MODEL PAPER - 1

MATHEMATICS

1. $A = \{x \in R : x \neq 0, -4 \le x \le 4\}$ $f : A \to R$ is defined as $f(x) = \frac{|x|}{x}$ then the range of f is

1) {1,-1} 2) $\{x : 0 \le x \le 4\}$ 3) {1} 4) $\{x : -4 \le x \le 0\}$

2. The number of relations from A={1,2,3} to B = {4,6,8,10} is
1) 4^3 2) 2^7 3) 2^{12} 4) 3^4

3. Let S (k) = 1 + 3 + 5 + ... + (2k-1) =3 +k^2. Then which of the following is true ? ***(Mathematical Induction)***

1) S(1) is correct 2) Principle of mathematical induction can be used to prove the formula
3) S(k) $\not\Rightarrow$ S(k+1) 4) S(k)$\Rightarrow$ S(k+1)

4. If D = $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$ for x ≠ 0, y ≠ 0, then D is ***(Matrices)***

1) divisible by neither x nor y 2) divisible by both x and y
3) divisible by x but not y 4) divisible by y but not x

5. If A = $\begin{bmatrix} a & b \\ b & a \end{bmatrix}$ and A^2 = $\begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}$, then ***(Matrices)***
1) $\alpha = a^2 + b^2$, $\beta = ab$ 2) $\alpha = a^2 + b^2$, $\beta = 2ab$ 3) $\alpha = a^2 + b^2$, $\beta = a^2 - b^2$ 4) $\alpha = 2ab$, $\beta = a^2 + b^2$

6. If the system of equations x + 2y + 3z = λx, 3x + y + 2z = λy, 2x + 3y + z = λz has nontrivial solution, then λ = ***(Matrices)***
1) 6 2) 12 3) 18 4) 16

7. If i + 2j + 3k, 3i + 2j + k are sides of a parallelogram, then a unit vector parallel to one of the diagonals of the parallelogram is ***(Addition of Vectors)***

1) $\frac{i+j+k}{\sqrt{3}}$ 2) $\frac{i-j+k}{\sqrt{3}}$ 3) $\frac{i+j-k}{\sqrt{3}}$ 4) $\frac{-i+j+k}{\sqrt{3}}$

8. The vector $\bar{a}$ =αi + 2j + βk lies in the plane of the vectors $\bar{b}$ = i + j and $\bar{c}$ = j + k and bisects the angle between $\bar{b}$ and $\bar{c}$. Then, which one of the following gives possible values of α and β? ***(Addition of Vectors)***
1) α =1, β = 1 2) α = 2, β = 2 3) α =1, β = 2 4) α = 2, β = 1

9. The value of a, for which the points A, B, C with position vectors 2i - j + k, i - 3j - 5k and ai - 3j + k respectively are the vertices of a right angled triangle with C = π/2 are ***(Dot and cross product)***
1) -2 and -1 2) -2 and 1 3) 2 and -1 4) 2 and 1

10. Let $\bar{a}$ =j - k and $\bar{c}$ = i - j - k. Then the vector $\bar{b}$ satisfying $(\bar{a} \times \bar{b}) + \bar{c} = \bar{0}$ and $\bar{a}.\bar{b}$ =3, is ***(Dot and cross product)***
1) -i + j - 2k 2) 2i - j + 2k 3) i - j - 2k 4) i + j - 2k

11. Let $\bar{a}, \bar{b}$ and $\bar{c}$ be non-zero vectors such that $(\bar{a} \times \bar{b}) \times \bar{c} = \frac{1}{3}|\bar{b}|\,|\bar{c}|\,\bar{a}$. If θ is the acute angle between the vectors $\bar{b}$ and $\bar{c}$, then sin θ equals ***(Triple product)***

1) $\frac{1}{3}$ 2) $\frac{\sqrt{2}}{3}$ 3) $\frac{2}{3}$ 4) $\frac{2\sqrt{2}}{3}$

12. $\frac{\cos^3 A + \sin^3 A}{\cos A + \sin A} + \frac{\cos^3 A - \sin^3 A}{\cos A - \sin A} =$ ***(Trignomentric Function)***
1) 0 2) -1 3) 1 4) 2

13. tan α = $\frac{m}{m+1}$, tan β = $\frac{1}{2m+1}$ then tan (α + β) = ***(Compound Angles)***
1) -1 2) 0 3) 1 4) 2

14. Let cos (α + β) = $\frac{4}{5}$ and sin (α - β) = $\frac{5}{13}$, where $0 \le \alpha, \beta \le \frac{\pi}{4}$. Then tan2α equals ***(Multiple and submultiple)***

1) $\frac{25}{16}$ 2) $\frac{56}{33}$ 3) $\frac{19}{12}$ 4) $\frac{20}{7}$

15. $m \tan(\theta - 30^0) = n \tan(\theta + 120^0)$ then $\frac{m+n}{m-n} =$ *(Transformation)*

1) $\cos 2\theta$ 2) $2 \cos 2\theta$ 3) $\sin 2\theta$ 4) $2 \sin 2\theta$

16. The period of the function $f(x) = \sin^4 x + \cos^4 x$ is *(Periodicity & Extreme values)*

1) π 2) $\frac{\pi}{2}$ 3) 2π 4) None of these

17. If sinx, sin 2x, sin 3x are in A.P., then x = *(Trignometric Equations)*

1) $\frac{n\pi}{2}, 2n\pi$ 2) $n\pi, n\pi \pm \frac{\pi}{3}$ 3) $n\pi, n\pi \pm \frac{\pi}{4}$ 4) $(2n+1)\frac{\pi}{2}, n\pi \pm \frac{\pi}{6}$

18. If $\text{Cos}^{-1}x - \text{Cos}^{-1}\frac{y}{2} = \alpha$, then $4x^2 - 4xy \cos \alpha + y^2$ is equal to *(Inverse Triangle)*

1) $-4 \sin^2 \alpha$ 2) $4 \sin^2 \alpha$ 3) 4 4) $2 \sin 2\alpha$

19. If $\cosh x = \sec\theta$, then $\coth^2\left(\frac{x}{2}\right) =$ *(Hyperbolic Functions)*

1) $\tan^2\left(\frac{\theta}{2}\right)$ 2) $\tan^2 \theta$ 3) $\cot^2\left(\frac{\theta}{2}\right)$ 4) $\cot^2 \theta$

20. In a triangle ABC, $\tan\frac{A}{2} = \frac{5}{6}$, $\tan\frac{C}{2} = \frac{2}{5}$, then *(Properties of triangle)*

1) a, c, b are in A.P 2) a, b, c are in A.P 3) b, a, c are in A.P 4) a, b, c are in G.P

21. $\frac{r_1}{bc} + \frac{r_2}{ca} + \frac{r_3}{ab} =$ *(Properties of triangle)*

1) $\frac{1}{r} - \frac{1}{2R}$ 2) $1 + \frac{r}{R}$ 3) $2 + \frac{r}{2R}$ 4) $1 - \frac{r}{2R}$

22. The locus of the point $\left(a + bt, b - \frac{a}{t}\right)$ where t is the parameter *(Locus)*

1) $(x - a)(y - b) = ab$ 2) $(x + a)(y - b) = ab$ 3) $(x - a)(y + b) = ab$ 4) $(x - a)(b - y) = ab$

23. The angle of rotation of axes to remove xy term in the equation $xy = c^2$ is *(Transformation of axis)*

1) $\pi/12$ 2) $\pi/6$ 3) $\pi/3$ 4) $\pi/4$

24. The line L given by $\frac{x}{5} + \frac{y}{b} = 1$ passes through the point (13, 32). The line K is parallel to L and has the equation $\frac{x}{c} + \frac{y}{3} = 1$. Then the distance between K and L is *(Straight Lines)*

1) $\frac{23}{\sqrt{15}}$ 2) $\sqrt{17}$ 3) $\frac{17}{\sqrt{15}}$ 4) $\frac{23}{\sqrt{17}}$

25. The point on the line $2x - 3y = 5$ which is equidistant from (1,2) and (3,4) is *(Straight Lines)*

1) (2,3) 2) (4,1) 3) (1,-1) 4) (4,6)

26. If the sum of the slopes of the lines given by $x^2 - 2cxy - 7y^2 = 0$ is four times their product, then c has the value *(Pair of straight line)*

1) 1 2) -1 3) 2 4) -2

27. The area of the triangle formed by the lines $x^2 + 4xy + y^2 = 0$, $x + y = 1$ is *(Pair of straight line)*

1) $\sqrt{3}$ 2) 2 3) 1 4) $\sqrt{3}/2$

28. The line passing through the points (5, 1, a) and (3, b, 1) crosses the yz-plane at the point $\left(0, \frac{17}{2}, \frac{-13}{2}\right)$. Then *(Three dimentional theorem)*

1) a = 2, b = 8 2) a = 4, b = 6 3) a = 6, b = 4 4) a = 8, b = 2

29. A line makes the same angle θ with each of the x and z axes. If the angle β, which it makes with y - axis, is such that $\sin^2\beta = 3 \sin^2\theta$, then $\cos^2\theta$ equals *(Directions Consines)*

1) $\frac{2}{3}$ 2) $\frac{1}{5}$ 3) $\frac{3}{5}$ 4) $\frac{2}{5}$

30. Distance between two parallel planes $2x + y + 2z = 8$ and $4x + 2y + 4z + 5 = 0$ is *(Plane)*

1) $\frac{3}{2}$ 2) $\frac{5}{2}$ 3) $\frac{7}{2}$ 4) $\frac{9}{2}$

31. $\lim_{x\to 2}\frac{\sqrt{1-\cos 2(x-2)}}{x-2}$ *(Limits)*

1) does not exist 2) equals $\sqrt{2}$ 3) equals $-\sqrt{2}$ 4) equal $\frac{1}{\sqrt{2}}$

32. $\lim_{x\to 0}\frac{\sqrt{4+x}-\sqrt[3]{8-3x}}{x}$ = *(Limits)*

1) $-\frac{1}{2}$ 2) $\frac{1}{2}$ 3) -3 4) 0

33. The function f : R/{0} → R given by f(x) = $\frac{1}{x}-\frac{2}{e^{2x}-1}$ can be made continuous at x = 0 by defining f(0) as *(Continuity)*

1) 2 2) -1 3) 0 4) 1

34. Let f(a) = g(a) = k and their nth derivatives $f^n(a)$, $g^n(a)$ exist and are not equal for some n. Further if $\lim_{x\to a}\frac{f(a)g(x)-f(a)-g(a)f(x)+g(a)}{g(x)-f(x)}$ =4, then the value of k is equal to *(Differentiations)*

1) 4 2) 2 3) 1 4) 0

35. Let y be implicit function of x defined by $x^{2x} - 2.x^x \cot y - 1 = 0$. Then $y'(1)$ equals *(Differentiations)*

1) -1 2) 1 3) log2 4) - log2

36. Approximate value of $\sqrt{63}$ is *(Error's)*

1) 3.972 2) 1.028 3) 3.979 4) 7.982

37. The equation of the tangent to the curve $y = x + \frac{4}{x^2}$, that is parallel to the x - axis is *(Tangents and normal)*

1) y = 0 2) y = 1 3) y = 2 4) y = 3

38. If the rate of change in the radius of a circle is 0.2cm/sec, then the rate of change in the area of the circle when the radius is 15cm is *(Rate of Change)*

1) 2π sq.cm/sec 2) 3π sq.cm/sec 3) 6π sq.cm/sec 4) 8π sq.cm/sec

39. The constant *c* of Rolle's theorem for the function f(x) = $\log\frac{x^2+ab}{(a+b)x}$ in [a, b] where 0 Ï [a, b] is *(Mean Values Theorem)*

1) $\sqrt{ab}$ 2) $\frac{a+b}{2}$ 3) $\frac{a-b}{2}$ 4) $\frac{b-a}{2}$

40. The function f (x) = $\frac{x}{2}+\frac{2}{x}$ has a local minimum at *(Maxima and Minima)*

1) x = -2 2) x = 0 3) x = 1 4) x = 2

41. The value of $\sum_{k=1}^{10}\left(\sin\frac{2k\pi}{11}+i\cos\frac{2k\pi}{11}\right)$ is *(Complex Numbers)*

1) 1 2) -1 3) - i 4) i

42. If |z - 4| < |z - 2|, its solution is given by *(Complex Numbers)*

1) Re z > 0 2) Re z < o 3) Re z > 3 4) Re z > 2

43. If x satisfies the equation $x^2 - 2x\cos\theta + 1 = 0$, then the value of $x^n + 1/x^n$ is *(De-Moiver's Theorem)*

1) $2^n \cos n\theta$ 2) $2^n \cos^n\theta$ 3) $2\cos n\theta$ 4) $2\cos^n\theta$

44. If the difference between the roots of the equation $x^2 + ax + 1 = 0$ is less than $\sqrt{5}$, then the set of possible values of a is *(Quadratic Expression)*

1) (-3, 3) 2) (-3, ∞) 3) (3, ∞) 4) (-∞, -3)

45. If the sum of the roots of the quadratic equation $ax^2 + bx + c = 0$ is equal to the sum of the squares of their reciprocals, then $\frac{a}{c}, \frac{b}{a}$ and $\frac{c}{b}$ are in *(Quadratic Expression)*

1) A.P 2) G.P 3) H.P 4) A.G.P

46. If α, β, γ are the roots of $x^3 + 3x + 2 = 0$ then the equation whose roots $\alpha(\beta+\gamma), \beta(\gamma+\alpha), \gamma(\alpha+\beta)$ is *(Theory of Equations)*

1) $x^3 - 6x^2 + 9x + 4 = 0$ 2) $x^3 - 3x^2 + 9x + 4 = 0$ 3) $x^3 + 6x^2 - 9x + 4 = 0$ 4) $x^3 - 6x^2 - 9x - 4 = 0$

47. If the letters of the word "SACHIN" are arranged in all possible ways and these words are written out as in dictionary, then the word SACHIN appears at serial number *(Permutation & Combination)*

1) 602 2) 603 3) 600 4) 601

48. The value of $^{50}C_4 + \sum_{r=1}^{6} {}^{56-r}C_3$ is *(Permutation & Combination)*

1) $^{55}C_4$ 2) $^{55}C_3$ 3) $^{56}C_3$ 4) $^{56}C_4$

49. The number of permutations that can be made using all the letters of the word BANANA is *(Permutation & Combination)*

1) 60 2) 120 3) 360 4) 720

50. If $^nC_{20} = {}^nC_{50}$ then n = *(Permutation & Combination)*

1) 20 2) 30 3) 50 4) 70

51. In the binomial expansion of $(a - b)^n$, $n \geq 5$, the sum of 5^{th} and 6^{th} terms is zero, then $\frac{a}{b}$ equals *(Binomial Theorem)*

1) $\frac{5}{n-4}$ 2) $\frac{6}{n-5}$ 3) $\frac{n-5}{6}$ 4) $\frac{n-4}{5}$

52. The coefficient of x^5 in $(1 + 2x + 3x^2 + ...)^{-3/2}$ is *(Binomial Theorem)*

1) 21 2) 25 3) 26 4) None of these

53. If $\frac{2x-5}{(x-3)^2} = \frac{a}{x-3} + \frac{b}{(x-3)^2}$ then a + b = *(Partial Fraction)*

1) 1 2) 2 3) 3 4) 4

54. Standard deviation of first 'n' natural numbers is *(Measure of Dispersion)*

1) $\sqrt{\frac{n-1}{nm}}$ 2) $\sqrt{\frac{n^2+1}{12}}$ 3) $\sqrt{\frac{n^2-1}{12}}$ 4) None

55. It is given that the events A and B are such that $P(A) = \frac{1}{4}$, $P(A/B) = \frac{1}{2}$ and $P(B/A) = \frac{2}{3}$. Then P(B) is *(Probability)*

1) $\frac{1}{2}$ 2) $\frac{1}{6}$ 3) $\frac{1}{3}$ 4) $\frac{2}{3}$

56. Five horses are in a race. Mr. A selects two of the horses at random and bets on them . The probability that Mr. A selected the winning horse, is *(Probability)*

1) $\frac{4}{5}$ 2) $\frac{3}{5}$ 3) $\frac{1}{5}$ 4) $\frac{2}{5}$

57. A problem in mathematics is given to three students A,B,C and their respective probabilities of solving the problem are $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$. Probability that the problem is solved, is *(Probability)*

1) 3/4 2) 1/2 3) 2/3 4) 1/3

58. A random variable X has Poisson distribution with mean 2. Then $P(X > 1.5)$ equals. *(Random Variables)*

1) $\frac{3}{e^2}$ 2) $1 - \frac{3}{e^2}$ 3) 0 4) $\frac{2}{e^2}$

59. The probability of answering 6 out of 10 questions correctly in a true or false examination is *(Random Variables)*

1) $^{10}C_4\left(\frac{1}{2}\right)^4$ 2) $^{10}C_6\left(\frac{1}{2}\right)^6$ 3) $^{10}C_6\left(\frac{1}{2}\right)^{10}$ 4) $^{10}C_6\left(\frac{1}{2}\right)^8$

60. The lines 2x - 3y = 5 and 3x - 4y = 7 are diameters of a circle having area as 154 sq. units. Then, the equation of the circle is *(Circles)*

1) $x^2 + y^2 + 2x - 2y = 62$ 2) $x^2 + y^2 + 2x - 2y = 47$
3) $x^2 + y^2 - 2x + 2y = 47$ 4) $x^2 + y^2 - 2x + 2y = 62$

61. A variable circles passes through the fixed point A (p, q) and touches x - axis. The locus of the other end of the diameter through A is *(Circles)*

1) $(x - p)^2 = 4qy$ 2) $(x - q)^2 = 4py$ 3) $(y - p)^2 = 4qx$ 4) $(y - q)^2 = 4px$

62. The two circles $x^2 + y^2 = ax$ and $x^2 + y^2 = c^2$ (c > 0) touch each other if *(Circles)*

1) 2 |a| = c 2) |a| = c 3) a = 2c 4) |a| = 2c

63. The intercept on the line y = x by the circle $x^2 + y^2 - 2x = 0$ is AB. Equation of the circle on AB as a diameter is *(System of Circles)*

1) $x^2 + y^2 - x - y = 0$ 2) $x^2 + y^2 - x + y = 0$ 3) $x^2 + y^2 + x + y = 0$ 4) $x^2 + y^2 + x - y = 0$

64. The intercept on the line y = x by the circle $x^2 + y^2 - 2x = 0$ is AB. Equation of the circle on AB as a diameter is *(System of Circles)*

1) $x^2 + y^2 - x - y = 0$ 2) $x^2 + y^2 + x - y = 0$ 3) $x^2 + y^2 + x + y = 0$ 4) $x^2 + y^2 - x + y = 0$

65. The equation of a tangent to the parabola $y^2 = 8x$ is y = x + 2. the point on this line from which the other tangent to the parabola is perpendicular to the given tangent is *(Parabola)*

1) (-1, 1) 2) (0, 2) 3) (2, 4) 4) (-2, 0)

66. The length of the normal chord at $(at^2, 2at)$ is least then t^2 = *(Parabola)*
1) 1 2) 2 3) 3 4) 4

67. The equation $\frac{x^2}{10-a}+\frac{y^2}{4-a}=1$ represents an ellipse if *(Ellipse)*
1) a < 4 2) a > 4 3) 4 < a < 10 4) a > 10

68. The foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{b^2}=1$ and the hyperbola $\frac{x^2}{144}+\frac{y^2}{81}=\frac{1}{25}$ coincide then the value of b^2 is *(Ellipse)*
1) 1 2) 5 3) 7 4) 9

69. For the hyperbola $\frac{x^2}{\cos^2\alpha}-\frac{y^2}{\sin^2\alpha}=1$ which of the following remains constant when α varies? *(Hyperbola)*
1) Eccentricity 2) Directrix 3) Abscissae of vertices 4) Abscissae of foci

70. If $\int \frac{3^{2x}}{2^{3x}}dx=\frac{1}{K}\left(\frac{3^{2x}}{2^{3x}}\right)+c$ then K = *(Integeration)*
1) $2\ \ell n\ 3 - 3\ \ell n\ 2$ 2) $3\ \ell n\ 2 - 2\ \ell n\ 3$ 3) $\ell n\ 9 + \ell n\ 8$ 4) $\ell n\ \sqrt{72}$

71. If $\int f(x) . \cos x . dx = \frac{1}{2} f^2(x) + c$, then f(x) can be *(Integeration)*
1) x 2) sin x 3) cos x 4) x sin x

72. $\int \frac{dx}{\cos x - \sin x}$ is equal to *(Integeration)*
1) $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2}-\frac{\pi}{8}\right)\right| + c$
2) $\frac{1}{\sqrt{2}}\log\left|\cot\left(\frac{x}{2}\right)\right| + c$
3) $I\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2}-\frac{3\pi}{8}\right)\right| + c$
4) $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2}+\frac{3\pi}{8}\right)\right| + c$

73. $\int_0^\infty \frac{dx}{\left(x+\sqrt{x^2+1}\right)^3} =$ *(Definite Integration)*
1) 3/8 2) 1/8 3) -3/8 4) -1/8

74. If $I_1 = \int_0^1 2^{x^2} dx$, $I_2 = \int_0^1 2^{x^3} dx$, $I_3 = \int_1^2 2^{x^2} dx$ and $I_4 = \int_1^2 2^{x^3} dx$ then *(Definite Integration)*
1) $I_3 > I_4$ 2) $I_3 = I_4$ 3) $I_1 > I_2$ 4) $I_2 > I_1$

75. $I_n = \int_0^{\pi/4} \tan^n x\, dx$, then $\lim_{n\to\infty} n[I_n + I_{n+2}]$ equals *(Definite Integration)*
1) 1/2 2) 1 3) ∞ 4) 0

76. Area bounded by $\frac{|x|}{a} + \frac{|y|}{b} = 1$ where a, b > 0 is *(Areas)*
1) $\frac{1}{2ab}$ 2) ab 3) 2ab 4) 4ab

77. The area bounded by the curves y = cos x and y = sin x between the ordinates x = 0 and $x = \frac{3\pi}{2}$ is *(Areas)*
1) $(4\sqrt{2} - 2)$ square units 2) $(4\sqrt{2} + 2)$ square units
3) $(4\sqrt{2} - 1)$ square units 4) $(4\sqrt{2} + 1)$ square units

78. The differential equation for the family of curves $x^2 + y^2 - 2ay = 0$, where a is an arbitary constant is *(Differential Equations)*
1) $2(x^2 - y^2)\ y' = xy$ 2) $2(x^2 + y^2)\ y' = xy$ 3) $(x^2 - y^2)\ y' = 2xy$ 4) $(x^2 + y^2)\ y' = 2xy$

79. The solution of the differential equation $ydx + (x + x^2y)\, dy = 0$ is *(Differential Equations)*
1) $\frac{-1}{xy} = c$ 2) $\frac{-1}{xy} + \log y = c$ 3) $\frac{1}{xy} + \log y = c$ 4) log y = cx

80. The solution of $\frac{dy}{dx} + 2y \cot x = 3x^2 \operatorname{cosec}^2 x$ is *(Differential Equations)*
1) $y \sin x = x^3 + c$ 2) $y \sin^2 x = x^2 + c$ 3) $y \sin^2 x = x^3 + c$ 4) $y \sin^3 x = x^3 + c$