

# Aimstutorial MODEL PAPER - 6

## MATHS - 1A

### SECTION - A

#### I. Answer ALL the following Very Short Answer Questions:

[10 x 2 = 20]

1. If  $f: \mathbb{R} \rightarrow \mathbb{R}$ ,  $g: \mathbb{R} \rightarrow \mathbb{R}$  are defined by  $f(x) = 4x - 1$ ,  $g(x) = x^2 + 2$  then find (i)  $(g \circ f)(x)$  (ii)  $f \circ g(x)$
2. Find the domain of the real function  $f(x) = \sqrt{x^2 - 25}$ .
3. If  $A = \begin{bmatrix} 2 & -1 & 2 \\ 1 & 3 & -4 \end{bmatrix}$  and  $B = \begin{bmatrix} 1 & -2 \\ -3 & 0 \\ 5 & 4 \end{bmatrix}$  then verify that  $(AB)^T = B^T A^T$ .
4. If  $A = \begin{bmatrix} a+ib & c+id \\ -c+id & a-ib \end{bmatrix}$ ,  $a^2 + b^2 + c^2 + d^2 = 1$ , then find the inverse of A.
5. Let  $\vec{a} = 2\vec{i} + 4\vec{j} - 5\vec{k}$ ,  $\vec{b} = \vec{i} + \vec{j} + \vec{k}$ ,  $\vec{c} = \vec{j} + 2\vec{k}$ . Find unit vector in the opposite direction of  $\vec{a} + \vec{b} + \vec{c}$ .
6. Find the vector equation of the line passing through the points  $2\vec{i} + \vec{j} + 3\vec{k}$ , and parallel to the vector  $4\vec{i} - 2\vec{j} + 3\vec{k}$ .
7. Find the unit vector perpendicular to the plane containing the vectors  $\vec{a} = 4\vec{i} + 3\vec{j} - \vec{k}$ ,  $\vec{b} = 2\vec{i} - 6\vec{j} - 3\vec{k}$ .
8. Prove that  $(1 + \cos\theta - \operatorname{cosec}\theta)(1 + \tan\theta + \sec\theta) = 2$
9. Find the extreme values of  $3\sin^2x + 5\cos^2x$ .
10. If  $\sinh x = 3$  then show that  $x = \log(3 + \sqrt{10})$ .

[www.Aimstutorial.in](http://www.Aimstutorial.in)

### SECTION - B

#### II. Answer any FIVE of the following Short Answer Questions:

[5 x 4 = 20]

11. If  $\theta - \phi = \frac{\pi}{2}$ , then show that  $\begin{bmatrix} \cos^2 \theta & \cos \theta \sin \theta \\ \cos \theta \sin \theta & \sin^2 \theta \end{bmatrix} \begin{bmatrix} \cos^2 \phi & \cos \phi \sin \phi \\ \cos \phi \sin \phi & \sin^2 \phi \end{bmatrix} = 0$ .
12. In the two dimensional plane, prove by using vector methods, the equation of the line whose intercepts on the axes are 'a' and 'b' is  $\frac{x}{a} + \frac{y}{b} = 1$ .
13. If  $\vec{a} = 2\vec{i} + \vec{j} - \vec{k}$ ,  $\vec{b} = -\vec{i} + 2\vec{j} - 4\vec{k}$ ,  $\vec{c} = \vec{i} + \vec{j} + \vec{k}$ , find  $(\vec{a} \times \vec{b}) \cdot (\vec{b} \times \vec{c})$ .
14. Show that  $\frac{1}{\sin 10^\circ} - \frac{\sqrt{3}}{\cos 10^\circ} = 4$ .
15. Solve  $1 + \sin^2\theta = 3 \sin\theta \cos\theta$ .
16. Prove that  $\cos\left(2 \tan^{-1} \frac{1}{7}\right) = \sin\left(2 \tan^{-1} \frac{3}{4}\right)$ .
17. In  $\Delta ABC$ , prove that  $\tan\left(\frac{B-C}{2}\right) = \frac{b-c}{b+c} \cot \frac{A}{2}$ .

**SECTION - C**

**III. Answer any FIVE of the following Long Answer Questions. :**

**[5 x 7 = 35]**

18. If  $f : A \rightarrow B$  is a function and  $I_A, I_B$  are identity functions on  $A, B$  respectively then prove that  $f \circ I_A = f = I_B \circ f$ .

19. Using the principle of finite Mathematical Induction prove that

$$1^2 + (1^2 + 2^2) + (1^2 + 2^2 + 3^2) + \dots \text{upto } n \text{ terms} = \frac{n(n+1)^2(n+2)}{12}, \forall n \in \mathbb{N}.$$

20. Show that 
$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} \begin{vmatrix} 2bc - a^2 & c^2 & b^2 \\ c^2 & 2ac - b^2 & a^2 \\ b^2 & a^2 & 2ab - c^2 \end{vmatrix} = (a^3 + b^3 + c^3 - 3abc)^2.$$

21. Solve the following system of equations by Gauss-Jordan method  
 $x + y + z = 3, 2x + 2y - z = 3, x + y - z = 1.$

22. If  $\vec{a} = \vec{i} - 2\vec{j} + 3\vec{k}, \vec{b} = 2\vec{i} + \vec{j} + \vec{k}, \vec{c} = \vec{i} + \vec{j} + 2\vec{k}$ , then find  $|(\vec{a} \times \vec{b}) \times \vec{c}|$ .

23. If  $A + B + C = \pi$ , then prove that  $\cos^2 \frac{A}{2} + \cos^2 \frac{B}{2} + \cos^2 \frac{C}{2} = 2 \left( 1 + \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2} \right).$

24. If  $r_1 = 2, r_2 = 3, r_3 = 6$  and  $r = 1$ , prove that  $a = 3, b = 4$  and  $c = 5$ .

[www.Aimstutorial.in](http://www.Aimstutorial.in)

\* \* \* \* \*