1. $\boldsymbol{Resolve\ \frac{3x+7}{x^2-3x+2}\ into\ partial\ fractions.}$

$\boldsymbol{Sol:}$

$$\frac{3x+7}{x^2-3x+2} = \frac{3x+7}{(x-1)(x-2)}$$

$$\boldsymbol{let}\ \frac{3x+7}{(x-1)(x-2)} = \frac{A}{x-1} + \frac{B}{x-2}$$

$$\Rightarrow \frac{3x+7}{(x-1)(x-2)} = \frac{A(x-2)+B(x-1)}{(x-1)(x-2)}$$

$$3x + 7 = A(x-2) + B(x-1) \ldots.(1)$$

$\boldsymbol{put\ x = 1}$	$\boldsymbol{put\ x = 2}$
$\Rightarrow 3 + 7 = A(1-2)$	$\Rightarrow 6 + 7 = B(2-1)$
10= -A $\Rightarrow A = -10$	$13 = B$

$$\frac{3x+7}{(x-1)(x-2)} = \frac{-10}{x-1} + \frac{13}{x-2}$$

2. $\boldsymbol{Resolve\ \frac{x+4}{(x^2-4)(x+1)}\ into\ partial\ fractions.}$

$\boldsymbol{Sol:}$

$$\frac{x+4}{(x^2-4)(x+1)} = \frac{x+4}{(x+2)(x-2)(x+1)}$$

$$\boldsymbol{let}\ \frac{x+4}{(x^2-4)(x+1)} = \frac{A}{x+2} + \frac{B}{x-2} + \frac{C}{x+1}$$

$$\frac{x+4}{(x^2-4)(x+1)}$$

$$= \frac{A(x-2)(x+1)+B(x+2)(x+1)+C(x-2)(x+2)}{(x+2)(x-2)(x+1)}$$

$x + 4 = A(x-2)(x+1) + B(x+2)(x+1)$
$+C(x-2)(x+2)$ **... (1)**

$\boldsymbol{put\ x = -2}$ in (1), we get
$\Rightarrow -2 + 4 = A(-2-2)(-2+1)$
$\Rightarrow 2 = A(-4)(-1) \Rightarrow 4A = 2 \Rightarrow A = \frac{1}{2}$

$\boldsymbol{put\ x = 2}$ in (1), we get
$\Rightarrow 2 + 4 = B(2+2)(2+1)$
$\Rightarrow 6 = A(4)(3) \Rightarrow 12A = 6 \Rightarrow A = \frac{1}{2}$

$\boldsymbol{put\ x = -1}$ in (1), we get
$\Rightarrow -1 + 4 = C(1-2)(1+2)$
$\Rightarrow 3 = C(-1)(3) \Rightarrow -3C = 3 \Rightarrow C = -1$

$$\frac{x+4}{(x^2-4)(x+1)} = \frac{\frac{1}{2}}{x+2} + \frac{\frac{1}{2}}{x-2} - \frac{1}{x+1}$$

3. $\boldsymbol{Resolve\ \frac{x^2+13x+15}{(2x+3)(x+3)^2}\ into\ partial\ fractions.}$

$\boldsymbol{Sol:\ let}\ \frac{x^2+13x+15}{(2x+3)(x+3)^2} = \frac{A}{2x+3} + \frac{B}{x+3} + \frac{C}{(x+3)^2}$

$$\Rightarrow \frac{x^2+13x+15}{(2x+3)(x+3)^2}$$

$$= \frac{A(x+3)^2+B(x+3)(2x+3)+C(2x+3)}{(2x+3)(x+3)^2}$$

$\Rightarrow \boldsymbol{x^2 + 13x + 15} = A(x+3)^2$
$+B(x+3)(2x+3)$+C(2x+3) ....(1)

$put\ x = -3\ in\ (1), we\ get$
$(-3)^2 + 13(-3) + 15 = C(2(-3)+3)$
$\Rightarrow$9-39+15=C(-3)
$\Rightarrow$-3C=-15
$\therefore$C=5

$put\ x = -\frac{3}{2}\ in\ (1), we\ get$
$\left(-\frac{3}{2}\right)^2 + 13\left(-\frac{3}{2}\right) + 15 = A\left(-\frac{3}{2} + 3\right)^2$
$\Rightarrow \frac{9}{4} - \frac{39}{2}$+15=A $(\frac{9}{4})$
$\Rightarrow \frac{9-72+60}{4} = \frac{9A}{4}$
$\Rightarrow -\frac{9}{4} = \frac{9A}{4}$
$\therefore$A=-1

$equating\ the\ coeffiecient\ of\ x^2, we\ get$

$A + 2B = 1$
$\Rightarrow 2B = 1 - A = 1 - -1 = 2$
$\therefore\ \boldsymbol{B = 1}$

$$\frac{x^2+13x+15}{(2x+3)(x+3)^2} = \frac{-1}{2x+3} + \frac{1}{x+3} + \frac{5}{(x+3)^2}$$

(H/W)

$\boldsymbol{Resolve\ \frac{x-1}{(x+1)(x-2)^2}\ into\ partial\ fractions.}$

$\boldsymbol{Resolve\ \frac{x^2-x+1}{(x+1)(x-1)^2}\ into\ partial\ fractions.}$

4. ***Resolve*** $\frac{3x-18}{x^3(x+3)}$ ***into partial fractions.***

Sol:

let $\frac{3x-18}{x^3(x+3)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{x+3}$

$\frac{3x-18}{x^3(x+3)} = \frac{A(x^2)(x+3)+Bx(x+3)+C(x+3)+D(x^3)}{x^3(x+3)}$

$3x-18 = A(x^2)(x+3) + Bx(x+3) + C(x+3) + D(x^3)$ **.... (1)**

$3x-18 = A(x^3+3x^2) + B(x^2+3x) + C(x+3) + D(x^3)$ **.... (2)**

put x $= \mathbf{0}$ in (1), we get

⇨ $3(0) - 18 = C(0+3)$

⇨ $-18 = 3C$ ⇨ $C = -6$

put x $= \mathbf{-3}$ in (1), we get

⇨ $3(-3) - 18 = D(-3)^3$

⇨ $-9 - 18 = D(-27)$ ⇨ $-27D = -27$

⇨ $D = 1$

equating the coeffiecient of x^3*, we get*

$A + D = 0$

⇨$A = -D = -1$

∴ $\boldsymbol{A = -1}$

equating the coeffiecient of x^2*, we get*

$\boldsymbol{3A + B = 0}$

⇨B = - 3A= $-3(-1)$

∴B=3

$\frac{3x-18}{x^3(x+3)} = \frac{-1}{x} + \frac{3}{x^2} - \frac{6}{x^3} + \frac{1}{x+3}$

Aims

5. ***Resolve*** $\frac{x^2+5x+7}{(x-3)^3}$ ***into partial fractions.***

Sol:

put $x - 3 = y$

⇨ $x = y + 3$

$\frac{x^2+5x+7}{(x-3)^3}$

$= \frac{(y+3)^2+5(y+3)+7}{y^3}$

$= \frac{y^2+6y+9+5y+15+7}{y^3}$

$= \frac{y^2+11y+31}{y^3}$

$= \frac{y^2}{y^3} + \frac{11y}{y^3} + \frac{31}{y^3}$

$= \frac{1}{y} + \frac{11}{y^2} + \frac{31}{y^3}$

$= \frac{1}{x-3} + \frac{11}{(x-3)^2} + \frac{31}{(x-3)^3}$

(H/W)

Resolve $\frac{3x^3-8x^2+10}{(x-1)^4}$ ***into partial fractions.***

Sol:

put $x - 3 = y$

⇨ $x = y + 3$

$\frac{3x^3-8x^2+10}{(x-1)^4}$

$= \frac{3}{x-1} + \frac{1}{x-1} - \frac{7}{x-1} + \frac{5}{x-1}$

6. ***Resolve*** $\frac{x^2-3}{(x+2)(x^2+1)}$ ***into partial fractions.***

Sol:

$$\frac{x^2-3}{(x+2)(x^2+1)} = \frac{A}{x+2} + \frac{(Bx+C)}{x^2+1}$$

$$\frac{x^2-3}{(x+2)(x^2+1)} = \frac{A(x^2+1)+(Bx+C)(x+2)}{(x+2)(x^2+1)}$$

$x^2 - 3 = A(x^2+1) + (Bx+C)(x+2)\dots(1)$

$x^2 - 3 = A(x^2+1) + B(x^2+2x)$
$+c(x+2)\dots(2)$

put $\boldsymbol{x = -2}$ in (2), we get
$\Rightarrow (-2)^2 - 3 = A((-2)^2+1)$
$\Rightarrow 4 - 3 = A(4+1) \Rightarrow 5A = 1$
$\therefore A = \frac{1}{5}$

equating the coeffiecient of x^2*, we get*

$A + B = 1$
$\Rightarrow B = 1 - B = 1 - \frac{1}{5} = \frac{4}{5}$
$\therefore \boldsymbol{B = \frac{4}{5}}$
equating the coeffiecient of x, we get

$\boldsymbol{2B + C = 0}$
$\Rightarrow$C = -2B= $-2\left(\frac{4}{5}\right) = -\frac{8}{5}$
$\therefore$C=-8/5

$$\frac{x^2-3}{(x+2)(x^2+1)} = \frac{\frac{1}{5}}{x+2} + \frac{\left(\frac{4}{5}x-\frac{8}{5}\right)}{x^2+1} = \frac{1}{5}\left[\frac{1}{x+2} + \frac{(4x-8)}{x^2+1}\right]$$

(H/W)

Resolve $\frac{2x^2+3x+4}{(x-1)(x^2+2)}$ ***into partial fractions.***

Sol:

$$\boldsymbol{let}\ \frac{2x^2+3x+4}{(x-1)(x^2+2)} = \frac{3}{x-1} + \frac{-1x+2}{x^2+2}$$

7. ***Resolve*** $\frac{x^3}{(x-a)(x-b)(x-c)}$ ***into partial fractions.***

Sol:

$$\boldsymbol{let}\ \frac{x^3}{(x-a)(x-b)(x-c)} = \frac{1}{1} + \frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c}$$

$$\frac{x^3}{(x-a)(x-b)(x-c)} = \frac{(x-a)(x-b)(x-c)+A(x-b)(x-c)+B(x-a)(x-c)+C(x-a)(x-b)}{(x-a)(x-b)(x-c)}$$

$x^3 = (x-a)(x-b)(x-c) + A(x-b)(x-c)$
$+B(x-a)(x-c) + C(x-a)(x-b)\ \dots.(1)$

put $x = a$ *in* (1)
$a^3 = A(a-b)(x-c)$

$\therefore A = \frac{a^3}{(a-b)(a-c)}$

put $x = b$ *in* (1)
$a^3 = B(b-a)(b-c)$

$\therefore B = \frac{b^3}{(b-a)(b-c)}$

put $x = c$ *in* (1)
$a^3 = C(c-a)(c-b)$

$\therefore C = \frac{c^3}{(c-a)(c-b)}$

$$\frac{x^3}{(x-a)(x-b)(x-c)} = \frac{1}{1} + \frac{\frac{a^3}{(a-b)(a-c)}}{x-a} + \frac{\frac{b^3}{(b-a)(b-c)}}{x-b} + \frac{\frac{c^3}{(c-a)(c-b)}}{x-c}$$

8. ***Resolve*** $\frac{x^3}{(2x-1)(x+2)(x-3)}$ ***into partial fractions.***

Sol:

$$\boldsymbol{let}\ \frac{x^3}{(2x-1)(x+2)(x-3)} = \frac{1}{2} + \frac{A}{2x-1} + \frac{B}{x+2} + \frac{C}{x-3}$$

(H/W)

$$\frac{x^3}{(2x-1)(x+2)(x-3)} = \frac{1}{2} + \frac{-\frac{1}{50}}{2x-1} + \frac{-\frac{8}{25}}{x+2} + \frac{\frac{27}{25}}{x-3}$$

9. ***Resolve*** $\frac{x^4}{(x-1)(x-2)}$ ***into partial fractions.***

Sol:

$$\frac{x^4}{(x-1)(x-2)} = (Ax^2+Bx+C)+\frac{D}{x-1}+\frac{E}{x-2}$$

$$\frac{x^4}{(x-1)(x-2)} = \frac{(Ax^2+Bx+C)(x-1)(x-2)+D(x-2)+E(x-1)}{(x-1)(x-2)}$$

$$x^4 = (Ax^2+Bx+C)(x-1)(x-2) + D(x-2)+E(x-1) \;....(1)$$

$$x^4 = (Ax^2+Bx+C)(x^2-3x+2) + D(x-2)+E(x-1)$$

$$x^4 = A(x^4-3x^3+2x^2)+B(x^3-3x^2+2x)\; C(x^2-3x+2)\; +D(x-2)+E(x-1)...(2)$$

put $x = 1$ in (1), we get

$\Rightarrow (1)^4 = D(1-2)$

$\Rightarrow 1 = -D \Rightarrow D = -1$

put $x = 2$ in (1), we get

$\Rightarrow (2)^4 = E(2-1)$

$\Rightarrow E = 16$

equating the coeffiecient of x^4, *we get*

$A = 1$

equating the coeffiecient of x^3, *we get*

$-3A + B = 0$

$\Rightarrow B = 3A = 3(1)$

$\therefore$B=3

equating the coeffiecient of x^2, *we get*

$2A - 3B + C = 0$

$\Rightarrow C = 3B - 2A$

$C = 3(3) - 2(1) = 9 - 2$

$\therefore$C=7

$$\frac{x^4}{(x-1)(x-2)} = (1x^2+3x+7)+\frac{-1}{x-1}+\frac{16}{x-2}$$

10. Find the coefficient of x^4 *in the expantion of* $\frac{3x}{(x-2)(x+1)}$.

Sol:

$\frac{3x+7}{(x-2)(x+1)}$

let $\frac{3x}{(x-2)(x+1)} = \frac{A}{x-2}+\frac{B}{x+1}$

$\Rightarrow \frac{3x}{(x-2)(x+1)} = \frac{A(x+1)+B(x-2)}{(x-2)(x+1)}$

$$\mathbf{3x = A(x+1)+B(x-2)\;....(1)}$$

put $x = -1$	**put $x = 2$**
$\Rightarrow 3(-1) = B(-1-2)$	$\Rightarrow 3(2) = A(2+1)$
$-3 = -3B$	$6 = 3A$
$\therefore$B=1	$\therefore$A=2

$$\frac{3x}{(x-2)(x+1)} = \frac{2}{x-2}+\frac{1}{x+1}$$

$$= \frac{2}{-2(1-\frac{x}{2})}+\frac{1}{(1+x)}$$

$$= \frac{2}{-2}\left(1-\frac{x}{2}\right)^{-1}+(1+x)^{-1}$$

$$= -\left(1-\frac{x}{2}\right)^{-1}+(1+x)^{-1}$$

$$= -\left[1+\frac{x}{2}+\left(\frac{x}{2}\right)^2+\left(\frac{x}{2}\right)^3+\left(\frac{x}{2}\right)^4...\infty\right]$$
$$+[1-x+x^2-x^3+x^4 ......\infty]$$

Now, the coefficient of x^4 *in the above expantion* is $= -\frac{1}{16}+1 = \frac{15}{16}$

Aims

11. Find the coefficient of x^n in the expantion of $\frac{x-4}{x^2-5x+6}$.

Sol:

$$\frac{x-4}{x^2-5x+6}=\frac{x-4}{(x-2)(x-3)}$$

let $\frac{x-4}{(x-2)(x-3)}=\frac{A}{x-2}+\frac{B}{x-3}$

$$\Rightarrow\frac{x-4}{(x-2)(x-3)}=\frac{A(x-3)+B(x-2)}{(x-2)(x-3)}$$

$$x-4=A(x-3)+B(x-2)\;....(1)$$

put $x=2$ $\Rightarrow 2-4=A(2-3)$ -2= -A $\Rightarrow A=2$	put $x=3$ $\Rightarrow 3-4=B(3-2)$ $-1=B$

$$\frac{x-4}{(x-2)(x-3)}=\frac{2}{x-2}+\frac{-1}{x-3}$$

$$=\frac{2}{-2(1-\frac{x}{2})}+\frac{-1}{-3(1-\frac{x}{3})}$$

$$=\frac{2}{-2}\left(1-\frac{x}{2}\right)^{-1}+\frac{1}{3}\left(1-\frac{x}{3}\right)^{-1}$$

$$=-\left(1-\frac{x}{2}\right)^{-1}+\frac{1}{3}\left(1-\frac{x}{3}\right)^{-1}$$

$$=-\left[1+\frac{x}{2}+\left(\frac{x}{2}\right)^2+\left(\frac{x}{2}\right)^3+\left(\frac{x}{2}\right)^4...\infty\right]$$

$$+\frac{1}{3}\left[1+\frac{x}{3}+\left(\frac{x}{3}\right)^2+\left(\frac{x}{3}\right)^3+\left(\frac{x}{3}\right)^4...\infty\right]$$

Now, the coefficient of x^n in the above expantion is $=-\frac{1}{2^n}+\frac{1}{3}.\frac{1}{3^n}$

$$=\frac{1}{3^{n+1}}-\frac{1}{2^n}$$

Aims