

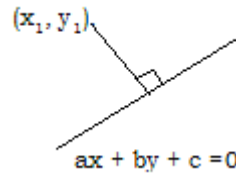
## LENGTH OF THE PERPENDICULAR FROM A POINT TO A STRAIGHT LINE AND DISTANCE BETWEEN TWO PARALLEL LINES

### THEOREM

The perpendicular distance from a point  $P(x_1, y_1)$  to the line  $ax + by + c = 0$  is

$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

**Proof:**



Let the axes be translated to the point  $P(x_1, y_1)$ .

Let  $(X, Y)$  be the new coordinates of  $(x, y)$ . Then  $x = X + x_1$ ,  $y = Y + y_1$ . The transformed equation of the given line is

$$a(X + x_1) + b(Y + y_1) + c = 0$$

$$\Rightarrow aX + bY + (ax_1 + by_1 + c) = 0$$

The perpendicular distance from the new origin  $P$  to the line is  $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$ .

(from normal form) The perpendicular distance from a point

$P(x_1, y_1)$  to the line  $ax + by + c = 0$  is  $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$ .

### DISTANCE BETWEEN PARALLEL LINES THEOREM

The distance between the two parallel lines  $ax + by + c_1 = 0$  and  $ax + by + c_2 = 0$  is

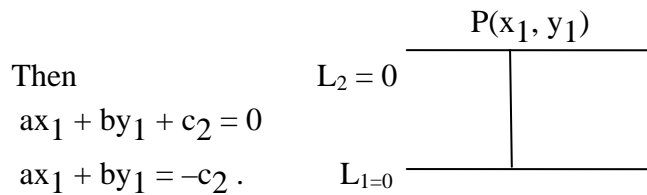
$$\frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}$$

**Proof:**

$$\text{Given lines are } ax + by + c_1 = 0 \quad \text{--- (1)}$$

$$ax + by + c_2 = 0 \quad \text{--- (2)}$$

Let  $P(x_1, y_1)$  be a point on the line (2).



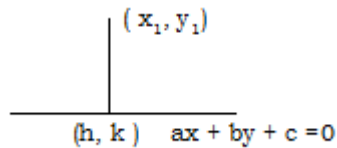
Distance between the parallel lines = Perpendicular distance from P to line (1)

$$= \frac{|ax_1 + by_1 + c_1|}{\sqrt{a^2 + b^2}} = \frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}$$

### FOOT OF THE PERPENDICULAR THEOREM

If  $(h, k)$  is the foot of the perpendicular from  $(x_1, y_1)$  to the line  $ax + by + c = 0$

$(a_0, b_0)$  then  $\frac{h - x_1}{a} = \frac{k - y_1}{b} = \frac{-(ax_1 + by_1 + c)}{a^2 + b^2}$ .



**Proof :**

Let  $A = (x_1, y_1)$   $P = (h, k)$

P lies on  $ax + by + c = 0$

$$ah + bk + c = 0$$

$$ah + bk = -c$$

Slope of  $\overline{AP}$  is  $\frac{k - y_1}{h - x_1}$

Slope of given line is  $-\frac{a}{b}$

$\overline{AP}$  is perpendicular to the given line

$$\Rightarrow \left( \frac{k - y_1}{h - x_1} \right) \left( -\frac{a}{b} \right) = -1$$

$$\Rightarrow \frac{k - y_1}{b} = \frac{h - x_1}{a}$$

By the law of multipliers in ratio and proportion

$$\frac{h - x_1}{a} = \frac{k - y_1}{b} = \frac{a(h - x_1) + b(k - y_1)}{a^2 + b^2}$$

$$= \frac{ah + bk - ax_1 - by_1}{a^2 + b^2} = \frac{-ax_1 - by_1 - c}{a^2 + b^2}$$

$$= \frac{-(ax_1 + by_1 + c)}{a^2 + b^2}$$

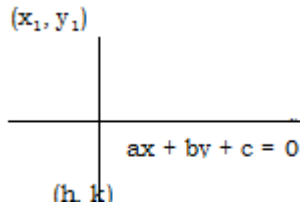
$$\text{Hence } \frac{h - x_1}{a} = \frac{k - y_1}{b} = \frac{-(ax_1 + by_1 + c)}{a^2 + b^2}$$

## IMAGE OF A POINT

### THEOREM

If  $(h, k)$  is the image of  $(x_1, y_1)$  w.r.t the line  $ax + by + c = 0$  ( $a \neq 0, b \neq 0$ ),

$$\text{then } \frac{h - x_1}{a} = \frac{k - y_1}{b} = \frac{-2(ax_1 + by_1 + c)}{a^2 + b^2} .$$



**Proof:**

Let  $A(x_1, y_1)$ ,  $B(h, k)$

$$\text{Mid point of } \overline{AB} \text{ is } P = \left( \frac{x_1 + h}{2}, \frac{y_1 + k}{2} \right)$$

Since  $B$  is the image of  $A$ , therefore mid point  $P$  lies on  $ax + by + c = 0$ .

$$\Rightarrow a \left( \frac{x_1 + h}{2} \right) + b \left( \frac{y_1 + k}{2} \right) + c = 0$$

$$\Rightarrow ax_1 + by_1 + ah + bk + 2c = 0$$

$$\Rightarrow ah + bk = -ax_1 - by_1 - 2c.$$

$$\text{Slope of } \overline{AB} \text{ is } \frac{k - y_1}{h - x_1}$$

$$\text{And Slope of given line is } -\frac{a}{b}$$

$\overline{AB}$  is perpendicular to the given line

$$\Rightarrow \left( \frac{k - y_1}{h - x_1} \right) \left( -\frac{a}{b} \right) = -1$$

$$\Rightarrow \frac{k - y_1}{b} = \frac{h - x_1}{a}$$

By the law of multipliers in ratio and proportion

$$\begin{aligned}
\frac{h-x_1}{a} &= \frac{k-y_1}{b} = \frac{a(h-x_1)+b(k-y_1)}{a^2+b^2} \\
&= \frac{ah+bk-ax_1-by_1}{a^2+b^2} \\
&= \frac{-ax_1-by_1-2c-ax_1-by_1}{a^2+b^2} \\
&= \frac{-2(ax_1+by_1+c)}{a^2+b^2}
\end{aligned}$$

$$\text{Hence } \frac{h-x_1}{a} = \frac{k-y_1}{b} = \frac{-2(x_1+by_1+c)}{a^2+b^2}$$

**Note 1:** The image of  $(x_1, y_1)$  w.r.t the line  $x = y$  is  $(y_1, x_1)$

**Note 2:** The image of  $(x_1, y_1)$  w.r.t the line  $x + y = 0$  is  $(-y_1, -x_1)$

### THEOREM

**If the four straight lines  $ax + by + p = 0$ ,  $ax + by + q = 0$ ,  $cx + dy + r = 0$  and  $cx + dy + s = 0$  form a parallelogram. Then the area of the parallelogram so formed is**

$$\left| \frac{(p-q)(r-s)}{bc-ad} \right|$$

**Proof:**

$$\text{Let } L_1 = ax + by + p = 0$$

$$L_2 = ax + by + q = 0$$

$$L_3 = cx + dy + r = 0$$

$$L_4 = cx + dy + s = 0$$

Clearly

$L_1 \parallel L_2$  and  $L_3 \parallel L_4$ . So  $L_1$  and  $L_3$  are nonparallel. Let  $\theta$  be the angle between  $L_1$  and  $L_3$ .

$$\text{Let } d_1 = \text{distance between } L_1 \text{ and } L_2 = \frac{|p-q|}{\sqrt{a^2+b^2}}$$

$$\text{Let } d_2 = \text{distance between } L_3 \text{ and } L_4 = \frac{|r-s|}{\sqrt{c^2+d^2}}$$

$$\begin{aligned}
\text{Now } \cos \theta &= \frac{|ac+bd|}{\sqrt{(a^2+b^2)(c^2+d^2)}} \quad \text{and} \quad \sin \theta = \frac{\sqrt{(a^2+b^2)(c^2+d^2)-(ac+bd)^2}}{(a^2+b^2)(c^2+d^2)} \\
&= \frac{|bc-ad|}{\sqrt{(a^2+b^2)(c^2+d^2)}}
\end{aligned}$$

$$\text{Now area of the parallelogram is } \frac{d_1 d_2}{\sin \theta} = \left| \frac{(p-q)(r-s)}{bc-ad} \right|$$

## ANGLE BETWEEN TWO LINES THEOREM

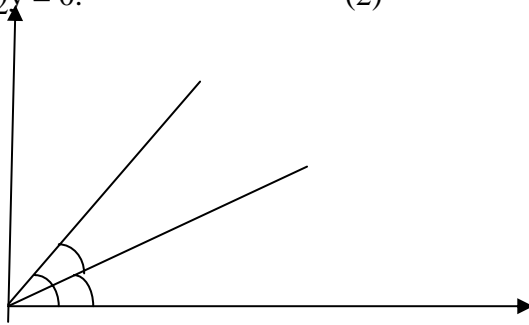
If  $\theta$  is an angle between the lines  $a_1x + b_1y + c_1 = 0$ ,  $a_2x + b_2y + c_2 = 0$  then

$$\cos \theta = \pm \frac{a_1a_2 + b_1b_2}{\sqrt{a_1^2 + b_1^2} \sqrt{a_2^2 + b_2^2}}$$

**Proof:** The lines passing through the origin and parallel to the given lines are

$$a_1x + b_1y = 0, \quad \text{-- (1)}$$

$$a_2x + b_2y = 0. \quad \text{-- (2)}$$



Let  $\theta_1, \theta_2$  be the inclinations of (1) and (2) respectively ( $\theta_1 > \theta_2$ )

Now  $\theta$  is an angle between (1) and (2)

$$\theta = \theta_1 - \theta_2$$

$P(-b_1, a_1)$  satisfies eq(1), the point lies on (1)

Similarly,  $Q(-b_2, a_2)$  lies on (2)

Let  $L, M$  be the projection of  $P, Q$  respectively on the  $x$  - axis.

$$\therefore \cos \theta_1 = \frac{OL}{OP} = \frac{-b_1}{\sqrt{a_1^2 + b_1^2}}, \quad \sin \theta_1 = \frac{PL}{OP} = \frac{a_1}{\sqrt{a_1^2 + b_1^2}}$$

$$\therefore \cos \theta_2 = \frac{OM}{OQ} = \frac{-b_2}{\sqrt{a_2^2 + b_2^2}}, \quad \sin \theta_2 = \frac{MQ}{OQ} = \frac{a_2}{\sqrt{a_2^2 + b_2^2}}$$

$$\theta = \theta_1 - \theta_2$$

$$\cos = \cos (\theta_1 - \theta_2)$$

$$= \cos \theta_1 \cos \theta_2 + \sin \theta_1 \sin \theta_2$$

$$= \frac{(-b_1)}{\sqrt{a_1^2 + b_1^2}} \frac{(-b_2)}{\sqrt{a_2^2 + b_2^2}} + \frac{a_1}{\sqrt{a_1^2 + b_1^2}} \frac{a_2}{\sqrt{a_2^2 + b_2^2}}$$

$$= \frac{a_1a_2 + b_1b_2}{\sqrt{a_1^2 + b_1^2} \sqrt{a_2^2 + b_2^2}}$$

**Note 1:** If  $\theta$  is the acute angle between the lines then

$$\cos \theta = \frac{|a_1a_2 + b_1b_2|}{\sqrt{a_1^2 + b_1^2} \sqrt{a_2^2 + b_2^2}}$$

**Note 2:** If  $\theta$  is an angle between two lines, then  $\phi$  is another angle between the lines.

**Note 3:** If  $\theta$  is an angle between two lines are not a right angle then the angle between the lines means the acute angle between the lines.

**Note 4:** If  $\theta$  is an angle between the lines  $a_1x + b_1y + c_1 = 0$ ,  $a_2x + b_2y + c_2 = 0$

$$\text{then } \tan \theta = \frac{a_1b_2 - a_2b_1}{a_1a_2 + b_1b_2}$$

**Note 5:** If  $\theta$  is the acute angle between the lines

$$a_1x + b_1y + c_1 = 0, a_2x + b_2y + c_2 = 0 \text{ then}$$

$$\begin{aligned} \tan \theta &= \frac{|a_1b_2 - a_2b_1|}{|a_1a_2 + b_1b_2|} = \frac{|a_1/b_1 - a_2/b_2|}{|(a_1a_2)/(b_1b_2) + 1|} \\ &= \frac{|(-a_1/b_1) - (-a_2/b_2)|}{|1 + (-a_1/b_1)(-a_2/b_2)|} \text{ where } m_1, m_2 \text{ are the slopes of the lines.} \end{aligned}$$

## THEOREM

**The equation of the line parallel to  $ax + by + c = 0$  and passing through  $(x_1, y_1)$  is  $a(x - x_1) + b(y - y_1) = 0$ .**

**Proof:**

Slope of the given line is  $-a/b$ .

$\Rightarrow$  Slope of the required line is  $-a/b$ . (lines are parallel)

Equation of the required line is

$$y - y_1 = -\frac{a}{b} (x - x_1)$$

$$b(y - y_1) = -a(x - x_1)$$

$$a(x - x_1) + b(y - y_1) = 0.$$

**Note 1:** The equation of a line parallel to  $ax + by + c = 0$  may be taken as  $ax + by + k = 0$ .

**Note 2:** The equation of a line parallel to  $ax + by + c = 0$  and passing through the origin is  $ax + by = 0$ .

## THEOREM

The equation of the line perpendicular to  $ax + by + c = 0$  and passing through  $(x_1, y_1)$  is  $b(x - x_1) - a(y - y_1) = 0$ .

### Proof:

Slope of the given line is  $-a/b$ .  $\Rightarrow$  Slope of the required line is  $b/a$ .  
(since product of slopes = -1)

Equation of the required line is  $y - y_1 = \frac{b}{a} (x - x_1)$

$$a(y - y_1) = b(x - x_1)$$

$$b(x - x_1) - a(y - y_1) = 0.$$

**Note 1:** The equation of a line perpendicular to  $ax + by + c = 0$  may be taken as

$$bx - ay + k = 0$$

**Note 2:** The equation of a line perpendicular to  $ax + by + c = 0$  and passing through the origin is  $bx - ay = 0$ .

## EXERCISE -3 (d)

I. Find the angle between the following straight lines.

1.  $y = 4 - 2x$ ,  $y = 3x + 7$

**Sol:** given lines are

$$y = 4 - 2x \Rightarrow 2x + y - 4 = 0 \text{ and } 3x - y + 7 = 0$$

Let  $\theta$  be the angle between the lines, then

$$\begin{aligned} \cos \theta &= \frac{|a_1 a_2 + b_1 b_2|}{\sqrt{a_1^2 + b_1^2} \sqrt{a_2^2 + b_2^2}} \\ &= \frac{|2 \cdot 3 + 1(-1)|}{\sqrt{4+1} \sqrt{9+1}} = \frac{5}{\sqrt{5} \sqrt{10}} = \frac{1}{\sqrt{2}} \end{aligned}$$

$$\therefore \theta = \frac{\pi}{4}$$

2.  $3x + 5y = 7$ ,  $2x - y + 4 = 0$

**Sol.** ans :  $\theta = \cos^{-1} \left( \frac{1}{\sqrt{170}} \right)$

3.  $y = -\sqrt{3}x + 5, y = \frac{1}{\sqrt{3}}x - \frac{2}{\sqrt{3}}$

**Sol.** slope of 1<sup>st</sup> line is  $m_1 = -\sqrt{3}$

Slope of 2<sup>nd</sup> line is  $m_2 = \frac{1}{\sqrt{3}}$ .

$$m_1 m_2 = (-\sqrt{3}) \frac{1}{\sqrt{3}} = -1.$$

The lines are perpendicular, hence angle between the lines is  $\theta = \frac{\pi}{2}$

4.  $ax + by = a + b, a(x - y) + b(x + y) = 2b$

**Sol.** given lines  $ax + by = a + b, (a + b)x + (-a + b)y = 2b$

let  $\theta$  be the angle between the lines, then

$$\begin{aligned} \cos \theta &= \frac{|a(a+b) + b(-a+b)|}{\sqrt{a^2 + b^2} \sqrt{(a+b)^2 + (-a+b)^2}} \\ &= \frac{|a^2 + ab - ab + b^2|}{\sqrt{a^2 + b^2} \sqrt{2(a^2 + b^2)}} = \frac{a^2 + b^2}{\sqrt{2}(a^2 + b^2)} = \frac{1}{\sqrt{2}} \end{aligned}$$

$$\therefore \theta = \frac{\pi}{4}$$

Find the length of the perpendicular drawn from the point given against the following straight lines.

5.  $5x - 2y + 4 = 0, (-2, -3)$

**Sol.** Length of the perpendicular  $= \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}} = \frac{|5(-2) - 2(-3) + 4|}{\sqrt{25 + 4}} = \frac{|-10 + 10|}{\sqrt{29}} = 0$

6.  $3x - 4y + 10 = 0, (3, 4)$

**Sol.** Length of the perpendicular  $= \frac{|3.3 - 4.4 + 10|}{\sqrt{9 + 16}} = \frac{3}{5}$

7.  $x - 3y - 4 = 0, (0, 0)$

**Sol.** Ans;  $\frac{4}{\sqrt{10}}$

Find the distance between the following parallel lines.

8.  $3x - 4y = 12, 3x - 4y = 7$

**Sol.** Given lines are  $3x - 4y = 12, 3x - 4y = 7$



$$\text{Distance between parallel lines} = \frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}} = \frac{|-12 + 7|}{\sqrt{9 + 16}} = \frac{5}{5} = 1$$

**9.  $5x - 3y - 4 = 0, 10x - 6y - 9 = 0$**

**Sol.** Equations of the lines can be taken as  $10x - 6y - 8 = 0, 10x - 6y - 9 = 0$

$$\text{Distance between parallel lines} = \frac{|-8 + 9|}{\sqrt{100 + 36}} = \frac{1}{2\sqrt{34}}$$

**10. Find the equation of the straight line parallel to the line  $2x + 3y + 7 = 0$  and passing through the point  $(5, 4)$ .**

**Sol.** Given line is  $2x + 3y + 7 = 0$

Equation of the parallel to  $2x + 3y + 7 = 0$  is  $2x + 3y = k$ .

This line is passing through P  $(5, 4)$

$$\Rightarrow 10 + 12 = k \Rightarrow k = 22$$

Equation of the required line is  $2x + 3y - 22 = 0$

**11. Find the equation of the straight line perpendicular to the line  $5x - 3y + 1 = 0$  and passing through the point  $(4, -3)$ .**

**Sol.** Equation of the given line is  $5x - 3y + 1 = 0$

Equation of the perpendicular to  $5x - 3y + 1 = 0$  is  $3x + 5y + k = 0$

This line is passing through P  $(4, -3)$

$$\Rightarrow 12 - 15 + k = 0 \Rightarrow k = 3$$

Equation of the required line is  $3x + 5y + 3 = 0$

**12. Find the value of k, if the straight lines  $6x - 10y + 3 = 0$  and  $kx - 5y + 8 = 0$  are parallel.**

**Sol.** Given lines are  $6x - 10y + 3 = 0$  and  $kx - 5y + 8 = 0$ .

$$\text{Since the lines are parallel} \Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} \Rightarrow \frac{6}{k} = \frac{-10}{-5} \Rightarrow -30 = -10k \Rightarrow k = 3$$

**13. Find the value of P, if the straight lines  $3x + 7y - 1 = 0$  and  $7x - py + 3 = 0$  are mutually perpendicular.**

**Sol.** Given lines are  $3x + 7y - 1 = 0, 7x - py + 3 = 0$

Since the lines are perpendicular

$$\Rightarrow a_1 a_2 + b_1 b_2 = 0 \Rightarrow 3 \cdot 7 + (-p) = 0 \Rightarrow 7p = 21 \Rightarrow p = 3$$

**14. Find the value of k, the straight lines  $y - 3kx + 4 = 0$  and  $(2k - 1)x - (8k - 1)y = 6$  are perpendicular.**

**Sol.** Given lines are  $-3kx + y + 4 = 0$

$$(2k - 1)x - (8k - 1)y - 6 = 0$$

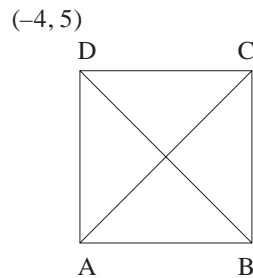
These lines are perpendicular  $\Rightarrow a_1a_2 + b_1b_2 = 0$

$$\Rightarrow -3k(2k - 1) - 1(8k - 1) = 0 \Rightarrow -6k^2 + 3k - 8k + 1 = 0$$

$$6k^2 + 5k - 1 = 0 \Rightarrow (k + 1)(6k - 1) = 0$$

$$k = -1 \text{ or } 1/6$$

15. **(-4, 5) is a vertex of a square and one of its diagonal is  $7x - y + 8 = 0$ . Find the equation of the other diagonal.**



**Sol.** let ABCD be the square.

Let the equation of the diagonal AC be  $7x - y + 8 = 0$

The point (-4, 5) is not satisfying the equation.

Let D = (-4, 5)

The other diagonal BD is perpendicular to AC.

Equation of BD is can be taken as  $x + 7y + k = 0$

$$D (-4, 5) \text{ is a point on this line } \Rightarrow -4 + 35 + k = 0 \Rightarrow k = 4 - 35 = -31$$

Equation of BD is  $x + 7y - 31 = 0$

II.

1. **Find the equation of the straight lines passing through (1, 3) and**

**i) parallel to ii) perpendicular to the line passing through the points (3, -5) and (-6, 1).**

**Sol.** Given points A (3, -5), B (-6, 1)

$$\text{Slope of AB} = \frac{-5 - 1}{3 - (-6)} = \frac{-6}{9} = \frac{-2}{3}$$

- i) slope of the line parallel to AB is  $\frac{-2}{3}$  equation of the line parallel to AB and passing through (1, 3) is

$$y - 3 = \frac{-2}{3}(x - 1) \Rightarrow 3y - 9 = -2x + 2 \Rightarrow 2x + 3y - 11 = 0$$

- ii) slope of the line perpendicular to AB is  $3/2$  .

Equation of the line passing through (1, 3) and having slope  $\frac{3}{2}$  is  $y - 3 = \frac{-2}{3}(x - 1)$   
 $\Rightarrow 2x + 3y - 5 = 0$ .

2. The line  $\frac{x}{a} - \frac{y}{b} = 1$  meets the X - axis at P. Find the equation of the line perpendicular to this line at P.

Sol. given line is  $\frac{x}{a} - \frac{y}{b} = 1$  -----(1)

On x- axis  $y = 0 \Rightarrow \frac{x}{a} - \frac{0}{b} = 1 \Rightarrow x = a$

Point P = (a, 0)

Equation of the line perpendicular to (1) is  $\frac{x}{b} + \frac{y}{a} = k$

This line is passing through P (a,0)  $\Rightarrow \frac{a}{b} + 0 = k \Rightarrow k = a/b$

Equation of the line is  $\frac{x}{b} + \frac{y}{a} = \frac{a}{b}$ .

3. Find the equation of the line perpendicular to the line  $3x + 4y + 6 = 0$  and making intercept -4 on X - axis .

Sol. Given line is  $3x + 4y + 6 = 0$ .

Equation of the perpendicular to  $3x + 4y + 6 = 0$  is  $4x - 3y = k$

$\Rightarrow \frac{4x}{k} - \frac{3y}{k} = 1 \Rightarrow \left[ \frac{x}{4} \right] + \left[ \frac{-y}{3} \right] = 1$

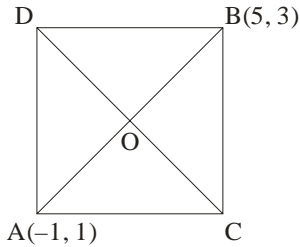
x - intercept =  $\frac{k}{4} = -4 \Rightarrow k = -16$

Equation of the required line is  $4x - 3y = -16 \Rightarrow 4x - 3y + 16 = 0$ .

4. A (-1, 1), B (5,3) are opposite vertices of a square in the XY plane. Find the equation of the other diagonal (not passing through A, B) of a square.

Sol. A(-1,1), B (5, 3) are opposite vertices of the square.

Slope of AB =  $\frac{1-3}{-1-5} = \frac{-2}{-6} = \frac{1}{3}$



The other diagonal is perpendicular to AB

$$\text{Slope of CD} = -\frac{1}{m} = -3$$

Let 'O' is the point of intersection of the diagonals then  $O = \left( \frac{-1+5}{2}, \frac{1+3}{2} \right) = (2, 2)$

Diagonal CD is passing through O (2,2), Equation of CD is  $y - 2 = -3(x - 2)$   
 $= -3x + 6 \Rightarrow 3x + y - 8 = 0.$

- 5. Find the foot of the perpendicular drawn from (4,1) upon the straight line  $3x - 4y + 12 = 0$ .**

**Sol.** Equation of the line is  $3x - 4y + 12 = 0$

If (h,k) is the foot of the perpendicular from  $(x_1, y_1)$  on the line  $ax + by + c = 0$ , then

$$\frac{h - x_1}{a} = \frac{k - y_1}{b} = -\frac{(ax_1 + by_1 + c)}{a^2 + b^2}$$

$$\frac{h - 4}{3} = \frac{k - 1}{-4} = -\frac{(12 - 4 + 12)}{9 + 16}$$

$$\frac{h - 4}{3} = \frac{k - 1}{-4} = -\frac{20}{25} = -\frac{4}{5}$$

$$h - 4 = -\frac{12}{5} \Rightarrow h = 4 - \frac{12}{5} = \frac{20 - 12}{5} = \frac{8}{5}$$

$$k - 1 = \frac{16}{5} \Rightarrow k = 1 + \frac{16}{5} = \frac{5 + 16}{5} = \frac{21}{5}$$

$$\therefore \text{Foot of the perpendicular} = \left( \frac{8}{5}, \frac{21}{5} \right)$$

- 6. Find the foot of the perpendicular drawn from (3,0) upon the straight line  $5x + 12y - 41 = 0$ .**

**Sol.** Ans:  $\left( \frac{49}{13}, \frac{24}{13} \right)$

7.  $x - 2y - 5 = 0$  is the perpendicular bisector of the line segment joining the points A,B. If A = (-1,-3), find the co-ordinates of B.

**Sol.** If PQ is the perpendicular bisector of AB, then B is the image of A in the line PQ.

Equation of the line is  $x - 3y - 5 = 0$

Given point A = (-1,-3) . Here B is the image of A w.r.t  $x - 3y - 5 = 0$ .

Let B(h,k) be the image of A (-1,-3), then

$$\frac{h - x_1}{a} = \frac{k - y_1}{b} = \frac{-2(ax_1 + by_1 + c)}{a^2 + b^2}$$

$$\frac{h + 1}{1} = \frac{k + 3}{-3} = \frac{-2(-1 + 9 - 5)}{1 + 9}$$

$$\frac{h + 1}{1} = \frac{y_2 + 3}{-3} = -\frac{3}{5}$$

$$\frac{h + 1}{1} = -\frac{3}{5} \Rightarrow h + 1 = -\frac{3}{5}$$

$$h = -1 - \frac{3}{5} = -\frac{8}{5} \Rightarrow \frac{y_2 + 3}{-3} = -\frac{3}{5}$$

$$k + 3 = \frac{9}{5} \Rightarrow k = \frac{9}{5} - 3 = -6/5$$

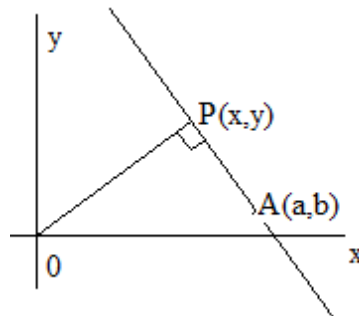
Co-ordinates of B are  $\left(\frac{-8}{5}, \frac{-6}{5}\right)$ .

8. Find the image of the point straight line  $3x + 4y - 1 = 0$ .

**Sol.** Ans:  $\left(-\frac{7}{5}, -\frac{6}{5}\right)$

9. Show that the distance of the point (6,-2) from the line  $4x + 3y = 12$  is half of the distance of the point (3,4) from the line  $4x - 3y = 12$ .

10. Find the locus of foot of the perpendicular from the origin to a variable straight line which always passes through a fixed point (a,b).



**Sol.**

Let P(x,y) be the foot of the perpendicular from O(0,0) to the line.

$$\text{Slope of OP} = \frac{y}{x}$$

Line is passing through A(a,b).

$$\text{Slope of AP} = \frac{y-a}{x-b}$$

Line AP is perpendicular to OP,  $\Rightarrow$  product of slopes = -1

$$\Rightarrow \frac{y}{x} \cdot \frac{y-a}{x-b} = -1 \Rightarrow y^2 - by = -(x^2 - ax)$$

$$\Rightarrow x^2 + y^2 - ax - by = 0$$

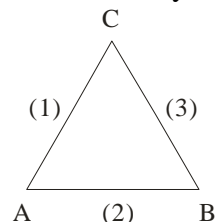
III.

1. Show that the lines  $x - 7y - 22 = 0$ ,  $3x + 4y + 9 = 0$  and  $7x + y - 54 = 0$  form a right angled isosceles triangle.

Sol. Given lines  $x - 7y - 22 = 0$  -----(1)

$3x + 4y + 9 = 0$  -----(2)

$7x + y - 54 = 0$  -----(3)



Let 'A' be the angle between (1), (2)

$$\cos A = \frac{|a_1 a_2 + b_1 b_2|}{\sqrt{a_1^2 + b_1^2} \sqrt{a_2^2 + b_2^2}} = \frac{|3 - 28|}{\sqrt{1 + 49} \sqrt{9 + 16}} = \frac{25}{5\sqrt{2} \cdot 5} = \frac{25}{25\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow A = 45^\circ$$

Let B be the angle between (2), (3) then  $\cos B = \frac{21 + 4}{\sqrt{9 + 16} \sqrt{49 + 1}} = \frac{25}{25\sqrt{2}} = \frac{1}{\sqrt{2}}$

$$\Rightarrow B = 45^\circ$$

Let 'C' be the angle between (3), (1)

$$\cos C = \frac{7 - 7}{\sqrt{1 + 49} \sqrt{49 + 1}} = 0 \Rightarrow C = 90^\circ$$

Since  $\angle A = \angle B = 45^\circ$  and  $\angle C = 90^\circ$

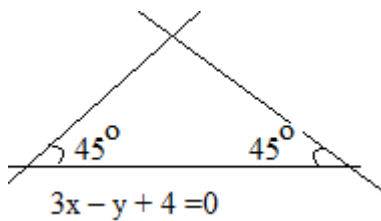
$\therefore$  Given lines form a right angled isosceles triangle.

2. Find the equation of the straight lines passing through the point (-3,2) and making an angle of  $45^\circ$  with the straight line  $3x - y + 4 = 0$ .

Sol. Given point P (-3,2)

Given line  $3x - y + 4 = 0$  -----(1)

Slope  $m_1 = -\frac{a}{b} = 3$



Let  $m$  be the slope of the required line.

Then  $\tan 45^\circ = \frac{m-3}{1+3m}$

$$\Rightarrow \left| \frac{m-3}{1+3m} \right| = 1 \Rightarrow \frac{m-3}{1+3m} = 1$$

$$\Rightarrow m-3 = 1+3m \Rightarrow 2m = -4 \text{ or } m = -2$$

$$\Rightarrow \frac{m-3}{1+3m} = -1 \Rightarrow m-3 = -1-3m$$

$$\Rightarrow 4m = 2 \Rightarrow m = \frac{1}{2}$$

**case (1)**  $m = -2$  and point  $(-3,2)$

Equation of the line is

$$y - 2 = -2(x + 3) = -2x - 6 \Rightarrow 2x + y + 4 = 0$$

**case (2)**  $m = \frac{1}{2}$ , point  $(-3,2)$

Equation of the line is

$$y - 2 = \frac{1}{2}(x + 3) \Rightarrow 2y - 4 = x + 3 \Rightarrow x - 2y + 7 = 0$$

**3. Find the angle of the triangle whose sides are  $x + y - 4 = 0$ ,  $2x + y - 6 = 0$ ,  $5x + 3y - 15 = 0$ .**

**Sol.** Ans:  $\cos^{-1} \left( \frac{4}{\sqrt{17}} \right), \cos^{-1} \left( \frac{13}{\sqrt{170}} \right), \cos^{-1} \left( \frac{3}{\sqrt{10}} \right)$

**4. Prove that the foot of the perpendiculars from the origin on the lines  $x + y = 4$ ,  $x + 5y = 26$  and  $15x - 27y = 424$  are collinear.**

**5. Find the equation of the lines passing through the point of intersection of the lines  $3x + 2y + 4 = 0$ ,  $2x + 5y = 1$  and whose distance from  $(2, -1)$  is 2.**

**Sol.** Equation of the lines passing through the point of intersection of the line

$L_1 \equiv 3x + 2y + 4 = 0, L_2 \equiv 2x + 5y - 1 = 0$  is

$$L_1 + \lambda L_2 = 0$$

$$(3x + 2y + 4) + \lambda (2x + 5y - 1) = 0$$

$$\Rightarrow (3 + 2\lambda)x + (2 + 5\lambda)y + (4 - \lambda) = 0 \text{ -----(1)}$$

Given distance from (2, -1) to (1) = 2

$$\frac{|(3 + 2\lambda)2 + (2 + 5\lambda)(-1) + (4 - \lambda)|}{\sqrt{(3 + 2\lambda)^2 + (2 + 5\lambda)^2}} = 2$$

$$\Rightarrow \frac{|-2\lambda + 8|}{\sqrt{(3 + 2\lambda)^2 + (2 + 5\lambda)^2}} = 2$$

$$\Rightarrow (-\lambda + 4)^2 = 9 + 4\lambda^2 + 12\lambda + 4 + 25\lambda^2 + 20\lambda$$

$$\Rightarrow 28\lambda^2 + 40\lambda - 3 = 0$$

$$\Rightarrow 28\lambda^2 - 2\lambda + 42\lambda - 3 = 0$$

$$\Rightarrow (2\lambda + 3)(14\lambda - 1) = 0 \Rightarrow \lambda = \frac{1}{14}, \lambda = -\frac{3}{2}$$

From (1)

If  $\lambda = \frac{1}{14}$ , then equation of the line is  $4x + 3y + 5 = 0$

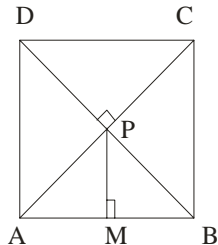
If  $\lambda = -\frac{3}{2}$ , then equation of the line is

$$y - 1 = 0.$$

- 6. Each side of a square is of length 4 units. The center of the square is (3, 7) and one of its diagonals is parallel to  $y = x$ . Find the Co-ordinates of its vertices.**

**Sol.** Let ABCD be the square. Side AB = 4

Point of intersection of the diagonals is the center P(3, 7)



From P drawn  $PM \perp AB$ . Then M is midpoint of AB

$$\therefore AM = MB = PM = 2$$

Since a diagonal is parallel to  $y = x$ , its sides are parallel to the co-ordinate axes.

$$M(3, 5)$$

$$\Rightarrow A(3-2, 5), B(3+2, 5), C(3+5, 7+2), D(3-2, 7+2)$$





