

CHAPTER 7
THE PLANE

TOPICS:

- 1. Equations of a plane**
- 2. Normal form**
- 3. perpendicular distance from a point to a plane.**
- 3. Intercept form**
- 4. Angle between two planes**
- 5. Distance between two parallel planes.**

PLANES

Definition:

A surface in space is said to be a plane surface or a plane if all the points of the straight line joining any two points of the surface lie on the surface.

THEOREM

The equation of the plane passing through a point (x_1, y_1, z_1) and perpendicular to a line whose direction ratios are a, b, c is $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

THEOREM

The equation of the plane passing through a point (x_1, y_1, z_1) is $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$ where a, b, c are constants.

THEOREM

The equation of the plane containing three points $(x_1, y_1, z_1), (x_2, y_2, z_2), (x_3, y_3, z_3)$

$$\text{is } \begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$

NORMAL FORM OF A PLANE

THEOREM

The equation of the plane which is at a distance of p from the origin and whose normal has the direction cosines (l, m, n) is $lx + my + nz = p$ (or) $x \cos \alpha + y \cos \beta + z \cos \gamma = p$.

NOTE. Equation of the plane through the origin is $lx + my + nz = 0$

Note: The equation of the plane $ax + by + cz + d = 0$ in the normal form is

$$\frac{a}{\sqrt{\sum a^2}}x + \frac{b}{\sqrt{\sum a^2}}y + \frac{c}{\sqrt{\sum a^2}}z = \frac{-d}{\sqrt{\sum a^2}} \text{ if } d \leq 0,$$

$$\frac{-a}{\sqrt{\sum a^2}}x + \frac{-b}{\sqrt{\sum a^2}}y + \frac{-c}{\sqrt{\sum a^2}}z = \frac{+d}{\sqrt{\sum a^2}} \text{ if } d > 0 \text{ where } \sum a^2 = a^2 + b^2 + c^2.$$

PERPENDICULAR DISTANCE FROM A POINT TO A PLANE

The perpendicular distance from the origin to the plane $ax + by + cz + d = 0$ is

$$\frac{|d|}{\sqrt{a^2 + b^2 + c^2}}$$

THEOREM

The perpendicular distance from $P(x_1, y_1, z_1)$ to the plane $ax + by + cz + d = 0$ is

$$\frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

THEOREM

Intercept form of the plane

The equation of the plane having a, b, c as x, y, z - intercepts respectively is

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

THEOREM

The intercepts of the plane $ax + by + cz + d = 0$ are respectively $\frac{-d}{a}, \frac{-d}{b}, \frac{-d}{c}$

ANGLE BETWEEN TWO PLANES

Definition: The angle between the normals to two planes is called the angle between the planes.

THEOREM

If θ is the angle between the planes $a_1x + b_1y + c_1z + d_1 = 0$, $a_2x + b_2y + c_2z + d_2 = 0$

$$\text{then } \cos \theta = \frac{a_1a_2 + b_1b_1 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \sqrt{a_2^2 + b_2^2 + c_2^2}}$$

Note

1. if θ is acute then $\cos \theta = \left| \frac{a_1a_2 + b_1b_1 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \sqrt{a_2^2 + b_2^2 + c_2^2}} \right|$
2. The planes $a_1x + b_1y + c_1z + d_1 = 0$, $a_2x + b_2y + c_2z + d_2 = 0$ are
 - (i) parallel iff $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

(ii) Perpendicular iff . $a_1a_2 + b_1b_2 + c_1c_2 = 0$

3. The given planes are perpendicular

$$\Leftrightarrow \theta = 90^\circ \Leftrightarrow \cos \theta = 0 \Rightarrow a_1a_2 + b_1b_2 + c_1c_2 = 0$$

THEOREM

The equation of the plane parallel to the plane $ax + by + cz + d = 0$ is

$ax + by + cz + k = 0$ where k is a constant.

THEOREM

The distance between the parallel planes $ax + by + cz + d_1 = 0, ax + by + cz + d_2 = 0$ is

$$\frac{|d_1 - d_2|}{\sqrt{a^2 + b^2 + c^2}} .$$

EXERCISE 7

I.

- 1. Find the equation of the plane if the foot of the perpendicular from origin to the plane is (1, 3, -5).**

Sol. Given point $P = (1, 3, -5)$.

\overline{op} is the normal to the plane

D. Rs of \overline{op} are 1, 3, -5.

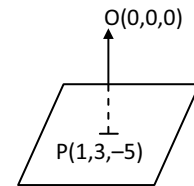
The plane passes through $P(1, 3, -5)$ equation of the plane is

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$$

$$1(x - 1) + 3(y - 3) - 5(z + 5) = 0$$

$$x - 1 + 3y - 9 - 5z - 25 = 0$$

$$x + 3y - 5z - 35 = 0$$



- 2. Reduce the equation $x + 2y - 3z - 6 = 0$ of the plane to the normal form.**

Sol. Equation of the plane is $x + 2y - 3z - 6 = 0$ i.e. $x + 2y - 3z = 6$

Dividing with $\sqrt{1^2 + 2^2 + (-3)^2} = \sqrt{1+4+9} = \sqrt{14}$, we get

$$\left(\frac{1}{\sqrt{14}}\right)x + \left(\frac{2}{\sqrt{14}}\right)y + \left(\frac{-3}{\sqrt{14}}\right)z = \frac{6}{\sqrt{14}}. \text{ Which is the normal form of the plane.}$$

- 3. Find the equation of the plane. Whose intercepts on X, Y, Z-axis are 1, 2, 4 respectively.**

Sol. Given X,Y,Z intercepts are $a = 1, b = 2, c = 4$

Equation of the plane in the intercept form is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

The equation of the plane in the intercept form is

$$\frac{x}{1} + \frac{y}{2} + \frac{z}{4} = 1 \Rightarrow 4x + 2y + z = 4.$$

- 4. Find the intercepts of the plane $4x + 3y - 2z + 2 = 0$ on the coordinate axes.**

Sol. Equation of the plane is $4x + 3y - 2z + 2 = 0$

$$-4x - 3y + 2z = -2 \Rightarrow -\frac{4x}{2} - \frac{3y}{z} + \frac{2z}{2} = -1 \Rightarrow \left(-\frac{1}{2}\right)x + \left(-\frac{2}{3}\right)y + \frac{z}{1} = -1$$

x-intercept = $-1/2$, y-intercept = $-2/3$, z-intercept = 1.

5. Find the d.c.'s of the normal to the plane $x + 2y + 2z - 4 = 0$.

Sol. Equation of the plane is $x + 2y + 2z - 4 = 0$

d.r.'s of the normal are $(1, 2, 2)$

Dividing with $\sqrt{1+4+4} = 3$

d.c.'s of the normal to the plane are $\left(\frac{1}{3}, \frac{2}{3}, \frac{2}{3}\right)$.

6. Find the equation of the plane passing through the point $(-2, 1, 3)$ and having $(3, -5, 4)$ as d.r.'s of its normal.

Sol. d.r.'s of the normal are $(3, -5, 4)$ and the plane passes through $(-2, 1, 3)$.

Equation of the plane is $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

$$3(x + 2) - 5(y - 1) + 4(z - 3) = 0$$

$$3x + 6 - 5y + 5 + 4z - 12 = 0$$

$$3x - 5y + 4z - 1 = 0.$$

7. Write the equation of the plane $4x - 4y + 2z + 5 = 0$ in the intercept form.

Sol. Equation of the plane is : $4x - 4y + 2z + 5 = 0$

$$-4x + 4y - 2z = 5$$

$$-\frac{4x}{5} + \frac{4y}{5} - \frac{2z}{5} = 1$$

Intercept form is $\frac{x}{\left(-\frac{5}{4}\right)} + \frac{y}{\left(\frac{5}{4}\right)} + \frac{z}{\left(-\frac{5}{2}\right)} = 1$

$$\text{x-intercept} = -\frac{5}{4}, \text{y-intercept} = \frac{5}{4} \quad \text{z-intercept} = -\frac{5}{2}$$

8. Find the angle between the planes $x + 2y + 2z - 5 = 0$ and $3x + 3y + 2z - 8 = 0$.

Sol. Equation of the plane are $x + 2y + 2z - 5 = 0$

$$3x + 3y + 2z - 8 = 0$$

Let θ be the angle between the planes, then

$$\cos \theta = \frac{|a_1a_2 + b_1b_2 + c_1c_2|}{\sqrt{a_1^2 + b_1^2 + c_1^2} \sqrt{a_2^2 + b_2^2 + c_2^2}} = \frac{|1-3+2-3+2-2|}{\sqrt{1+4+4}\sqrt{9+9+4}} = \frac{13}{3\sqrt{22}}$$

$$\theta = \cos^{-1}\left(\frac{13}{3\sqrt{22}}\right)$$

II.

1. Find the equation of the plane passing through the point (1, 1, 1) and parallel to the plane $x + 2y + 3z - 7 = 0$.

Sol. Equation of the given plane is $x + 2y + 3z - 7 = 0$

Equation of the plane parallel to this plane is $x + 2y + 3z = k$

This plane passing through the point P(1, 1, 1)

$$\Rightarrow 1 + 2 + 3 = k \Rightarrow k = 6$$

Equation of the required plane is $x + 2y + 3z = 6$

2. Find the equation of the plane passing through (2, 3, 4) and perpendicular to X-axis.

Sol. Since the plane is perpendicular to x-axis ,

\therefore x-axis is the normal to the plane

d.r.'s of x-axis are 1, 0, 0

\therefore Equation of the required plane is $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

$$\Rightarrow 1(x-2) + 0 + 0 = 0 \Rightarrow x-2 = 0$$

3. Show that $2x + 3y + 7 = 0$ represents a plane perpendicular to XY-plane.

Sol. Equation of the given plane is $2x + 3y + 7 = 0$

D,r,s if normal to the plane are 2,3,0

Equation of xy-plane is : $z = 0$

d.r.s of normal to the planes are 0,0,1

$$\text{now } a_1a_2 + b_1b_2 + c_1c_2 = 2 \cdot 0 + 3 \cdot 0 + 0 \cdot 1 = 0$$

The plane $2x + 3y + 7 = 0$ is perpendicular to xy-plane.

4. Find the constant k so that the planes $x - 2y + kz = 0$ and $2x + 5y - z = 0$ are at right angles. Find the equation of the plane through

(1, -1, -1) and perpendicular to these planes.

Sol. Equations of the given planes are

$$x - 2y + kz = 0 \text{ and } 2x + 5y - z = 0$$

since these planes are perpendicular , therefore

$$1 \cdot 2 - 2 \cdot 5 + k(-1) = 0$$

$$2 - 10 = k \Rightarrow k = -8$$

Equations of the planes are

$$x - 2y - 8z = 0 \quad \dots \text{(i)}$$

$$2x + 5y - z = 0 \quad \dots \text{(ii)}$$

Let a,b,c be the drs of normal to the required plane.

This plane is perpendicular to the planes (i) and (ii).

$$a - 2b - 8c = 0$$

$$2a + 5b - c = 0$$

$$\begin{array}{ccc} a & b & c \\ -2 & -8 & 1 \\ 5 & -1 & -2 \end{array}$$

$$\frac{a}{2+40} = \frac{b}{-16+1} = \frac{c}{5+4} \Rightarrow \frac{a}{42} = \frac{b}{-15} = \frac{c}{9}$$

The required plane passing through (1, -1, -1)

∴ Equation of the plane can be taken as

$$a(x - 1) + b(y + 1) + c(z + 1) = 0 \quad \dots \text{(iii)}$$

$$42(x - 1) - 15(y + 1) + 9(z + 1) = 0$$

$$42x - 42 - 15y - 15 + 9z + 9 = 0$$

$$42x - 15y + 9z - 48 = 0.$$

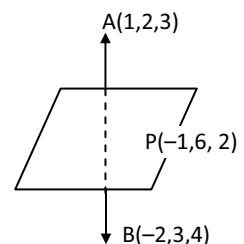
- 5. Find the equation of the plane through (-1, 6, 2) and perpendicular to the join of (1, 2, 3) and (-2, 3, 4).**

Sol. Given points are

A(1, 2, 3) and B(-2, 3, 4).

d.r.'s of AB are 1 + 2, 2 - 3, 3 - 4

i.e. 3, -1, -1



Since the plane is perpendicular to the line joining A(1, 2, 3) and B(-2, 3, 4),

AB is normal to the plane and the plane passes through the point P(-1, 6, 2).

Equation of the required plane is :

$$3(x + 1) - 1(y - 6) - 1(z - 2) = 0$$

$$3x + 3 - y + 6 - z + 2 = 0$$

$$3x - y - z + 11 = 0$$

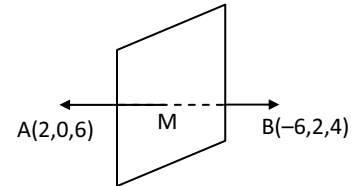
6, Find the equation of the plane bisecting the line segment joining (2, 0, 6) and (-6, 2, 4) and perpendicular to it.

Sol. A(2, 0, 6), B(-6, 2, 4) are the given points

Let 'M' be the mid point of AB.

Coordinates of M are :

$$\left(\frac{2-6}{2}, \frac{0+2}{2}, \frac{6+4}{2} \right) = (-2, 1, 5)$$



Since the plane is perpendicular to AB, d.r.'s of the normal to the plane are

$$2 + 6, 0 - 2, 6 - 4 \text{ i.e., } 8, -2, 2$$

Equation of the required plane is :

$$8(x + 2) - 2(y - 1) + 2(z - 5) = 0$$

$$\Rightarrow 8x + 16 - 2y + 2 + 2z - 10 = 0$$

$$\Rightarrow 8x - 2y + 2z + 8 = 0.$$

$$\Rightarrow 48x - y + z + 4 = 0$$

7. Find the equation of the plane passing through (0, 0, -4) and perpendicular to the line joining the point (1, -2, 2) and (-3, 1, -2).

Sol. Ans; $4x - 3y + 4z + 16 = 0$.

8. Find the equation of the plane through (4, 4, 0) and perpendicular to the planes $2x + y + 2z + 3 = 0$ and $3x + 3y + 2z - 8 = 0$.

Sol. Equation of the plane passing through P(4,4,0) is :

$$a(x - 4) + b(y - 4) + c(z - 0) = 0 \quad \dots(i)$$

This plane is perpendicular to

$$2x + y + 2z - 3 = 0$$

$$3x + 3y + 2z - 8 = 0$$

$$\therefore 2a + b + 2c = 0 \quad \dots(ii)$$

$$3a + 3b + 2c = 0 \quad \dots(iii)$$

$$\begin{array}{ccc}
 a & b & c \\
 1 & 2 & 2 \\
 3 & 2 & 3 \\
 3 & 3 & 3
 \end{array}$$

$$\frac{a}{2-6} = \frac{b}{6-4} = \frac{c}{6-3} \Rightarrow \frac{a}{-4} = \frac{b}{2} = \frac{c}{3}$$

Substituting in (i), equation of the required plane is :

$$-4(x - 4) + 2(y - 4) + 3(z - 0) = 0$$

$$\Rightarrow -4x + 16 + 2y - 8 + 3z = 0$$

$$\Rightarrow -4x + 2y + 3z + 8 = 0.$$

$$\Rightarrow 4x - 2y - 3z - 8 = 0$$

III.

- 1. Find the equation of the plane through the points (2, 2, -1), (3, 4, 2), (7, 0, 6).**

Sol. A(2, 2, -1), B(3, 4, 2), C(7, 0, 6) are the given points.

Let a, b, c be the d. rs of normal to the plane.

Equation of the plane passing through A(2, 2, -1) is $a(x - 2) + b(y - 2) + c(z + 1) = 0$... (i)

This plane is passing through B(3, 4, 2) and C(7, 0, 6).

$$a(3 - 2) + b(4 - 2) + c(2 + 1) = 0 \Rightarrow a + 2b + 3c = 0 \quad \dots \text{(ii)}$$

$$a(7 - 2) + b(0 - 2) + c(6 + 1) = 0 \Rightarrow 5a - 2b + 7c = 0 \quad \dots \text{(iii)}$$

From (ii) and (iii) :

$$\begin{array}{ccc}
 a & b & c \\
 2 & 3 & 1 \\
 -2 & 7 & 5 \\
 -2 & -2 & -2
 \end{array}$$

$$\frac{a}{14+6} = \frac{b}{15-7} = \frac{c}{-2-10}$$

$$\frac{a}{20} = \frac{b}{8} = \frac{c}{-12} \Rightarrow \frac{a}{5} = \frac{b}{2} = \frac{c}{-3}$$

Substituting in (i) equation of the required plane is

$$5(x - 2) + 2(y - 2) - 3(z + 1) = 0 \Rightarrow 5x - 10 + 2y - 4 - 3z - 3 = 0$$

$$\Rightarrow 5x + 2y - 3z - 17 = 0$$

2. Show that the points (0, -1, 0), (2, 1, -1), (1, 1, 1), (3, 3, 0) are coplanar.

Sol. Given points are A(0,-1,0) B(2,1,-1) c(1,1,1) and D(3,3,0)

The equation of the plane containing three points $(x_1, y_1, z_1), (x_2, y_2, z_2), (x_3, y_3, z_3)$

$$\text{is } \begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

The equation of the plane through A,B,C is

$$\begin{vmatrix} x-0 & y+1 & z-0 \\ 2-0 & 1+1 & -1-0 \\ 1-0 & 1+1 & 1-0 \end{vmatrix} = 0$$

$$\begin{vmatrix} x & y+1 & z \\ 2 & 2 & -1 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

$$\Rightarrow x(2+2) - (y+1)(2+1) + z(4-2) = 0$$

$$\Rightarrow 4x - 3y + 2z - 3 = 0$$

$$\text{Substituting D(3,3,0), } 4 \cdot 3 - 3 \cdot 3 + 2 \cdot 0 - 3 = 0 \Rightarrow 12 - 9 - 3 = 0 \Rightarrow 0 = 0.$$

Therefore D is a point of the plane ABC.

Hence given points are coplanar.

3. Find the equation of the plane through (6, -4, 3), (0, 4, -3) and cutting of intercepts whose sum is zero.

Sol. Suppose a, b, c are the intercepts of the plane.

$$\text{Equation of the plane is } \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

$$\text{Given } a + b + c = 0 \Rightarrow c = -(a + b)$$

The plane is passing through P(6, -4, 3), Q(0, 4, -3)

$$\Rightarrow \frac{6}{a} - \frac{4}{b} + \frac{3}{c} = 1 \text{ and } \frac{4}{b} - \frac{3}{c} = 1$$

$$\text{Adding these two } \frac{6}{a} = 2 \Rightarrow a = \frac{6}{2} = 3$$

$$\frac{4}{b} - \frac{3}{c} = 1 \Rightarrow 4c - 3b = bc \Rightarrow$$

$$\begin{aligned}
c &= -a - b = -3 - b \Rightarrow 4(-3 - b) - 3b = b(-3 - b) \\
&\Rightarrow -12 - 4b - 3b = -3b - b^2 \Rightarrow b^2 - 4b - 12 = 0 \\
&\Rightarrow (b - 6)(b + 2) = 0 \Rightarrow b = 6, -2
\end{aligned}$$

Case I :

$$b = 6 \Rightarrow c = -3 - b = -3 - 6 = -9$$

$$\text{Equation of the plane is : } \frac{x}{3} + \frac{y}{6} - \frac{z}{9} = 1$$

$$6x + 3y - 2z = 18$$

Case II :

$$b = -2 \Rightarrow c = -3 - b = -3 + 2 = -1$$

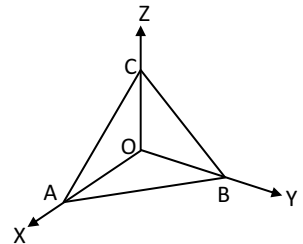
$$\text{Equation of the plane is : } \frac{x}{3} + \frac{y}{-2} + \frac{z}{-1} = 1$$

4. A plane meets the coordinate axes in A, B, C. If the centroid of ΔABC is

(a, b, c). Show that the equation of the plane is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 3$.

Sol. let α, β, γ be the intercepts of the plane ABC.

$$\text{Equation of the plane is } \frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 1 \quad \dots(i)$$



Coordinates of A are $(\alpha, 0, 0)$, B are $(0, \beta, 0)$ and C are $(0, 0, \gamma)$.

$$\text{Centroid of } \Delta ABC \text{ is } G = \left(\frac{\alpha}{3}, \frac{\beta}{3}, \frac{\gamma}{3} \right) = (a, b, c)$$

$$\frac{\alpha}{3} = a, \frac{\beta}{3} = b, \frac{\gamma}{3} = c \Rightarrow \alpha = 3a, \beta = 3b, \gamma = 3c$$

Substituting in (i), equation of the plane ABC is : $\frac{x}{3a} + \frac{y}{3b} + \frac{z}{3c} = 1 \Rightarrow \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 3$.

5. Show that the plane through (1, 1, 1), (1, -1, 1) and (-7, -3, -5) is parallel to y-axis.

Sol. Equation of the plane through (1, 1, 1), (1, -1, 1) and (-7, -3, -5) is

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} x-1 & y-1 & z-1 \\ 0 & -2 & 0 \\ -8 & -4 & -6 \end{vmatrix} = 0 \Rightarrow 3x-4z+1=0$$

D.rs of normal to the plane are 3, 0, -4

$$\text{d.rs of y axis are } 0, 1, 0 \Rightarrow a_1a_2 + b_1b_2 + c_1c_2 = 3 \cdot 0 + 0 \cdot 1 - 4 \cdot 0 = 0$$

Normal to the plane is perpendicular to the y-axis. hence plane is parallel to Y-axis.

6. Show that the equations $ax + by + r = 0$, $by + cz + p = 0$, $cz + ax + q = 0$ represent planes perpendicular to XY, YZ, ZX planes respectively.

Sol. Given plane is : $ax + by + c = 0$

d.r.'s of the normal are (a, b, 0)

Equation of XY-plane is $z = 0$

d.r.'s of the normal are (0, 0, 1)

$$\Rightarrow a_1a_2 + b_1b_2 + c_1c_2 = a \cdot b + b \cdot 0 + 0 \cdot 1 = 0$$

$\therefore ax + by + r = 0$ is perpendicular to xy-plane. Similarly we can show that

$by + cz + p = 0$ is perpendicular to yz-plane and $cz + ax + q = 0$ is perpendicular to zx-plane.

7. Find the equation of the plane passing through (2, 0, 1) and (3, -3, 4) and perpendicular to $x - 2y + z = 6$.

Sol. Equation of the plane passing through (2, 0, 1) is

$$a(x - 2) + by + c(z - 1) = 0 \quad \dots(i) \quad \text{where } a, b, c \text{ are d.rs of normal to the plane.}$$

This plane passes through B(3, -3, 4)

$$\Rightarrow a - 3b + 3c = 0 \quad \dots (ii)$$

The plane (i) is perpendicular to $x - 2y + z = 6$

$$\Rightarrow a - 2b + c = 0 \quad \dots (iii)$$

Solving (ii) and (iii)

$$\begin{array}{ccc} a & b & c \\ -3 & 3 & 1 \\ -2 & 1 & -2 \end{array}$$

$$\frac{a}{-3+6} = \frac{b}{3-1} = \frac{c}{-2+3}$$

$$\frac{a}{3} = \frac{b}{2} = \frac{c}{1}$$

Substituting in (i), equation of the required plane is :

$$3(x - 2) + 2y + 1(z - 1) = 0$$

$$3x - 6 + 2y + z - 1 = 0$$

$$3x + 2y + z - 7 = 0.$$

Problems for practice

1. Find the equation of the plane if the foot of the perpendicular from origin to the plane is $(2, 3, -5)$.

Ans : $2x + 3y - 5z - 38 = 0$.

2. Find the equation to the plane through the points $(0, -1, -1)$, $(4, 5, 1)$ and $(3, 9, 4)$.

Ans. $5x - 7y + 11z + 4 = 0$

3. Find the equation to the plane parallel to the ZX plane and passing through $(0, 4, 4)$.

Ans. $y = 4$

4. Find the equation of the plane through the point (α, β, γ) and parallel to the plane $ax + by + cz = 0$.

Sol. Equation of the given plane is $ax + by + cz = 0$

Equation of the parallel plane is $ax+by+cz = k$

This plane passes through $P(\alpha, \beta, \gamma) \Rightarrow a\alpha + b\beta + c\gamma = K$

\therefore Equation of the required plane is :

$$ax + by + cz = a\alpha + b\beta + c\gamma$$

$$\text{i.e. } a(x - \alpha) + b(y - \beta) + c(z - \gamma) = 0.$$

5. Find the angle between the plane $2x - y + z = 6$ and $x + y + 2z = 7$.

Ans. $\theta = \pi/3$

6. Find the equation of the plane passing through $(2, 0, 1)$ and $(3, -3, 4)$ and perpendicular to $x - 2y + z = 6$.

Ans. $3x + 2y + z - 7 = 0$