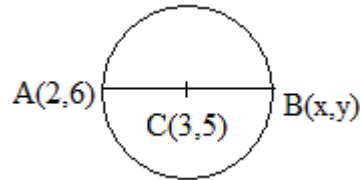
Midpoint of AC = Midpoint of BD

$$\Rightarrow \left(\frac{-2+4}{2}, \frac{-1+3}{2} \right) = \left(\frac{1+x}{2}, \frac{0+y}{2} \right) \Rightarrow \frac{x+1}{2} = \frac{2}{2} \Rightarrow x+1=2 \Rightarrow x=1$$

$$\Rightarrow \frac{y}{2} = \frac{2}{2} \Rightarrow y=2. \quad \text{Therefore, Coordinates of D are (1, 2).}$$

11. A (2, 6) is one of the extremities of a diameter of a circle with centre (3, 5), find the other point.

Sol. A (2, 6), B(x, y) are the ends of the diameter.



And C(3, 5) is the centre.

Now C is the mid point of AB. $\therefore \left(\frac{2+x}{2}, \frac{6+y}{2} \right) = (3,5)$

$$\frac{2+x}{2} = 3 \Rightarrow 2+x = 6 \Rightarrow x = 4 \quad \text{and} \quad \frac{6+y}{2} = 5 \Rightarrow 6+y = 10 \Rightarrow y = 4$$

Coordinates of the other point B are (4, 4).

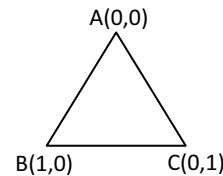
12. Find the area of the triangle formed by the following points.

i) (0, 0), (1, 0), (0, 1) ii) (5, 2), (-9, -3), (-3, -5)

Sol. i) A(0, 0), B(1, 0), C(0, 1) are the vertices of the triangle.

$$\text{Area of } \triangle OAB \text{ is } = \frac{1}{2} |x_1y_2 - x_2y_1|$$

$$= \frac{1}{2} |1 \cdot 1 - 0 \cdot 0| = \frac{1}{2} \text{ sq. units}$$

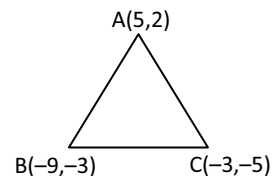


ii) A(5, 2), B(-9, -3), C(-3, -5) are the vertices of the triangle.

$$= \frac{1}{2} \begin{vmatrix} x_2 - x_1 & y_2 - y_1 \\ x_3 - x_1 & y_3 - y_1 \end{vmatrix} = \frac{1}{2} \begin{vmatrix} -9-5 & -3-2 \\ -3-5 & -5-2 \end{vmatrix}$$

$$= \frac{1}{2} \begin{vmatrix} -14 & -5 \\ -8 & -7 \end{vmatrix}$$

$$= \frac{1}{2} |98 - 40| = \frac{58}{2} = 29 \text{ sq. units}$$



13. Show that the following points are collinear.

i) (0, -2), (-1, 1), (-2, 4) ii) (-1, 7), (3, -5), (4, -8)

Sol. i) A(0, -2), B(-1, 1), C(-2, 4) are the given points.

$$\text{Area of } \triangle ABC = \frac{1}{2} |0(1-4) - 1(4+2) - 2(-2-1)| = \frac{1}{2} |0 - 6 + 6| = 0$$

\therefore A, B, C are collinear.

