1. ***Find the probability of drawing an ace or a spade from a well shuffled pack of 52 playing cards.***
Sol:
Let A, B be the events of drawing an ace, heart respectively, when a card is drawn from a pack of 52 cards.
$P(A) = \frac{4_{C_1}}{52_{C_1}} = \frac{4}{52}$,

$P(B) = \frac{13_{C_1}}{52_{C_1}} = \frac{13}{52}$

$P(A \cap B) = \frac{1_{C_1}}{52_{C_1}} = \frac{1}{52}$

By addition theorem
P $(A \cup B)$ =P (A) + P$(B) - P(A \cap B)$.

$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}$

2. ***If two numbers are selected randomly from 20 consecutive natural numbers, find the probability that the sum of the two numbers is (i) an even number (ii) an add number.***
Sol: out of 20 consecutive natural numbers we select '2' numbers in 20_{C_2} *ways*
$\Rightarrow 20_{C_2} = \frac{20\times19}{2\times1}$ =190
$\therefore n(s) = 190$
In 20 consecutive natural numbers; 10 are even numbers and 10 are odd numbers
We know that

even +even =even (or) odd + odd =even

$10_{C_2} + 10_{C_2} = \frac{10 \times 9}{2 \times 1} + \frac{10 \times 9}{2 \times 1} = 45 + 45 = 90$
$\therefore n$ (A)=90
$required\ probability\ P(A) = \frac{n\,(A)}{n(s)} = \frac{90}{190} = \frac{9}{19}$.
(ii)P (getting sum of two numbers are odd)
=1-P (getting sum of two numbers are even)
$=1-\frac{9}{19} = \frac{10}{19}$.

3. **In a class of 60 boys and 20 girls, half of the boys and half of the girls know cricket. Find the probability of a person selected from the class is either a boy or a girl who know cricket.**
Sol:
Number of boys =60
Number of girls =20
Let A be the event that the selected person is a boy and
B be the event that the selected person is a girl who knows cricket.
$P(A) = \frac{60}{80}$ *and* $P(B) = \frac{10}{20}$
A, B *are mutually exclusive events*
$A \cap B = \emptyset$
By addition theorem
P $(A \cup B)$ =P (A) + P$(B) - P(A \cap B)$.

$= \frac{60}{80} + \frac{10}{80} - 0$

$= \frac{70}{80} = \frac{7}{8}$

4. **A bag contains 12 two rupee coins, 7 one rupee coins, and 4 half rupee coins. If three coins are selected at random, then find the probability that (a) The sum of three coins is maximum (b) the sum of three coins is minimum.**
Sol:
Number of two rupee coins =12
Number of one rupee coins=7
Number of half rupee coins =4
(i)To have sum of three coins as maximum, we shall select all the three coins are from two rupee coins.
$\therefore$
probability that the sum of 3 *coins is max*
$= \frac{12_{C_3}}{23_{C_3}}$

$= \frac{\frac{12.11.10}{3.2.1}}{\frac{23.22.21}{3.2.1}}$

$= \frac{20}{161}$

5. ***A speaks truth in 75% of the cases and B in 80% of the cases. What is the probability that their statements about an incident do not match.***

Sol: let A and B be the events that the persons A, B respectively to speak truth about an incident.

Given that P (A) $=\frac{75}{100}=\frac{3}{4}\Rightarrow$ P (A) $=\frac{1}{4}$.

P (B) $=\frac{80}{100}=\frac{4}{5}\Rightarrow$ P $(\overline{B})=\frac{1}{5}$.

P (A) =1-P $(\overline{A})$

Clearly A, B are independent events.

Now probability that their statements about an incident do not match

$P(A\cap\overline{B})\cup P(\overline{A}\cap B)=P(A)P(\overline{B})+P(\overline{A})P(B)$

$=\frac{3}{4}.\frac{1}{5}+\frac{1}{4}.\frac{4}{5}$

$=\frac{7}{20}$

6. ***Two people A and B are rolling a die on the condition that the person who gets 3 will win the game. if A starts the game, then fined the probability of A and B respectively to win the game.***

Sol:

Let P be the event of getting 3 on a die$=\frac{1}{6}$

q be the event of not getting 3 on a die

$=1-\frac{1}{6}=\frac{5}{6}$

A, B be the events that A, B will win the game respectively.

Then A will win in 1st or 3rd or 5th .....Chances.

(i)The probability of A will win the game is

P (A) $=p+qqp+qqqqp+\cdots$

P (A) $=p+q^2p+q^2p+\cdots$

$=\frac{\mathbf{p}}{\mathbf{1-q^2}}=\frac{\frac{1}{6}}{1-(\frac{5}{6})^2}=\frac{1}{6}.\frac{6^2}{6^2-5^2}=\frac{6}{11}$

G.P series

$a=p \text{ and } r=q^2$

$S_\infty=\frac{\boldsymbol{a}}{\mathbf{1}-\boldsymbol{r}}=\frac{\boldsymbol{p}}{\mathbf{1}-\boldsymbol{q}^2}$

(ii) The probability of B will win the game is

=1-P (A) $=1-\frac{6}{11}=\frac{5}{11}$.

7. ***If A, B, C are three events in a sample space s, then S.T***
$P(A\cup B\cup C)=P(A)+P(B)+P(C)-P(A\cap B)-P(B\cap C)-P(A\cap C)+P(A\cap B\cap C)$

Sol: $P(A\cup B\cup C)=P[A\cup(B\cup C)]$

By Additional theorem

$\therefore P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$=P(A)+P(B\cup C)-P[A\cap(B\cup C)]$

$=P(A)+P(B)+P(C)-P(B\cap C)-P[(A\cap B)\cup(A\cap C)]$

$=P(A)+P(B)+P(C)-P(B\cap C)-P(A\cap B)-P(A\cap C)+P(A\cap B\cap A\cap C)$

$\therefore P(A\cup B\cup C)=P(A)+P(B)+P(C)-P(B\cap C)-P(A\cap B)-P(A\cap C)+P(A\cap B\cap C)$.

8. ***The probability that Australia wins a match against India in a cricket game is given to be 1/3.if India and Australia play 3 matches, what is the probability that (1) Australia will loose all the three matches? (ii) Australia will win at least one match?***

Sol: let E be the event of Australia win against India then P (E) $=\frac{1}{3}$

$\Rightarrow P(\overline{E})=1-P(E)=1-\frac{1}{3}=\frac{2}{3}$

(i) Probability that Australia will loose all the three matches= $P(\overline{E})P(\overline{E})P(\overline{E})=\frac{2}{3}.\frac{2}{3}.\frac{2}{3}=\frac{8}{27}$

(ii) Probability that Australia will win at least one match=1-(Probability that Australia will loose all the three matches)

(iii) $=1-\frac{8}{27}=\frac{19}{27}$.

9. ***State and prove multiplication theorem.***
Sol:
Statement: if
A, B *are two events of a random experiment with* $P(A) > 0, P(B) > 0$ *then prove that*

$$P(A\cap B) = \begin{cases} P(A)P\left(\frac{B}{A}\right) \; or \\ P\left(\frac{A}{B}\right)P(B) \end{cases}$$

Proof: let's' be the sample space. And A, B are two events by definition of conditional probability;

The probability of the event B after the event A has occurred is called conditional probability of B given by A and it is denoted by P(B/A) and define

As $P\left(\frac{B}{A}\right) = \frac{P(A\cap B)}{P(A)}$

$\Rightarrow P(A\cap B) = P(A)P\left(\frac{B}{A}\right)$ ....(1)

And also The probability of the event A after the event B has occurred is called conditional probability of A given by B and it is denoted by P(A/B) and define

As $P\left(\frac{A}{B}\right) = \frac{P(A\cap B)}{P(B)}$

$\Rightarrow P(A\cap B) = P\left(\frac{B}{A}\right)P(B)$ ....(2)

From (1)& (2)

$$P(A\cap B) = \begin{cases} P(A)P\left(\frac{B}{A}\right) \; or \\ P\left(\frac{A}{B}\right)P(B) \end{cases}$$

10. **Three screws are drawn at random from a lot of 50 screws, 5 of which are defective. Find the probability of the event that all the 3 screws are non-defective, assuming that the drawing is (a) with replacement (b) without replacement.**

Sol:
Number of defective screws =5
Number of good screws =45
Total number of screws =50

(i) With replacement:

$$=\left[\frac{45_{C_1}}{50_{C_1}}\right]\left[\frac{45_{C_1}}{50_{C_1}}\right]\left[\frac{45_{C_1}}{50_{C_1}}\right]$$

$$=\left[\frac{45}{50}\right]\left[\frac{45}{50}\right]\left[\frac{45}{50}\right]$$

$$=\frac{9.9.9}{10.10.10}=\left[\frac{9}{10}\right]^3$$

$(ii) Without replacement.$

$$=\left[\frac{45_{C_1}}{50_{C_1}}\right]\left[\frac{44_{C_1}}{49_{C_1}}\right]\left[\frac{43_{C_1}}{48_{C_1}}\right]$$

$$=\left[\frac{45}{50}\right]\left[\frac{44}{49}\right]\left[\frac{43}{48}\right]$$

$$=\left[\frac{9}{10}\right]\left[\frac{11}{49}\right]\left[\frac{43}{12}\right]=\frac{3}{10}.\frac{11}{49}.\frac{43}{4}=\frac{1419}{1960}$$

11. **If one card is drawn from a pack of cards, then show that the events of an ace and getting a heart card are independent events.**

Sol:
Let A, B be the events of getting that an ace, a heart card respectively and S be the sample space.
$n(A) = 4, n(B) = 13, n(S) = 52$

$$P(A) = \frac{n(A)}{n(S)} = \frac{4}{52} = \frac{1}{13}$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{13}{52} = \frac{1}{4}$$

$$n(A\cap B) = 1\{a\ ace\ heart\}$$

$$P(A\cap B)\frac{n(A\cap B)}{n(s)} = \frac{1}{52} = \frac{1}{13}.\frac{1}{4}$$

$=P(A).P(B)$

$\therefore A, B$ *are independent events.*

12. **A problem in calculus is given to two students A and B whose chances of solving it are1/3 and ¼ respectively. Find the probability of the problem being solved if both of them by independently.**

Sol:

Given that $P(A) = \frac{1}{3}, \ P(B) = \frac{1}{4}$

& A, B are independent events.

For independent events $P(A \cap B) = P(A).P(B)$

$\therefore$ *probability that the problem bieng solved is*

P $(A \cup B)$ =P $(A) +$ P$(B) - P(A \cap B)$.

$= \frac{1}{3} + \frac{1}{4} - \frac{1}{3}.\frac{1}{4}$

$= \frac{4+3-1}{12}$

$= \frac{6}{12} = \frac{1}{2}$.

*13. **If A and B are independent events with P(A)=0.6,P(B)=0.7 then compute I)P(A∩B) ii) P (A∪B) iii) P $(\frac{B}{A})$ iv) P (A∪B).***

Sol: given that A, B are independent events

then $P(A \cap B) = P(A)P(B)$

Given P (A) =0.6; P (B) =0.7

i. P(A∩B)= $P(A).P(B) = (0.6).(0.7) = 0.42$

ii. P $(A \cup B)$ =P (A) + P (B) –P $(A \cap B)$

$= 0.6 + 0.7 - 0.42 = 1.3 - 042 = 0.88$

iii. $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{P(A)P(B)}{P(A)} = P(B) = 0.7$

iv. P$(\overline{A} \cap \overline{B})$= P $(\overline{A \cup B})$

$= 1 - $ P $(A \ \cup B) = 1 - 0.88 = 0.12$

*14. **If A and B are independent events with P(A)=0.2,P(B)=0.5then compute (i)P(A∩B)ii) P $(\frac{A}{B})$ iii) P $(\frac{B}{A})$ iv) P (A∪B).***

Sol:

(i)P (A∩B) $= P(A).P(B)$

$= (0.2)(0.5) = 0.1$

$(ii)P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$

$= \frac{P(A)P(B)}{P(A)} = P(B) = 0.5$

$(iii)P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

$= \frac{P(A)P(B)}{P(B)} = P(A) = 0.2$

(iv)By addition theorem

P $(A \cup B)$ =P $(A) +$ P$(B) - P(A \cap B)$.

⇨P $(A \cup B)$ =0.2 + 0.5 − (0.2)(0.5)

⇨P $(A \cup B)$ =0.7 − (0.1) = 0.

*15. **A, B are two independent events such that the probability of both the events to occur is 1/6 and the probability of both the events do not occur is 1/3.findP (A).***

Sol: given that A, B are independent events

then $P(A \cap B) = P(A).P(B) = \frac{1}{6}$

Let P (A) $=x$ *and* $P(B) = y$

then $xy = \frac{1}{6} ⇨ 6xy = 1$ ....

given $P(\overline{A} \cap \overline{B}) = \frac{1}{3} ⇨ P(\overline{A \cup B}) = \frac{1}{3}$

⇨ $1 - P(A \cup B) = \frac{1}{3}$

⇨ $1 - \frac{1}{3} = P(A \cup B)$

⇨ $\frac{2}{3} =$ P (A $\cup B$)

⇨ $\frac{2}{3} =$ P (A) + P (B)– P (A $\cap B$)

⇨ $\frac{2}{3} = x + y - \frac{1}{6}$

⇨ $x + y = \frac{2}{3} + \frac{1}{6}$

⇨ $x + y = \frac{5}{6}$

⇨ $6x + 6y = 5$

⇨ $6y = 5 - 6x$ … …

From $x(5 - 6x) = 1$

⇨ $5x - 6x^2 = 1$

$\Rightarrow 6x^2 - 5x - 1 = 0$
$\Rightarrow 6x^2 - 3x - 2x - 1 = 0$
$\Rightarrow 3x(2x-1) - 1(2x-1) = 0$
$\Rightarrow (3x-1)(2x-1) = 0$
$x = \frac{1}{2}\ or \frac{1}{3} \Rightarrow P(A) = \frac{1}{2}\ or \frac{1}{3}$

16. A bag B_1 contains 4 white and 2 black balls; bag B_2 contains 3 white and 4 black balls, a bag is drawn at random and a ball is chosen at random from it. Then what is the probability that the ball is white.

Sol:

Let A_1, A_2 be the events of choosing bags B_1, B_2 respectively.
Here A_1, A_2 are equally likely events.
Then $P(A_1) = \frac{1}{2}, P(A_2) = \frac{1}{2}$
Let W be the event of drawing a white ball from the selected bag

P $\left(\frac{W}{A_1}\right)$ probability of drawing a white ball from bag $B_1 = \frac{4}{6} = \frac{2}{3}$
P $\left(\frac{W}{A_2}\right)$ probability of drawing a white ball from bag $B_1 = \frac{3}{7}$
By Total probability theorem,

$$P(W) = P(A_1)P\left(\frac{W}{A_1}\right) + P(A_2)P\left(\frac{W}{A_2}\right)$$

$$= \frac{1}{2}.\frac{2}{3} + \frac{1}{2}.\frac{7}{3}$$

$$= \frac{14+9}{42} = \frac{23}{42}$$