DEFINITIONS, CONCEPTS AND FORMULAE

Random variable:

1. Let S be the sample space of a radom experiment. A real valued function $X : S \to R$ is called a random variable.

2. Let $X : S \to R$ be a descrete random variable with range $\{x_1, x_2, x_3, ...\}$. If $\Sigma x_i P(X = x_i)$ exists, then $\Sigma x_i P(X = x_r)$ is called the 'mean' of the random vairable X. It is denoted by μ or $\overline{x}$. If $\Sigma(x_i - \mu)^2 P(X = x_r)$ exists, then $\Sigma(x_i - \mu)^2 P(X = x_r)$ is called variance of the random variable X. It is denoted by σ^2.

3. $\sum_{i=1}^{n} P(X = X_i) = 1$.

4. $\sum_{i=1}^{\infty} P(X = X_i) = 1$.

5. Mean of the random variable $\mu = \sum_{i=1}^{n} x_i P(X = x_i)$.

6. Variance $\sigma^2 = \sum_{i=1}^{n} x_i^2 P(X = x_i) - \mu^2$.

7. Standard deviation = $\sqrt{\text{Variance}}$.

Binomial distribution:

8. A discrete random variable X is said to follow a binomial distribution with parameters n and p where $0 < p < 1$ if $P(X = r) = {}^nC_r q^{n-r} p^r$ where r = 0, 1, 2, ..., n.
9. If X is a binomial variate with parameters n, p, then it is described by writing $X \sim B(n, p)$.
10. n, p are the parameters of the binomial distribution.
11. $P(X = r) = {}^nC_r q^{n-r} p^r$ where $p + q = 1$
12. B.D is $(q + p)^n$
13. Mean of X is np.
14. Variance of X is npq.
15. Standard deviation = $\sqrt{npq}$.

Poisson distribution:

16. Let $\lambda > 0$ be a real number. A random variable X with range {0, 1, 2, ...} is said to follow poisson distribution with parameter λ if

    $P(X = r) = \dfrac{e^{-\lambda} \lambda^r}{r!}$ for r =0, 1, 2, ...

AIMS

17. λ is the only parameter of the poisson distribution.
18. $P(X = r) = \dfrac{e^{-\lambda} \lambda^r}{r!}$ where r = 0, 1, 2 ....∞ .
19. Mean = λ.
20. Variance = λ.
21. Standard deviation = $\sqrt{\lambda}$.

LEVEL - I (VSAQ)

1. The probability distribution of a random variable X is

X = x	0	1	2	3
P(X = x)	k	3k	3k	k

Find the value of k and the mean of X.

A: Given that X is a random variable.

$\Rightarrow \sum_{i=1}^{3} P(X = x_i) = 1$
$\Rightarrow k + 3k + 3k + k = 1$
$\Rightarrow 8k = 1$
$\Rightarrow k = 1/8$

Let μ be the mean of X.

$\therefore \mu = \sum_{i=1}^{3} x_i P(x = x_i)$
$= 0(k) + 1(3k) + 2(3k) + 3(k)$
$= 12k$
$= \dfrac{12}{8} = \dfrac{3}{2}$

$\therefore$ k = 1/8, μ = 3/2

2. A random variable X has the range {1, 2, 3 ....} If $P(X = r) = \dfrac{C^r}{r!}$ for r = 1, 2, 3, ..., then find C.

A: Given that X is a common variable

$\Rightarrow \sum_{r=1}^{\infty} P(x = r) = 1$

$\Rightarrow \sum_{r=1}^{\infty} \frac{C^r}{r!} = 1$

$\frac{C}{1!} + \frac{C^2}{2!} + \frac{C^3}{3!} + ...\infty = 1$

Adding 1 on bothsides,

$1 + \frac{C}{1!} + \frac{C^2}{2!} + ...\infty = 1 + 1$

$\Rightarrow e^c = 2$

$\Rightarrow C = \log_e 2.$

3. Find the constant C, so that $f(x) = C\left(\frac{2}{3}\right)^x$, x = 1, 2, 3 ...... is the probability distribution function of a discrete random variable X.

A: Given that X is a discrete random variable

$\Rightarrow \sum_{x=1}^{\infty} f(x) = 1$

$\Rightarrow \sum_{x=1}^{\infty} C\left(\frac{2}{3}\right)^x = 1$

$\Rightarrow C\left[\frac{2}{3} + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^3 + ...\infty\right] = 1$

$\Rightarrow C\left[\frac{\frac{2}{3}}{1-\frac{2}{3}}\right] = 1 \qquad \because S_\infty = \frac{a}{1-r}$

$\Rightarrow 2C = 1$

$\therefore C = \frac{1}{2}.$

4. For a binomial ditribution with mean 6 and variance 2, Find the first two terms of hte distribution..

A: Let n, p be the paremeters of the binomial distribution.

np = 6, npq = 2.

$q = \frac{npq}{np} = \frac{2}{6} = \frac{1}{3}.$

$\Rightarrow$ p = 1 - q = 1 - 1/3 = 2/3.

Also np = 6

$\Rightarrow$n x 2/3 = 6

$\Rightarrow$n = 9

First two terms of the binomial distribution are

P(X = 0), P(X = 1)

$= {}^9C_0\left(\frac{1}{3}\right)^9\left(\frac{2}{3}\right)^0, {}^9C_1\left(\frac{1}{3}\right)^8\left(\frac{2}{3}\right)^1$

$= \frac{1}{3^9}, \frac{2}{3^7}.$

AIMS

5. The mean and variance of a binomial distribution are 4 and 3 respectively. Find P(X $\geq$ 1)

A: Let n, p be the parameters of the binomial distribution.

np = 4, npq = 3

$q = \frac{npq}{np} = \frac{3}{4}$

$\Rightarrow p = 1 - q = 1\ 1 - \frac{3}{4} = \frac{1}{4}$

Also np = 4

n.1/4 / 4

n = 16

Required probability P(X $\geq$ 1)

$= \sum_{r=1}^{16} P(X = r)$

$= \sum_{r=1}^{16} {}^{16}C_r \left(\frac{3}{4}\right)^{16-r}\left(\frac{1}{4}\right)^r.$

6. If the mean and variance of a binomial variate X are 2. 4 and 1. 44 respectively, then find p and n.

A: Let n, p be the parameter of the binomial distribution.

Given that np = 2.4, npq = 1.44

$q = \frac{npq}{np} = \frac{1.44}{2.4} = \frac{144}{240} = \frac{12}{20} = \frac{3}{5}$

$\Rightarrow p = 1 - q = 1 - \frac{3}{5} = \frac{2}{5}$

Also np = 2.4

$\Rightarrow n\left(\frac{2}{5}\right) = 2.4$

$\Rightarrow$n = 6 $\quad \because$ n = 6, p = 2/5

7. X follows Poisson distribution such that P(X = 1) = 3P (X = 2). Find the variance of X.

A: Let X be the parameter of the poisson distribution.
Given that P(X = 1) = 3P(X = 2)

$$\Rightarrow \frac{e^{-\lambda}\lambda^1}{1!} = 3\frac{e^{-\lambda}\lambda^2}{2!}$$

$$\Rightarrow 1 = \frac{3\lambda}{2}$$

$$\Rightarrow \lambda = \frac{2}{3}$$

Variance of X = $\frac{2}{3}$.

8. A poisson variate X satisties P(X = 1) = P(X = 2). Find P(X = 5).

A: Let λ be the parameter of the poisson distribution.

Given that P(X = 1) = P(X = 2)

$$\Rightarrow \frac{e^{-\lambda}\lambda^1}{1!} = \frac{e^{-\lambda}\lambda^2}{2!}$$

$$\Rightarrow 1 = \frac{\lambda}{2}$$

$$\Rightarrow \lambda = 2$$

Now P(X = 5) = $\frac{e^{-\lambda}\lambda^5}{5!}$

$$= \frac{e^{-2}.2^5}{5!}$$

$$= \frac{32e^{-2}}{120}$$

$$= \frac{4e^{-2}}{15}.$$

LEVEL - I (LAQ)

1. If X : S → R is a discrete random variable with range $\{x_1, x_2, x_3, ...\}$; μ is mean and σ^2 is variance of X, then prove that $\sigma^2 + \mu^2 = \Sigma x_i^2 P(X = x_i)$.

A: Proof:
Given that X : S → R is a discrete random variable with range $\{x_1, x_2, x_3, ...\}$ Since X is a discrete random variable, $\sum_{i=1}^{\infty} P(X = x_i) = 1$

By the definition, mean $\mu = \sum_{i=1}^{\infty} P(X = x_i)$

By the definition, variance

$$\sigma^2 = \sum_{i=1}^{\infty} (x_i - \mu)^2 P(X = x_i)$$

$$\sigma^2 = \sum_{i=1}^{\infty} (x_i^2 - 2\mu x_i + \mu^2)\ P(X = x_i)$$

$$= \sum_{i=1}^{\infty} x_i^2 P(X = x_i) - 2\mu \sum_{i=1}^{\infty} x_i P(X = x_i) + \mu^2 \sum_{i=1}^{\infty} P(X = x_i)$$

$$= \sum_{i=1}^{\infty} x_i^2 P(X = x_i) - 2\mu(\mu) + \mu^2 (1)$$

$$= \sum_{i=1}^{\infty} x_i^2 P(X = x_i) - \mu^2$$

$$\therefore \sigma^2 + \mu^2 = \sum_{i=1}^{\infty} x_i^2 P(X = x_i).$$

2.

$X = x_i$	-2	-1	0	1	2	3
$P(X = x_i)$	0.1	k	0.2	2k	0.3	k

is the probability distribution of a random variable X. Find the value of k and the variance of X.

A: Given that X is a random variable.
Let μ be the mean and σ^2 be the variance of X.

Here $\sum_{i=-2}^{3} P(X = x_i) = 1$

$\Rightarrow$ 0.1 + k + 0.2 + 2k + 0.3 + k = 1
$\Rightarrow$ 4k + 0.6 = 1
$\Rightarrow$ 4k = 1 - 0.6 = 0.4
$\Rightarrow$ k = 0.1.

Mean $\mu = \sum_{i=-2}^{3} P(X = x_i)$

$= (-2)(0.1) - 1(0.1) + 0(0.2) + 1(0.2) + 2(0.3) + 3(0.1)$
$= 0.8.$

We know that

$\sigma^2 + \mu^2 = \sum_{i=-2}^{3} P(X = x_i)$

$\sigma^2 + 0.8^2 = (-2)^2 (0.1) + (-1)^2 (0.1) + 0^2 (0.2) + (1^2)(0.2) + 2^2 (0.3) + 3^2 (0.1)$

$\sigma^2 = 2.80 - 0.64$

$\sigma^2 = 2.16$

Hence k = 0.1, σ^2 = 2.16.

3. The Probability distribution of a random variable X is given below.

$X = x_i$	1	2	3	4	5
$P(X = x_i)$	k	2k	3k	4k	5k

Find the value of k and the mean and variance of X.

A: Given that X is a random variable

$\Rightarrow \sum_{i=1}^{5} P(X = x_i) = 1$

$\Rightarrow k + 2k + 3k + 4k + 5k = 1$

$\Rightarrow 15k = 1$

$\Rightarrow k = \frac{1}{15}$

Let μ be the mean and σ^2 be the variance of X

$\mu = \sum_{i=1}^{5} P(X = x_i)$

$= 1(k) + 2(2k) + 3(3k) + 4(4k) + 5(5k)$

$= k + 4k + 9k + 16k + 25k$

$= 55k$

$= \frac{55}{15}$

$= \frac{11}{3}$.

$\sigma^2 = \sum_{i=1}^{5} x_i^2 P(X = x_i) - \mu^2$

$= 1(k) + 4(2k) + 9(3k) + 16(4k) + 25(5k) - \left(\frac{11}{3}\right)^2$

$= 225k - \frac{121}{9}$

$= 225\left(\frac{1}{15}\right) - \frac{121}{9}$

$= 15 - \frac{121}{9}$

$= \frac{135 - 121}{9}$

$= \frac{14}{9}$.

4. A random variable X has the following probability distribution.

$X = x_i$	0	1	2	3	4	5	6	7
$P(X = x_i)$	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

Find (i) k (ii) the mean (iii) P(0 < X < 5)

A: Given that X is a random variable

$\sum_{i=0}^{7} P(X = x_i) = 1$

$\Rightarrow 0 + k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$

$\Rightarrow 10k^2 + 9k - 1 = 0$

$\Rightarrow 10k^2 + 10k - k - 1 = 0$

$\Rightarrow 10k(k + 1) - 1(k + 1) = 0$

$\Rightarrow (k + 1)(10k - 1) = 0$

$k = -1$ is false

$\Rightarrow k = \frac{1}{10}$.

Let μ be the mean of X.

$\mu = \sum_{i=0}^{7} x_i P(X = x_i)$

$= 0(0) + 1(0.1) + 2(0.2) + 3(0.2) + 4(0.3) + 5(0.01) + 6(0.02) + 7(0.17)$

$= 0.1 + 0.4 + 0.6 + 1.2 + 0.05 + 0.12 + 1.19$

$= 3.66$

$P(0 < X < 5)$

$= P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4)$

$= k + 2k + 2k + 3k$

$= 8k$

$= 8(0.1)$

$= 0.8$.

5. The range of random variable X is {0, 1, 2}. Given that $P(X = 0) = 3C^3$, $P(X = 1) = 4C - 10C^2$, $P(X = 2) = 5C - 1$.

Find (i) the value of C,

(ii) $P(X < 1)$

(iii) $P(1 < X \le 2)$

(iv) $P(0 < X \le 3)$

A: Given that range of random variable X is {0, 1, 2}.

$P(X = 0) = 3C^3, P(X = 1) = 4C - 10C^2, P(X = 2) = 5C - 1$

Since X is a random variable,

$\sum_{i=0}^{2} P(X = x_i) = 1$

$\Rightarrow P(X = 0) + P(X = 1) + P(X = 2) = 1$

$\Rightarrow 3C^3 + 4C - 10C^2 + 5C - 1 = 1$

$\Rightarrow 3C^3 - 10C^2 + 9C - 2 = 0$

Clearly C = 1 satisfies this equation

By Synthetic division,

$$\begin{array}{c|cccc} & 3 & -10 & 9 & -2 \\ 1 & 0 & 3 & -7 & 2 \\ \hline & 3 & -7 & 2 & \lfloor 0 \end{array}$$

Now the above equation becomes,

$(C - 1)(3C^2 - 7C + 2) = 0$

$\Rightarrow C = 1, 3C^2 - 6C - C + 2 = 0$

$\Rightarrow 3C(C - 2) -1 (C - 2) = 0$

$\Rightarrow (C - 2)(3(-1) = 0$

$\therefore C = 1, 2, \frac{1}{3}$.

C = 1, 2 are not possible.

SO, the value of $C = \frac{1}{3}$.

ii) $P(X < 1) = P(X = 0)$

$= 3C^3$

$= 3\left(\frac{1}{3}\right)^3$

$= \frac{1}{9}$.

iii) $P(1 < x \leq 2) = P(X = 2)$

$= 5C - 1$

$= \frac{5}{3} - 1$

$= \frac{2}{3}$.

iv) $P(0 < X \leq 3) = P(X = 1) + P(X = 2)$

$= \frac{2}{9} + \frac{2}{3}$

$= \frac{8}{9}$.

6. If X is a random variable with the probablity distribution $P(X = k) = \frac{(k+1)C}{2^k}$ (k = 0, 1, 2, ...) then find C ?

A: Here, we use the folowing :

Sum of the infinite A.G.P ($|r| < 1$).

$a(1) + (a + d)r + (a + 2d)r^2 + \ldots\ldots \infty$

$= \frac{a}{1-r} + \frac{dr}{(1-r)^2}$

Given that X is a random variable with

$P(X = k) = \frac{(k+1)C}{2^k}$, k = 0, 1, 2....

$\Rightarrow \sum_{k=0}^{\infty} P(X = k)$

$\Rightarrow \sum_{k=0}^{\infty} \frac{(k+1)C}{2^k} = 1.$

$\Rightarrow C \sum_{k=0}^{\infty} (k+1) . \frac{1}{2^k} = 1$

$\Rightarrow C\left[1.1 + 2.\frac{1}{2} + 3.\frac{1}{2^2} + 4.\frac{1}{2^3} + \ldots\ldots\ldots\infty\right] = 1$

Here a = 1, d = 1, $r = \frac{1}{2}$

$\Rightarrow C\left[\frac{a}{1-r} + \frac{dr}{(1-r)^2}\right] = 1$

$\Rightarrow C\left[\frac{1}{1-\frac{1}{2}} + \frac{1.\frac{1}{2}}{\left(1-\frac{1}{2}\right)^2}\right] = 1$

$\Rightarrow C\left[\frac{1}{\frac{1}{2}} + \frac{\frac{1}{2}}{\frac{1}{4}}\right] = 1$

$\Rightarrow C[2 + 2] = 1$

$\Rightarrow 4C = 1$

$C = \frac{1}{4}$.

7. If the mean and variance of a binomial variate X are 2.4 and 1.44 respectively find $P(1 < X \leq 1.4)$.

A: Let n, p be the parameters of the binomial distribution.

Given that np = 24, npq = 1.44

Now $q = \frac{npq}{np} = \frac{1.44}{1.4} = \frac{144}{240} = \frac{12}{20} = \frac{3}{5}$

$\therefore p = 1 - q = 1 - \frac{3}{5} = \frac{2}{5}$

Also np = 2.4

$\Rightarrow n\left(\frac{2}{5}\right) = 2.4$

$\Rightarrow n = 6$

Required probablity $P(1 < X \leq 4)$

$= P(X = 2) + P(X = 3) + P(X = 4)$

$= {}^6C_2 \left(\frac{3}{5}\right)^4 \left(\frac{2}{5}\right)^2 + {}^6C_3 \left(\frac{3}{5}\right)^3 \left(\frac{2}{5}\right)^3 + {}^6C_4 \left(\frac{3}{5}\right)^2 \left(\frac{2}{5}\right)^4$

$= \frac{15(81)4}{5^6} + \frac{20(27)(8)}{5^6} + \frac{15(9)(16)}{5^6}$

$= \frac{972+864+432}{5^5}$

$= \frac{2268}{3125}$.

8. If the difference between the mean and the variance of a binomial distribution is $\frac{5}{9}$, then find the probablity for the event of 2 successes, when the experiment is conducted 5 times.

A: Let n, p be the parameters of the binomial distribution.

Given that n = 5,

$np - npq = \frac{5}{9}$

$5p(1 - q) = \frac{5}{9}$

$p^2 = \frac{1}{9}$

$p = \frac{1}{3}$

Required probablity is

$P(X = 2) = {}^5C_2 \left(\frac{2}{3}\right)^3 \left(\frac{1}{3}\right)^2$

$= 10 \left(\frac{8}{27}\right) \left(\frac{1}{9}\right)$

$= \frac{80}{243}$.

9. One in nine ships is likely to be wrecked when they set on sail. When 6 ships are set on sail, find the probablity for
(i) atleast one will arrive safely,
(ii) exactly three will arrive safely.

A: Let X be the number of ships arriving safely, when six ships are set on sail. X follows binomial distribution,

Let n, p be the parameter.

the n = 6, $q = \frac{1}{9}$

$\Rightarrow p = 1 - q = 1 - \frac{1}{9} = \frac{8}{9}$

(i) Probablity that atleast one ship will arrive safely

$= P(X \geq 1)$

$= 1 - P(X = 0)$

$= 1 - {}^6C_0 \left(\frac{1}{9}\right)^6 \left(\frac{8}{9}\right)^0$

$= 1 - \frac{1}{9^6}$

(ii) Probablity that exactly three ships will arrive safely

$= P(X = 3)$

$= {}^6C_3 \left(\frac{1}{9}\right)^3 \left(\frac{8}{9}\right)^3$

$= 20 \left(\frac{8^3}{9^6}\right)$.

AIMS

10. In the experiment of tossing a coin n times, if the variable X denotes the number of heads and P(X = 4), P(X = 5), P(X = 6) are in A.P, then find n.

A: Let X denotes the number of heads, when a coin is tossed for n times.

Clearly X follows binomial distribution.

Let n, p be the parameters of binomial distribution.

Here $p = \frac{1}{2}$

$q = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}$

Given that P(X = 4), P(X = 5), P(X = 6) are in A.P

$\Rightarrow {}^nC_4 \left(\frac{1}{2}\right)^{n-4} \left(\frac{1}{2}\right)^4, {}^nC_5 \left(\frac{1}{2}\right)^{n-5} \left(\frac{1}{2}\right)^5,$

${}^nC_6 \left(\frac{1}{2}\right)^{n-6} \left(\frac{1}{2}\right)^6$ are in A.P.

$\Rightarrow {}^nC_4 . \frac{1}{2^n}, {}^nC_5 . \frac{1}{2^n}, {}^nC_6 \frac{1}{2^n}$ are in A.P

$\Rightarrow {}^nC_4, {}^nC_5, {}^nC_6$, are in A.P

$\Rightarrow 2.{}^nC_5 = {}^nC_4 + {}^nC_6$

$\Rightarrow 2 . \frac{n!}{(n-5)!\,5!} = \frac{n!}{(n-4)!\,4!} + \frac{n!}{(n-6)!\,6!}$

$\Rightarrow \frac{2}{(n-5)(n-6)!\,5(4!)} = \frac{1}{(n-4)(n-5)(n-6)!\,4!}$

$+ \frac{1}{(n-6)!\,6(5)(4!)}$

$\Rightarrow \frac{2}{5(n-5)} = \frac{1}{(n-4)(n-5)} + \frac{1}{30}$

$$\Rightarrow \frac{2}{5(n-5)} = \frac{30+n^2-9n+20}{(n-4)(n-5)}$$

$\Rightarrow 12(n-4) = 30 + n^2 - 9n + 20$

$\Rightarrow 12n - 48 = 30 + n^2 - 9n + 20$

$\Rightarrow n^2 - 21n + 98 = 0$

$\Rightarrow n^2 - 7n - 14n + 98 = 0$

$\Rightarrow n(n-7) - 14(n-7) = 0$

$\Rightarrow n = 7$ or 14.

11.

X = x	- 3	- 2	-1	0	1	2	3
P(X = x)	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$

is the probability distribution of a random variable X. Find the variance of X.

A: Let μ be the mean and σ^2 be the variance of random variable X.

$$\text{Mean } \mu = \sum_{i=-3}^{3} x_i P(X = x_i)$$

$$=(-3)\left(\frac{1}{9}\right)+(-2)\left(\frac{1}{9}\right)+(-1)\left(\frac{1}{9}\right)+0\left(\frac{1}{3}\right)+1\left(\frac{1}{9}\right)+2\left(\frac{1}{9}\right)+3\left(\frac{1}{9}\right)$$

$= 0$.

$\therefore$ Mean $\mu = 0$

We know that

$$\sigma^2 = \sum_{i=-3}^{3} x_i^2 P(X = x_i) - \mu^2$$

$$= (-3)^2\left(\frac{1}{9}\right) + \left(-2^2\right)\left(\frac{1}{9}\right) + \left(-1^2\right)\left(\frac{1}{9}\right) + 0^2\left(\frac{1}{3}\right)$$

$$+\left(1^2\right)\left(\frac{1}{9}\right) + \left(2^2\right)\left(\frac{1}{9}\right) + 3^2\left(\frac{1}{9}\right) - 0^2$$

$$= 9.\frac{1}{9} + 4.\frac{1}{9} + 1.\frac{1}{9} + 0 + 1.\frac{1}{9} + 4.\frac{1}{9} + 9.\frac{1}{9} - 0$$

$$= \frac{1}{9}(9+4+1+1+4+9)$$

$$= \frac{28}{9}.$$

Hence variance of X is $\frac{28}{9}$

LEVEL - II (VSAQ)

1. The probability that a person choosen at random is left handed (in hand writing) is 0.1. What is the probability that in a group of 10 people, there is one who is left handed.

A: Here $n = 10$, $p = 0.1 = \frac{1}{10}$, $q = 0.9 = \frac{9}{10}$.

$\therefore$ The required probability that exactly one out of 10 is left handed is $P(X = 1) = {}^{10}C_1\, p^1\, q^{10-1}$.

AIMS

$$= 10.\left(\frac{1}{10}\right)^1 . \left(\frac{9}{10}\right)^9 = \left(\frac{9}{10}\right)^9.$$

2. It is given that 10% of the electric bulbs manufactured by a company are defective. In a sample of 20 bulbs, find the probability that more than 2 are defective.

A: Let X be number of defective bulbs in the sample of 20 bulbs.

The probability that a bulb will be defective is

$p = \frac{10}{100} = \frac{1}{10}$, Hence $q = 1 - \frac{1}{10} = \frac{9}{10}$

Now, X follows the binomial distribution with parameters $n = 20$, $p = \frac{1}{10}$, $q = \frac{9}{10}$.

$\therefore$ The required probability is

$$P(X > 2) = \sum_{k=3}^{20} {}^{20}C_k \left(\frac{1}{10}\right)^k \left(\frac{9}{10}\right)^{20-k}$$

$$= \sum_{k=3}^{20} {}^{20}C_k . \frac{9^{20-k}}{10^{20}}.$$

3. On an average, rain falls on 12 days in every 30 days, find the probability that, rain will fall on just 3 days of a given week.

A: Let X be the number of days rain falls in a week.

The probability that rain will fall on a day.

$p = \frac{12}{30} = \frac{2}{5}$, Hence $q = 1 - \frac{2}{5} = \frac{3}{5}$.

Now, X follows the binomial distribution with parameters $n = 7$, $p = \frac{2}{5}$, $q = \frac{3}{5}$

$\therefore$ The required probability is

$$P(X = 3) = {}^7C_3\left(\frac{2}{5}\right)^3\left(\frac{3}{5}\right)^4 = 35.\frac{2^3.3^4}{5^7}.$$

4. In a book of 450 pages, there are 400 typographical errors. Assuming that the number of errors per page follow the Poisson law, find the probability that a random sample of 5 pages will contain no typographical error.

A: Let the average number of errors per page in the book is $\lambda = \frac{400}{450} = \frac{8}{9}$.

The probability that a page contain 'r' errors is

$$P(X = r) = \frac{e^{-\lambda}\lambda^r}{r!}$$

The probability that a page contain no errors is

$$P(X = 0) = \frac{e^{-\lambda}\lambda^r}{r!} = e^{-8/9}.$$

$\therefore$ The required probability that a random sample of 5 pages will contain no error is

$$\left[P(X = 0)\right]^5 = \left[e^{-8/9}\right]^5.$$

5. Deficiency of red cells in the blood cells is determined by examining a specimen of blood under a microscope, Suppose a small volume contains on an average 20 red cells for normal persons. Using the poisson distribution, find the probability that a specimen of blood taken from a normal person will contain less than 15 red cells.

A: Here $\lambda = 20$.

Let P(X = r) denote the probability that a specimen taken from a normal person will contain 'r' red cells.

$\therefore$ The required probability is

$$P(X < 15) = \sum_{r=0}^{14} P(X = r) = \sum_{r=0}^{14}\frac{e^{-20}20^r}{r!}.$$

6. In a city, 10 accidents take place in a span of 50 days. Assuming that the number of accidents follows the Poisson distribution, find the probability that there will be 3 or more accidents in a day.

A: Here $\lambda = \frac{10}{50} = 0.2$

The required probability is

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$$P(X \geq 3) = 1 - \left[P(X = 0) + P(X = 1) + P(X = 2)\right]$$

$$= 1 - \left[\frac{e^{-0.2}(0.2)^0}{0!} + \frac{e^{-0.2}(0.2)^1}{1!} + \frac{e^{-0.2}(0.2)^2}{2!}\right]$$

$$= 1 - \left[\frac{1}{e^{0.2}} + \frac{1}{5e^{0.2}} + \frac{1}{50e^{0.2}}\right] = 1 - \frac{61}{50e^{0.2}}.$$

LEVEL - II (LAQ)

1. The range of a random variable X is {1, 2, 3,...} and P(X = k) = $\frac{C^k}{k!}$, k = 1, 2, 3,...... Find the value of C and P(0 < X < 3).

A. Given that X is a random variable

$$\Rightarrow \sum_{k=1}^{\infty} P(X = k) = 1$$

$$\Rightarrow \sum_{k=1}^{\infty}\frac{C^k}{k!} = 1$$

$$\Rightarrow \frac{C}{1!} + \frac{C^2}{2!} + \frac{C^3}{3!} + ...\infty = 1$$

Adding 1 on both sides

$$\Rightarrow 1 + \frac{C}{1!} + \frac{C^2}{2!} + \frac{C^3}{3!} + ...\infty = 1 + 1$$

$$\Rightarrow \quad e^C = 2$$

$$\Rightarrow \quad C = \log_e 2$$

ii) P(0 < X < 3)

$\Rightarrow$ P(X = 1) + P(X = 2)

$$= \frac{C}{1!} + \frac{C^2}{2!}$$

$$= \log_e 2 + (\log_e 2)^2.$$

2. A cubical die is thrown. Find the mean and variance of X, giving the number on the face that shows up.

A: Let X be the number on the face that shows up when a die is thrown. Here X is a random variable. Probability distribution of X is shown below.

$X = x_i$	1	2	3	4	5	6
$P(X = x_i)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

Let μ be the mean and σ^2 be the variance of X

$$\mu = \sum_{i=1}^{6} x_i P(X = x_i)$$

$$= 1\left(\frac{1}{6}\right) + 2\left(\frac{1}{6}\right) + 3\left(\frac{1}{6}\right) + 4\left(\frac{1}{6}\right) + 5\left(\frac{1}{6}\right) + 6\left(\frac{1}{6}\right)$$

$$= \frac{21}{6}$$

$$= \frac{7}{2}.$$

$$\sigma^2 = \sum_{i=1}^{6} x_i^2 P(X = x_i) - \mu^2$$

$$= 1^2\left(\frac{1}{6}\right) + 2^2\left(\frac{1}{6}\right) + 3^2\left(\frac{1}{6}\right) + 4^2\left(\frac{1}{6}\right) + 5^2\left(\frac{1}{6}\right) + 6^2\left(\frac{1}{6}\right) - \left(\frac{7}{2}\right)^2$$

$$= \frac{91}{6} - \frac{49}{4}$$

$$= \frac{182 - 147}{12}$$

$$= \frac{35}{12}.$$

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