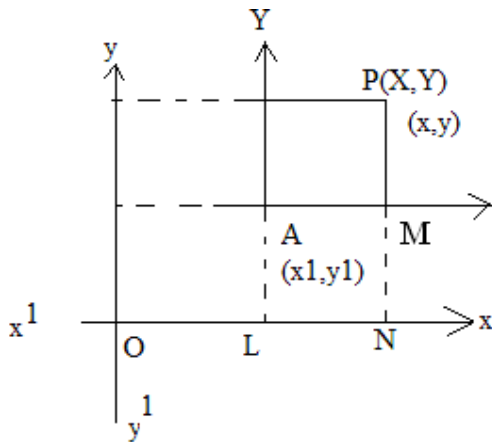


TRANSLATION OF AXES.

1. **Definition:** Without changing the direction of the axes, the transformation in which the origin is shifted to another position or point is called translation of axes.
2. **Theorem:** To find the co-ordinates of a point (x, y) are translated by shifting the origin to a point (x_1, y_1)

Proof :



Let ox^1 oy^1 be the original axes and $A(x_1, y_1)$ be a point to which the origin is shifted.

Let AX, AY be the new axes which are parallel to the original axes as in figure.

Let P be any point in the plane whose coordinates w.r.t old system are (x, y)

And w.r.t new axes are (X, Y) .

From figure, $A = (x_1, y_1)$ then $AL = y_1$, $OL = x_1$,

$P(x, y)$ then $x = ON = OL + LN = OL + AM = x_1 + X = X + x_1$

Hence $x = X + x_1$.

$y = PN = PM + MN = Y + AL = Y + y_1$

therefore, $y = Y + y_1$

hence $x = X + x_1$, $y = Y + y_1$

3.Theorem: To find the point to which the origin is to be shifted by translation of axes to eliminate x, y terms(first degree terms) in the equation $ax^2 + 2xhy + by^2 + 2gx + 2fy + c = 0$ ($h^2 = ab$)

Proof : given equation is $ax^2 + 2xhy + by^2 + 2gx + 2fy + c = 0$

Let (x_1, y_1) be a point to which the origin is shifted by translation

Let (X, Y) be the new co-ordinates of the point (x, y) .

\therefore the equations of the transformation are $x = X + x_1, y = Y + y_1$

Now the transformed equation is

$$a(X + x_1)^2 + 2h(X + x_1)(Y + y_1) + b(Y + y_1)^2 + 2g(X + x_1) + 2f[Y + y_1] + c = 0$$

$$\Rightarrow a(X^2 + 2x_1X + x_1^2) + 2h(XY + x_1Y + y_1X + x_1y_1) + b(Y^2 + 2y_1Y + y_1^2) + 2g(X + x_1) + 2f(Y + y_1) + c = 0$$

$$\Rightarrow aX^2 + 2hXY + bY^2 + 2X(ax_1 + hy_1 + g) + 2Y(hx_1 + by_1 + f) + (ax_1^2 + 2x_1x_1 + by_1^2 + 2gx_1 + 2fy_1 + c) = 0$$

Since x, y terms (the first degree terms) are to be eliminated

$$ax_1 + hy_1 + g = 0$$

$$hx_1 + by_1 + f = 0$$

Solving these two equations by the method of cross pollination

$$\begin{array}{ccc} \underbrace{x_1} & \underbrace{y_1} & \underbrace{1} \\ \begin{array}{cc} h & g \\ b & f \end{array} & \begin{array}{cc} a & h \\ h & b \end{array} & \begin{array}{cc} h & b \end{array} \end{array}$$

$$\frac{x_1}{hf - bg} = \frac{y_1}{gh - af} = \frac{1}{ab - h^2} \Rightarrow x_1 = \frac{hf - bg}{ab - h^2}, y_1 = \frac{gh - af}{ab - h^2}$$

$$\text{New origin is } \Rightarrow (x_1, y_1) = \left(\frac{hf - bg}{ab - h^2}, \frac{gh - af}{ab - h^2} \right)$$

Note: (i) The point to which the origin has to be shifted to eliminate x, y terms by translation in the equation $ax^2 + by^2 + 2gx + 2fy + c = 0$ is $\left(\frac{-g}{a}, \frac{-f}{b} \right)$

If $b = a$, then the new origin is $\left(\frac{-g}{a}, \frac{-f}{a} \right)$

(ii) The point to which the origin has to be shifted to eliminate x, y terms by translation of axes in the equation $a(x + x_1)^2 + b(y + y_1)^2 = c$ is $(-x_1, -y_1)$

iii) The point to which the origin has to be shifted to eliminate x, y terms by translation in the equation $2hxy + 2gx + 2fy + c = 0$ is $\left(\frac{-f}{h}, \frac{-g}{h}\right)$

Theorem : To find the condition that the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ to be in the form $aX^2 + 2hXY + bY^2 = 0$ when the axes are translated. $(h^2 \neq ab)$

Proof : From theorem 3, we get

$$ax_1 + hy_1 + g = 0 \text{ _____(1)}$$

$$hx_1 + by_1 + f = 0 \text{ _____(2)}$$

Solving (1) and (2),

$$(x_1, y_1) = \left(\frac{hf - bg}{ab - h^2}, \frac{gf - af}{ab - h^2}\right)$$

Since the equation is to be in the form of $aX^2 + 2hXY + bY^2 = 0$, then for this we should have

$$ax_1^2 + 2hx_1y_1 + by_1^2 + 2gx_1 + 2fy_1 + c = 0$$

$$\Rightarrow (ax_1 + hy_1 + g)x_1 + (x_1 + by_1 + f)y_1 + (gx_1 + fy_1 + c) = 0$$

$$\Rightarrow (0) \cdot x_1 + (0) \cdot y_1 + (gx_1 + fy_1 + c) = 0 \text{ from (1) and (2)} \Rightarrow gx_1 + fy_1 + c = 0 \text{ _____ (3)}$$

Substituting x_1, y_1 in (3), we get

$$g\left(\frac{hf - bg}{ab - h^2}\right) + f\left(\frac{gh - af}{ab - h^2}\right) + c = 0$$

$$\Rightarrow fgh - bg^2 + fgh - af^2 + abc + -ch^2 = 0$$

$$\Rightarrow abc + 2fgh - af^2 - bg^2 - ch^2 = 0$$

PROBLEMS.

1. When the origin is shifted to (4,-5) by the translation of axes, find the co-ordinates of the following points with reference to new axes.?

Sol. i) (0,3)

New origin = (4,-5) = (h,k)

Old co-ordinates are (0,3) = (x,y)

$$X = x - h = 0 - 4 = -4$$

$$Y = y - k = 3 + 5 = 8$$

New co-ordinates are (-4,8)

ii) (-2,4) ANS. (-6,9)

iii) (4,-5)

ans. (0,0)

2. The origin is shifted to (2,3) by the translation of axes. If the co-ordinates of a point P changes as follows, find the co-ordinates of P in the original system.?

i) (4,5)

ii) (-4,3)

iii) (0,0)

Sol. New origin = (2,3) = (h,k)

i) New co-ordinates are (4,5)

$$x = 4, y = 5$$

$$x = x + h = 4 + 2 = 6$$

$$y = y + k = 5 + 3 = 8$$

Old co-ordinates are (6,8)

ii) New co-ordinates are (-4,3)

ANS. (-2,6)

iii) New co-ordinates are (0,0)

ANS. (2,3)

3. Find the point to which the origin is to be shifted so that the point (3,0) may change to (2,-3)?

Sol. $(x, y) = (3, 0)$

$$(x^1, y^1) = (2, -3)$$

Let (h, k) be the new origin

$$\therefore h = x - x^1 = 3 - 2 = 1$$

$$k = y - y^1 = 0 + 3 = 3$$

$$\therefore (h, k) = (1, 3)$$

4. When the origin is shifted to (-1,2) by the translation of axes, find the transformed equations of the following.?

i) $x^2 + y^2 + 2x - 4y + 1 = 0$

ii) $2x^2 + y^2 - 4x + 4y = 0$

Sol. i) The given equation is $x^2 + y^2 + 2x - 4y + 1 = 0$ -----(1)

Origin is shifted to (-1,2) $\therefore h = -1, k = 2$

Equations of transformations are $x = X + h, y = Y + k$

i.e, $x = X - 1, y = Y + 2$

The new equation is

$$(X - 1)^2 + (Y + 2)^2 + 2(X - 1) - 4(Y + 2) + 1 = 0$$

$$\Rightarrow X^2 + 1 - 2X + Y^2 + 4 + 4Y + 2X - 2 - 4Y - 8 + 1 = 0$$

$$\Rightarrow X^2 + Y^2 - 4 = 0$$

ii) Old equation is $2x^2 + y^2 - 4x + 4y = 0$

Ans. $2X^2 + Y^2 - 8X + 8Y + 18 = 0$

5. The point to which the origin is shifted and the transformed equation are given below. Find the original equation.?

i) $(3, -4); x^2 + y^2 = 4$ ii) $(-1, 2); x^2 + 2y^2 + 16 = 0$

Sol. i) New origin = $(3, -4) = (h, k)$

$$\therefore X = x - h, \quad Y = y - k$$

$$= x - 3 \quad = y + 4$$

The original equation of $(X)^2 + (Y)^2 = 4$ is $(x - 3)^2 + (y + 4)^2 = 4$

$$x^2 - 6x + 9 + y^2 + 8y + 16 = 4$$

$$\therefore x^2 + y^2 - 6x + 8y + 21 = 0$$

ii) New origin = $(h, k) = (-1, 2)$

$$X = x - h, \quad Y = y - k$$

$$= x + 1 \quad = y - 2$$

The original equation of $X^2 + 2Y^2 + 16 = 0$

$$(x + 1)^2 + 2(y - 2)^2 + 16 = 0$$

$$x^2 + 2x + 1 + 2y^2 - 8y + 8 + 16 = 0 \Rightarrow x^2 + 2y^2 + 2x - 8y + 25 = 0$$

6. Find the point to which the origin is to be shifted so as to remove the first degree terms from the equation, $4x^2 + 9y^2 - 8x + 36y + 4 = 0$

Sol. The given equation is

$$4x^2 + 9y^2 - 8x + 36y + 4 = 0 \text{ comparing with } ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

$$a = 4 \quad h = 0 \quad g = -4 \quad b = 9 \quad f = 18$$

$$-\frac{g}{a} = \frac{4}{4} = 1, \quad -\frac{f}{b} = -2$$

Origin should be shifted to $\left(\frac{-g}{a}, \frac{-f}{b}\right) = (1, -2)$

SHORT ANSWER QUESTIONS

1. When the origin is shifted to the point (2,3), the transformed equation of a curve is $x^2 + 3xy - 2y^2 + 17x - 7y - 11 = 0$. Find the original equation of the curve?

Sol. New origin $= (2,3) = (h,k)$

Equations of transformation are

$$X = x + h, y = Y + k$$

$$X = x - h = x - 2, Y = y - 3$$

Transformed equation is

$$x^2 + 3xy - 2y^2 + 17x - 7y - 11 = 0$$

Original equation is

$$(x-2)^2 + 3(x-2)(y-3) - 2(y-3)^2 + 17(x-2) - 7(y-3) - 11 = 0$$

$$x^2 + 4x + 4 + 3xy - 9x - 6y + 18 - 2y^2 + 12y - 18 + 17x - 34 - 7y + 21 - 11 = 0$$

$$\Rightarrow x^2 + 3xy - 2y^2 + 4x - y - 20 = 0 \quad \text{is the required original equation.}$$

3. If the transformed equation of a curve is $3x^2 + xy - y^2 - 7x + y - 7 = 0$ when the origin is shifted to the point (1,2) by translation of axes, find the original equation of the curve?

Sol. Same as above problem.

$$\text{Ans. } 3x^2 + xy - y^2 - 15x + 4y + 13 = 0.$$