POINT OF INTERSECTION OF TWO STRAIGHT LINES

THEOREM

The point of intersection of the two non parallel lines

$a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$ is $\left(\frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}, \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}\right)$ **.**

Proof:

The lines are not parallel $\Rightarrow$ Slopes are not equal

$$\Rightarrow \frac{-a_1}{b_1} \neq \frac{-a_2}{b_2} \Rightarrow a_1b_2 \neq a_2b_1 \Rightarrow a_1b_2 - a_2b_1 \neq 0$$

Let $P(\alpha, \beta)$ be the point of intersection. Then $a_1\alpha + b_1\beta + c_1 = 0$ and $a_2\alpha + b_2\beta + c_2 = 0$.

Solving by the method of cross multiplication

$$\begin{array}{cccc} b_1 & c_1 & a_1 & b_1 \\ b_2 & c_2 & a_2 & b_2 \end{array}$$

$$\frac{\alpha}{b_1c_2 - b_2c_1} = \frac{\beta}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

$$\Rightarrow \alpha = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}, \quad \beta = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$$

Point of intersection is P = $\left(\frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}, \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}\right)$

THEOREM

The ratio in which the line L = ax + by + c = 0 divides the line segment joining $A(x_1, y_1)$, $B(x_2, y_2)$ is $-L_{11} : L_{22}$, where $L_{11} = L(x_1, y_1) = ax_1 + by_1 + c$ and $L_{22} = L(x_2, y_2) = ax_2 + by_2 + c$.

Proof:

Let k : 1 be the ratio in which the line divides the line segment.

The point which divides in the ratio k : 1 is $P = \left(\frac{kx_2 + x_1}{k+1}, \frac{ky_2 + y_1}{k+1}\right)$

Since P lies on the line ax + by + c = 0 $\Rightarrow a\left(\frac{kx_2 + x_1}{k+1}\right) + b\left(\frac{ky_2 + y_1}{k+1}\right) + c = 0$

$\Rightarrow a(kx_2 + x_1) + b(ky_2 + y_1) + c(k + 1) = 0$

$\Rightarrow k(ax_2 + by_2 + c) = -(ax_1 + by_1 + c)$

$$\Rightarrow k = -\frac{(ax_1 + by_1 + c)}{ax_2 + by_2 + c}$$

Required Ratio = $-(ax_1 + by_1 + c) : (ax_2 + by_2 + c)$

$= -L_{11} : L_{22}$.

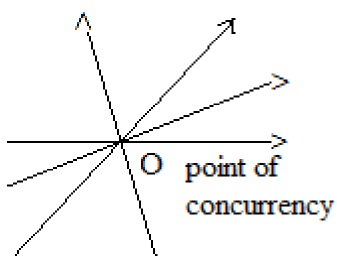
Note: The points A,B lie in the same side or opposite side of the line L = 0 according as L_{11}, L_{22} have the same sign or opposite signs.

1.The points A,B are opposite sides of the line L = 0 iff L_{11} and L_{22} have opposite signs.

2. The points A,B are same side of the line L = 0 iff L_{11} and L_{22} have same sign.

CONCURRENT LINES

Three or more lines are said to be concurrent if they have a point in common. The common point is called the point of concurrence.



Concurrent lines

CONDITION FOR CONCURRENCY OF THREE STRAIGHT LINES.

THEOREM

The condition that the lines $a_1x + b_1y + c_1 = 0, a_2x + b_2y + c_2 = 0$, $a_3x + b_3y + c_3 = 0$ to be concurrent is $a_3(b_1c_2 - b_2c_1) + b_3(c_1a_2 - c_2a_1) + c_3(a_1b_2 - a_2b_1) = 0$.

Proof:

Suppose the given lines are concurrent.

Let $P(\alpha, \beta)$ be the point of concurrence.

Then $a_1\alpha + b_1\beta + c_1 = 0$ -- (1)

$a_2\alpha + b_2\beta + c_2 = 0$ -- (2)

$a_3\alpha + b_3\beta + c_3 = 0$ -- (3)

By solving (1) and (2) we get

$$\alpha = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1},\ \beta = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$$

Therefore P = $\left(\frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}, \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1} \right)$

Substituting P in eq.(3), we get

$$a_3\left(\frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}\right) + b_3\left(\frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}\right) + c_3 = 0$$

$\Rightarrow a_3(b_1c_2 - b_2c_1) + b_3(c_1a_2 - c_2a_1) + c_3(a_1b_2 - a_2b_1) = 0$ which is required condition.

Above condition can be written in a determinant form as $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0.$

Problem:

Find the condition that the lines ax + hy + g = 0, hx + by + f = 0, gx + fy + c = 0 to be concurrent.

Sol: Let P be the point of concurrence

$a\alpha + h\beta + g = 0$ -- (1)

$h\alpha + b\beta + f = 0$ -- (2)

$g\alpha + f\beta + c = 0$ ---(3)

solving (1) and (2) we get

$$\begin{array}{cccc} h & g & a & h \\ b & f & h & b \end{array}$$

$$\frac{\alpha}{hf - bg} = \frac{\beta}{gh - af} = \frac{1}{ab - h^2} \Rightarrow \alpha = \frac{hf - bg}{ab - h^2}, \beta = \frac{gh - af}{ab - h^2}$$

Sub these values in eq.(3)

$$g\left(\frac{hg - bg}{ab - h^2}\right) + f\left(\frac{gh - af}{ab - h^2}\right) + c = 0$$

$\Rightarrow g(hf - bg) + f(gh - af) + c(ab - h^2) = 0$

$\Rightarrow fgh - bg^2 + fgh - af^2 + abc - ch^2 = 0$

$\Rightarrow abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

The condition is $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

FAMILY OF STRAIGHT LINES – CONCURRENT STRAIGHT LINES

THEOREM

Suppose $L_1 = a_1x + b_1 y + c_1 = 0$, $L_2 = a_2x + b_2 y + c_2 = 0$ are two intersection lines.

(i) If $(\lambda_1, \lambda_2) \neq (0,0)$ then $\lambda_1 L_1 + \lambda_2 L_2 = 0$ represents a straight line passing through the point of intersection of $L_1 = 0$ and $L_2 = 0$.

(ii) The equation of any line passing through the point of intersection of

$L_1 = 0$, $L_2 = 0$ is of the form where $(\lambda_1, \lambda_2) \neq (0,0)$.

Note: If $L_1 = 0$, $L_2 = 0$ are two intersecting lines, then the equation of any line other than $L_2 = 0$ passing through their point of intersection can be taken as $L_1 + \lambda L_2 = 0$ where λ is a parameter.

THEOREM

If there exists three constants p,q,r not all zero such that $p(a_1x + b_1y + c_1) + q(a_2x + b_2y + c_2) + r(a_3x + b_3y + c_3) = 0$ for all x and y then the three lines $a_1x + b_1 y + c_1 = 0$, $a_2x + b_2 y + c_2 = 0$ and $a_3x + b_3 y + c_3 = 0$ in which no two of them are parallel, are concurrent.

Theorem

Let $L_1 = a_1x + b_1 y + c_1 = 0$, $L_2 = a_2x + b_2 y + c_2 = 0$ represent two parallel lines.Then the straight line represented by $\lambda_1 L_1 + \lambda_2 L_2 = 0$ is parallel to each of the straight line $L_1=0$ and $L_2=0$.

A SUFFICIENT CONDITION FOR CONCURRENCY OF THREE STRAIGHT LINES

THEOREM

If $L_1 = a_1x + b_1y + c_1 = 0$, $L_2 = a_2x + b_2y + c_2 = 0$, $L_3 = a_3x + b_3y + c_3 = 0$ are three straight lines, no two of which are parallel, and if non-zero real numbers $\lambda_1, \lambda_2, \lambda_3$ exist such that $\lambda_1 L_1 + \lambda_2 L_2 + \lambda_3 L_3 = 0$ then the straight lines $L_1=0$, $L_2 = 0$ and $L_3= 0$are concurrent.

EXERCISE – 3(C)

I

1. Find the ratio in which the following straight lines divided the line segment joining the given points. state whether the points lie on the same side or on either side of the straight line. i) 3x – 4y = 7, (2, -7) and (-1, 3)

Sol. Equation of the line is 3x – 4y = 7

Given points are A(2, -7) andB(-1, 3). The ratio in which the line L=0 divides the line join of point A and B is - $\frac{L_{11}}{L_{22}} = -\frac{(ax_1+by_1+c)}{(ax_2+by_2+c)}$

$$= -\frac{[3.2-4(-7)-7]}{[3-(-1)-4.3-7]} = -\frac{(6+28-27)}{-3-12-7} = \frac{-27}{-22} = \frac{27}{22}$$

The ratio is positive , therefore given points lie on opposite sides of the line.

ii) L ≅ 3x + 4y = 6, A(2, -1) and B(1, 1)

Sol. Equation of the line is 3x + 4y – 6 = 0

Ratio is - $\frac{L_{11}}{L_{22}} = -\frac{(ax_1+by_1+c)}{(ax_2+by_2+c)}$ $= \frac{-\left(3.2+4(-1)-6\right)}{3.1+4.1-6} = \frac{-(-4)}{1} = \frac{4}{1}$

Ratio is positive, the points lie on the opposite side of the line.

iii) 2x + 3y = 5, (0, 0) and (-2, 1)

Sol. 2x + 3y – 5 = 0

$\Rightarrow$ - $\frac{L_{11}}{L_{22}} = -\frac{(ax_1+by_1+c)}{(ax_2+by_2+c)} = \frac{-(0+0-5)}{-4+3-5} = \frac{-5}{6}$

Ratio is –ve, the points lie on same side of the line.

2. Find the point of intersection of the following lines.

i) 7x + y + 3 = 0, x + y = 0

Sol. Solving the equations 7x + y + 3 = 0, x + y = 0

$$\begin{array}{cccccccc} & x & & y & & 1 & & \\ 1 & & +3 & & 7 & & 1 & \\ 1 & & 0 & & 1 & & 1 & \end{array}$$

$\Rightarrow \frac{x}{0-3} = \frac{y}{3} = \frac{1}{7-1} \Rightarrow x = \frac{-3}{6} = -\frac{-1}{2}$, $y = \frac{+3}{6} = \frac{+1}{2}$

point of intersection is $\left(\frac{-1}{2}, \frac{1}{2}\right)$

ii) 4x + 8y – 1 = 0, 2x – y + 1 = 0

Ans : $\left(\frac{-7}{20}.\frac{3}{10}\right)$

3. **Show that the straight lines (a –b) x + (b –c) y = c – a, (b-c) x+ (c-a)y= (a – b) and (c –a)x + (a – b)y b - c are concurrent.**

Sol. Equations of the given lines are

$L_1=(a - b)\,x + (b - c)\,y - c + a=0$ --- (1)

$L_2=(b - c)\,x + (c - a)\,y - a + b=0$ --- (2)

$L_3=(c - a)\,x + (a - b)\,y - b + c=0$ --- (3)

If three lines L_1, L_2, L_3 are concurrent , then there exists non zero real numbers $\lambda_1, \lambda_2, \lambda_3$, such that $\lambda_1 L_1 + \lambda_2 L_2 + \lambda_3 L_3 = 0$.

Let $\lambda_1 = 1, \lambda_2 = 1, \lambda_3 = 1$, then $1.L_1 + 1.L_2 + 1.L_3 = 0$

Hence the given lines are concurrent.

4. **Transform the following equation into form $L_1 + \lambda L_2 = 0$ and find the point of concurrency of the family of straight lines represented by the equation.**

i) **(2 + 5k) x – 3 (1 + 2k) y + (2 – k) = 0**

Sol. Given equation is

$(2 + 5k)\,x - 3(1 + 2k)\,y + (2 - k) = 0 \Rightarrow (2x - 3y + 2) + k\,(5x - 6y - 1) = 0$ which is of the form $L_1 + \lambda L_2 = 0$ where $L_1 = 2x - 3y + 2 = 0$ and $L_2 = 5x - 6y - 1 = 0$.

Therefore given equation represents a family of straight lines.

Solving above two lines,

$$\begin{array}{cccccccc} & x & & y & & 1 & & \\ -3 & & 2 & & 2 & & -3 \\ -6 & & -1 & & 5 & & -6 \end{array}$$

$$\frac{x}{3+12} = \frac{y}{10+2} = \frac{1}{-12+15} \Rightarrow x = \frac{15}{3} = 5, y = \frac{12}{3} = 4$$

The point of concurrency is P(5, 4).

ii) **(k +1) x + (k + 2) y + 5 = 0**

Sol. Point of concurrency is P(5, -5).

5. **Find the value of P. If the straight lines x + p = 0, y + 2 = 0 and 3x + 2y + 5 = 0 are concurrent.**

Sol. Equations of the given lines are

x + p = 0---(1), y + 2 = 0---(2) and 3x + 2y + 5 = 0 ---(3)

From (2), y = - 2

Substituting in (3), 3x – 4 + 5 = 0

$3x = 4 - 5 = -1 \Rightarrow \quad x = -\frac{1}{3}$

Point of intersection of (2) and (3) is $P\left(-\frac{1}{3}, -2\right)$

Since the given lines are concurrent, therefore P is a point on x + p = 0

$\Rightarrow -\frac{1}{3}+p=0 \Rightarrow p=\frac{1}{3}$

6. Find the area of the triangle formed by the following straight lines and the co-ordinates axes.

i) x – 4y + 2= 0

Sol. Equation of the line is x – 4y + 2 = 0 $\Rightarrow$-x + 4y = 2

$\Rightarrow \frac{x}{-2}+\frac{y}{\left(\frac{1}{2}\right)}=1 \Rightarrow$ a = - 2, b = $\frac{1}{2}$

Area of $\Delta OAB=\frac{1}{2}|ab|$

$=\frac{1}{2}|-2\times\frac{1}{2}|=\frac{1}{2}$ sq. units

ii) 3x – 4y + 12 = 0

Sol. Equation of the line is 3x – 4y + 12 = 0

$\Rightarrow$-3x + 4y = 12 $\Rightarrow$ $\frac{x}{-4}+\frac{t}{3}=1$

a = - 4, b = 3

Area of $\Delta OAB=\frac{1}{2}|ab|$

$=\frac{1}{2}|(-4)(3)|=\frac{1}{2}(12)$ = 6 sq. units

II.

1. A straight line meets the co-ordinates axes at A and B. Find the equation of the straight line, when i) $\overline{AB}$ is divided in the ratio 2 : 3 at (-5, 2). ii) $\overline{AB}$ is divided in the ratio 1 : 2 at (-5, 4) iii) (p, q) bisects $\overline{AB}$

Sol. i) Point P=(-5, 2).Let OA = a and OB = b

Then A (a, 0) and B (0,b). Given P divides AB in the ratio 2 : 3

P = $\left(\frac{3a}{5},\frac{2b}{5}\right)=(-5,2) \Rightarrow \frac{3a}{5}=-\frac{2b}{5}=2 \Rightarrow a=-\frac{25}{3}=b=5$

Equation of AB is $\frac{x}{a}+\frac{y}{b}=1$

$\frac{x}{\left(-\frac{25}{3}\right)}+\frac{y}{5}=1 \Rightarrow \frac{-3x}{25}+\frac{y}{5}=1 \Rightarrow$ -3x + 5y = 25 $\Rightarrow$3x – 5y + 25 = 0

ii) Ans: $8x-5y+60=0$

iii) (p, q) bisects $\overline{AB}$

Sol: let P=(p,q).let OA = a and OB = b then A (a, 0) and B (0,b)

Given P=Mid point of AB is $= \left(\frac{a}{b}, \frac{b}{2}\right) = (p, q)$

$\Rightarrow \frac{a}{2} = p, \frac{b}{2} = q \Rightarrow a = 2p, b = 2q$

Equation of AB is $\frac{x}{a} + \frac{y}{b} = 1 \Rightarrow \frac{x}{2p} + \frac{y}{2q} = 1 \Rightarrow \frac{x}{p} + \frac{y}{q} = 2$

2. Find the equation of the straight line passing through the points (-1, 2) and (5,-1) and also find the area of the triangle formed by it with the axes of co-ordinates.

Sol. Let P (-1, 2) and Q (5, -1)

Equation of the line through P and Q is

$(y - y_1)(x_1 - x_2) = (x - x_1)(y_1 - y_2)$

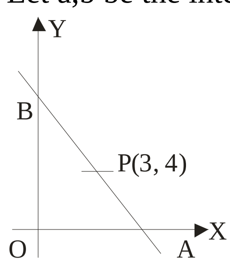
$\Rightarrow$ (y – 2) (-1 -5) = (x + 1) (2 + 1) $\Rightarrow$-6 (y – 2) = 3 (x + 1)

$\Rightarrow$-2y + 4 = x + 1$\Rightarrow$x + 2y – 3 = 0

Area of Δ OAB is= $\frac{c^2}{2|ab|} = \frac{9}{2|1.2|} = \frac{9}{4}$ sq. units

3. A triangle is of area 24 sq. units is formed by a straight line and the co-ordinate axes is the first quadrant. Find the equation of the straight line, if it passes through (3, 4).

Sol. Let a,b be the intercepts of the line.



Equation of the line is $\frac{x}{a} + \frac{y}{b} = 1$

This line is passing through P (3, 4)

$\Rightarrow \frac{3}{a} + \frac{4}{b} = 1 \Rightarrow \frac{4}{b} = 1 - \frac{3}{a} = \frac{a-3}{a} \Rightarrow b = \frac{4a}{a-3}$

Given area of $\Delta AB = 24 \Rightarrow \frac{1}{2}|ab| = 24$

$$\Rightarrow \frac{1}{2}\frac{4a^2}{a-3} = 24 \Rightarrow a^2 = 12(a-3) = 12a - 36$$

$$\Rightarrow a^2 - 12a + 36 = 0 \Rightarrow (a-6)^2 = 0 \Rightarrow a = 6$$

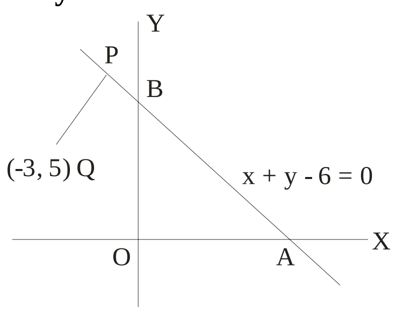
$$b = \frac{4a}{a-3} = \frac{24}{3} = 8$$

Equation of the line is $\frac{x}{6} + \frac{y}{8} = 1$

$\Rightarrow 4x + 3y = 24$

$\Rightarrow 4x + 3y - 24 = 0$

4. A straight line with slope 1 passes through Q (-3, 5) and meets the straight line x + y – 6 = 0 at P. Find the distance PQ.



Sol. Slope m = 1. Point on the line is Q (-3, 5)

Equation of the line is $y-5=1(x+3) \Rightarrow x-y+8=0$-----(1)

Given line is x+y-6 =0.----(2)

Solving (1) and (2), point of intersection is P(-1, 7)

$$PQ = \sqrt{(-1+3)^2 + (7-5)^2} = \sqrt{4+4} = \sqrt{8}\ 2\sqrt{2}$$

5. Find the set of values of 'a' if the points (1, 2) and (3, 4) lie to the same side of the straight line 3x – 5y + a = 0

Sol. Let P (1, 2), Q (3, 4)

Equation of the given line is L≈ 3x – 5y + a = 0

$L_{11} = 3.1 - 5.2 + a = a - 7$ and$L_{22} = 3.3 - 5.4 + a = a - 11$

Since the points are on the same side of L=0

$$\Rightarrow -\frac{L_{11}}{L_{22}} < o \Rightarrow -\frac{a-7}{a-11} < 0 \Rightarrow (a-7)(a-11) > 0$$

$\Rightarrow (a-7) < 0$ and $(a-11) > 0 \Rightarrow a < 7 and a > 11$

6. Show that the lines 2x + y – 3 = 0, 3x + 2y – 2 = 0 and 2x – 3y – 23 = 0 are concurrent and find the point of con-currency.

Sol. Equations of the given lines are

2x + y – 3 = 0 ---(1)
3x + y – 2 = 0 ---(2)
2x – 3y – 23 = 0 ---(3)

Solving (1) and (2)

$$\begin{array}{cccccc} & x & & y & & 1 \\ 1 & & -3 & & 2 & & 1 \\ 2 & & -2 & & 3 & & 2 \end{array}$$

$$\frac{x}{-2+6} = \frac{y}{-9+4} = \frac{1}{4-3}$$

x = 4, y = - 5

Therefore point of intersection is P (4, -5).

Substituting P in eq. (3), we get

2x – 3y -23 = 2(4) – 3(-5) – 23 = 8 + 15 – 23 = 0

$\Rightarrow$P lies on (3)

Hence the given lines are concurrent and point of concurrency is P (4, -5)

7. Find the value of p, if the following lines are concurrent.

i) 3x + 4y = 5, 2x + 3y = 4, p x + 4y = 6

ii) 4x – 3y = 7, 2x + p y + 2 = 0,
6x + 5y – 1 = 0

Sol. i) Given lines are 3x + 4y – 5 =0----(1) and 2x + 3y – 4 =0------(2)

Solving,

$$\begin{array}{cccccc} & x & & y & & 1 \\ 4 & & -5 & & 3 & & 4 \\ 3 & & -4 & & 2 & & 3 \end{array}$$

$$\frac{x}{-16+15} = \frac{y}{-10+12} = \frac{1}{9-8}$$

x = - 1, y = 2

Point of intersection of (1) and (2) is P (1, 2)

Since the lines are concurrent,P lies on px+4y=6

$\Rightarrow -p+8=6 \Rightarrow p=8-6=2$

ii) Ans: p = 4

8. Determine whether or not the four straight lines with equations x + 2y – 3 =0, 3x + 4y – 7 = 0, 2x + 3y – 4 = 0 and 4x + 5y – 6 = 0 are concurrent.

Sol. Equation of the given lines are

x + 2y – 3 = 0 ---(1)
3x + 4y – 7 = 0 ---(2)

$2x + 3y - 4 = 0$ ---(3)

$4x + 5y - 6 = 0$ ---(4)

Solving (1) and (2)

Point of intersection of (1) and (2) is P (1,1).

Substituting P in (3) and (4),

$2x + 3y - 4 = 2.1 + 3.1 - 4 = 5 - 4 - 1 \neq 0$

$4x + 5y - 6 = 4.1 + 5.1 - 6 = 9 - 6 = 3 \neq 0$

$\therefore$ P (1, 1) is not a point on (3) and (4)

$\therefore$ The given line is not concurrent.

9. If 3a + 2b +4c = 0, then show that the equation ax + by + c = 0 represents a family of concurrent straight lines and find the point of concurrency.

Sol. Ans: $\left(\frac{3}{4}, \frac{1}{2}\right)$

10. If non-zero numbers a, b, c are in harmonic progression then show that the equation $\frac{x}{a} + \frac{y}{b} + \frac{1}{c} = 0$ represents a family of concurrent lines and find the point of concurrency.

Sol. Given equation is $\frac{x}{a} + \frac{y}{b} + \frac{1}{c} = 0$

Given a, b, c are in H.P. $\Rightarrow \frac{2}{b} = \frac{1}{a} + \frac{1}{c} \Rightarrow \frac{1}{a} + \frac{(-2)}{b} + \frac{1}{c} = 0$

From above equation we can say the the line is $\frac{x}{a} + \frac{y}{b} + \frac{1}{c} = 0$ passing through the point (1, -2) .

$\therefore$ The equation $\frac{x}{a} + \frac{y}{b} + \frac{1}{c} = 0$ represent a family of concurrent lines and the point of concurrency is P (1, -2)

III.

1. Find the point on the straight lines 3x + y + 4 = 0which is equidistant form the points (-5, 6) and (3, 2).

Sol. Given points A(-5, 6) and B(3, 2).

Let $P(x_1, y_1)$ be the point on 3x + y + 4 = 0.

$\Rightarrow 3x_1 + y_1 + 4 = 0$ ---(1)

Given PA = PB $\Rightarrow PA^2 = PB^2$

$(x_1+5)^2+(y_1-6)^2=(x_1-3)^2+(y_1-2)^2$

$\Rightarrow x_1^2+10x_1+25+y_1^2-12y_1+36 \;= x_1^2-6x_1+9+y_1^2-4y_1+4$

$\Rightarrow 16x_1-8y_1+48=0 \Rightarrow 2x_1-y_1+6=0$ --------(2)

Solving (1) and (2), $P(x_1, y_1)$ = (-2 , 2)

2. A straight line through P(3, 4) makes an angle of 60º with the positive direction of the x-axes. Find the co-ordinates of the points on that line which are 5 units away from P.

Sol. Inclination Θ=60º , P(3, 4) = (x_1, y_1) and r =5.

The coordinates of the points which are at a distance 5 units from P are

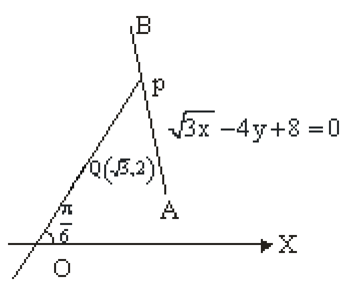
$(x_1 \pm r\cos\theta, y_1 \pm r\sin\theta) \;= (3\pm 5\cos 60, 4\pm 6\sin 60)$

Co-ordinates of Q are $\left(3\pm 5\frac{1}{2}, 4\pm 5\frac{\sqrt{3}}{2}\right)=\left(3-5\frac{1}{2}, 4-5\frac{\sqrt{3}}{2}\right)$ and $\left(3+5\frac{1}{2}, 4+5\frac{\sqrt{3}}{2}\right)$

$=\left(\frac{11}{2}, \frac{8+5\sqrt{3}}{2}\right)$ and $\left(\frac{1}{2}, \frac{8-5\sqrt{3}}{2}\right)$

3. A straight line through Q $(\sqrt{3}, 2)$ makes an angle of $\frac{\pi}{6}$ with X-axis in positive direction. If this straight line intersects $\sqrt{3}x-4y+8=0$ at P; find the distance of PQ.

Sol.



Inclination of the line is $\theta=\frac{\pi}{6}$

Slope m = tan 30º = $\frac{1}{\sqrt{3}}$ and Line is passing through Q$(\sqrt{3}, 2)$

$\Rightarrow$Equation of the line is y – 2 = $\frac{1}{\sqrt{3}}(x-\sqrt{3})$

$\Rightarrow \sqrt{3}y-2\sqrt{3}=x-3 \quad \Rightarrow x - \sqrt{3}y=-\sqrt{3}$ ---------(1)

Equation of given line is $\sqrt{3}x - 4y + 8 = 0$ ----(2)

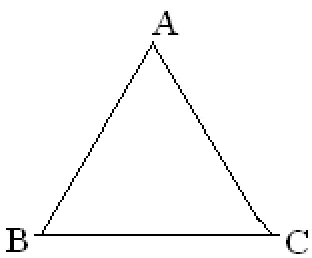
Solving (1) and (2), point of intersection is P=$\left(4\sqrt{3}, 5\right)$

$$PQ = \sqrt{\left(4\sqrt{3} - \sqrt{3}\right)^2 + (5-2)^2} = = \sqrt{27 + 9} = 6$$

$\Rightarrow$ PQ = 6 units.

4. Show that the origin is with in the triangle whose angular points are (2, 1), (3, 2) and (-4, -1).

Sol.



The vertices of the Δl e ABC are A = (2, 1), = (3, -2), = (-4, -1)

The equation of BC is $L = x + 7y + 11 = 0$ ---(1)

The equation of CA is $\Rightarrow L' = x - 3y + 1 = 0$ ---(2)

Equation of AB is L" = - 3x + y – 7 = 0 ---(3)

L" (-4, -1) = 3 (-4) -1 -7 = - 20 is negative.

L" (0, 0) = 3(0) + 0 – 7 = - 7 is negative

So (-4, -1), (0. 0) lie on the same side of AB

Hence O (0, 0) lies to the left of AB ---(4)

L' (3, -2) = 3 – 3(-2) + 1 = 10 is positive

L' (0, 0) =0 – 3(0) + 1 = 1 is positive

So (0, 0), (3, -2) lie on the same side of AC both down of AC ---(5)

L (2, 1) = 2 + 7 (1) + 11 = 20 is positive

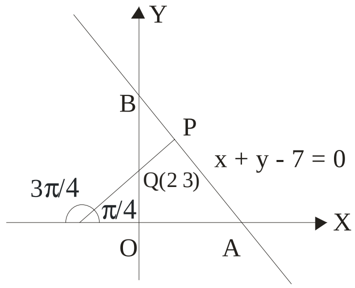
L (0, 0) = 0 + 7 (0) + 11 = 11 is positive

So (0, 0) and (2, 1) lie on the same side of BC.

So (0, 0) lie upwards form BC ---(6)

From (4), (5), (6), it follows O (0, 0) lies down-wards of AC, upwards of BC, and to the left of AB. So O (0, 0) will lie inside the Δ ABC.

5. A straight line through Q(2, 3) makes an angle $\frac{3\pi}{4}$ with negative direction of the X-axis. If the straight line intersects the line x + y – 7 = 0 at P, find the distance PQ.



Sol. The line PQ makes an angle $\frac{3\pi}{4}$ with the negative direction of X- axis i.e., PQ makes and angle $\pi - \frac{3\pi}{4} = \frac{\pi}{4}$ with the positive direction of X- axis.

Co-ordinates of Q are (2, 3)

Let the Co-ordinates of P be $(x_1 + r\cos\theta, y_1 + r\sin\theta)$

$= \left(2 + r.\cos\frac{\pi}{4}, 3 + r.\sin\frac{\pi}{4}\right) = \left(2 + \frac{r}{\sqrt{2}}, 3 + \frac{r}{\sqrt{2}}\right)$

P is a point on the line x + y – 7 = 0

$\Rightarrow 2 + \frac{r}{\sqrt{2}} + 3 + \frac{r}{\sqrt{2}} - 7 = 0 \Rightarrow 2.\frac{r}{\sqrt{2}} = 7 - 2 - 3 = 2$

$\therefore r = \sqrt{2}$

PQ = r = $\sqrt{2}$ units.

6. Show that the straight lines x + y = 0, 3x + y – 4 = 0 and x + 3y – 4 = 0 form an isosceles triangle.

Hint : solve the given equations and find the vertices A,B,C of the triangle. Then find sides AB,BC and CA. Then show that two sides are equal.

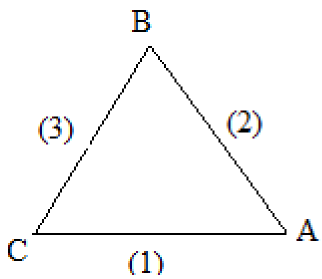
7. Find the area of the triangle formed by the straight lines 2x – y – 5 = 0, x – 5y + 11 = 0 and x + y – 1 = 0.

Sol. Given lines

2x – y – 5 = 0 ---(1)

x – 5y + 11 = 0 ---(2)

x + y – 1 = 0 ---(3)



SOLVING (1) and (2) , vertex A = (4, 3)
By solving (2), (3) vertex B = (-1, 2)
By solving (3), (1) vertex $c = (2, -1)$

$$= \frac{1}{2}\begin{vmatrix} 4+1 & 3-2 \\ 2+1 & -1-2 \end{vmatrix}$$

Area of Δ ABC $= \frac{1}{2}\begin{vmatrix} 5 & 1 \\ 3 & -3 \end{vmatrix}$

$$= \frac{1}{2}\left|-15-3\right|$$

$$= \frac{1}{2}\times 18 = 9 \text{ sq. units}$$