## CHAPTER 7

## THE PLANE

## TOPICS:

1. Equations of a plane
2. Normal form
3.perpendicula distance from a point to a plane.

3 .Intercept form
4. Angle between two planes
5. Distance between two parallel planes.

## PLANES

## Definition:

A surface in space is said to be a plane surface or a plane if all the points of the straight line joining any two points of the surface lie on the surface.

## THEOREM

The equation of the plane passing through a point $\left(\mathrm{x}_{1}, \mathrm{y}_{1}, \mathrm{z}_{1}\right)$ and perpendicular to a line whose direction ratios are $\mathrm{a}, \mathrm{b}, \mathrm{c}$ is $a\left(x-x_{1}\right)+b\left(y-y_{1}\right)+c\left(z-z_{1}\right)=0$

## THEOREM

The equation of the plane passing through a point $\left(\mathrm{x}_{1}, \mathrm{y}_{1}, \mathrm{z}_{1}\right)$ is $a\left(x-x_{1}\right)+b\left(y-y_{1}\right)+c\left(z-z_{1}\right)=0 \quad$ where $\mathrm{a}, \mathrm{b}, \mathrm{c}$ are constants.

## THEOREM

The equation of the plane containing three points $\left(\mathrm{x}_{1}, \mathrm{y}_{1}, \mathrm{z}_{1}\right),\left(\mathrm{x}_{2}, \mathrm{y}_{2}, \mathrm{z}_{2}\right),\left(\mathrm{x}_{3}, \mathrm{y}_{3}, \mathrm{z}_{3}\right)$
is $\left|\begin{array}{ccc}x-x_{1} & y-y_{1} & z-z_{1} \\ x_{2}-x_{1} & y_{2}-y_{1} & z_{2}-z_{1} \\ x_{3}-x_{1} & y_{3}-y_{1} & z_{3}-z_{1}\end{array}\right|=\mathbf{0}$

## NORMAL FORM OF A PLANE <br> THEOREM

The equation of the plane which is at a distance of $p$ from the origin and whose normal


NOTE. Equation of the plane through the origin is $1 x+m y+n z=0$
Note: The equation of the plane $\mathrm{ax}+\mathrm{by}+\mathrm{cz}+\mathrm{d}=0$ in the normal form is
$\frac{a}{\sqrt{\sum a^{2}}} x+\frac{b}{\sqrt{\sum a^{2}}} y+\frac{c}{\sqrt{\sum a^{2}}} z=\frac{-d}{\sqrt{\sum a^{2}}}$ if $d \leq 0$,
$\frac{-a}{\sqrt{\sum a^{2}}} x+\frac{-b}{\sqrt{\sum a^{2}}} y+\frac{-c}{\sqrt{\sum a^{2}}} z=\frac{+d}{\sqrt{\sum a^{2}}}$ if $d>0$ where $\sum a^{2}=a^{2}+b^{2}+c^{2}$.

## PERPENDICULAR DISTANCE FROM A POINT TO A PLALNE

The perpendicular distance from the origin to the plane $a x+b y+c z+d=0$ is

$$
\frac{|d|}{\sqrt{a^{2}+b^{2}+c^{2}}}
$$

## THEOREM

The perpendicular distance from $\mathrm{P}\left(\mathrm{x}_{1}, \mathrm{y}_{1}, \mathrm{z}_{1}\right)$ to the plane $\mathrm{ax}+\mathrm{by}+\mathrm{cz}+\mathrm{d}=0$ is $\frac{\left|a x_{1}+b y_{1}+c z_{1}+d\right|}{\sqrt{a^{2}+b^{2}+c^{2}}}$

## THEOREM

Intercept form of the plane
The equation of the plane having $\mathrm{a}, \mathrm{b}, \mathrm{c}$ as $\mathrm{x}, \mathrm{y}, \mathrm{z}$ - intercepts respectively is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

## THEOREM

The intercepts of the plane $\mathrm{ax}+\mathrm{by}+\mathrm{cz}+\mathrm{d}=0$ are respectively $\frac{-\boldsymbol{d}}{\boldsymbol{a}}, \frac{-\boldsymbol{d}}{\boldsymbol{b}}, \frac{-\boldsymbol{d}}{\boldsymbol{c}}$

## ANGLE BETWEEN TWO PLANES

Definition: The angle between the normals to two planes is called the angle between the planes.

## THEOREM

If $\theta$ is the angle between the planes $a_{1} x+b_{1} y+c_{1} z+d_{1}=0, a_{2} x+b_{2} y+c_{2} z+d_{2}=0$

$$
\text { then } \quad \cos \theta=\frac{a_{1} a_{2}+b_{1} b_{1}+c_{1} c_{2}}{\sqrt{a_{1}^{2}+b_{1}^{2}+c_{1}^{2}} \sqrt{a_{2}^{2}+b_{2}^{2}+c_{2}^{2}}}
$$

## Note

1. if $\theta$ is acute then $\quad \cos \theta=\left|\frac{a_{1} a_{2}+b_{1} b_{1}+c_{1} c_{2}}{\sqrt{a_{1}^{2}+b_{1}^{2}+c_{1}^{2}} \sqrt{a_{2}^{2}+b_{2}^{2}+c_{2}^{2}}}\right|$
2. The planes $a_{1} x+b_{1} y+c_{1} z+d_{1}=0, a_{2} x+b_{2} y+c_{2} z+d_{2}=0$ are
(i) parallel iff $\frac{a_{1}}{a_{2}}=\frac{b_{1}}{b_{2}}=\frac{c_{1}}{c_{2}}$
(ii) Perpendicular iff . $a_{1} a_{2}+b_{1} b_{2}+c_{1} c_{2}=0$
3. The given planes are perpendicular
$\Leftrightarrow \theta=90^{\circ} \Leftrightarrow \cos \theta=0 \Rightarrow a_{1} a_{2}+b_{1} b_{2}+c_{1} c_{2}=0$

## THEOREM

The equation of the plane parallel to the plane $a x+b y+c z+d=0$ is $a x+b y+c z+k=0$ where $k$ is a constant.

## THEOREM

The distance between the parallel planes $a x+b y+c z+d_{1}=0, a x+b y+c z+d_{2}=0$ is $\frac{\left|d_{1}-d_{2}\right|}{\sqrt{a^{2}+b^{2}+c^{2}}}$.

## EXERCISE 7

I.

1. Find the equation of the plane if the foot of the perpendicular from origin to the plane is $(\mathbf{1}, \mathbf{3}, \mathbf{- 5})$.
Sol. Given point $\mathrm{P}=(1,3,-5)$.
$\overrightarrow{o p}$ is the normal to the plane
D. Rs of $\overrightarrow{o p}$ are $1,3,-5$.

The plane passes through $\mathrm{P}(1,3,-5)$ equation of the plane is $a\left(x-x_{1}\right)+b\left(y-y_{1}\right)+c\left(z-z_{1}\right)=0$
$1(\mathrm{x}-1)+3(\mathrm{y}-3)-5(\mathrm{z}+5)=0$
$x-1+3 y-9-5 z-25=0$
$x+3 y-5 z-35=0$

2. Reduce the equation $x+2 y-3 z-6=0$ of the plane to the normal form.

Sol. Equation of the plane is $x+2 y-3 z-6=0$ i.e. $x+2 y-3 z=6$
Dividing with $\sqrt{1^{2}+2^{2}+(-3)^{2}}=\sqrt{1+4+9}=\sqrt{14}$, we get
$\left(\frac{1}{\sqrt{14}}\right) x+\left(\frac{2}{\sqrt{14}}\right) y+\left(\frac{-3}{\sqrt{14}}\right) z=\frac{6}{\sqrt{14}}$. Which is the normal form of the plane.
3. Find the equation of the plane. Whose intercepts on $X, Y, Z$-axis are $1,2,4$ respectively.
Sol.
Given $\mathrm{X}, \mathrm{Y}, \mathrm{Z}$ intercepts are $\mathrm{a}=1, \mathrm{~b}=2, \mathrm{c}=4$
Equation of the plane in the intercept form is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$
The equation of the plane in the intercept form is

$$
\frac{x}{1}+\frac{y}{2}+\frac{z}{4}=1 \Rightarrow 4 x+2 y+z=4
$$

4. Find the intercepts of the plane $4 x+3 y-2 z+2=0$ on the coordinate axes.

Sol. Equation of the plane is $4 x+3 y-2 z+2=0$

$$
-4 x-3 y+2 z=2 \Rightarrow-\frac{4 x}{2}-\frac{3 y}{z}+\frac{2 z}{2}=1 \Rightarrow \frac{x}{\left(-\frac{1}{2}\right)}+\frac{y}{\left(-\frac{2}{3}\right)}+\frac{z}{1}=1
$$

x -intercept $=-1 / 2, \mathrm{y}$-intercept $=-2 / 3, \quad \mathrm{z}$-intercept $=1$.
5. Find the d.c.'s of the normal to the plane $x+2 y+2 z-4=0$.

Sol. Equation of the plane is $x+2 y+2 z-4=0$
d.r.'s of the normal are $(1,2,2)$

Dividing with $\sqrt{1+4+4}=3$
d.c.'s of the normal to the plane are $\left(\frac{1}{3}, \frac{2}{3}, \frac{2}{3}\right)$.
6. Find the equation of the plane passing through the point $(-2,1,3)$ and having $(3,-5,4)$ as d.r.'s of its normal.
Sol. d.r.'s of the normal are $(3,-5,4)$ and the plane passes through $(-2,1,3)$.
Equation of the plane is $a\left(x-x_{1}\right)+b\left(y-y_{1}\right)+c\left(z-z_{1}\right)=0$

$$
\begin{aligned}
& 3(x+2)-5(y-1)+4(z-3)=0 \\
& 3 x+6-5 y+5+4 z-12=0 \\
& 3 x-5 y+4 z-1=0
\end{aligned}
$$

7. Write the equation of the plane $4 x-4 y+2 z+5=0$ in the intercept form.

Sol. Equation of the plane is : $4 x-4 y+2 z+5=0$

$$
-4 x+4 y-2 z=5
$$

$$
-\frac{4 x}{5}+\frac{4 y}{5}-\frac{2 z}{5}=1
$$

Intercept form is

$$
\frac{x}{\left(-\frac{5}{4}\right)}+\frac{y}{\left(\frac{5}{4}\right)}+\frac{z}{\left(-\frac{5}{2}\right)}=1
$$

x -intercept $=-\frac{5}{4}, \mathrm{y}$-intercept $=\frac{5}{4} \quad \mathrm{z}$-intercept $=-\frac{5}{2}$
8. Find the angle between the planes $x+2 y+2 z-5=0$ and $3 x+3 y+2 z-8=0$.

Sol. Equation of the plane are $x+2 y+2 z-5=0$

$$
3 x+3 y+2 z-8=0
$$

Let $\theta$ be the angle between the planes, then

$$
\begin{aligned}
& \cos \theta=\frac{\left|a_{1} a_{2}+b_{1} b_{2}+c_{1} c_{2}\right|}{\sqrt{a_{1}^{2}+b_{1}^{2}+c_{1}^{2}} \sqrt{a_{2}^{2}+b_{2}^{2}+c_{2}^{2}}}=\frac{|1-3+2-3+2-2|}{\sqrt{1+4+4} \sqrt{9+9+4}}=\frac{13}{3 \sqrt{22}} \\
& \theta=\cos ^{-1}\left(\frac{13}{3 \sqrt{22}}\right)
\end{aligned}
$$

II.

1. Find the equation of the plane passing through the point $(1,1,1)$ and parallel to the plane $x+2 y+3 z-7=0$.
Sol. Equation of the given plane is $x+2 y+3 z-7=0$ Equation of the plane parallel to this plane is $\quad x+2 y+3 z=k$
This plane passing through the point $\mathrm{P}(1,1,1)$

$$
\Rightarrow \quad 1+2+3=k \Rightarrow k=-6
$$

Equation of the required plane is $x+2 y+3 z=6$
2. Find the equation of the plane passing through $(2,3,4)$ and perpendicular to X -axis.
Sol. Since the plane is perpendicular to x -axis,
$\therefore \mathrm{x}$-axis is the normal to the plane
d.r.'s of x -axis are $1,0,0$
$\therefore$ Equation of the required plane is $a\left(x-x_{1}\right)+b\left(y-y_{1}\right)+c\left(z-z_{1}\right)=0$
$\Rightarrow 1(\mathrm{x}-2)+0+0=0 \Rightarrow \mathrm{x}-2=0$
3. Show that $2 x+3 y+7=0$ represents a plane perpendicular to XY-plane.

Sol. Equation of the given plane is $2 \mathrm{x}+3 \mathrm{y}+7=0$
D,rs if normal to the plane are $2,3,0$
Equation of $x y$-plane is : $z=0$
d.rs of normal to the planes are $0,0,1$
now $\mathrm{a}_{1} \mathrm{a}_{2}+\mathrm{b}_{1} \mathrm{~b}_{2}+\mathrm{c}_{1} \mathrm{c}_{2}=2 \Rightarrow 2 \cdot 0+3 \cdot 0+0 \cdot 1=0$
The plane $2 \mathrm{x}+3 \mathrm{y}+7=0$ is perpendicular to xy -plane.
4. Find the constant $k$ so that the planes $x-2 y+k z=0$ and $2 x+5 y-z=0$ are at right angles. Find the equation of the plane through
$(1,-1,-1)$ and perpendicular to these planes.
Sol. Equations of the given planes are

$$
x-2 y+k z=0 \text { and } 2 x+5 y-z=0
$$

since these planes are perpendicular, therefore

$$
\begin{aligned}
& 1 \cdot 2-2 \cdot 5+\mathrm{k}(-1)=0 \\
& 2-10=\mathrm{k} \Rightarrow \mathrm{k}=-8
\end{aligned}
$$

Equations of the planes are

$$
\begin{align*}
& x-2 y-8 z=0  \tag{i}\\
& 2 x+5 y-z=0 \tag{ii}
\end{align*}
$$

Let $\mathrm{a}, \mathrm{b}, \mathrm{c}$ be the drs of normal to the required plane.
This plane is perpendicular to the planes (i) and (ii).

$$
\begin{aligned}
& a-2 b-8 c=0 \\
& 2 a+5 b-c=0
\end{aligned}
$$



$$
\frac{a}{2+40}=\frac{b}{-16+1}=\frac{c}{5+4} \Rightarrow \frac{a}{42}=\frac{b}{-15}=\frac{c}{9}
$$

The required plane passing through $(1,-1,-1)$
$\therefore$ Equation of the plane can be taken as

$$
\begin{aligned}
& a(x-1)+b(y+1)+c(z+1)=0 \\
& 42(x-1)-15(y+1)+9(z+1)=0 \\
& 42 x-42-15 t-15+9 z+9=0 \\
& 42 x-15 y+9 z-48=0
\end{aligned}
$$

## 5. Find the equation of the plane through $(-1,6,2)$ and perpendicular to the join of $(1,2,3)$ and $(-2,3,4)$.

Sol. Given points are
$\mathrm{A}(1,2,3)$ and $\mathrm{B}(-2,3,4)$.
d.r.'s of AB are $1+2,2-3,3-4$
i.e. $3,-1,-1$


Since the plane is perpendicular to the line joining $\mathrm{A}(1,2,3)$ and $\mathrm{B}(-2,3,4)$, AB is normal to the plane and the plane passes through the point $\mathrm{P}(-1,6,2)$. Equation of the required plane is :

$$
\begin{aligned}
& 3(x+1)-1(y-6)-1(z-2)=0 \\
& 3 x+3-y+6-z+2=0 \\
& 3 x-y-z+11=0
\end{aligned}
$$

6, Find the equation of the plane bisecting the line segment joining $(2,0,6)$ and $(-6,2,4)$ and perpendicular to it.
Sol. $\quad \mathrm{A}(2,0,1), \mathrm{B}(-6,2,4)$ are the given points
Let ' M ' be the mid point of AB .
Coordinates of M are :

$\left(\frac{2-6}{2}, \frac{0+2}{2}, \frac{6+4}{2}\right)=(-2,1,5)$
Since the plane is perpendicular to AB , d.r.'s of the normal to the plane are $2+6,0-2,6-4$ i.e., $\quad 8,-2,2$
Equation of the required plane is :

$$
\begin{aligned}
& 8(x+2)-2(y-1)+2(z-5)=0 \\
& \Rightarrow 8 x+16-2 y+2+2 z-10=0 \\
& \Rightarrow 8 x-2 y+2 z+8=0 \\
& \Rightarrow 48 x-y+z+4=0
\end{aligned}
$$

7. Find the equation of the plane passing through $(0,0,-4)$ and perpendicular to the line joining the point $(1,-2,2)$ and $(-3,1,-2)$.
Sol. Ans; $4 \mathrm{x}-3 \mathrm{y}+4 \mathrm{z}+16=0$.
8. Find the equation of the plane through $(4,4,0)$ and perpendicular to the planes $2 x+y+2 z+3=0$ and $3 x+3 y+2 z-8=0$.
Sol. Equation of the plane passing through $\mathrm{P}(4,4,0)$ is :

$$
\begin{equation*}
a(x-4)+b(y-4)+c(z-0)=0 \tag{i}
\end{equation*}
$$

This plane is perpendicular to

$$
\begin{gather*}
2 x+y+2 z-3=0 \\
3 x+3 y+2 z-8=0 \\
\therefore 2 a+b+2 c=0 \quad .  \tag{ii}\\
3 a+3 b+2 c=0 \quad . . \tag{iii}
\end{gather*}
$$

$$
\begin{gathered}
c \stackrel{b}{c}{ }_{3}^{c}{ }_{3}^{c}=\frac{\mathrm{b}}{2-6}=\frac{\mathrm{c}}{6-3} \Rightarrow \frac{a}{-4}=\frac{b}{2}=\frac{\mathrm{c}}{3}
\end{gathered}
$$

Substituting in (i), equation of the required plane is :

$$
\begin{aligned}
&-4(\mathrm{x}-4)+2(\mathrm{y}-4)+3(\mathrm{z}-0)=0 \\
& \Rightarrow-4 \mathrm{x}+16+2 \mathrm{y}-8+3 \mathrm{z}=0 \\
& \Rightarrow-4 \mathrm{x}+2 \mathrm{y}+3 \mathrm{z}+8=0 \\
& \Rightarrow 4 \mathrm{x}-2 \mathrm{y}-3 \mathrm{z}-8=0
\end{aligned}
$$

III.

1. Find the equation of the plane through the points $(2,2,-1),(3,4,2)$, $(7,0,6)$.
Sol. $\mathrm{A}(2,2,-1), \mathrm{B}(3,4,2), \mathrm{C}(7,0,6)$ are the given points.
Let $\mathrm{a}, \mathrm{b}, \mathrm{c}$ be the d . rs of normal to the plane.
Equation of the plane passing through
$\mathrm{A}(2,2,-1)$ is $\mathrm{a}(\mathrm{x}-2)+\mathrm{b}(\mathrm{y}-2)+\mathrm{c}(\mathrm{z}+1)=0$
This plane is passsing through $\mathrm{B}(3,4,2)$ and $\mathrm{C}(7,0,6)$.

$$
\begin{align*}
& a(3-2)+b(4-2)+c(2+1)=0 \Rightarrow a+2 b+3 c=0  \tag{ii}\\
& a(7-2)+b(0-2)+c(6+1)=0 \Rightarrow 5 a-2 b+7 c=0 \tag{iii}
\end{align*}
$$

From (ii) and (iii) :

$\frac{a}{14+6}=\frac{b}{15-7}=\frac{c}{-2-10}$
$\frac{\mathrm{a}}{20}=\frac{\mathrm{b}}{8}=\frac{\mathrm{c}}{-12} \Rightarrow \frac{\mathrm{a}}{5}=\frac{\mathrm{b}}{2}=\frac{\mathrm{c}}{-3}$
Substituting in (i) equation of the required plane is

$$
\begin{aligned}
& 5(x-2)+2(y-2)-3(z+1)=0 \Rightarrow 5 x-10+2 y-4-3 z-3=0 \\
& \Rightarrow 5 x+2 y-3 z-17=0
\end{aligned}
$$

2. Show that the points $(0,-1,0),(2,1,-1),(1,1,1),(3,3,0)$ are coplanar.

Sol. Given points are $\mathrm{A}(0,-1,0) \mathrm{B}(2,1,-1) \mathrm{c}(1,1,1)$ and $\mathrm{D}(3,3,0)$
The equation of the plane containing three points $\left(\mathrm{x}_{1}, \mathrm{y}_{1}, \mathrm{z}_{1}\right),\left(\mathrm{x}_{2}, \mathrm{y}_{2}, \mathrm{z}_{2}\right),\left(\mathrm{x}_{3}, \mathrm{y}_{3}, \mathrm{z}_{3}\right)$
is $\left|\begin{array}{ccc}x-x_{1} & y-y_{1} & z-z_{1} \\ x_{2}-x_{1} & y_{2}-y_{1} & z_{2}-z_{1} \\ x_{3}-x_{1} & y_{3}-y_{1} & z_{3}-z_{1}\end{array}\right|=\mathbf{0}$
The equation of the plane through $\mathrm{A}, \mathrm{B}, \mathrm{C}$ is

$$
\begin{aligned}
& \left|\begin{array}{ccc}
x-0 & \mathrm{y}+1 & \mathrm{z}-0 \\
2-0 & 1+1 & -1-0 \\
1-0 & 1+1 & 1-0
\end{array}\right|=0 \\
& \left|\begin{array}{ccc}
x & \mathrm{y}+1 & \mathrm{z} \\
2 & 2 & -1 \\
1 & 2 & 1
\end{array}\right|=0 \\
& \Rightarrow x(2+2)-(y+1)(2+1)+z(4-2)=0 \\
& \Rightarrow 4 x-3 y+2 z-3=0
\end{aligned}
$$

Substituting $D(3,3,0), 4.3-3.3+2.0-3=0 \Rightarrow 12-9-3=0 \Rightarrow 0=0$.
Therefore D is a point of the plane ABC .
Hence given points are coplanar.

## 3. Find the equation of the plane through

$(6,-4,3),(0,4,-3)$ and cutting of intercepts whose sum is zero.
Sol. Suppose a, b, c are the intercepts of the plane.
Equation of the plane is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$
Given $\mathrm{a}+\mathrm{b}+\mathrm{c}=0 \Rightarrow \mathrm{c}=-(\mathrm{a}+\mathrm{b})$
The plane is passing through $\mathrm{P}(6,-4,3), \mathrm{Q}(0,4,-3)$

$$
\Rightarrow \frac{6}{\mathrm{a}}-\frac{4}{\mathrm{~b}}+\frac{3}{\mathrm{c}}=1 \text { and } \frac{4}{\mathrm{~b}}-\frac{3}{\mathrm{c}}=1
$$

Adding these two $\frac{6}{a}=2 \Rightarrow a=\frac{6}{2}=3$
$\frac{4}{b}-\frac{3}{c}=1 \Rightarrow 4 c-3 b=b c \Rightarrow$

$$
\begin{aligned}
& c=-a-b=-3-b \Rightarrow 4(-3-b)-3 b=b(-3-b) \\
& \Rightarrow-12-4 b-3 b=-3 b-b^{2} \Rightarrow b^{2}-4 b-12=0 \\
& \Rightarrow(b-6)(b+2)=0 \Rightarrow b=6,-2
\end{aligned}
$$

## Case I :

$\mathrm{b}=6 \Rightarrow \mathrm{c}=-3-\mathrm{b}=-3-6=-9$
Equation of the plane is : $\frac{x}{3}+\frac{y}{6}-\frac{z}{9}=1$

$$
6 x+3 y-2 z=18
$$

## Case II :

$\mathrm{b}=-2 \Rightarrow \mathrm{c}=-3-\mathrm{b}=-3+2=-1$
Equation of the plane is: $\frac{x}{3}+\frac{y}{-2}+\frac{z}{-1}=1$
4. A plane meets the coordinate axes in $A, B, C$. If the centroid of $\triangle A B C$ is ( $\mathbf{a}, \mathbf{b}, \mathbf{c}$ ). Show that the equation of the plane is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=3$.

Sol. let $\alpha, \beta, \gamma$ be the intercepts of the plane ABC.
Equation of the plane is $\frac{x}{\alpha}+\frac{y}{\beta}+\frac{z}{\beta}=1$


Coordinates of A are $(\alpha, 0,0), \mathrm{B}$ are $(0, \beta, 0)$ and C are $(0,0, \gamma)$.

$$
\begin{aligned}
& \text { Centroid of } \triangle \mathrm{ABC} \text { is } \mathrm{G}=\left(\frac{\alpha}{3}, \frac{\beta}{3}, \frac{\gamma}{3}\right)=(\mathrm{a}, \mathrm{~b}, \mathrm{c}) \\
& \frac{\alpha}{3}=\mathrm{a}, \frac{\beta}{3}=\mathrm{b}, \frac{\gamma}{3}=\mathrm{c} \Rightarrow \alpha=3 \mathrm{a}, \beta=3 \mathrm{~b}, \gamma=3 \mathrm{c}
\end{aligned}
$$

Substituting in (i), equation of the plane $A B C$ is: $\frac{x}{3 a}+\frac{y}{3 b}+\frac{z}{3 c}=1 \Rightarrow \frac{x}{a}+\frac{y}{b}+\frac{z}{c}=3$.
5. Show that the plane through $(1,1,1),(1,-1,1)$ and $(-7,-3,-5)$ is parallel to y-axis.
Sol. Equation of the plane through $(1,1,1),(1,-1,1)$ and $(-7,-3,-5)$ is

$$
\left|\begin{array}{ccc}
x-x_{1} & y-y_{1} & z-z_{1} \\
x_{2}-x_{1} & y_{2}-y_{1} & z_{2}-z_{1} \\
x_{3}-x_{1} & y_{3}-y_{1} & z_{3}-z_{1}
\end{array}\right|=\mathbf{0} \quad \Rightarrow\left|\begin{array}{ccc}
x-1 & y-1 & z-1 \\
0 & -2 & 0 \\
-8 & -4 & -6
\end{array}\right|=0 \Rightarrow 3 x-4 z+1=0
$$

D.rs of normal to the plane aer $3,0,-4$
d.rs of y axis are $0,1,0 \Rightarrow a_{1} a_{2}+b_{1} b_{2}+c_{1} c_{2}=3.0+0.1-4.0=0$

Normal to the plane is perpendicular to the y -axis. hence plnae is parallel to Y -axis.
6. Show that the equations $a x+b y+r=0, b y+c z+p=0, c z+a x+q=0$ represent planes perpendicular to $X Y, Y Z, Z X$ planes respectively.
Sol. Given plane is: $\mathrm{ax}+\mathrm{by}+\mathrm{c}=0$
d.r.'s of the normal are ( $\mathrm{a}, \mathrm{b}, 0$ )

Equation of XY-plane is $\mathrm{z}=0$
d.r.'s of the normal are $(0,0,1)$
$\Rightarrow a_{1} a_{2}+b_{1} b_{2}+c_{1} c_{2}=\mathrm{a} \cdot \mathrm{b}+\mathrm{b} \cdot 0+0 \cdot 1=0$
$\therefore \mathrm{ax}+\mathrm{by}+\mathrm{r}=0$ is perpendicular to xy -plane. Similarly we can show that
by $+\mathrm{cz}+\mathrm{p}=0$ is perpendicular to yz-plane and $\mathrm{cz}+\mathrm{ax}+\mathrm{q}=0$ is perpendicular to zx -plane.
7. Find the equation of the plane passing through $(2,0,1)$ and $(3,-3,4)$ and perpendicular to $\mathrm{x}-2 \mathrm{y}+\mathrm{z}=6$.
Sol. Equation of the plane passing through $(2,0,1)$ is
$a(x-2)+b y+c(z-1)=0 \quad \ldots$ (i) where $a, b, c$ are d.rs of normal to the plane.
This plane passes through $\mathrm{B}(3,-3,4)$

$$
\begin{equation*}
\Rightarrow \mathrm{a}-3 \mathrm{~b}+3 \mathrm{c}=0 \tag{ii}
\end{equation*}
$$

The plane (i) is perpendicular to $\mathrm{x}-2 \mathrm{y}+\mathrm{z}=6$

$$
\begin{equation*}
\Rightarrow a-2 b+c=0 \tag{iii}
\end{equation*}
$$

Solving (ii) and (iii)
a
$\frac{\mathrm{a}}{-3}-\frac{\mathrm{a}}{-3+6}=\frac{\mathrm{b}}{3-1}=\frac{\mathrm{c}}{-2+3}$
$\frac{a}{3}=\frac{b}{2}=\frac{c}{1}$

Substituting in (i), equation of the required plane is :

$$
\begin{aligned}
& 3(x-2)+2 y+1(z-1)=0 \\
& 3 x-6+2 y+z-1=0 \\
& 3 x+2 y+z-7=0
\end{aligned}
$$

## Problems for practice

1. Find the equation of the plane if the foot of the perpendicular from origin to the plane is $(2,3,-5)$.
Ans: $2 \mathrm{x}+3 \mathrm{y}-5 \mathrm{z}-38=0$.
2. Find the equation to the plane through the points $(0,-1,-1),(4,5,1)$ and $(3,9,4)$.
Ans. $5 \mathrm{x}-7 \mathrm{y}+11 \mathrm{z}+4=0$
3. Find the equation to the plane parallel to the $\mathbf{Z X}$ plane and passing through $(0,4,4)$.

Ans. $\mathrm{y}=4$
4. Find the equation of the plane through the point $(\alpha, \beta, \gamma)$ and parallel to the plane $\mathbf{a x}+\mathrm{by}+\mathbf{c z}=\mathbf{0}$.
Sol. Equation of the given plane is $\mathrm{ax}+\mathrm{by}+\mathrm{cz}=0$
Equation of the parallel plane is $\mathrm{ax}+\mathrm{by}+\mathrm{cz}=\mathrm{k}$
This plane passes through $\mathrm{P}(\alpha, \beta, \gamma) \Rightarrow \mathrm{a} \alpha+\mathrm{b} \beta+\mathrm{c} \gamma=\mathrm{K}$
$\therefore$ Equation of the required plane is :
$a x+b y+c z=a \alpha+b \beta+c \gamma$
i.e. $a(x-\alpha)+b(y-\beta)+c(z-\gamma)=0$.
5. Find the angle between the plane $2 x-y+z=6$ and $x+y+2 z=7$.

Ans. $\theta=\pi / 3$
6. Find the equation of the plane passing through $(2,0,1)$ and $(3,-3,4)$ and perpendicular to $x-2 y+z=6$.
Ans. $3 \mathrm{x}+2 \mathrm{y}+\mathrm{z}-7=0$

