MODEL PAPER - 3

MATHEMATICS

1. Domain of the function $f(x) = \frac{3}{4-x^2} + \log_{10}(x^3 - x)$ is *(Functions)*

1) (1, 2) 2) $(-1, 0) \cup (1, 2)$ 3) $(1, 2) \cup (2, \infty)$ 4) $(-1, 0) \cup (1, 2) \cup (2, \infty)$

2. If $f(x) = \frac{1}{\sqrt{x+2\sqrt{2x-4}}} + \frac{1}{\sqrt{x-2\sqrt{2x-4}}}$ for x > 2, then f(11) = *(Functions)*

1) $\frac{7}{6}$ 2) $\frac{5}{6}$ 3) $\frac{6}{7}$ 4) $\frac{5}{7}$

3. Statement 1: The sum of the series 1 + (1 + 2 + 4) + (4 + 6 + 9) + ... + (361 + 380 + 400) is 8000.

Statement 2: $\sum_{k=1}^{n}\left[k^3 - (k-1)^3\right] = n^3$ for any natural number n. *(Mathematical Induction)*

1) Statement 1 is true, statement 2 is true, statement 2 is not a correct explanation for statement 1.
2) Statement 1 is true, statement 2 is false
3) Statement 1 is false, statement 2 is true
4) Statement 1 is true, statement 2 is true, statement 2 is a correct explanation for statement 1

4. Let $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$ and $10\,B = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$. If B is the inverse of matrix A, then a is *(Matrices)*

1) -2 2) 1 3) 2 4) 5

5. Let a,b,c be such that b+c $\neq$ 0. If $\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c-1 \end{vmatrix} + \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ (-1)^{n+2}a & (-1)^{n+1}b & (-1)^n c \end{vmatrix} = 0$, then the value of n is *(Matrices)*

1) any even integer 2) any odd integer 3) any integer 4) zero

6. If the system of linear equations, x + 2ay + az = 0, x + 3by + bz = 0 and x + 4cy + cz = 0 non - zero solution, then a,b,c satisty *(Matrices)*

1) 2ab = ac + bc 2) 2b = a + c 3) $b^2 = ac$ 4) 2ac = ab + bc

7. If C is the midpoint of AB and P is any point outside AB, then *(Addition of Vectors)*

1) $\overline{PA} + \overline{PB} = 2\overline{PC}$ 2) $\overline{PA} + \overline{PB} = \overline{PC}$ 3) $\overline{PA} + \overline{PB} + 2\overline{PC} = \overline{0}$ 4) $\overline{PA} + \overline{PB} + \overline{PC} = \overline{0}$

8. The vectors $\overline{AB}$ =3i+4k, and $\overline{AC}$ =5i-2j+4k are the sides of a triangle ABC. The length of the median through A is *(Addition of Vectors)*

1) $\sqrt{18}$ 2) $\sqrt{72}$ 3) $\sqrt{33}$ 4) $\sqrt{288}$

9. A particle is acted upon by constant forces 4i + j - 3k and 3i + j - k which displace it from a point i + 2j + 3k to the point 5i + 4j + k. The work done in standard units by the forces is given by *(Dot and cross product)*

1) 40 units 2) 30 units 3) 25 units 4) 15 units

10. Let $\overline{a}$ and $\overline{b}$ be two unit vectors. If the vectors $\overline{c} = \overline{a} + 2\overline{b}$ and $\overline{d} = 5\overline{a} - 4\overline{b}$ are perpendicular to each other, then the angle between $\overline{a}$ and $\overline{b}$ is *(Dot and cross product)*

1) $\frac{\pi}{3}$ 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{6}$ 4) $\frac{\pi}{2}$

11. If $\overline{a} = \frac{1}{\sqrt{10}}(3i + k)$ and $\overline{b} = \frac{1}{7}(2i+3j-6k)$, then the value of $(2\overline{a} - \overline{b}).[(\overline{a} \times \overline{b}) \times (\overline{a} + 2\overline{b})]$ is *(Triple product)*

1) -5 2) -3 3) 5 4) 3

12. The expression $\frac{\tan A}{1-\cot A}+\frac{\cot A}{1-\tan A}$ can be written as *(Trignomentric Function)*

1) secA cosecA + 1 2) tan A + cot A 3) sec A + cosec A 4) sin A cos A + 1

13. $\tan\left(\frac{\pi}{4}+\theta\right) . \tan\left(\frac{3\pi}{4}+\theta\right) =$ *(Compound Angles)*

1) 0 2) -1 3) 1 4) 2

14. $\tan^6\frac{\pi}{9} - 33\tan^4\frac{\pi}{9} + 27\tan^2\frac{\pi}{9} =$ *(Multiple and submultiple)*

1) $\tan\frac{\pi}{3}$ 2) $\tan^2\frac{\pi}{3}$ 3) $\tan\frac{\pi}{6}$ 4) $\tan^2\frac{\pi}{6}$

15. If a, b are acute angles and cos 2a = $\frac{3\cos 2\beta - 1}{3-\cos 2\beta}$, then *(Transformation)*

1) tana = 2 tanb 2) tana = $\sqrt{2}$ tanb 3) tanb = $2\sqrt{2}$ tana 4) tanb = $\sqrt{2}$ tana

16. $\frac{\sin^2 A + \sin A + 1}{\sin A} \geq k$ then k = *(Periodicity & Extreme values)*

1) 2 2) 1 3) 3 4) 4

17. If tan pq = tan qq, then the values of q form a progression, which is *(Trignometric Equations)*

1) A.P 2) G.P 3) H.P 4) A.G.P

18. 2 $\cot^{-1}$ (7) + $\cos^{-1}\left(\frac{3}{5}\right)$, in principal value, is equal to *(Inverse Triangle)*

1) $\text{cosec}^{-1}\left(\frac{117}{125}\right)$ 2) $\cos^{-1}\left(\frac{44}{125}\right)$ 3) $\cos^{-1}\left(\frac{44}{117}\right)$ 4) $\text{Tan}^{-1}\left(\frac{41}{117}\right)$

19. log (cosh 4x - sinh 4x) = *(Hyperbolic Functions)*

1) -2x 2) -3x 3) -4x 4) -8x

20. If the angles of a triangle are in the ratio 1 : 5 : 6, then the ratio of its sides is *(Properties of triangle)*

1) $(\sqrt{3}-1) : (\sqrt{3}+1) : 2\sqrt{2}$ 2) $(\sqrt{2}-1) : (\sqrt{2}+1) : 2\sqrt{2}$

8) $(\sqrt{3}-1) : (\sqrt{2}+1) : 2\sqrt{2}$ 4) $(\sqrt{2}-1) : (\sqrt{3}+1) : 2\sqrt{2}$

21. In ABC, right angled at A, $\cos^{-1}\left(\frac{R}{r_2+r_3}\right)$ is *(Properties of triangle)*

1) 30^0 2) 60^0 3) 90^0 4) 45^0

22. Locus of the point (2+3cost, -5 + 3 sint) is *(Locus)*

1) $(x-2)^2+(y-5)^2=9$ 2) $(x+2)^2+(y-5)^2=9$ 3) $(x-2)^2+(y+5)^2=9$ 4) $(x+2)^2+(y+5)^2=9$

23. The angle of rotation of the axes so that the equation ax + by + c = 0 may be reduced to the form X = constant is *(Transformation of axis)*

1) Tan^{-1} b/a 2) Tan^{-1} a/b 3) $-\text{Tan}^{-1}$ b/a 4) $-\text{Tan}^{-1}$ a/b

24. The medians AD and BE of a triangle with vertices A (0,b), B(0,0) and C (a,0) are perpendicular to each other if *(Straight Lines)*

1) $b=\sqrt{2}a$ 2) $a=\pm\sqrt{3}b$ 3) $a=\pm\sqrt{2}b$ 4) $b=\sqrt{3}a$

25. The area of the quadrilateral formed by the lines 2x - y - 11 =0, 2x + 3y - 7 = 0, 3x - 5y - 1 = 0, x + 2y + 7 = 0 is *(Straight Lines)*

1) 7/2 2) 7 3) 49 4) 49/2

26. If $ax^2+by^2+2fy+c=0$ (a ¹ 0) represents a pair of lines then *(Pair of straight line)*

1) f is A.M. between a and c 2) f is A.M. between b and c

3) f is G.M. between a and c 4) f is G.M. between b and c

27. If two lines $2x^2+2(K-1)xy-3y^2=0$ are equally inclined to the axes then K = *(Pair of straight line)*

1) 1 2) 2 3) -1 4) -2

28. If origin is the orthocentre of a triangle formed by the points (cosa, sina, 0), (cosb, sinb, 0), (cosg, sing, 0) then $\sum\cos(2\alpha-\beta-\gamma)=$ *(Three dimentional theorem)*

1) 0 2) 1 3) 2 4) 3

29. If the direction cosines of a line are $\frac{1}{3},\frac{1}{3}$, n and n < 0 then n = *(Directions Consines)*

1) $-\frac{\sqrt{7}}{3}$ 2) $-\frac{7}{3}$ 3) $-\frac{\sqrt{3}}{7}$ 4) $-\frac{3}{7}$

30. The perpendicular distance from the plane passing through (1, 2, 2), (-5, -1, 2), (1, 0, -4) to the point (2, 2, 4) is *(Plane)*

1) 1 2) 1/2 3) 7 4) 3/7

31. $\underset{x\to 0}{\text{Lt}} \frac{\sin x - x + x^3/6}{x^5}$ *(Limits)*

1) 1/120 2) 1/110 3) 1/100 4) 1/90

32. $\lim_{x\to 0} \frac{(1-\cos 2x)(3+\cos x)}{x \tan 4x} =$ *(Limits)*

1) $\frac{1}{2}$ 2) 1 3) 2 4) $\frac{-1}{4}$

33. f is define on [-5, 5] as f(x) = $\begin{cases} x, & \text{if } x \text{ is rational} \\ -x, & \text{if } x \text{ is irrational} \end{cases}$ then *(Continuity)*

1) f(x) is continuous at every x, except x = 0
2) f(x) is discontinuous at every x, except x = 0
3) f(x) is continuous at every where
4) f(x) is discontinuous at every where

34. If y = sec (Tan^{-1}x), then $\frac{dy}{dx}$ at x = 1 is equal to *(Differentiations)*

1) $\frac{1}{2}$ 2) 1 3) $\sqrt{2}$ 4) $\frac{1}{\sqrt{2}}$

35. Let $f(x) = \begin{cases} x & \forall x < 1 \\ 2-x & \text{for } 1 \le x \le 2 \\ -2+3x-x^2 & \forall x > 2 \end{cases}$ then f(x) is *(Differentiations)*

1) differentiable at x =1 2) differentiable at x = 2 3) differentiable at x = 1 & x = 2 4) $f'(0) = 0$

36. If the percentage error in the surface area of sphere is k, then the percentage error in its volume is

(Error's)

1) 3k/2 2) 2k/3 3) k/3 4) 4k/3

37. If y = 4x - 5 is a tangent to the a curve $y^2 = px^3 + q$ at (2, 3), then *(Tangents and normal)*

1) p = 2, q = -7 2) p = -2, q = 7 3) p = -2, q = -7 4) p = 2, q = 2

38. The side of a square is equal to the diameter of a circle. If the side and radius change at the same rate, then the ratio of the change of their areas is *(Rate of change)*

1) 1 : π 2) π : 1 3) 2 : π 4) 1 : 2

39. Rolle's theorem can not applicable for *(Mean values theorem)*

1) f(x) = $\sqrt{1-x^2}$ in [-1,1]
2) f(x) = |x| in [-1,1]
3) f(x) = x^2- 1 in [-1, 1]
4) f(x) = $x^3 + x^2$ -x-1 in [-1, 1]

40. The shortest distance between the line y - x = 1 and the curve x = y^2 is *(maximum and Minimum)*

1) $\frac{3\sqrt{2}}{8}$ 2) $\frac{2\sqrt{3}}{8}$ 3) $\frac{3\sqrt{2}}{5}$ 4) $\frac{\sqrt{3}}{4}$

41. If $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix}$ = x + iy, then *(Complex Numbers)*

1) x = 3, y = 1 2) x = 1, y = 3 3) x = 0, y = 3 4) x = 0, y = 0

42. If z is a complex number of unit modulus and argument θ, then arg $\left(\frac{1+z}{1+z}\right)$ equals *(Complex Numbers)*

1) $\frac{\pi}{2}$ - θ 2) θ 3) π - θ 4) - θ

43. If $x_n = \cos(\pi/4^n) + i \sin(\pi/4^n)$ then $x_1 . x_2 . x_3 ..... \infty$ = *(De-Moiver's Theorem)*

1) $\frac{1+i\sqrt{3}}{2}$ 2) $\frac{-1+i\sqrt{3}}{2}$ 3) $\frac{1-i\sqrt{3}}{2}$ 4) $\frac{-1-i\sqrt{3}}{2}$

44. If one root of the equation $x^2 + px + 12 = 0$ is 4, while the equation $x^2 + px + q = 0$ has equal roots, then the value of q is *(Quadratic Expression)*

1) $\frac{49}{4}$ 2) 12 3) 3 4) 4

45. If the quadratic equation $3x^2 + 2(a^2 + 1) x + (a^2 - 3a + 2) = 0$ possesses roots of opposite signs, then 'a' lies in the interval *(Quadratic Expression)*
1) (2,3) 2) $(-\infty,-1)$ 3) (-1,1) 4) (1,2)

46. If the roots of $32x^3 - 48x^2 + 22x - 3 = 0$ are in A.P. then the middle root is *(Theory of Equations)*
1) 2 2) $\frac{1}{2}$ 3) 4 4) $\frac{1}{4}$

47. The number of ways of distributing 8 identical balls in 3 distinct boxes, so that none of the boxes is empty is *(Permutation & Combination)*
1) 5 2) 21 3) 3^8 4) ${}^8C_3$

48. Assuming the balls to be identical except for difference in colours, the number of ways in which one or more balls can be selected from 10 white, 9 green and 7 black balls is *(Permutation & Combination)*
1) 630 2) 879 3) 880 4) 629

49. Ten candidates $A_1, A_2, A_3, ... A_{10}$ can be arranged in a row. If A_1 is just above A_2 then the number of ways are *(Permutation & Combination)*
1) 9! 2! 2) 10! 3) 10! 2! 4) 9!

50. If ${}^nC_4, {}^nC_5, {}^nC_6$ are in A.P, then the value of n is *(Permutation & Combination)*
1) 11 2) 17 3) 8 4) 14 or 7

51. The number of integral terms in the expansion of $(\sqrt{3} + \sqrt[8]{5})^{256}$ is *(Binomial Theorem)*
1) 32 2) 33 3) 34 4) 35

52. The coefficient of t^{24} in $(1 + t^2)^{12} (1 + t^{12}) (1 + t^{24})$ is *(Binomial Theorem)*
1) ${}^{12}C_6$ 2) ${}^{12}C_6 + 13$ 3) ${}^{12}C_6 + 2$ 4) ${}^{12}C_6 + 1$

53. $\frac{3x - 1}{(1 - x + x^2)(2 + x)} =$ *(Partial Fractions)*
1) $\frac{x}{x^2 - x + 1} + \frac{1}{x + 2}$ 2) $\frac{x}{x^2 + x + 1} + \frac{2}{x + 2}$ 3) $\frac{x}{x^2 - x + 1} - \frac{1}{x + 2}$ 4) $\frac{x}{x^2 - x + 1} - \frac{2}{x + 2}$

54. If standard deviation of a data is 3, arithmetic mean is 20, then coefficient of variation is *(Measure of Dispersion)*
1) 15 2) 3/20 3) 20/3 4) None

55. Three houses are available in a locality. Three persons apply for the houses. Each applies for one house without consulting others. The probability that all the three apply for the same house is *(Probability)*
1) $\frac{7}{9}$ 2) $\frac{8}{9}$ 3) $\frac{1}{9}$ 4) $\frac{2}{9}$

56. If A and B are two mutually exclusive events, then *(Probability)*
1) $P(A) < P(\overline{B})$ 2) $P(A) > P(\overline{B})$ 3) $P(A) < P(B)$ 4) None of these

57. Let A and B be two events such that $P(A \cup B) \geq \frac{3}{4}$ and $\frac{1}{8} \leq P(A \cap B) \leq \frac{3}{8}$. *(Probability)*

Statement 1: $P(A) + P(B) \geq \frac{7}{8}$.

Statement 2: $P(A) + P(B) \leq \frac{11}{8}$.

1) Statement 1 is true, Statement 2 is true; statement 2 is not a correct explanation for statement 1
2) Statement 1 is true, Statement 2 is false
3) Statement 1 is false, Statement 2 is true
4) Statement 1 is true, Statement 2 is true; Statement 2 is a correct explanation for statement 1

58. A fair die is tossed eight times. The probability that a third six is observed on the eight throw, is *(Random Variables)*
1) $\frac{{}^7C_2 \times 5^5}{6^7}$ 2) $\frac{{}^7C_2 \times 5^5}{6^8}$ 3) $\frac{{}^7C_2 \times 5^5}{6^6}$ 4) None of these

59. If X is a Poisson variate such that $P(X = 0) = \frac{1}{2}$, then the variance of X is *(Random Variables)*
1) $\frac{1}{2}$ 2) 2 3) $\log_e 2$ 4) 3

60. The area of equilateral triangle inscrible in the circle $x^2 + y^2 + 6x - 8y + 4 = 0$ is *(Circles)*
1) $63\sqrt{3}$ 2) $\frac{63\sqrt{3}}{4}$ 3) $\frac{9\sqrt{3}}{4}$ 4) $\frac{3\sqrt{63}}{4}$

61. The circle $x^2 + y^2 = 4x + 8y + 5$ intersects the line $3x - 4y = m$ at two distinct points, if *(Circles)*
1) $-85 < m < -35$ 2) $-35 < m < 15$ 3) $15 < m < 65$ d) $35 < m < 85$

62. The length of the diameter of the circle which touches the x - axis at the point (1,0) and passes through the point (2,3) is *(Circles)*
1) 6/5 2) 5/3 3) 10/3 4) 3/5

63. If P and Q are the points of intersection of the circle $x^2 + y^2 + 3x + 7y + 2p - 5 = 0$ and $x^2 + y^2 + 2x + 2y - p^2 = 0$ then there is a circle passing through P,Q and (1,1) and *(System of Circles)*
1) all values of p 2) all except one value of p
3) all except two values of p 4) exactly one value of p.

64. Two circles of radii 3, 4 intersect orthogonally. Then the length of the common chord is *(System of Circles)*
1) 12/5 2) 24/25 3) 24/5 4) 25/24

65. The equation of the directrix of the parabola whose vertex (3, 2) and focus (2, -1) is *(Parabola)*
1) $x + 3y - 19 = 0$ b) $x - 2y - 9 = 0$ 3) $2x + 6y - 24 = 0$ 4) $x - 3y - 19 = 0$

66. If the vertex of the parabola $y = x^2 - 8x + c$ lies on x-axis, then the value of c is
(Parabola) 1) -16 2) -4 3) 4 4) 16

67. An ellipse has OB as semi minor axis, F and F' its foci and the angle FBF' is a right angle. Then the eccentricity of the ellipse is *(Ellipse)*
1) $\frac{1}{\sqrt{3}}$ 2) $\frac{1}{4}$ 3) $\frac{1}{2}$ 4) $\frac{1}{\sqrt{2}}$

68. If the minor axis of an ellipse subtends an angle 60^0 at each foci its eccentricity is *(Ellipse)*
1) $\cos 15^0$ 2) $\cos 30^0$ 3) $\cos 45^0$ 4) $\cos 60^0$

69. The equation of the chord joining two points (x_1, y_1) and (x_2, y_2) on the rectangular hyperbola $xy = c^2$ is *(Hyperbola)*
1) $\frac{x}{x_1+x_2}+\frac{y}{y_1+y_2}=1$ 2) $\frac{x}{x_1-x_2}+\frac{y}{y_1-y_2}=1$
3) $\frac{x}{y_1+y_2}+\frac{y}{x_1+x_2}=1$ 4) $\frac{x}{y_1-y_2}+\frac{y}{x_1-x_2}=1$

70. $\int\left(\frac{\tan\frac{1}{x}}{x}\right)^2 dx =$ *(Integration)*
1) $x - \tan x + c$ 2) $\frac{1}{x} - \tan\frac{1}{x} + c$ 3) $\frac{1}{x} + \tan\frac{1}{x} + c$ 4) $x + \tan x + c$

71. The value of $\sqrt{2}\int\frac{\sin x}{\sin\left(x-\frac{\pi}{4}\right)}dx$ is *(Integration)*
1) $x - \log\left|\cos\left(x-\frac{\pi}{4}\right)\right| + c$ 2) $x + \log\left|\cos\left(x-\frac{\pi}{4}\right)\right| + c$
3) $x - \log\left|\sin\left(x-\frac{\pi}{4}\right)\right| + c$ 4) $x + \log\left|\sin\left(x-\frac{\pi}{4}\right)\right| + c$

72. $\int e^{-x}(\sec^2 x - \tan x).dx =$ *(Integration)*
1) $e^x \tan x + c$ 2) $e^{-x} \tan x + c$ 3) $e^x \cot x + c$ 4) $e^{-x} \cot x + c$

73. The value of $\lim_{x\to 0}\frac{\int_a^{x^2}\sec^2 t\,dt}{x\sin x}$ is *(Definite Integrals)*
1) 3 2) 2 3) 1 4) -1

74. The value of the integral $\int_3^6\frac{\sqrt{x}}{\sqrt{9-x}+\sqrt{x}}dx$ is *(Definite Integrals)*
1) 3/2 2) 2 3) 1 4) 1/2

75. $\int_0^1 \log\sin\left(\frac{\pi x}{2}\right)dx =$ *(Definite Integrals)*
1) $\log\frac{1}{2}$ 2) $\log\frac{1}{3}$ 3) $\log\frac{1}{4}$ 4) 0

76. The area of the plane region bounded by the curves $x + 2y^2 = 0$ and $x + 3y^2 = 1$ is equal to *(Areas)*
1) $\frac{4}{3}$ square units 2) $\frac{5}{3}$ square units 3) $\frac{1}{3}$ square units 4) $\frac{2}{3}$ square units

77. The area bounded between the parabolas $x^2 = \frac{y}{4}$ and $x^2 = 9y$ and the straight line y = 2 is *(Areas)*

1) $\frac{20\sqrt{2}}{3}$ 2) $10\sqrt{2}$ 3) $20\sqrt{2}$ 4) $\frac{10\sqrt{2}}{3}$

78. Form the differential equation by eliminating the arbitrary constant from the equation $y = e^x(a \cos 2x + b \sin 2x)$ *(Differential Equations)*

1) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 5y = 0$ 2) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = 0$ 3) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 5y = 0$ 4) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 5y = 0$

79. Solution of $\frac{dy}{dx} = \frac{x - y + 2}{x + y - 1}$ *(Differential Equations)*

1) $x^2 + y^2 + xy - 4y - 2x = c$ 2) $x^2 - y^2 - 2xy + 4x + 2y = c$
3) $x^2 - y^2 + xy + 2x - 4y = c$ 4) $x^2 + y^2 - xy + 4x - 2y = c$

80. Linear form of $\frac{dy}{dx} + x \sin 2y = x^3 \cos^2 y$ *(Differential Equations)*

1) $\frac{du}{dx} + \frac{u}{x^2} = x$ 2) $\frac{du}{dx} + ux = \frac{du}{dx}$ 3) $\frac{du}{dx} + 2ux = x^3$ 4) $\frac{du}{dx} + \frac{u}{x} = x^2$