Aimstutorial MODEL PAPER - 4

MATHS - 1A

SECTION - A

I. Answer ALL the following Very Short Answer Questions: **[10 x 2 = 20]**

1. If f = {(1, 2), (2, -3), (3, -1)} then find (i) 2 + f (ii) $\sqrt{f}$.
2. Find the domain of $\sqrt{9-x^2}$.
3. For any square matrix A, show that AA' is symmetric.
4. Find the determinant of the matrix $\begin{bmatrix} 1^2 & 2^2 & 3^2 \\ 2^2 & 3^2 & 4^2 \\ 3^2 & 4^2 & 5^2 \end{bmatrix}$.
5. Find the unit vector in the direction of vector $\bar{a} = 2\bar{i} + 3\bar{j} + \bar{k}$.
6. If vectors $-3\bar{i} + 4\bar{j} + \lambda\bar{k}, \mu\bar{i} + 8\bar{j} + 6\bar{k}$ are collinear vectors then find λ & μ.
7. Find the angle between the planes $\bar{r}.(2\bar{i} - \bar{j} + 2\bar{k}) = 3$, $\bar{r}.(3\bar{i} + 6\bar{j} + \bar{k}) = 4$.
8. If $0 < A < \frac{\pi}{4}$ and $\cos A = \frac{4}{5}$, then find the values of sin 2A and cos 2A.
9. Find the maximum and minimum value of f(x) = 3sinx - 4cos x.
10. Prove that $\cosh^2 x + \sinh^2 x = \cosh 2x$.

SECTION - B

II. Answer any FIVE of the following Short Answer Questions: **[5 x 4 = 20]**

11. Examine whether the following system of equations are consistent or inconsistent and if consistent, find the complete solution x + y + z = 1, 2x + y + z = 2, x + 2y + 2z = 1.
12. $\bar{a}, \bar{b}, \bar{c}$ are non-coplanar vectors. Prove that the following four points are coplanar.

    $6\bar{a} + 2\bar{b} - \bar{c}$, $2\bar{a} - \bar{b} + 3\bar{c}$, $\bar{a} + 2\bar{b} - 4\bar{c}$, $-12\bar{a} - \bar{b} - 3\bar{c}$.
13. Find the volume of the tetrahedron, whose vertices are (1,2,1), (3,2,5), (2, -1,0) and (-1,0,1).
14. Prove that $\left(1+\cos\frac{\pi}{5}\right)\left(1+\cos\frac{\pi}{5}\right)\left(1+\cos\frac{7\pi}{5}\right)\left(1+\cos\frac{9\pi}{5}\right) = \frac{1}{16}$.
15. Solve $\sqrt{3}\sin\theta - \cos\sigma = \sqrt{2}$.
16. Prove that $\text{Sin}^{-1}\frac{3}{5} + \text{Cos}^{-1}\frac{12}{13} = \text{Cos}^{-1}\frac{33}{65}$.
17. If $\sin\theta = \frac{a}{(b+c)}$ then show that $\cos\theta = \frac{2\sqrt{bc}}{b+c}\cos\left(\frac{A}{2}\right)$.

SECTION - C

III. Answer any FIVE of the following Long Answer Questions. : **[5 x 7 = 35]**

18. If f : A$\to$B is a bijective function then prove that (i) $fof^{-1} = I_B$ (ii) f^{-1} of $= I_A$.

19. Using Mathematical induction, prove that statement for all n$\in$N

$$\left(1+\frac{3}{1}\right)\left(1+\frac{5}{4}\right)\left(1+\frac{7}{9}\right).....\left(1+\frac{2n+1}{n^2}\right) = (n + 1)^2.$$

20. Show that $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$ = abc (a - b)(b-c)(c-a).

21. Solve the following system of equations by using Cramer's rule.
x - y + 3z = 5, 4x + 2y - z = 0, -x + 3y + z = 5.

22. If $\bar{a} = 2\bar{i} + \bar{j} - 3\bar{k}$, $\bar{i} - 2\bar{j} + \bar{k}$, $\bar{c} = -\bar{i} - \bar{j} + 4\bar{k}$ and $\bar{d} = \bar{i} + \bar{j} + \bar{k}$, then compute $\left|\left(\bar{a} \times \bar{b}\right) \times \left(\bar{c} \times \bar{d}\right)\right|$.

23. If A, B, C are angles of a triangle, prove that cos2A + cos2B + cos 2C = -1-4cosA cosB cosC.

24. Show that $\frac{A}{2} + \sin^2\frac{B}{2} - \sin^2\frac{C}{2} = 1 - \frac{r}{2R}$.

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