MODEL PAPER - 4

MATHEMATICS

1. Locus of complex numbers satisfying $z^2 + \bar{z}^2 = 2$ constitute a *(Complex Numbers)*
   1) Hyperbola 2) Parabola 3) Ellipse 4) Circle

2. If n is a positive integer and $(1 + i)^n + (1 - i)^n = k \cos\frac{n\pi}{4}$, then k = *(De-Moiver's Theorem)*
   1) $2^{\frac{n}{2}}$ 2) $2^{\frac{n}{2}+1}$ 3) $2^{\frac{n}{2}-1}$ 4) 2^{n+1}

3. If f(x) = $\frac{x}{x-1}$, then $(fofo\ldots\ldots\ldots of_{19\ times})(x)$ is equal to *(Functions)*
   1) $\frac{x}{x-1}$ 2) $\left(\frac{x}{x-1}\right)^{19}$ 3) $\frac{19x}{x-1}$ 4) x

4. The length of latusrectum of the parabola $x^2 - 4x - 8y + 12 = 0$ is *(Parabola)*
   1) 4 2) 6 3) 8 4) 10

5. The locus of the point intersection of the lines $x\cos\alpha - y\sin\alpha = a$ and $x\sin\alpha + y\cos\alpha = b$ is a *(Locus)*
   1) ellipse 2) pair of lines 3) hyperbola 4) circle

6. The real number which most exceeds its cube, is
   1) $\frac{1}{2}$ 2) $\frac{1}{\sqrt{3}}$ 3) $\frac{1}{\sqrt{2}}$ 4) $\frac{-1}{\sqrt{3}}$

7. $\int_0^{\frac{\pi}{2}} \frac{\sin^{2/3} x}{\sin^{2/3} x + \cos^{2/3} x} dx =$ *(Integration)*
   1) $\frac{3\pi}{4}$ 2) π 3) $\frac{\pi}{2}$ 4) $\frac{\pi}{4}$

8. The ratio in which $\bar{i} + 2\bar{j} + 3\bar{k}$ divides the join of $-2\bar{i} + 3\bar{j} + 5\bar{k}$ and $7\bar{i} - \bar{k}$ is *(Vectors)*
   1) - 3 : 2 2) 1 : 2 3) 2 : 3 4) - 4 : 3

9. If $f(x) = \begin{cases} \frac{x^2 - 9}{x - 3} & \text{if } x \neq 3 \\ 2x + k, & \text{otherwise} \end{cases}$ is continuous at x = 3, then k is equal to *(Continuity)*
   1) 3 2) 6 3) - 6 4) 0

10. $\frac{\sin^2 A - \sin^2 B}{\sin A\cos A - \sin B\cos B} =$ *(Trignomentry)*
    1) tan (A + B) 2) tan (A - B) 3) cot (A + B) 4) cos (A - B)

11. $\frac{\cos^2 33^0 - \cos^2 57^0}{\sin 21^0 - \cos 21^0} =$ *(Trignomentry)*
    1) $\frac{1}{\sqrt{2}}$ 2) $-\frac{1}{\sqrt{2}}$ 3) $\frac{1}{2}$ 4) $-\frac{1}{2}$

12. Maximum value of $5\cos x + 3\cos(x - 60^0) + 7$ is *(Trignomentry)*
    1) 7 2) 7 + $\sqrt{34}$ 3) 14 4) 15

13. In a ΔABC, a tan A + b tan B = (a + b) tan $\left(\frac{A+B}{2}\right)$, then *(Trignomentry)*
    1) A = B 2) A = -B 3) A = 2B 4) B = 2A

14. The general solution of x satisfying the equation $\sqrt{3}\sin x + \cos x = \sqrt{3}$ is given by *(Trignomentry Equations)*
    1) $x = n\pi \pm \frac{\pi}{3}$ 2) $x = n\pi \pm \frac{\pi}{6}$ 3) $x = 2n\pi + \frac{\pi}{2}, 2n\pi + \frac{\pi}{6}$ 4) $x = 2n\pi \pm \frac{\pi}{2}$

15. The value of $\sin\left[\frac{1}{2}\text{Cot}^{-1}\left(\frac{-3}{4}\right)\right] =$ *(Inverse Trignomentry)*
    1) $\frac{1}{\sqrt{5}}$ 2) $\frac{2}{\sqrt{5}}$ 3) $\frac{-1}{\sqrt{5}}$ 4) $\frac{-2}{\sqrt{5}}$

16. If tanh x = $\frac{2}{3}$, then tanh 3x = *(Hyperbolic)*

1) $\frac{8}{9}$ 2) $\frac{8}{27}$ 3) $\frac{62}{63}$ 4) 2

17. In ΔABC, angles A, B, C are in arithmetic progression, then the value of $\frac{a+c}{\sqrt{a^2-ac+c^2}}$ will be *(Properties of Triangles)*

1) $2\sin\left(\frac{A+C}{2}\right)$ 2) $2\sin\left(\frac{A-C}{2}\right)$ 3) $2\cos\left(\frac{A+C}{2}\right)$ 4) $2\cos\left(\frac{A-C}{2}\right)$

18. In ΔABC, $a^2\cot A + b^2\cot B + c^2\cot C$ = *(Properties of Triangles)*

1) $\frac{abc}{R}$ 2) $\frac{abc}{r}$ 3) $\frac{abc}{s}$ 4) $\frac{abc}{\Delta}$

19. If ABCDEF is a regular hexagon with $\overline{AB}=\overline{a}, \overline{BC}=\overline{b}$ then match the following *(Vectors)*

List - I	**List - II**
A) $\overline{BE}$	1) $4\overline{a}$
B) $\overline{FA}$	2) $3\overline{a}$
C) $\overline{AC}+\overline{AD}+\overline{EA}+\overline{FA}$	3) $\overline{a}-\overline{b}$
D) $\overline{AB}+\overline{AC}+\overline{AD}+\overline{EA}+\overline{FA}$	4) $2(\overline{b}-\overline{a})$

The correct match is

1) A - 1, B - 3, C - 2, D - 4 2) A - 4, B - 3, C - 2, D - 1
3) A - 3, B - 2, C - 1, D - 4 4) A - 2, B - 3, C - 4, D - 1

20. A = (1, 1, 1), B = (1, 2, 3), C = (2, -1, 1) then the length of the internal bisector of $\underline{|A}$ is *(3D)*

1) $\frac{\sqrt{3}}{2}$ 2) $\frac{\sqrt{2}}{3}$ 3) $\sqrt{\frac{3}{2}}$ 4) $\frac{2}{\sqrt{3}}$

21. If A, B, C, D are four points in space then $|\overline{AB}\times\overline{CD}+\overline{BC}\times\overline{AD}+\overline{CA}\times\overline{BD}|$ = k (area of ΔABC) then k =

1) 2 2) 3 3) 4 4) 1 *(Vectors)*

22. Observe the following statements : *(Vectors)*

I : Three vectors are coplanar, if one of them is a linear combination of the other two.
II : Any four coplanar vectors are linearly dependent.
Then which of the following is true?

1) Both I and II are true and II is a correct explanation of I
2) Both I and II are true but II is not a correct explanation of I
3) I is true, but II is false 4) I is false and II is true

23. The cartesian equation of the plane passing through the point (-2, 1, 3) and perpendicular to the vector $3\overline{i}+\overline{j}+5\overline{k}$ is *(Plane)*

1) 2x - y - 3z - 10 = 0 2) 2x - y - 3z + 10 = 0 3) 3x + y + 5z - 10 = 0 4) 3x + y + 5z + 10 = 0

24. If A = $\begin{bmatrix}\alpha & 0\\ 1 & 1\end{bmatrix}$ and B = $\begin{bmatrix}1 & 0\\ 5 & 1\end{bmatrix}$, then the value of α for which $A^2 = B$, is *(Matrices)*

1) 1 2) -1 3) 4 4) no real value

25. $\begin{vmatrix}x^n & x^{n+2} & x^{n+3}\\ y^n & y^{n+2} & y^{n+3}\\ z^n & z^{n+2} & z^{n+3}\end{vmatrix}=(x-y)(y-z)(z-x)\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$, then n equals to *(Matrices)*

1) 1 2) -1 3) 2 4) -2

26. If adjoint of the matrix $\begin{bmatrix}1 & 2 & 1\\ 3 & 2 & 3\\ 1 & 1 & 2\end{bmatrix}$ is $\begin{bmatrix}\alpha & \beta & \gamma\\ -3 & 1 & 0\\ 1 & 1 & -4\end{bmatrix}$, then the ascending order of α, β, γ is *(Matrices)*

1) α, β, γ 2) γ, β, α 3) β, γ, α 4) β, α, γ

27. For the equations x + 2y + 3z = 1, 2x + y + 3z = 2, 5x + 5y + 9z = 4 *(Matrices)*

1) no solution 2) unique solution 3) infinitely many solution 4) only two solutions

28. Let g(x) = 1 + x - [x] and f(x) = $\begin{cases} -1, & x<0 \\ 0, & x=0 \\ 1, & x>0 \end{cases}$ then for all x $\in$ R, f{g(x)} is equal to *(Functions)*

1) x 2) 1 3) f(x) 4) g(x)

29. If A = {1, 2, 3, 4, 5}, B = {a, b, c, d}, then the number of onto functions that can be defined from A to B is

1) 505 2) 36 3) 240 4) 245 *(Functions)*

30. $2.4^{2n+1} + 3^{3n+1}$ is divisible by *(Mathematical Inductions)*

1) 7 2) 5 3) 11 4) 13

31. One value of $\left[\dfrac{1+\sin\frac{\pi}{8}+i\cos\frac{\pi}{8}}{1+\sin\frac{\pi}{8}-i\cos\frac{\pi}{8}}\right]^{\frac{8}{3}}$ is *(De-Moivers Theorem)*

1) 1 2) i 3) - i 4) - 1

32. If x is integer satisfying $x^2 - 6x + 5 \leq 0$ and $x^2 - 2x > 0$ then the number of possible values of x is

1) 3 2) 4 3) 2 4) infinity *(Quadratic Expressions)*

33. If $20^{3-2x^2} = \left(40\sqrt{5}\right)^{3x^2-2}$, then x = *(Quadratic Expressions)*

1) ± 1 2) $\pm\sqrt{\frac{4}{5}}$ 3) $\pm\sqrt{\frac{12}{13}}$ 4) $\pm\sqrt{\frac{13}{12}}$

34. If the roots of the equation $x^3 - 3px^2 - 3qx - r = 0$ are in H.P., then the mean root is *(Theory fo Equations)*

1) $\frac{3r}{p}$ 2) $\frac{2r}{q}$ 3) $\frac{r}{q}$ 4) $\frac{-r}{q}$

35. The value of 'a' for which the equations $x^3 + ax + 1 = 0$ and $x^4 + ax^2 + 1 = 0$ have a common root is

1) 1 2) - 1 3) 2 4) - 2 *(Theory fo Equations)*

36. **Statement I** : The number of terms in the expansion of $(x + y + z)^n$ are ${}^{(n+2)}C_2$. *(Binomial Theorem)*

Statement II : The number of terms in the expansion of $(x + y + z)^{10}$ are 45.

1) only I is true 2) only II is true 3) both I and II are true 4) neither I nor II is true

37. $1-\frac{1}{5}+\frac{1.4}{5.10}-\frac{1.4.7}{5.10.15}+\ldots\ldots\ldots\ldots\infty =$ *(Binomial Theorem)*

1) $\sqrt{\frac{1}{8}}$ 2) $\sqrt{\frac{3}{8}}$ 3) $\sqrt[3]{\frac{5}{8}}$ 4) $\sqrt{\frac{5}{8}}$

38. The number of ways of arranging the letters of the word 'SUCCESSFUL' so that all S's will come together is

1) $\frac{8!}{2!2!}$ 2) $\frac{10!}{2!2!}$ 3) $\frac{9!}{3!}$ 4) 10! *(Permutation & Combination)*

39. The number of combinations of 3 letters from the word "ELLIPSE" is *(Permutation & Combination)*

1) 10 2) 8 3) 18 4) 14

40. The number of palindromes with 6 digits that can be formed using the digits 0, 2, 4, 6, 8 is *(Permutation & Combination)*

1) 4×5^2 2) 5^3 3) 4^5 4) 4×5^4

41. Coefficient of x^4 in the expansion of $\frac{2x-1}{(x-2)(x+1)}$ is *(Partial Fractions)*

1) $\frac{17}{32}$ 2) $\frac{29}{32}$ 3) $\frac{31}{32}$ 4) $\frac{33}{32}$

42. The variance of 20 observations is 5. If each observation is multiplied by 2, the variance of the resulting observations is *(Measure of Dispersions)*

1) 5 2) 10 3) $5\sqrt{2}$ 4) 20

43. The probabilities of solving a problem by 3 students A,B,C are independently are $\frac{1}{3},\frac{1}{4},\frac{1}{5}$. The probability that the problem will be solved is *(Probability)*

1) $\frac{1}{5}$ 2) $\frac{4}{5}$ 3) $\frac{3}{5}$ 4) $\frac{2}{5}$

44. A box contains 24 identical balls of which 12 are white and 12 are black. The balls are drawn at random from the box one at a time with replacement. The probability that a white ball is drawn for the 4^{th} time on the 7^{th} draw is *(Probability)*

1) $\frac{5}{64}$ 2) $\frac{27}{32}$ 3) $\frac{5}{32}$ 4) $\frac{1}{2}$

45. One hundred identical coins, each with probability p, of showing up heads are tossed once. If $0 < p < 1$ and the probability of heads showing on 50 coins is equal to that of heads showing on 51 coins, then the value of p is

1) $\frac{1}{2}$ 2) $\frac{49}{101}$ 3) $\frac{50}{101}$ 4) $\frac{51}{101}$ *(Probability)*

46. In a binomial distribution, mean = $\frac{10}{3}$ and sum of mean and variance is $\frac{40}{9}$ then parameter p = *(Binomial Theorem)*

1) $\frac{1}{3}$ 2) $\frac{2}{3}$ 3) $\frac{1}{4}$ 4) $\frac{1}{2}$

47. The range of random variable X = {1, 2, 3, ...} and the probability are given by $P(X = k) = \frac{3^{ck}}{k!}$ and c is constant, then c = *(Random Variables)*

1) $\frac{1}{2}\log(\log 2)$ 2) $\log_3(\log_e 2)$ 3) $\frac{\log(\log 2)}{\log_3 e}$ 4) $\log_2(\log 3)$

48. If f(5) = 7 and $f'(5) = 7$, then $\underset{x\to 5}{\text{Lt}}\frac{xf(5)-5f(x)}{x-5}$ is given by *(Limits)*

1) -28 2) 28 3) 35 4) -35

49. $\underset{x\to 0}{\text{Lt}}\left[\text{cosec}^3 x.\cot x - 2\cot^3 x.\text{cosec}\, x + \frac{\cot^4 x}{\sec x}\right]$ is equal to *(Limits)*

1) 1 2) -1 3) 0 4) 2

50. Let $3f(x) - 2f\left(\frac{1}{x}\right) = x$, then $f'(2)$ is equal to *(Differentiation)*

1) $\frac{2}{7}$ 2) $\frac{1}{2}$ 3) 2 4) $\frac{7}{2}$

51. Let f be a twice differentiable function such that $f''(x) = -f(x)$ and $f'(x) = g(x)$. $h'(x) = [f(x)]^2 + [g(x)]^2$, h(1) = 8, h(0) = 2, then h(2) = *(Differentiation)*

1) 1 2) 2 3) 3 4) 14

52. If the relative error in the radius of a sphere is 0.1, then the relative error in its surface area is *(Errors)*

1) 0.3 2) 0.4 3) 0.2 4) 0.1

53. The length of a pair of parallel sides of a rectangle is increasing at a rate of 1 cm/sec, keeping the area constant to 16 cm². If the length of the parallel sides is 2 cm, the rate of change in other pair of sides is *(Rate of Change)*

1) 4 cm/sec 2) - 4 cm/sec 3) - 2 cm/sec 4) 2 cm/sec

54. If the tangent at P on the curve $x^m y^n = a^{m+n}$ meets the axes at A, B then P divides AB in the ratio *(Tangents & Normals)*

1) m : n 2) n : m 3) 1 : 1 4) m + n : n

55. **Assertion(A)** : For the function $f(x) = x^2 + 3x + 2$, Lagrange's mean value theorem is applicable in [1, 2] and the value of c is $\frac{3}{2}$.

Reason(R) : If Lagrange's mean value theorem is applicable for any quadratic polynomial on [a, b] then value of c is $\frac{a+b}{2}$ *(Rolle's Theorem)*

1) Both A and R are true and R is a correct explanation of A.
2) Both A and R are true but R is not a correct explanation of A.
3) A is true, but R is false
4) A is false and R is true

56. The function xe^x is decreasing in *(Maxima & Minima)*

1) $(-\infty, 0)$ 2) $(0, \infty)$ 3) $(-\infty, -1)$ 4) $\left(\frac{1}{e}, \infty\right)$

57. $\int \frac{(x-x^5)^{1/5}}{x^6} dx$ is equal to *(Integration)*

1) $\frac{5}{24}\left(\frac{1}{x^4}-1\right)^{6/5}+c$ 2) $\frac{5}{24}\left(1+\frac{1}{x^4}\right)^{6/5}+c$ 3) $-\frac{5}{24}\left(\frac{1}{x^4}-1\right)^{6/5}+c$ 4) $\frac{-5}{24}\left(\frac{1}{x^4}+1\right)^{6/5}+c$

58. $\int \frac{\log x}{(1+\log x)^2} dx =$ *(Integration)*

1) $\frac{1}{(1+\log x)^2}+c$ 2) log (1 + logx) + c 3) $\frac{\log x}{1+\log x}+c$ 4) $\frac{x}{1+\log x}+c$

59. $\int_0^{\frac{\pi}{2}} \frac{2\tan x+3\cot x}{\tan x+\cot x} dx =$ *(Definite Integrals)*

1) $\frac{\pi}{2}$ 2) $\frac{\pi}{4}$ 3) $\frac{5\pi}{4}$ 4) $\frac{5\pi}{2}$

60. $\lim_{n\to\infty}\sum_{r=1}^{n}\frac{r}{n^2+r^2}=$ *(Definite Integrals)*

1) $\frac{\pi}{4}$ 2) 0 3) log 2 4) log $\sqrt{2}$

61. Statement I: The general solution of $\frac{ydx-xdy}{y^2}=0$ represents a family of straight lines.

Statement II: The general solution of xdy + ydx = 0 represents a hyperbola.
Which of the above is correct ? *(Differenttial Equations)*
1) Only I 2) Only II 3) Both I & II 4) Neither I nor II

62. The solution of $x^3 - y^3 \frac{dy}{dx} = 2x$ is *(Differenttial Equations)*

1) $x^4 + y^4 = 4x^2 + c$ 2) $y^4 = x^4 - 4x^2 + c$ 3) $y^4 = x^4 + 4x^2 + c$ 4) $x^4 + y^4 + 4x^2 = c$

63. $y = Ae^x + Be^{2x} + Ce^{3x}$ satisfies the differential equation *(Differenttial Equations)*
1) $y_3 - 6y_2 + 11y_1 - 6y = 0$ 2) $y_3 + 6y_2 + 11y_1 + 6y = 0$
3) $y_3 + 6y_2 - 11y_1 + 6y = 0$ 4) $y_3 - 6y_2 - 11y_1 + 6y = 0$

64. The circumcentre of the triangle with vertices (0, 30), (4, 0), (30, 0) is *(Straight Lines)*
1) (10, 10) 2) (12, 12) 3) (15, 15) 4) (17, 17)

65. The incentre of the traingle formed by the lines $x + y\sqrt{3} = 0$, $x - y\sqrt{3} = 0$ and x = 3 is *(Straight Lines)*

1) $(0, \sqrt{3})$ 2) (3, 0) 3) (0, 2) 4) (2, 0)

66. The equation of the bisector of the acute angle between the lines 2x - y + 4 = 0 and x - 2y = 1 is *(Straight Lines)*

1) x - y + 1 = 0 2) x + y + 5 = 0 3) x - y = 5 4) x + y - 1 = 0

67. The lines x - y - 2 = 0, x + y - 4 = 0 and x + 3y = 6 are concurrent at *(Straight Lines)*
1) (1, 2) 2) (3, 1) 3) (2, 2) 4) (1, 1)

68. The quadrilateral formed by the pairs of lines xy + x + y + 1 = 0, xy + 3x + 3y + 9 = 0 is *(Pair of Straight Lines)*
1) parallelogram 2) rhombus 3) rectangle 4) square

69. If the pair of lines $3x^2 - 5xy + py^2 = 0$ and $6x^2 - xy - 5y^2 = 0$ have one line in common, then p = *(Pair of Straight Lines)*

1) $2, \frac{25}{4}$ 2) $-2, \frac{25}{4}$ 3) $-2, \frac{-25}{4}$ 4) $2, \frac{-25}{4}$

70. The equation to the locus of points which are equidistant from the points (1, -3, 4), (1, 3, 4) is *(3D)*
1) xy = 0 2) y = 0 3) z = 0 4) x = 0

71. If (2, 1, -1) and (1, -1, -1) are the direction ratio's of two lines then the direction cosines of the line perpendicular to the lines are *(3D)*

1) $\frac{1}{3}, -\frac{1}{3}, \frac{2}{3}$ 2) $\frac{2}{3}, \frac{-1}{3}, \frac{2}{3}$ 3) $\frac{-2}{3}, \frac{-1}{3}, \frac{2}{3}$ 4) $\frac{2}{\sqrt{14}}, \frac{-1}{\sqrt{14}}, \frac{3}{\sqrt{14}}$

72. The equation of the plane through the line of intersection of the planes x + y + z - 1=0, 2x + 3y + 4z - 5 = 0 and perpendicular to the plane is x - y + z = 0 is *(Planes)*
1) 7x - y - 6z - 17 = 0 2) x - z + 2 = 0 3) 7x + y + 6z - 27 = 0 4) x - z + 1 = 0

73. If the polar the point (2, 2) to the circle $x^2 + y^2 = 16$ meets the coordinate axes in A and B, then the circum centre of ΔOAB is ***(Circles)***

1) $\left(\frac{8}{3}, \frac{8}{3}\right)$ 2) $\left(\frac{16}{3}, \frac{8}{3}\right)$ 3) (8, 8) 4) (4, 4)

74. The points (4, -2), (3, b) are conjugate w.r.t the circle $x^2 + y^2 = 24$ if b = ***(Circles)***

1) -6 2) 6 3) 4 4) 12

75. The locus of the centre of the circle which cuts the circles $x^2 + y^2 + 4x - 6y + 9 = 0$ and $x^2 + y^2 - 5x + 4y + 2 = 0$ orthogonally is ***(Circles)***

1) $9x + 10y + 7 = 0$ 2) $9x + 10y - 7 = 0$ 3) $9x - 10y - 7 = 0$ 4) $9x - 10y + 7 = 0$

76. The distance of the point (1, 2) to the common chord of the circles $x^2 + y^2 - 2x + 3y - 5 = 0$, $x^2 + y^2 + 10x + 8y - 1 = 0$ is

1) 2 2) 1 3) $\sqrt{2}$ 4) $\sqrt{3}$ ***(Circles)***

77. If $2y = 5x + k$ is a tangent to the parabola $y^2 = 6x$, then k = ***(Parabola)***

1) $\frac{2}{3}$ 2) $\frac{4}{5}$ 3) $\frac{3}{5}$ 4) $\frac{6}{5}$

78. Observe the following lists. For the parabola $(y - 1)^2 = 4(x - 2)$ ***(Parabola)***

List - I	**List - II**
A) Equation of axis is	1) $x - 2 = 0$
B) Equation of latusrectum is	2) $y - 1 = 0$
C) Equation of directrix is	3) $x - 3 = 0$
D) Equation of tangent at vertex	4) $x - 1 = 0$

The correct match for list - I from list - II is

1) A - 2, B - 3, C - 4, D - 1 2) A - 2, B - 1, C - 4, D - 3
3) A - 3, B - 2, C - 1, D - 4 4) A - 4, B - 1, C - 3, D - 2

79. Tangent drawn at any point on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ cuts the major axis at P and minor axis at D. C is centre then $\frac{a^2}{CP^2} + \frac{b^2}{CD^2} =$ ***(Ellipse)***

1) 1 2) 2 3) 4 4) $\frac{1}{2}$

80. Product of perpendiculars drawn from a point on hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$ to its asymptotes is ***(Hyperbola)***

1) $\frac{144}{9}$ 2) $\frac{144}{25}$ 3) $\frac{25}{9}$ 4) $\frac{144}{45}$