MODEL PAPER - 2

MATHEMATICS

1. If $f : R \to R$ satisfies f(x+y) = f(x) + f(y) for all x, y Î R and f(1) = 7, then $\sum_{r=1}^{n} f(r)$ is *(Functions)*

1) $\frac{7n}{2}$ 2) $\frac{7(n+1)}{2}$ 3) n(n+1) 4) $\frac{7n(n+1)}{2}$

2. The domain of the function f(x) = $\frac{\sin^{-1}(x-3)}{\sqrt{9-x^2}}$ is *(Functions)*

1) [2, 3] 2) [2, 3) 3) [1, 2] 4) [1, 2)

3. The sum of the first n terms of the series $1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + ...$ is $\frac{n(n+1)^2}{2}$ when n is even. When n is odd the sum is *(Mathematical Induction)*

1) $\frac{3n(n+1)}{2}$ 2) $\left[\frac{n(n+1)}{2}\right]^2$ 3) $\frac{n(n+1)^2}{4}$ 4) $\frac{n^2(n+1)}{2}$

4. Let $A = \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}$ If $|A^2| = 25$, then |a | equals *(Matrices)*

1) 5^2 2) 1 3) $\frac{1}{5}$ 4) 5

5. If w (¹ 1) is a cubic root of unity, then $\begin{vmatrix} 1 & 1+i+\omega^2 & \omega^2 \\ 1-i & -1 & \omega^2-1 \\ -i & -1+\omega-i & -1 \end{vmatrix}$ equals *(Matrices)*

1) 0 2) 1 3) i 4) w

6. The number of values of k, for which the system of equations (k + 1)x + 8y = 4k, kx + (k + 3)y = 3k - 1 has no solution, is *(Matrices)*

1) 1 2) 2 3) 3 4) Infinite

7. Let $\bar{a}, \bar{b}$ and $\bar{c}$ be three non-zero vectors such that no two of these are collinear. If the vector $\bar{a} + 2\bar{b}$ is collinear with $\bar{c}$ and $\bar{b} + 3\bar{c}$ is collinear with $\bar{a}$ (I being some non-zero scalar) then $\bar{a} + 2\bar{b} + 6\bar{c}$ = *(Addition of Vectors)*

1) $\lambda\bar{a}$ 2) $\bar{0}$ 3) $\lambda\bar{c}$ 4) $\lambda\bar{b}$

8. Consider the points A, B, C and D with position vectors 7i - 4j + 7k, i - 6j + 10k, -i - 3j + 4k and 5i - j + 5k respectively. Then ABCD is a *(Addition of Vectors)*

1) Square 2) Rhombus 3) Rectangle 4) None of these

9. The distance between the line $\bar{r}$ = 2i - 2j + 3k + l(i - j + 4k) and the plane $\bar{r}$. (i + 5j + k) = 5 is *(Dot and cross product)*

1) $\frac{10}{3}$ 2) $\frac{3}{10}$ 3) $\frac{10}{3\sqrt{3}}$ 4) $\frac{10}{9}$

10. If, in a right angled triangle ABC, the hypotenuse AB = p, then $\overline{AB} \cdot \overline{AC} + \overline{BC} \cdot \overline{BA} + \overline{CA} \cdot \overline{CB}$ = *(Dot and cross product)*

1) $2p^2$ 2) $p^2/2$ 3) p^2 4) $4p^2$

11. Let $\bar{u}$ = i + j, $\bar{v}$ = i - j and $\bar{w}$ = i + 2j + 3k. If $\bar{n}$ is a unit vector such that $\bar{u}.\bar{n}$ = 0 and $\bar{v}.\bar{n}$ = 0, then $|\bar{w}.\bar{n}|$ is equal to *(Triple product)*

1) 0 2) 1 3) 2 4) 3

12. If $\frac{\sin x + \cos x}{\cos^3 x}$ = a $\tan^3 x$ + b $\tan^2 x$ + c tanx + d, then a + b + c + d = *(Trignomentric Function)*

1) 4 2) -4 3) 1 4) -1

13. If tan A = 18/17, tan B = 1/35 then A - B = *(Compound Angles)*

1) 0 2) $\pi/4$ 3) $\pi/3$ 4) $\pi/2$

14. A quadratic equation whose roots are tan $22\frac{1}{2}^0$ and cot $22\frac{1}{2}^0$ is *(Multiple and submultiple)*

1) $x^2 - 2\sqrt{2}x + 1 = 0$ 2) $2x^2 - \sqrt{2} + 1 = 0$ 3) $x^2 + 2\sqrt{2}x - 1 = 0$ 4) $x^2 - 2\sqrt{2}x - 1 = 0$

15. The value of sinx + siny = a and cosx + cosy = b, then cos (x - y) = *(Transformation)*

1) $\frac{a^2+b^2+2}{2}$ 2) $\frac{a^2-b^2+2}{2}$ 3) $\frac{a^2+b^2-2}{2}$ 4) $\frac{b^2-a^2+2}{2}$

16. Range of $\log_{1/5}[\sqrt{2}(\cos x + \sin x) + 3]$ is *(Periodicity & Extreme values)*

1) [-1, 0] 2) [-2, -1] 3) [0, 1] 4) [0, ∞)

17. If tan 2θ tan θ = 1, then θ = *(Trignometric Equations)*

1) $n\pi + \frac{\pi}{6}$ 2) $n\pi \pm \frac{\pi}{6}$ 3) $2n\pi \pm \frac{\pi}{6}$ 4) $n\pi \pm \frac{\pi}{3}$

18. $\text{Cot}^{-1}\left(\sqrt{\cos\alpha}\right) - \text{Tan}^{-1}\left(\sqrt{\cos\alpha}\right) = x$, then sin x equal to *(Inverse Triangle)*

1) $\tan^2\frac{\alpha}{2}$ 2) $\cot^2\frac{\alpha}{2}$ 3) tanα 4) $\cot\frac{\alpha}{2}$

19. $\tanh^{-1}\left(\frac{1}{3}\right) + \coth^{-1}(3) =$ *(Hyperbolic Functions)*

1) log 2 2) log 3 3) log $\sqrt{3}$ 4) log $\sqrt{2}$

20. In a triangle ABC, $2ca\sin\left(\frac{A-B+C}{2}\right)$ is equal to *(Properties of triangle)*

1) $a^2 + b^2 - c^2$ 2) $c^2 + a^2 - b^2$ 3) $b^2 + c^2 - a^2$ 4) $c^2 - a^2 - b^2$

21. $\frac{r_1(r_2+r_3)}{\sqrt{r_1r_2+r_2r_3+r_3r_1}} =$ *(Properties of triangle)*

1) a 2) b 3) c 4) a^2

22. The locus represented by $x = \frac{1}{2}a\left(t+\frac{1}{t}\right)$, $y = \frac{1}{2}a\left(t-\frac{1}{t}\right)$ is *(Locus)*

1) $x^2 + y^2 = a^2$ 2) $x^2 - y^2 = a^2$ 3) $2x^2 - y^2 = a^2$ 4) $x^2 - 2y^2 = a^2$

23. If the axes are rotated through an angle 180^0 the equation 2x - 3y + 4 = 0 becomes *(Transformation of axis)*

1) 2X - 3Y - 4 = 0 2) 2X + 3Y - 4 = 0 3) 3X - 2Y + 4 = 0 4) 3X + 2Y + 4 = 0

24. Perpendicular bisector of the line segment joining P(1, 4) and Q(k, 3) has y - intercept -4, then a possible value of k is *(Straight Lines)*

1) -4 2) 1 3) 2 4) -2

25. The line segment joining the points (1, 2) and (k, 1) is divided by the line 3x + 4y - 7 = 0 in the ratio 4 : 9 then k is *(Straight Lines)*

1) 2 2) -2 3) 3 4) -3

26. If one of the lines given by $6x^2 - xy + 4cy^2 = 0$ is 3x + 4y = 0 then c equals *(Pair of straight line)*

1) 1 2) -1 3) 3 4) -3

27. The quadrilateral formed by the pair of lines $x^2 - 5x + 6 = 0$, $9x^2 + 24xy + 16y^2 + 3x + 4y - 6 = 0$ is *(Pair of straight line)*

1) parallelogram 2) rhombus 3) rectangle 4) square

28. The centroid of the tetrahedron ABCD divides the line joining the vertex A to the centroid of triangle BCD in the ratio *(Three dimentional theorem)*

1) 1 : 2 2) 2 : 1 3) 1 : 3 4) 3 : 1

29. The projections of a line of length ‘d’ on the co-ordinate planes are 3, 4, 5 then d = *(Directions Consines)*

1) 50 2) $5/\sqrt{2}$ 3) 5 4) $5\sqrt{2}$

30. A plane which passes through the point (3,2,0) and the line $\frac{x-4}{1} = \frac{y-7}{5} = \frac{z-4}{4}$ is *(Plane)*

1) x - y + z = 1 2) x + y + z = 5 3) x - 2y - z = 1 4) 2x - y + z = 5

31. $\lim_{x\to\infty}\left(\frac{x^2+5x+3}{x^2+x+2}\right)^x$ is equal to *(Limits)*

1) e^4 2) e^2 3) e^3 4) e

32. $\lim_{x\to\infty}\left(\frac{3x-4}{3x+2}\right)^{\frac{x+1}{3}} =$ *(Limits)*

1) $e^{-2/3}$ 2) $e^{3/2}$ 3) $e^{2/3}$ 4) e

33. The function $f(x) = \begin{cases} 0, & x \text{ is rational} \\ 1, & x \text{ is irrational is} \end{cases}$ *(Continuity)*

1) Continuous only at x = 1
2) Discontinuous only at 0
3) Discontinuous at 0, 1
4) Discontinuous every where

34. If $y = (x+\sqrt{1+x^2})^n$, then $(1 + x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx}$ is *(Differentiations)*

1) n^2y 2) $-n^2y$ 3) $-y$ 4) $2x^2y$

35. Let $f(x) = \begin{cases} x^2, & \text{if } x \le 1 \\ ax+b, & \text{if } x > 1 \end{cases}$. If f is differentiable at x = 1 then *(Differentiations)*

1) a = 1, b = -1 2) a = 2, b = -1 3) a = 2, b = 1 4) a = 1, b = 1

36. If $y = x^n$ then the ratio of the relative errors in y and x are *(Error's)*

1) 1:1 2) 2:1 3) n:1 4) 1:n

37. If the line ax + by + c = 0 is a normal to xy = 1 then *(Tangents and normal)*

1) a > 0, b < 0 2) a > 0, b > 0 3) a > b = 0 4) a < 0, b < 0

38. The rate of change of area of a square plate is equal to that of the rate of change of its perimeter. Then length of the side is *(Rate of change)*

1) 2 units 2) 3 units 3) 4 units 4) 6 units

39. The constant 'c' of Rolle's theorem for the function f(x) = sin x in [0, 2p] is *(Mean values theorem)*

1) $\pi/6$ 2) $\pi/3$ 3) $\pi/4$ 4) $\pi/2$

40. A point on the parabola $y^2 = 18x$ at which the ordinare increases at twice the rate of the abscissa, is *(maximum and Minimum)*

1) (2,4) 2) (2,-4) 3) $\left(-\frac{9}{8}, \frac{9}{2}\right)$ 4) $\left(\frac{9}{8}, \frac{9}{2}\right)$

41. If z = x - iy and $z^{\frac{1}{3}} = p + iq$, then $\left(\frac{x}{p}+\frac{y}{q}\right) / (p^2 + q^2) =$ *(Complex Number)*

1) 1 2) -1 3) 2 4) -2

42. If $z = i(i + \sqrt{2})$, then the value of $z^4 + 4z^3 + 6z^2 + 4z$ is *(Complex Number)*

1) -9 2) - 5 3) 3 4) 6

43. If $(\sqrt{3}+i)^{100} = 2^{99}(a + ib)$ then b = *(De-Moiver's Theorem)*

1) 1 2) $\sqrt{3}$ 3) $\sqrt{2}$ 4) 2

44. The value of a for which the sum of the squares of the roots of the equation $x^2 - (a - 2)x - a - 1 = 0$ assume the least value is *(Quadratic Expression)*

1) 2 2) 3 3) 0 4) 1

45. If α and β are the roots of the equation $x^2 - x + 1 = 0$, then $\alpha^{2009} + \beta^{2009}$ is equal to *(Quadratic Expression)*

1) -2 2) -1 3) 1 4) 2

46. The non - repeated root of $x^3 + 4x^2 + 5x + 2 = 0$ is *(Theory of Equation)*

1) $-\frac{5}{3}$ 2) -2 3) -1 4) 1

47. The range of the function $f(x) = {}^{7-x}P_{x-3}$ is *(Permutation & Combination)*

1) {1,2,3} 2) {1,2,3,4,5,6} 3) {1,2,3,4} 4) {1,2,3,4,5}

48. The least positive integral value of x which satisfies the inequality ${}^{10}C_{x-1} > 2.{}^{10}C_x$ is *(Permutation & Combination)*

1) 6 2) 5 3) 8 4) 9

49. The number of ways in which 8 boys and 5 girls can sit around a round table so that no two girls come together is *(Permutation & Combination)*

1) $8!\ {}^9P_5$ 2) $7!\ {}^8P_5$ 3) $8!\ {}^8P_5$ 4) $7!\ {}^9P_5$

50. If ${}^nP_r = 720$, ${}^nC_r = 120$, then (n,r) is *(Permutation & Combination)*

1) (7,4) 2) (6,2) 3) (8,4) 4) (10,3)

51. The sum of the series ${}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + \ldots\ldots + {}^{20}C_{10}$ is *(Binomial Theorem)*

1) $-{}^{20}C_{10}$ 2) $\frac{1}{2}\ {}^{20}C_{10}$ 3) 0 4) ${}^{20}C_{10}$

52. The remainder left out when $8^{2n} - (62)^{2n+1}$ is divided by 9 is *(Binomial Theorem)*

1) 2 2) 7 3) 8 4) 0

53. If $\frac{1}{(x+1)(x^2+x+1)} = \frac{k}{x+1} - \frac{x}{x^2+x+1}$ then k = *(Partial Fraction)*

1) 0 2) 1 3) -1 4) -2

54. The measure of dispersion which is used to find more consistent data is *(Measure of Dispersion)*
1) Standard deviation 2) Mean deviation 3) Quartile deviation 4) Range

55. A pair of fair dice is thrown independently three times. The probability of getting a score of exactly 9 twice is *(Probability)*
1) 1/729 2) 8/9 3) 8/729 4) 8/243

56. Events A,B,C are mutually exclusive events such that P (A) = $\frac{3x+1}{3}$, P (B) = $\frac{1-x}{4}$ and P (C) = $\frac{1-2x}{2}$. The set of possible values of x are in the interval. *(Probability)*
1) $\left[\frac{1}{3},\frac{1}{2}\right]$ 2) $\left[\frac{1}{3},\frac{2}{3}\right]$ 3) $\left[\frac{1}{3},\frac{13}{3}\right]$ 4) [0,1]

57. A and B play a game where each is asked to select a number from 1 to 25. If the two numbers match, both of them win a prize. The probability that they will not win a prize in a single trial is *(Probability)*
1) $\frac{1}{25}$ 2) $\frac{24}{25}$ 3) $\frac{2}{25}$ 4) None of these

58. The mean and variance of a random variable X having a binomial distribution are 4 and 2 respectively, then P (X = 1) is *(Random Variable)*
1) $\frac{1}{32}$ 2) $\frac{1}{16}$ 3) $\frac{1}{8}$ 4) $\frac{1}{4}$

59. A random variable X has the range {1, 2, 3, 4}. If P (X = 1) = 1/k, P (X =2) = 2/k, P (X=3) = 3/k, P(X=4)=4/k then k = *(Random Variable)*
1) 1 2) 10 3) 1/10 4) 1/5

60. The circle described on the line joining the points (0, 1) (a, b) as diameter cuts the x - axis in points whose abscissae are roots of the equation *(Circles)*
1) $x^2 + ax + b = 0$ 2) $x^2 + ax - b = 0$ 3) $x^2 - ax + b = 0$ 4) $x^2 - ax - b = 0$

61. A circles touches the x - axis and also touches the circle with centre at (0,3) and radius 2. The locus of the centre of the circle is *(Circles)*
1) a parabola 2) a hyperbola 3) a circle 4) an ellipse

62. The equation of the circum circle of the triangle formed by the lines $y + \sqrt{3}x = 6$, $y - \sqrt{3}x = 6$ and y = 0 is *(Circles)*
1) $x^2 + y^2 - 4y = 0$ 2) $x^2 + y^2 + 4x = 0$ 3) $x^2 + y^2 + 4y - 12 = 0$ 4) $x^2 + y^2 - 4y - 12 = 0$

63. If A, B are the centres of two intersecting circles at P by acute angle, and each passes through the centres of the other circle. Then ΔPAB is *(System of Circles)*
1) Equilateral 2) Right angled 3) Isosceles 4) Right angled and Isosceles

64. Two circles whose radii are r and R and whose distance between the centres is 'd' cut each other orthogonally. Then the length of their common chord is *(System of Circles)*
1) $\frac{2rR}{\sqrt{r+R}}$ 22) $\frac{rR}{\sqrt{r^2+R^2}}$ 3) $\frac{2rR}{\sqrt{r^2+R^2}}$ 4) $\frac{2rR}{r^2+R^2}$

65. If $a \neq 0$ and the line 2bx + 3cy + 4d = 0 passing through the points of intersection of the parabolas $y^2 = 4ax$ and $x^2 = 4ay$, then *(Parabola)*
1) $d^2 + (2b + 3c)^2 = 0$ 2) $d^2 + (3b + 2c)^2 = 0$ 3) $d^2 + (2b - 3c)^2 = 0$ 4) $d^2 + (3b - 2c)^2 =0$

66. If two tangents drawn from the point (α, β) to the parabola $y^2 = 4x$ be such that the slope of one tangent is double of the other, then *(Parabola)*
1) $\beta = (2/9)\alpha^2$ 2) $\alpha = (2/9)\beta^2$ 3) $2\alpha = 9\beta^2$ 4) $\alpha^2 = 9\beta^2$

67. A focus of an ellipse is at the origin. The directrix is the line x = 4 and the eccentricity is $\frac{1}{2}$, then the length of the semi major axis is *(Ellipse)*
1) $\frac{5}{3}$ 2) $\frac{8}{3}$ 3) $\frac{2}{3}$ 4) $\frac{4}{3}$

68. The radius of the circle passing through the foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ and having its centre at (0,3) is *(Ellipse)*
1) 4 unit 2) 3 unit 3) $\sqrt{12}$ unit 4) $\frac{7}{2}$ unit

69. The locus of a point P(α, β) moving under the condition that the line y = $\alpha x + \beta$ is a tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, is *(Hyperbola)*
1) hyperbola 2) a parabola 3) a circle 4) an ellipse

70. $\int \frac{x^5}{\sqrt{1+x^3}} dx =$ *(Integration)*
1) $\frac{2}{9}(1+x^3)^{5/2} + \frac{2}{3}(1+x^3)^{3/2} + c$ 2) $\log |\sqrt{x} + \sqrt{1+x^3}| + c$
3) $\log |\sqrt{x} - \sqrt{1+x^3}| + c$ 4) $\frac{2}{9}(1+x^3)^{3/2} - \frac{2}{3}(1+x^3)^{1/2} + c$

71. $\int \frac{(x^2-2)}{(x^4+5x^2+4)\tan^{-1}\left(\frac{x^2+2}{x}\right)} dx =$ *(Integration)*

2) $\log|\tan^{-1}\sqrt{x+2}|+c$ 2) $\log|\tan^{-1}\frac{x^2+2}{x}|+c$ 3) $\sin^{-1}\left(\frac{x+2}{x}\right)+c$ 4) $\tan^{-1}\left(\frac{x+2}{x}\right)+c$

72. $\int \frac{1}{9\sin^2x-16\cos^2x} dx$ *(Integration)*

1) $\frac{1}{24}\log\left|\frac{3\tan x-4}{3\tan x+4}\right|+c$ 2) $\frac{1}{24}\log\left|\frac{4\tan x-3}{4\tan x+3}\right|+c$

3) $\frac{1}{24}\log\left|\frac{3\tan x+4}{3\tan x-4}\right|+c$ 4) $\frac{1}{24}\log\left|\frac{4\tan x+3}{4\tan x-3}\right|+c$

73. Let $\frac{d}{dx}f(x)=\left(\frac{e^{\sin x}}{x}\right)$, $x>0$. If $\int_1^4 \frac{3}{x}e^{\sin(x^3)}dx = F(k)-F(1)$, then one of the possible values of k, is *(Definite Integration)*

1) 15 2) 16 3) 63 4) 64

74. Let $I=\int_0^1 \frac{\sin x}{\sqrt{x}}dx$ and $J=\int_0^1 \frac{\cos x}{\sqrt{x}}dx$ then which one of the following is true? *(Definite Integration)*

1) $I>\frac{2}{3}$ and $J<2$ 2) $I>\frac{2}{3}$ and $J>2$ 3) $I<\frac{2}{3}$ and $J<2$ 4) $I<\frac{2}{3}$ and $J>2$

75. $\int_0^a x^3(ax-x^2)^{3/2}dx =$ *(Definite Integration)*

1) $-\frac{9\pi a^7}{2048}$ 2) $\frac{3\pi a^7}{2048}$ 3) $\frac{9\pi a^7}{2048}$ 4) $\frac{9\pi a^7}{2345}$

76. The area of the region bounded by the curves y = |x - 1| and y = 3 - |x| is *(Areas)*
1) 2 square units 2) 3 square units c) 4 square units 4) 6 square units

77. The area of the portion of the circular disc of radius a cut off between the chords of the length a and a/2 is *(Areas)*

1) $\frac{\sqrt{15}}{16}a^2-\frac{\sqrt{3}}{4}a^2-\frac{\pi}{3}a^2+a^2\text{Sin}^{-1}\frac{\sqrt{15}}{4}$ 2) $\frac{\sqrt{15}}{32}a^2-\frac{\sqrt{3}}{8}a^2$

3) $\frac{\pi}{6}a^2-\frac{a^2}{2}\text{Sin}^{-1}\frac{\sqrt{15}}{4}$ 4) None

78. The differential equation of the family of circles with fixed radius 5 units and centre on the line y = 2 is *(Differential Equations)*
1) $(y-2)^2(y')^2=25-(y-2)^2$ 2) $(y-2)^2(y')^2=25+(y-2)^2$
3) $(y-2)(y')^2=25-(y-2)^2$ 4) $(y-2)(y')^2=25+(x-2)^2$

79. Solution of the differential equation cos x dy = y (sin x - y) dx, $-0<x\frac{\pi}{2}$, is *(Differential Equations)*

1) sec x = (tan x + c)y 2) y sec x = tan x + c 3) y tan x = sec x + c 4) tan x = (sec x + c) y

80. The solution of the differential equation $\frac{dy}{dx}-y\tan x=e^x\sec x$ is *(Differential Equations)*

1) $y=e^x\cos x+c$ 2) $y\cos x=e^x+c$ 3) $y=e^x\sin x+c$ 4) $y\sin x=e^x+c$