MODEL PAPER - 8

MATHEMATICS

1. If 2sin x + 5cos y + 7sin z = 14, then $7\tan\frac{x}{2} + 4\cos y - 6\cos z =$ *(Trigonometric)*
   1) 4   2) -3   3) 11   4) 5
2. $\tan 70^0 - \tan 20^0 - 2\tan 40^0 = k \tan\theta$, $(k,\theta) =$ *(Trigonometric)*
   1) $(2,10^0)$ 2) $(4,10^0)$ 3) $(2,20^0)$ 4) $(4,20^0)$
3. $\frac{d}{dx}\left\{\sin^2 \cot^{-1}\sqrt{\frac{1+x}{1-x}}\right\} =$ *(Differentiation)*
   1) 0   2) 1/2   3) -1/2   4) -1
4. If $\theta_1, \theta_2, \theta_3 ....\theta_n$ are in A.P, then $\frac{\sin\theta_1 + \sin\theta_2 + ...+\sin\theta_n}{\cos\theta_1 + \cos\theta_2 + ...+\cos\theta_n} =$ *(Differentiation)*
   1) 0   2) $\tan(\theta_1 + \theta_n)$ 3) $\tan\left(\frac{\theta_1+\theta_n}{2}\right)$ 4) $\cot\left(\frac{\theta_1+\theta_n}{2}\right)$
5. A monument ABCD stands on a level ground, A being on the ground. At a point P on the ground the segments AB, AC and AD subtends angles α, β and γ respectively. If AB = a, AC = b, AD = c, AP = x and $\alpha + \beta + \gamma = 180^0$, then $x^2 =$ *(Properties of Triangle)*
   1) $\frac{b-a}{b+a}$ 2) $\frac{abc}{a+b+c}$ 3) $\frac{a+b+c}{abc}$ 4) $\frac{2abc}{a+b+c}$
6. Number of different products that can be formed with 8 different prime numbers is
   1) 256   2) 248   3) 247   4) 255
7. In a triangle ΔABC, $6r = R$, $r_1 = 7r$ then $\angle A =$ *(Properties of Triangle)*
   1) $\frac{\pi}{4}$ 2) $\frac{\pi}{6}$ 3) $\frac{\pi}{2}$ 4) $\frac{\pi}{3}$
8. If in a ΔABC, cos A cos B + sin A sin B sin C = 1, then a : b : c = *(Properties of Triangle)*
   1) $1 : 1 : \sqrt{2}$ 2) $1 : \sqrt{2} : \sqrt{3}$ 3) 1 : 2 : 3   4) $1 : 1 : \sqrt{3}$
9. If in a ΔABC, a = 6, b = 3, $\cos(A - B) = \frac{4}{5}$, then the area of ΔABC = *(Properties of Triangle)*
   1) 6   2) 8   3) 10   4) 9
10. If $a^2 + b^2 + c^2 = 1$, then the range of ab + bc + ca is *(Functions)*
    1) $[1,0)$ 2) $\left[\frac{-1}{2},0\right)$ 3) $\left[\frac{-1}{2},1\right]$ 4) $[-1,1]$
11. Let α,β are the roots of the quadratic equation $x^2 - (a - 2)x - (a + 1) = 0$ where a is variable. Then the least value of $\alpha^2 + \beta^2 =$ *(Quadratic Expression)*
    1) 3   2) 5   3) 74) 10
12. If α is a root of $x^7 = 1$ and $\alpha \neq 1$, then the value of $\alpha^{101} + \alpha^{102} + \alpha^{103} + ... \alpha^{205} =$ *(De-Moiver's Theorem)*
    1) 1   2) 0   3) -104   4) 104
13. $\bar{b}$ and $\bar{c}$ are two unit vectors along positive x and y axes and $\bar{a}$ is any vector, then *(Vectors)*
    $(\bar{a}.\bar{b})\bar{b} + (\bar{a}.\bar{c})\bar{c} + \frac{\{\bar{a}.(\bar{b}x\bar{c})\}(\bar{b}x\bar{c})}{|\bar{b}x\bar{c}|}$
    1) $\bar{a}$ 2) $\bar{b}$ 3) $\bar{c}$ 4) $\bar{a}+\bar{b}+\bar{c}$
14. If |A| = 4, |adj A| = 64, then the number of elements in the matrix A is *(Matrices)*
    1) 4   2) 9   3) 12   4) 16
15. The least number of negative roots of the equation $x^5 - 3x^4 + 4x^3 - 5x^2 + 6x - 7 = 0$ is *(Theory of Equations)*
    1) 0   2) 1   3) 2   4) 3
16. If α, β, γ are the roots of the equation $ax^3 + bx^2 + cx + d = 0$, then the value of $\sum\alpha^2\beta^2 =$ *(Theory of Equations)*
    1) $\frac{c^2+2bd}{a^2}$ 2) $\frac{2bd-c^2}{a^2}$ 3) $\frac{c^2-2bd}{a^2}$ 4) $\frac{c^2+bd}{a^2}$
17. If $g(x) = 1 + \sqrt{x}$, $f[g(x)] = 3 + 2\sqrt{x} + x$, then f(x) = *(Functions)*
    1) $1 + 2x^2$ 2) $2 + x^2$ 3) 1 + x   4) $1 + x^2$

18. One of the roots of x if $\begin{vmatrix} 3-x & -6 & 3 \\ -6 & 3-x & 3 \\ 3 & 3 & -6-x \end{vmatrix} = 0$ is *(Matrices)*

1) 6 2) 3 3) -3 4) 0

19. If $\bar{a} = 2\bar{i} + 3\bar{j} + 4\bar{k}$, $\bar{b} = \bar{i} + \bar{j} + 5\bar{k}$ and $\bar{a}, \bar{b}, \bar{c}$ form a left handed system then $\bar{c}$ = *(Vectors)*

1) $11\bar{i} - 6\bar{j} - \bar{k}$ 2) $-11\bar{i} + 6\bar{j} + \bar{k}$ 3) $11\bar{i} - 6\bar{j} + \bar{k}$ 4) $11\bar{i} + 6\bar{j} + \bar{k}$

20. $\bar{a}.\{(\bar{b}+\bar{c})x(\bar{a}+\bar{b}+\bar{c})\}$ *(Vectors)*

1) 0 2) $[\bar{a}\ \bar{b}\ \bar{c}]$ 3) $-[\bar{a}\ \bar{b}\ \bar{c}]$ 4) $2[\bar{a}\ \bar{b}\ \bar{c}]$

21. sin [cot^{-1} {tan (cos^{-1}x)}] = *(Inverse Trignomentry)*

1) $\frac{1}{x}$ 2) $\frac{-1}{x^2}$ 3) x 4) $\sqrt{1-x^2}$

22. If $\bar{a}, \bar{b}, \bar{c}$ are non coplanar unit vectors such that $\bar{a} \times (\bar{b} \times \bar{c}) = \frac{\bar{b}+\bar{c}}{\sqrt{2}}$, then the angle between $\bar{a}, \bar{b}$ is

1) π 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{2}$ 4) $\frac{3\pi}{4}$ *(Vectors)*

23. **Assertion(A) :** Number of unit vectors perpendicular to the plane containing three non collinear points is two. *(Vectors)*

Reason(R): A unit vector perpendicular to both $\bar{a}$ and $\bar{b}$ is $\pm \frac{\bar{a} \times \bar{b}}{|\bar{a} \times \bar{b}|}$.

Which of the following is true?
1) Both A and R are true but R is the correct explanation of A
2) Both A and R are true but R is not the correct explanation of A
3) A is true R is false 4) A is false R is true

24. A man is known to speak truth 3 times out of 4 times. He throws a die and reports that it is six. Then the probability that it is actually six is *(Probability)*

1) $\frac{1}{3}$ 2) $\frac{3}{5}$ 3) $\frac{3}{8}$ 4) $\frac{1}{2}$

25. Consider the following two statements: *(Probability)*

I: The probability that randomly chosen day of a month is monday, is $\frac{1}{84}$.

II: If A is any event of a random experiment, then $0 \leq P(\bar{A}) \leq 1$.

Then the which of the above statement is true?
1) I only 2) II only 3) Both I and II 4) Neither I nor II

26. A coin is tossed once. If head comes up, then it is tossed again and if a tail comes up, a die is thrown. Then the number of points in the sample space of the experiment is *(Probability)*
1) 24 2) 12 3) 7 4) 8

27. The range of a random variable X is {0,1,2,3...........} and its probabilities are given by P(X = k) = $\frac{c(k+1)}{2^k}$, k = 0,1,2,......... then c = *(Random Variables)*

1) $\frac{1}{2}$ 2) $\frac{1}{3}$ 3) $\frac{1}{4}$ 4) $\frac{3}{4}$

28. A five digit number divisible by 3 is to be formed using 0,1,2,3,4,5 without repetition. The total number of 5 digit numbers we get is *(Permutation & Combinations)*
1) 72 2) 84 3) 216 4) 228

29. There are 6 '+' signs and 4 '-' signs. The number of ways of arranging them so that no two '-' signs are together is *(Permutation & Combinations)*
1) 35 2) 42 3) 48 4) 56

30. If $A = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$ and $|A^3| = 125$, then α = *(Matrices)*

1) 0 2) ± 2 3) ± 3 4) ± 5

31. The value of λ for which the system of equations 2x - y - z = 12, x - 2y + z = -4, x + y + λz = 4 has no solution is *(Matrices)*
1) 3 2) -3 3) 2 4) -2

32. If in the expansion of $(1 + x)^n$, 9^{th}, 10^{th}, 11^{th} terms coefficients are in A.P, then n = *(Binomial Theorem)*
1) 11 2) 12 3) 14 4) 16

33. If $\frac{3x}{(x-6)(x+\alpha)} = \frac{2}{x-6} + \frac{1}{x+\alpha}$ then α = *(Partial Fraction)*
1) 6 2) 3 3) 2 4) 1

34. $\frac{(1+\tan 22^0)(1+\tan 23^0)}{(1-\cot 22^0)(1-\cot 23^0)}$ = *(Trignometry)*
1) -1 2) 2 3) 1 4) 4

35. If $f(x) = \int_0^{\sin x} t^2 dt$, then period of $f'(x)$ is *(Trignometry)*
1) $\frac{\pi}{2}$ 2) $\frac{2\pi}{3}$ 3) 2π 4) π

36. If z = x + iy satisfies amp (z - 1) = amp (z + 3i) then the value of (x - 1) : y *(Complex Numbers)*
1) 1 : 2 2) 1 : 3 3) 2 : 3 4) 1: 4

37. The value of $\tan^2(\sec^{-1} 3) + \cot^2(\text{cosec}^{-1} 4)$ =
1) 20 2) 21 3) 22 4) 23

38. A man from the top of a 100 m high tower sees a car moving towards the tower at an angle of 30^0. After some time the angle of depression becomes 60^0. The distance in metres travelled by the car during this time is
1) $\frac{100\sqrt{3}}{3}$ 2) $\frac{200\sqrt{3}}{3}$ 3) $100\sqrt{3}$ 4) $200\sqrt{3}$ *(Properties of Triangle)*

39. If $a_k = \frac{1}{k(k+1)}$ for k = 1,2,.........., n then $\left(\sum_{k=1}^{n} a_k\right)^2$ = *(Mathematical Induction)*
1) $\frac{n}{n+1}$ 2) $\frac{n^2}{n+1}$ 3) $\frac{n}{(n+1)^2}$ 4) $\frac{n^2}{(n+1)^2}$

40. If $\cosh\alpha = \sec x$, then $\tan^2\frac{x}{2}$ = *(Hyperbolic Functions)*
1) $\coth^2\frac{\alpha}{2}$ 2) $\text{sech}^2\frac{\alpha}{2}$ 3) $\tanh^2\frac{\alpha}{2}$ 4) $\text{cosech}^2\frac{\alpha}{2}$

41. $\underset{x\to 0}{Lt}\frac{\tan x - x}{x^2 \tan x}$ = *(Limits)*
1) 0 2) ½ 3) 1/3 4) 1

42. $\underset{x\to\infty}{Lt}\frac{6x^2 - \cos 3x}{x^2+5}$ = *(Limits)*
1) $\frac{6}{5}$ 2) 6 3) 1 4) $\frac{-3}{5}$

43. If $y = \frac{\sin^2 x}{1+\cot x} + \frac{\cos^2 x}{1+\tan x}$ then $\frac{dy}{dx}$ at $x = \frac{\pi}{4}$ is *(Differentiation)*
1) 0 2) -1 3) 1 4) $\sqrt{2}$

44. $\frac{d}{dx}\sin^{-1}\left(\frac{3x}{2} - \frac{x^3}{2}\right)$ = *(Differentiation)*
1) $\frac{3}{\sqrt{4-x^2}}$ 2) $\frac{6}{\sqrt{4-x^2}}$ 3) $\frac{3}{\sqrt{4+x^2}}$ 4) $\frac{1}{\sqrt{4-x^2}}$

45. $f(x) = \begin{cases} \left(\frac{a^x+b^x+c^x}{3}\right)^{1/x}, & x \neq 0 \\ k, & x = 0 \end{cases}$ is continuous x = 0 then k = *(Continunity)*
1) abc 2) 1 3) $(abc)^3$ 4) $(abc)^{1/3}$

46. If $f(x) = e^x$, $g(x) = \sin^{-1}x$ and and h(x) = f(g(x)) then $\frac{h'(x)}{h(x)}$ = *(Differentiation)*
1) $\frac{1}{\sin^{-1}x.\sqrt{1-x^2}}$ 2) $\frac{1}{\sqrt{1-x^2}}$ 3) $\frac{e^x}{\sin^{-1}e^x.\sqrt{1-e^{2x}}}$ 4) $\frac{e^x}{\sqrt{1-e^{2x}}}$

47. In a cube the percentage increase in the side is 1. The percentage increase in volume of cube is *(Error)*
1) 2 2) $\frac{1}{2}$ 3) $\frac{1}{3}$ 4) 3

48. The area of the triangle formed by the tangent $xy = c^2$ with co-ordinate axes is *(Tangents & Normals)*
1) c^2 2) 2c 3) $2c^2$ 4) c

49. Water is being poured into the inverted conical vessel at the rate 1.5 cubic meter/min. Its depth is always equal to twice its radius. The level of water is rising at the rate of $\frac{3}{8\pi}$ meter/min when its depth is
1) 1 mt 2) 2 mt 3) 3 mt 4) 4 mt *(Rate of Change)*

50. The function $f(x) = \tan^{-1}(\sin x + \cos x)$ is an increasing function in *(Maxima & Minima)*
1) $\left(\frac{-\pi}{2}, \frac{\pi}{4}\right)$ 2) $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ 3) $\left(0, \frac{\pi}{2}\right)$ 4) $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

51. The height of the cylinder of maximum volume which can be inscribed in a sphere of radius 'r' is
1) $\sqrt{2}\,r$ 2) $\frac{2r}{\sqrt{3}}$ 3) $\frac{4r}{\sqrt{3}}$ 4) $\sqrt{\frac{2}{3}}\,r$ *(Maxima & Minima)*

52. Lagrange's theorem can not applicable for *(Mean Value Theorem)*
1) $f(x) = x^2$ in [1,2] 2) $f(x) = x^3$ in [-1,1] 3) $f(x) = x$ in [-1,1] 4) $f(x) = \frac{1}{x}$ in [-1,1]

53. Standard deviation of 27, 35, 40, 35, 36, 36, 29 is *(Measure of Dispersion)*
1) 17.14 2) 4.14 3) 34 4) None

54. $\int \frac{1}{(1+e^x)(1+e^{-x})} dx =$ *(Integration)*
1) $\frac{1}{1+e^x} + c$ 2) $\frac{2}{1+e^x} + c$ 3) $\frac{-2}{1+e^x} + c$ 4) $\frac{-1}{1+e^x} + c$

55. $\int \sec^{-1} x \, dx =$ *(Integration)*
1) $x \sec^{-1} x + \cosh^{-1} x + c$ 2) $x \sec^{-1} x - \cosh^{-1} x + c$
3) $x \sec^{-1} x - \sinh^{-1} x + c$ 4) $x \sec^{-1} x + \sinh^{-1} x + c$

56. $\int_{-1}^{1} \frac{1}{(1+x^2)^2} dx =$ *(Definite Integrals)*
1) $\frac{\pi}{4} - \frac{1}{2}$ 2) $\frac{\pi}{4} + \frac{1}{2}$ 3) $\frac{\pi}{8}$ 4) $\frac{\pi}{8} + \frac{1}{2}$

57. $\int_0^{\pi} x \sin^5 x \cos^6 x \, dx$ *(Definite Integrals)*
1) $\frac{8\pi}{693}$ 2) $\frac{8\pi^2}{693}$ 3) $\frac{16\pi}{693}$ 4) $\frac{16\pi^2}{693}$

58. The area of the region bounded by the curves $y = |x-2|$, $x = 1$, $x=3$ and x-axis is *(Areas)*
1) 2 2) 4 3) 1/2 4) 1

59. If a, b, c are the orders of the differential equations $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$, $\left(\frac{dy}{dx}\right)^3 + \left(\frac{dy}{dx}\right)^2 + y^4 = 0$, $\frac{d^3y}{dx^3} + 2\left(\frac{dy}{dx}\right)^4 = \cos x$ then the ascending order of a, b, c is *(Definite Integrals)*
1) a, b, c 2) b, c, a 3) b, a, c 4) c, b, a

60. The solution of $xdy = (y + x\cos^2 \frac{y}{x})dx$ is *(Definite Integrals)*
1) $\tan\left(\frac{y}{x}\right) = \log cx$ 2) $\tan \frac{y}{x} = \log cy$ 3) $\sin\left(\frac{y}{x}\right) = \log cx$ 4) $\cot \frac{y}{x} = \log cx$

61. O(0, 0) A(6, 0) B(0, 4) are three points. If P is a point such that area of ΔPOB is twice the area of ΔPOA, then the locus of P is *(Locus)*
1) $x^2 - 3y^2 = 0$ 2) $x^2 - 9y^2 = 0$ 3) $y^2 - 9x^2 = 0$ 4) $y^2 - 3x^2 = 0$

62. The angle of ration of axes to remove xy term in the equation $7x^2 + 2\sqrt{3}xy + 9y^2 = 10$ is *(Transformations)*
1) $\frac{\pi}{12}$ 2) $\frac{\pi}{6}$ 3) $\frac{\pi}{3}$ 4) $\frac{5\pi}{12}$

63. If the straight line drawn through the point P($\sqrt{3}$, 2) making an angle $\frac{\pi}{6}$ with x-axis meets the line $\sqrt{3}x - 4y + 8 = 0$ at Q. Then PQ is *(Straight Lines)*
1) 4 2) 5 3) 6 4) 9

64. If the lines $ax + by + c = 0$, $bx + cy + a = 0$ and $cx + ay + b = 0$, $(a \neq b \neq c)$ are concurrent then the point of concurrence is *(Straight Lines)*
1) (0, 0) 2) (1, 1) 3) (2, 2) 4) (-1, -1)

65. Area of triangle formed by angle bisectors of co-ordinate axes and the line x = 6 in sq. units is
1) 36 2) 18 3) 72 4) 9 *(Straight Lines)*

66. If $2x + 3y = 7$ make equal angles with $ax^2 + 12xy + ky^2 = 0$ then k = *(Pair of Straight Lines)*
1) 3 2) 7 3) 14 4) 21

67. If the line $3x + 4y = 1$ cuts $25x^2 + 25y^2 = 4$ in P and Q and O is the origin then $\angle POQ$ = *(Pair of Straight Lines)*
1) $\frac{\pi}{6}$ 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{3}$ 4) $\frac{\pi}{2}$

68. If (p, q, r) is equidistant from (1, 2, -3) (2, -3, 1) and (-3, 1, 2) then p + q + r = *(3D)*
1) -1 2) 0 3) 1 4) 6

69. If a line makes angles 60^0, 45^0, 45^0 and θ with four diagonals of a cube then $\sin^2\theta$ = *(Direction Cosines)*
1) $\frac{1}{12}$ 2) $\frac{11}{12}$ 3) $\sqrt{\frac{11}{12}}$ 4) $\frac{31}{12}$

70. If the plane $2x - 3y + 5z - 2 = 0$ divides the line segment joining (1, 2, 3) and (2, 1, k) in the ratio 9 : 11 then k = *(Plane)*
1) -2 2) 1 3) -10 4) -1/2

71. O is the origin and OA, OB are a pair of tangents to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$, c > 0, then the equation to the circumcircle of the ΔOPQ is *(Circles)*
1) $x^2+y^2-gx-fy = 0$ 2) $x^2+y^2+\frac{g}{2}x+\frac{f}{2}y = 0$ 3) $x^2+y^2-\frac{g}{2}x-\frac{f}{2}y = 0$ 4) $x^2+y^2+gx+fy = 0$

72. If the pole of a straight line with respect to the circle $x^2+y^2 = a^2$ lies on the circle $x^2+y^2=9a^2$. If the straight line touches the circle $x^2+y^2 = r^2$, then *(Circles)*
1) $9a^2 = r^2$ 2) $9r^2 = a^2$ 3) $3r^2 = a^2$ 4) $r^2 = a^2$

73. The condition that the circles $x^2+y^2+2g_1x + 2f_1y = 0$, $x^2+y^2+2g_2x + 2f_2y = 0$ may touch each other is *(Circles)*
1) $g_1g_2 = f_1f_2$ 2) $g_1f_2 = g_2f_1$ 3) $g_1+g_2 = f_1+f_2$ 4) $g_1 - g_2 = f_1 - f_2$

74. The locus of the centre of the circle which cuts the circles $x^2+y^2+4x-6y+9 = 0$ and $x^2+y^2-4x+6y+4=0$ orthogonally is *(System of Circles)*
1) $4x - 6y + 5 = 0$ 2) $4x - 6y - 5 = 0$ 3) $8x - 12y + 5 = 0$ 4) $8x + 12y + 5 = 0$

75. If only one common tangent can be drawn to the circles $x^2 + y^2 - 2x - 4y - 20 = 0$ and $(x + 3)^2 + (y + 1)^2 = p^2$, then p = *(System of Circles)*
1) 20 2) 16 3) 9 4) 10

76. The locus of the point of intersection of two tangents to the parabola $y^2 = 4ax$ which make an angle 30^0 with one another is *(Parabola)*
1) $(x+a)^2 = 3(y^2-4ax)$ 2) $(x+a)^2 = y^2 - 4ax$ 3) $y^2 - 4ax = 3(x+a)^2$ 4) $y^2-4ax = 9(x+a)^2$

77. If the lines $2x + 3y + 12 = 0$, $x - y + k = 0$ are conjugate lines with respect to the parabola $y^2=8x$ then k = *(Parabola)*
1) - 12 2) 7/2 3) 12 4) -2

78. The equation of the ellipse with its axes as the coordinate axes and whose latusrectum is 10 and distance between the foci is the length of minor axis *(Ellipse)*
1) $x^2 + 2y^2 = 16$ 2) $x^2 + 2y^2 = 32$ 3) $x^2 + 2y^2 = 64$ 4) $x^2 + 2y^2 = 100$

79. The locus of point of intersection of perpendicular tangents to $\frac{x^2}{25}-\frac{y^2}{9}=1$ is *(Hyperbola)*
1) $x^2 + y^2 = 16$ 2) $x^2 + y^2 = 25$ 3) $x^2 + y^2 = 34$ 4) $x^2 + y^2 = 9$

80. The area (in square units) bounded by the curves $x = - 2y^2$ and $x = 1 - 3y^2$ is *(Areas)*
1) $\frac{2}{3}$ 2) 1 3) $\frac{4}{3}$ 4) $\frac{5}{3}$