

CHAPTER 3

STRAIGHT LINES – 1

TOPICS

1. Inclination, slope of a line when inclination is given, slope of a line through two points.
2. Equation of a line in point slope form, two point form , Slope intercept form, intercepts form.
3. Equation of a line in normal form
4. Equation of a line in symmetric form and parametric form.
5. Reduction of a line into various forms
6. Point of intersection of two lines
7. Family of straight lines .
8. Condition for concurrency of three straight lines.
9. Angle between two lines
10. Length of perpendicular from a point to line
11. Distance between two parallel lines
12. Foot of the perpendicular from a point to a straight Line
13. Image of a line w.r.t a line
14. Concurrent lines- properties related to a triangle.

STRAIGHT LINES

INCLINATION OF A LINE

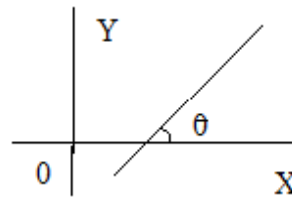
If a line L makes an angle θ with the positive direction of the x - axis, then θ is called the inclination of the line L .

The range of θ is $0 \leq \theta \leq \pi$.

If $\theta = 0^0$, then the line L is parallel to the x - axis.

If $\theta = 90^0$, then the line L is perpendicular to the x - axis.

Two lines have same inclination if and only if they are parallel.



Slope of a line:

If θ is the inclination of a line L then $\tan \theta$ is called the slope (or gradient) of the line L .

The slope of a line L is denoted by m . $m = \tan \theta$ ($\theta \neq 90^0$)

e.g., i) If the inclination of a line is 45^0 , then its slope is 1.

ii) If the slope of a line is $\sqrt{3}$, then its inclination is 60^0 .

Note 1: The slope of x - axis is 0 . ($\because \theta=0$)

Note 2: The slope of a horizontal line is 0 . ($\because \theta=0$)

Note 3 : The slope of y - axis is not defined .

Note 4 : The slope of a vertical line is not defined.

Note 5 : if 1. $\theta = 0^0$ $m = 0$.

2. $0^0 < \theta < 90^0$ $m > 0$.

3. $\theta = 90^0$ m is not defined.

4. $90^0 < \theta < 180^0$ $m < 0$.

THEOREM

The slope of the line passing through the points $A(x_1, y_1)$ and $B(x_2, y_2)$ is $\frac{y_2 - y_1}{x_2 - x_1}$, $x_1 \neq x_2$.

(If $x_1 = x_2$, then the line is vertical and hence its slope is not defined.)

Note: Two non vertical lines are parallel iff their slopes are equal.

Note: Two non vertical lines are perpendicular iff the product of their slopes is -1 .

EQUATIONS OF A LINE

Equation of horizontal line

The equation of the horizontal line passing through the point (x_1, y_1) is $y = y_1$

Note 1: The equation of x - axis is $y = 0$.

Note 2: The equation of any line parallel to x -axis is $y = k$, k is an unknown constant.

Equation of vertical line

The equation of the vertical line passing through the point (x_1, y_1) is $x = x_1$.

Note 1: The equation of y - axis is $x = 0$.

Note 2: The equation of any line parallel to y -axis is $x = k$, k is an unknown constant.

POINT -SLOPE FORM

The equation of the line passing through $A(x_1, y_1)$ and having slope m is $y - y_1 = m(x - x_1)$.

Note: The equation of the line passing through the origin and having slope m is $y = mx$.

TWO POINT FORM

The equation of the line passing through $A(x_1, y_1)$, $B(x_2, y_2)$ is

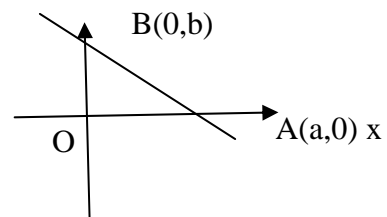
$$(y - y_1)(x_2 - x_1) = (x - x_1)(y_2 - y_1).$$

Note: The equation of the line passing through (x_1, y_1) and (x_2, y_2) ($x_1 \neq x_2$) is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) \quad (\text{or}) \quad \frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1}$$

INTERCEPTS:

If a line cuts x - axis at $A(a, 0)$ and the y - axis at $B(0, b)$ then a is called the x - intercept and b is called the y - intercept of the line. y



SLOPE INTERCEPT FORM

The equation of the line having slope m and y - intercept c is $y = mx + c$.

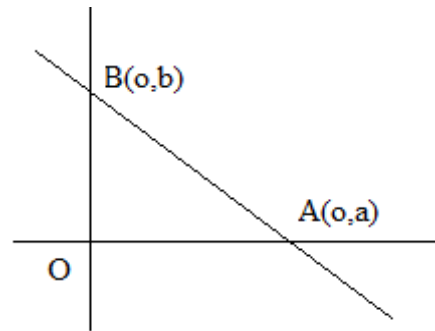
Note: The equation of the line having slope m and x - intercept a is $y = m(x - a)$.

INTERCEPT FORM

The equation of the line having x - intercept $a(\neq 0)$ and y - intercept $b(\neq 0)$ is $\frac{x}{a} + \frac{y}{b} = 1$.

Proof:

Since the x - intercept of the line is a , the line cuts the x - axis at $(a, 0)$ and the y - intercept of the line is b , the line cuts the y - axis at $(0, b)$.



Equation of the line is $y - 0 = \frac{b-0}{0-a}(x - a)$

$$ay = -bx + ab \Rightarrow bx + ay = ab$$

$$\Rightarrow \frac{bx}{ab} + \frac{ay}{ab} = 1 \Rightarrow \frac{x}{a} + \frac{y}{b} = 1$$

EXERCISE – 3(A)

1. Find the slope of the line $x + y = 0$ and $x - y = 0$.

Sol. Given line is $x + y = 0 \Rightarrow y = -x$ which is of the form $y = mx$.

Therefore slope $m = -1$

Equation of the line is $x - y = 0 \Rightarrow y = x$.

\Rightarrow slope of the line is $m = 1$.

2. Find the equation of the line containing the points (2, -3) and (0, -3).

Sol. Equation of the line is $(y - y_1) = \frac{(y_2 - y_1)}{(x_2 - x_1)}(x - x_1)$

$$(y + 3) = \frac{(-3 + 3)}{(2 - 0)}(x - 2)$$

$$2(y + 3) = 0 \Rightarrow y + 3 = 0$$

3. Find the equation of the line containing the points (1, 2) and (1, -2).

Ans; $x - 1 = 0$

4. Find the angle which the straight line $y = \sqrt{3}x - 4$ makes with the Y-axis.

Sol. Equation of the line is $y = \sqrt{3}x - 4$

$$\text{Slope } m = \sqrt{3} = \tan \frac{\pi}{6}$$

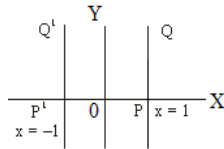
$$\text{Angle made with } x\text{-axis} = \frac{\pi}{6}$$

$$\therefore \text{Angle made with } y\text{-axis} = \frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3}$$

5. Write the equation of the reflection of the line $x = 1$ in the Y-axis.

Sol. Equation of PQ is $x = 1$

Reflection about y-axis is $x = -1$ i.e., $x + 1 = 0$



6. Find the condition for the points (a, 0), (h, k) and (0, b) where $ab \neq 0$ to be collinear.

Sol. A(a, 0), B(h, k), C(0, b) are collinear.

\Rightarrow Slope of AB = Slope of AC

$$\Rightarrow \frac{k - 0}{h - a} = \frac{-b}{a} \Rightarrow ak = -bh + ab$$

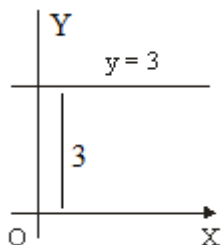
$$\Rightarrow bh + ak = ab \Rightarrow \frac{h}{a} + \frac{k}{b} = 1$$

7. Write the equation of the straight lines parallel to X- axis is

i) at a distance of 3 units above the x-axis and

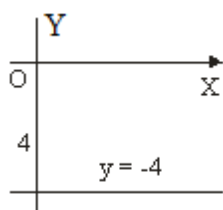
ii) at a distance of 4 units below the x- axis.

Sol. i)



Equation of the line which is at distance 3 units above the x- axis is $y = 3$

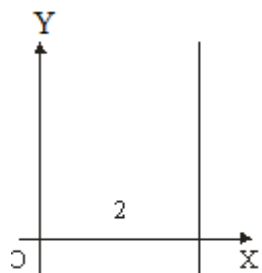
ii)



Equation of the line which is at distance 3 units below the x- axis is $y = -3$

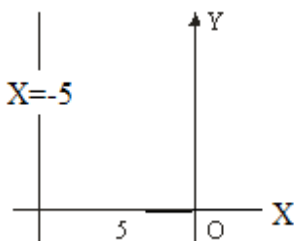
8. Write the equation of the straight line parallel to Y-axis and i) at a distance of 2 units from the Y - axis to the right of it.ii) at a distance of 5 units from the Y - axis to the left of it.

Sol. i)



Equation of the required line is $x = 2$ or $x - 2 = 0$

ii)



Equation of the required line is $x = -5 \Rightarrow x + 5 = 0$

II.

1. Find the slope of the straight line passing through the following points.

i) $(-3, 8)$ $(10, 5)$ ii) $(3, 4)$ $(7, -6)$ iii) $(8, 1), (-1, 7)$ iv) $(-p, q)$ $(q, -p)$ ($p, q \neq 0$)

Sol. i) Slope = $\frac{y_1 - y_2}{x_1 - x_2} = \frac{8 - 5}{-3 - 10} = \frac{-3}{13}$

ii) Slope = $\frac{4 + 3}{3 - 7} = \frac{10}{-4} = \frac{-5}{2}$

iii) Slope = $\frac{1 - 7}{8 + 1} = \frac{-6}{9} = -\frac{2}{3}$

iv) Slope = $\frac{q + p}{-p - q} = \frac{(p + q)}{-(p + q)} = -1$

2. Find the value of x , if the slope of the line passing through $(2, 5)$ and $(x, 3)$ is 2.

Sol. Slope of the line is 2.

But slope = $\frac{y_1 - y_2}{x_1 - x_2} \Rightarrow \frac{5 - 3}{2 - x} = 2 \Rightarrow 2 = 2(2 - x) \Rightarrow x = 2 - 1 = 1$

3. Find the value of y if the line joining the points $(3, y)$ and $(2, 7)$ is parallel to the line joining the points $(-1, 4)$ and $(0, 6)$.

Sol. Let A $(3, y)$, B $(2, 7)$, P $(-1, 4)$ and Q $(0, 6)$ be the given points.

Slope of AB = $\frac{y - 7}{3 - 2} = y - 7$ AND slope of PQ = $\frac{4 - 6}{-1 - 0} = \frac{-2}{-1} = 2$

lines AB and PQ are parallel \Rightarrow their slopes are equal $\Rightarrow y - 7 = 2 \Rightarrow y = 2 + 7 = 9$

4. Find the slopes of the lines i) parallel to and ii) is perpendicular to the line passing through $(6, 3)$ and $(-4, 5)$.

Sol. Let A $(6, 3)$ and B $(-4, 5) \Rightarrow$ Slope of AB = $\frac{3 - 5}{6 + 4} = \frac{-2}{10} = -\frac{1}{5}$

i) Slope of the line parallel to line AB = slope of AB = $-\frac{1}{5}$

ii) slope of a line perpendicular to AB = $\frac{-1}{\text{slope of AB}} = 5$

5. Find the equation of the straight line which makes the following angles with positive X-axis and which passes through the points given below

i) $\frac{\pi}{4}$ and $(0, 0)$ ii) $\frac{\pi}{3}$ and $(1, 2)$ iii) 135° and $(3, -2)$ iv) 150° and $(-2, -1)$

Sol. i) inclination $\theta = \frac{\pi}{4}$ and point is $(0, 0)$.

Slope of the line $m = \tan 45^\circ = 1$

Equation of the line is $y - y_1 = m(x - x_1)$

$$y - 0 = 1(x-0) \quad \text{i.e., } y = x \text{ or } x - y = 0$$

ii) **ANS:** $\sqrt{3}x - y + (2 - \sqrt{3}) = 0$

iii) inclination $\theta = 135^\circ$ and point $(3, -2)$

$$\text{Slope } m = \tan 135^\circ = -\tan 45^\circ = -1$$

$$\text{Equation of the line is } y - y_1 = m(x - x_1)$$

$$\Rightarrow y + 2 = -1(x - 3) = -x + 3 \text{ i.e., } x + y - 1 = 0$$

iv) **ANS:** $x + \sqrt{3}y + (2 + \sqrt{3}) = 0$

6. Find the equation of the straight line passing through the origin and making equal angles with co-ordinates axes.

Sol. since require lines are passing through origin and making equal angles with the axes, therefore angles are 45° and 135° .

i) $\theta = 45^\circ$ and point $(0, 0)$

$$\text{slope } m = \tan 45^\circ = 1$$

$$\Rightarrow \text{equation of the line is } y = mx \Rightarrow y = x$$

ii) $\theta = 135^\circ$ and point $(0, 0) \Rightarrow \text{slope } m = \tan 135^\circ = -1$

$$\Rightarrow \text{equation of the line is } y = mx \Rightarrow y = -x$$

7. The angle made by a straight line with the positive X-axis in the positive direction and the Y-intercept cut of by it are given below. Find the equation of the straight line.

i) $60^\circ, 3$ ii) $150^\circ, 2$ iii) $45^\circ, -2$ iv) $\tan^{-1}\left(\frac{2}{3}\right), 3$

Sol. i) Slope $m = \tan 60^\circ = \sqrt{3}$ and y intercept is $c = 3$

$$\text{Equation of the line is } y = mx + c \Rightarrow y = \sqrt{3}x + 3$$

$$\sqrt{3}x - y + 3 = 0$$

ii) **ANS:** $x + \sqrt{3}y - 2\sqrt{3} = 0$

iii) **ANS:** $x - y - 2$

iv) inclination $\theta = \tan^{-1}\left(\frac{2}{3}\right) \Rightarrow \tan \theta = \frac{2}{3}$

Slope $m = \tan \theta = \frac{2}{3}$ and y intercept $c = 3$ Equation of the line is $y = mx + c$

$$\Rightarrow y = \frac{2}{3}x + 3 \Rightarrow 3y = 2x + 9 \Rightarrow 2x - 3y + 9 = 0$$

- 8. Find the equation of the straight line passing through (-4, 5) and cutting off equal non-zero intercepts on the co-ordinate axes?**

Sol. Line is making equal intercepts, let the intercepts be a, a

Equation of the line in the intercept form is

$$\frac{x}{a} + \frac{y}{b} = 1 \quad \Rightarrow \quad \frac{x}{a} + \frac{y}{a} = 1 \quad \Rightarrow \quad x + y = a$$

This line is passing through p (-4, 5) \Rightarrow $-4 + 5 = a \Rightarrow a = 1$

Equation of the line is $x + y = 1$

- 9. Find the equation of the straight line passing through (-2, 4) and making non-zero intercepts whose sum is zero. ?**

Sol. Let the intercepts be a, b.

Given $a + b = 0 \Rightarrow b = -a$

Equation of the line in the intercept form is

$$\frac{x}{a} + \frac{y}{b} = 1 \Rightarrow \frac{x}{a} - \frac{y}{a} = 1 \Rightarrow x - y = a$$

This line is passing through P(-2, 4)

$\therefore -2 - 4 = a \Rightarrow a = -6$

Equation of the required line is $x - y = -6 \Rightarrow x - y + 6 = 0$

III.

- 1. Find the equation of the straight line passing through the point (3, -4) and making X and Y- intercepts which are in the ratio 2 : 3. ?**

Sol. Let the intercepts be a, b.

Given $\frac{a}{b} = \frac{2}{3} \Rightarrow b = \frac{3a}{2}$

Equation of the line in the intercept form is

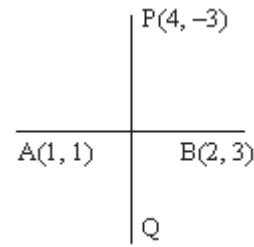
$$\frac{x}{a} + \frac{y}{b} = 1 \Rightarrow \frac{x}{a} + \frac{2y}{3a} = 1 \Rightarrow 3x + 2y = 3a$$

This line passes through p(3, -4) \Rightarrow $9 - 8 = 3a \Rightarrow 3a = 1$

Equation of the required line is $3x + 2y = 1$ $3x + 2y - 1 = 0$

2. Find the equation of the straight line passing through the point (4, -3) and perpendicular to the line passing through the points (1, 1) and (2, 3).

Sol. Let A(1, 1), B(2, 3)



$$\text{Slope of AB is } m = \frac{1-3}{1-2} = \frac{-2}{-1} = 2$$

Since required line is perpendicular to AB,

$$\Rightarrow \text{Slope of required line} = -\frac{1}{m} = -\frac{1}{2}$$

Required line is passing through p(4, -3)

$$\Rightarrow \text{Equation of the line is } y - y_1 = m(x - x_1)$$

$$\Rightarrow y + 3 = -\frac{1}{2}(x - 4) \Rightarrow 2y + 6 = -x + 4 \Rightarrow x + 2y + 2 = 0$$

3. Show that the following sets of points are collinear and find the equation of the line L containing them.

i) (-5, 1), (5, 5), (10, 7) ii) (1, -3), (-2, -6), (2, 6) iii) (a, b + c), (b, c + a), (c, a + b)

Sol. i) Given points A(-5, 1), B(5, 5), C(10, 7)

Equation of AB is

$$(y - y_1)(x_1 - x_2) = (x - x_1)(y_1 - y_2)$$

$$(y - 1)(-5 - 5) = (x + 5)(1 - 5)$$

$$\Rightarrow -10 + 10 = -4x - 20$$

$$\Rightarrow 4x - 10y + 30 = 0$$

$$\text{or } 2x - 5y + 15 = 0$$

substituting c (10, 7),

$$2x - 5y + 15 = 2 \cdot 10 - 5 \cdot 7 + 15$$

$$= 20 - 35 + 15 = 0$$

Therefore C is a point on the line AB.

A, B, C are collinear and Equation of the line containing them is $2x - 5y + 15 = 0$

ii) ANS: $3x - y = 0$

iii) Given points A(a, b + c), B(b, c + a), C(c, a + b)

$$\text{Equation of AB is } (y - (b + c))(a - b) = (x - a)(b + c - c - a)$$

$$\Rightarrow (y - b - c)(a - b) = -(a - b)(x - a)$$

$$\Rightarrow x + y - (a + b + c) = 0$$

Substituting C(c, a + b) in above equation

$$c + a + b - a - b - c = 0 \Rightarrow C \text{ lies on } AB$$

Therefore, A, B, C are collinear.

Equation of the line containing them is $x + y = a + b + c$

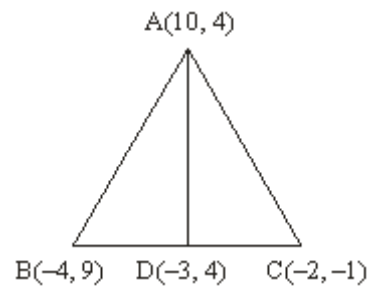
4. A(10, 4), B(-4, 9) and C(-2, -1) are the vertices of a triangle ABC. Find the equations of
 i) \overline{AB} ii) the median through A iii) the altitude through B
 IV) the perpendicular bisector of AB

Sol. i) vertices of the triangle are A(10, 4), B(-4, 9), C(-2, -1)
 \Rightarrow Equation of AB is $(y - 4)(10 + 4) = (x - 10)(4 - 9)$
 $\Rightarrow 14y - 56 = -5x + 50 \Rightarrow 5x + 14y - 106 = 0$

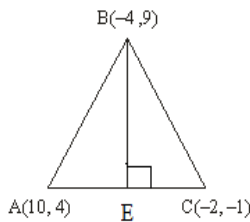
ii) let D be the mid-point of BC

$$\Rightarrow D = \left(\frac{-4 - 2}{2}, \frac{9 - 1}{2} \right) = (-3, 4)$$

Equation of AD is $(y - 4)(10 + 3) = (x + 3)(4 - 4)$
 $\Rightarrow 13(y - 4) = 0 \Rightarrow y - 4 = 0 \Rightarrow y = 4$



iii)



$$\text{Slope of AC is } m = \frac{4 - 1}{10 - (-2)} = \frac{5}{12}$$

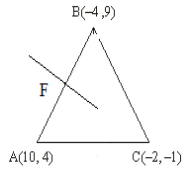
since BE is perpendicular to AC, Slope of BE = $\frac{-1}{m} = \frac{-12}{5}$

\Rightarrow Equation of the altitude BE is $y - 9 = \frac{-12}{5}(x + 4)$

$$\Rightarrow 5y - 45 = -12x - 48 \Rightarrow 12x + 5y + 3 = 0$$

iv) let F be the mid-point of AB

$$\Rightarrow F = \left(\frac{10 - 4}{2}, \frac{4 + 9}{2} \right) = \left(3, \frac{13}{2} \right)$$



$$\text{Slope of AB } m = \frac{4-9}{10+4} = \frac{-5}{14}$$

$$\Rightarrow \text{Slope of the line perpendicular to AB} = \frac{-1}{m} = \frac{14}{5}$$

$$\text{Point is } = \left(3, \frac{13}{2}\right) \text{ and slope } \frac{14}{5}$$

$$\text{Equation of the perpendicular bisector AB is } y - \frac{13}{2} = \frac{14}{5}(x - 3)$$

$$\Rightarrow 5y - \frac{65}{2} = 14x - 42$$

$$\Rightarrow 14x - 5y - \left(\frac{65}{2} - 42\right) = 0$$

$$\Rightarrow 14x - 5y - \frac{19}{2} = 0$$

$$\Rightarrow 28x - 10y - 19 = 0$$