## Equations with roots are in A.P, G.P, H.P

1. **Solve the equation roots being in A.P**

$$4x^3 - 24x^2 + 23x + 18 = 0$$

***Sol:*** Given

$f(x) = 4x^3 - 24x^2 + 23x + 18 = 0$

$\{a = 4, b = -24, c = 23, d = 18\}$

Given that roots are in A.P

Let the roots $a - d, a, a + d$ in A.P

$S_1 = \alpha + \beta + \gamma = -\frac{b}{a}$

$\Rightarrow (a - d + a + a + d) = -\frac{-24}{4}$

$\Rightarrow 3a = 6 \Rightarrow a=2$

∴(x-2) is a factor of $4x^3 - 24x^2 + 23x + 18 = 0$

| | | | | |
|---|---|---|---|---|
| 2 | 4 | -24 | 23 | 18 |
| | 0 | 8 | -32 | -18 |
| | 4 | -16 | -9 | 0 |

$\Rightarrow 4x^2 - 16x - 9 = 0$

$\Rightarrow 4x^2 - 18x + 2x - 9 = 0$

$\Rightarrow 2x(2x - 9) + 1(2x - 9) = 0$

$\Rightarrow (2x - 9)(2x + 1) = 0$

$\Rightarrow (2x - 9) = 0, (2x + 1) = 0$

$\Rightarrow x = \frac{9}{2}, x = -\frac{1}{2}$

∴ The roots are $-\frac{1}{2}$, 2, $\frac{9}{2}$

**(H/W)**

$$8x^3 - 36x^2 - 18x + 81 = 0$$

$$x^3 - 3x^2 - 6x + 8 = 0$$

2. **Solve the equation roots being in G.P**

$$3x^3 - 26x^2 + 52x - 24 = 0$$

***Sol:*** Given

$f(x) = 3x^3 - 26x^2 + 52x - 24 = 0$

$\{a = 3, b = -26, c = 52, d = -24\}$

Given that roots are in G.P

Let the roots $\frac{a}{r}, a, ar$ in G.P

$S_3 = \alpha\beta\gamma = -\frac{d}{a}$

$\frac{a}{r}.a.ar = -(-\frac{24}{3})$

$\Rightarrow a^3 = 8 = 2^3$

a=2

∴(x-2) is a factor of $3x^3 - 26x^2 + 52x - 24 = 0$

| | | | | |
|---|---|---|---|---|
| 1 | 3 | -26 | 52 | -24 |
| | 0 | 6 | -40 | 24 |
| | 3 | -20 | 12 | 0 |

$\Rightarrow 3x^2 - 20x + 12 = 0$

$\Rightarrow 3x^2 - 18x - 2x + 12 = 0$

$\Rightarrow 3x(x - 6) - 2(x - 6) = 0$

$\Rightarrow (x - 6)(3x - 2) = 0$

$\Rightarrow (x - 6) = 0, (3x - 2) = 0$

$\Rightarrow x = 6, x = \frac{2}{3}$

∴The roots are 2/3, 2, 6.

**(H/W)**

$$54x^3 - 39x^2 - 26x + 16 = 0$$

3. Solve the equation, given that roots are in H.P

$$15x^3 - 23x^2 + 9x - 1 = 0$$

***Sol:*** Given

$f(x) = \mathbf{15x^3 - 23x^2 + 9x - 1 = 0} \ldots (1)$

Put $y = \frac{1}{x}$ so that $\frac{15}{y^3} - \frac{23}{y^2} + \frac{9}{y} - 1 = 0$

$\Rightarrow \mathbf{y^3 - 9\,y^2 + 23y - 15 = 0 \ldots (2)}$

Given that roots of (1) are in H.P, so the roots of (2) are in A.P

Let the roots $a-d, a, a+d$ in A.P

$\{a = 1, b = -9, c = 23, d = -15\}$

$$S_1 = \alpha + \beta + \gamma = -\frac{b}{a}$$

$\Rightarrow (a - d + a + a + d) = -(-\frac{9}{1})$

$\Rightarrow 3a = 9 \quad \Rightarrow a=3$

$\therefore$(y-3) is a factor of $\mathbf{y^3 - 9\,y^2 + 23y - 15 = 0}$

| | | | | |
|---|---|---|---|---|
| 3 | 1 | -9 | 23 | -15 |
| | 0 | 3 | -18 | 15 |
| | 1 | -6 | 5 | 0 |

$\Rightarrow y^2 - 6y + 5 = 0$

$\Rightarrow y^2 - 5y - 1y + 5 = 0$

$\Rightarrow y(y-5) - 1(y-5) = 0$

$\Rightarrow (y-5)(y-11) = 0$

$\Rightarrow (y-5) = 0, (y-1) = 0$

$\Rightarrow y = 5, y = 1$

$\therefore$ The roots are $1, 3, 5$ *in A.P*

$\therefore$ The roots are $1, \frac{1}{3}, \frac{1}{5}$ *in H.P*

**(H/W)**

$$\mathbf{6x^3 - 11x^2 + 6x - 1 = 0}$$

4. Solve the equation

$\mathbf{18x^3 + 81x^2 + 121x + 60 = 0}$ **Given that one root is equal to half the sum of the remaining roots.**

***Sol:*** Given

$f(x) = \mathbf{18x^3 + 81x^2 + 121x + 60 = 0}$

$\{a = 18, b = 81, c = 121, d = 60\}$

Let the roots $\alpha, \beta, \gamma$

Given that $\beta = \frac{\alpha+\gamma}{2} \Rightarrow 2\beta = \alpha + \gamma \ldots (1)$

$$S_1 = \alpha + \beta + \gamma = -\frac{b}{a}$$

$\Rightarrow (a - d + a + a + d) = -(\frac{81}{18})$

$\Rightarrow 3a = -\frac{9}{2} \quad \Rightarrow a=-3/2$

$\therefore (x + 3/2)$ is a factor of $\mathbf{18x^3 + 81x^2 + 121x + 60 = 0}$

| | | | | |
|---|---|---|---|---|
| $-\frac{3}{2}$ | 18 | 81 | 121 | 60 |
| | 0 | -27 | -81 | -60 |
| | 18 | -54 | 40 | 0 |

$(\div\ by\ 2)$

$\Rightarrow 9x^2 - 27x - 20 = 0$

$\Rightarrow 9x^2 - 15x - 12x - 20 = 0$

$\Rightarrow 3x(3x-5) + 4(3x-5) = 0$

$\Rightarrow (3x-5)(3x+4) = 0$

$\Rightarrow (3x-5) = 0, (3x+4) = 0$

$\Rightarrow x = \frac{5}{3}, x = -\frac{4}{3}$

$\therefore$ The roots are $-\frac{3}{2}, -\frac{4}{3}, -\frac{5}{3}$

# Reciprocal Equations

5. **Solve the eq"n**

$x^4 - 10x^3 + 26x^2 - 10x + 1 = 0.$

*Sol*: Given eq"n

$x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$.... (1)

*it is a reciprocal eq"n of class – I*

Dividing both sides by " $x^2$ "

$\Rightarrow \frac{x^4}{x^2} - \frac{10x^3}{x^2} + \frac{26x^2}{x^2} - \frac{10x}{x^2} + \frac{1}{x^2} = 0$

$\Rightarrow x^2 - 10x + 26 - \frac{10}{x} + \frac{1}{x^2} = 0$

$\Rightarrow \left(x^2 + \frac{1}{x^2}\right) - 10\left(x + \frac{1}{x}\right) + 26 = 0$

Let $x + \frac{1}{x} = p \Rightarrow x^2 + \frac{1}{x^2} = p^2 - 2$

$\Rightarrow p^2 - 2 - 10p + 26 = 0$

$\Rightarrow p^2 - 10p + 24 = 0$

$\Rightarrow p^2 - 4p - 6p + 24 = 0$

$\Rightarrow (p-4) - 6(p-4) = 0$

$\Rightarrow (p-4)(p-6) = 0$

$\Rightarrow (p-4) = 0\ ,\quad (p-6) = 0$

$\Rightarrow p = 4 \quad and \quad p = 6$

Case (1)

$x + \frac{1}{x} = 4$

$\Rightarrow \frac{x^2+1}{x} = 4$

$\Rightarrow x^2 + 1 = 4x$

$\Rightarrow x^2 - 4x + 1 = 0$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{4 \pm \sqrt{4^2-4.1.1}}{2.1}$

$= \frac{4 \pm \sqrt{16-4}}{2}$

$= \frac{4 \pm \sqrt{12}}{2}$

$= \frac{4 \pm 2\sqrt{3}}{2}$

$= 2\frac{(2 \pm \sqrt{3})}{2}$

$= (2 \pm \sqrt{3})$

case (2)

$x + \frac{1}{x} = 6$

$\frac{x^2+1}{x^2} = 6$

$x^2 + 1 = 6x$

$x^2 - 6x + 1 = 0$

$= \frac{6 \pm \sqrt{6^2-4.1.1}}{2.1}$

$= \frac{6 \pm \sqrt{36-4}}{2}$

$= \frac{6 \pm \sqrt{32}}{2}$

$= \frac{6 \pm 4\sqrt{2}}{2}$

$= \frac{2(3 \pm 2\sqrt{2})}{2}$

$= (3 \pm 2\sqrt{2})$

(H/W)

$\mathbf{2x^4 + x^3 - 11x^2 + x + 2 = 0.}$

$Ans = 2, \quad \frac{1}{2}, \quad \frac{-3 \pm \sqrt{5}}{2}$

6. Solve

$\mathbf{2x^5 + x^4 - 12x^3 - 12x^2 + x + 2 = 0.}$

***Sol***: Given eq"n is a reciprocal eq"n of first class and add degree. So -1 is a root of it.

***By synthetic division***

| | | | | | |
|---|---|---|---|---|---|
| 2 | 1 | -12 | -12 | 1 | 2 |
| 0 | -2 | 1 | 11 | 1 | -2 |
| 2 | -1 | -11 | -1 | 2 | 0 |

Now the given eq"n can be written as

(x+1)( $\mathbf{2x^4 - 1x^3 - 11x^2 - 1x + 2}$) = **0.**

Consider $2x^4 - 1x^3 - 11x^2 - 1x + 2 = 0.$

$\div x^2$

$\Rightarrow \frac{2x^4}{x^2} - \frac{x^3}{x^2} - 11\frac{x^2}{x^2} - \frac{x}{x^2} + \frac{2}{x^2} = 0$

$\Rightarrow 2x^2 - 1x - 11 - \frac{1}{x} + \frac{2}{x^2} = 0$

$\Rightarrow 2\left(x^2 + \frac{1}{x^2}\right) - 1\left(x + \frac{1}{x}\right) - 11 = 0$

Let $x + \frac{1}{x} = p \Rightarrow x^2 + \frac{1}{x^2} = p^2 - 2$

$\Rightarrow 2(p^2 - 2) - 1p - 11 = 0$

$\Rightarrow 2p^2 - 4 - p - 11 = 0$

$\Rightarrow 2p^2 - p - 15 = 0$

$\Rightarrow 2p^2 - 6p + 5p - 15 = 0$

$\Rightarrow 2p(p-3) + 5(p-3) = 0$

$\Rightarrow (p-3)(2p+5) = 0$

$\Rightarrow (p-3) = 0\ ,\quad (2p+5) = 0$

$\Rightarrow p = 3 \quad and \quad p = -\frac{5}{2}$

Case (1)

$x + \frac{1}{x} = 3$

$\Rightarrow \frac{x^2+1}{x} = 3$

$\Rightarrow x^2 + 1 = 3x$

$\Rightarrow x^2 - 3x + 1 = 0$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{3 \pm \sqrt{3^2-4.1.1}}{2.1}$

$= \frac{3 \pm \sqrt{9-4}}{2}$

$= \frac{3 \pm \sqrt{5}}{2}$

case (2)

$x + \frac{1}{x} = \frac{-5}{2}$

$\frac{x^2+1}{x^2} = -\frac{5}{2}$

$2x^2 + 2 = -5x$

$2x^2 + 5x + 2 = 0$

$= \frac{-5 \pm \sqrt{5^2-4.2.2}}{2.2}$

$= \frac{-5 \pm \sqrt{25-16}}{4}$

$= \frac{-5 \pm \sqrt{9}}{4}$

$= \frac{-5 \pm 3}{4}$

$= \frac{-5+3}{4} \; or \; \frac{-5-3}{4} = -\frac{1}{2} \; or -2$

$\left(\frac{H}{W}\right) \mathbf{x^5 - 5x^4 + 9x^3 - 9x^2 + 5x - 1 = 0.}$

$Ans = 1, \frac{3 \pm \sqrt{5}}{2}, \frac{1 \pm i\sqrt{3}}{2}$

7. Solve

$\mathbf{6x^6 - 25x^5 + 31x^4 - 31x^2 + 25x - 62 = 0.}$

***Sol***: Given eq''n is a reciprocal eq''n of 2nd class and even degree. So 1, -1 are the root of it.

***By synthetic division***

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 6 | -25 | 31 | 0 | -31 | 25 | -6 | |
| 0 | 6 | -19 | 12 | 12 | -19 | 6 | |
| 6 | -19 | 12 | 12 | -19 | 6 | 0 | |
| 0 | -6 | 25 | -37 | 25 | -6 | | |
| 6 | -25 | 37 | -25 | 6 | 0 | | |

Consider $6x^4 - 25x^3 + 37x^2 - 25x + 6 = 0.$

$\div x^2$

$\Rightarrow \frac{6x^4}{x^2} - 25\frac{x^3}{x^2} + 37\frac{x^2}{x^2} - \frac{25x}{x^2} + \frac{6}{x^2} = 0$

$\Rightarrow 6x^2 - 25x + 37 - \frac{25}{x} + \frac{6}{x^2} = 0$

$\Rightarrow 6\left(x^2 + \frac{1}{x^2}\right) - 25\left(x + \frac{1}{x}\right) + 37 = 0$

Let $x + \frac{1}{x} = p \Rightarrow x^2 + \frac{1}{x^2} = p^2 - 2$

$\Rightarrow 6(p^2 - 2) - 25p + 37 = 0$

$\Rightarrow 6p^2 - 12 - 25p + 37 = 0$

$\Rightarrow 6p^2 - 25p + 25 = 0$

$\Rightarrow 6p^2 - 15p - 10p + 25 = 0$

$\Rightarrow 3p(2p - 5) - 5(2p - 5) = 0$

$\Rightarrow (2p - 5)(3p - 5) = 0$

$\Rightarrow (2p - 5) = 0\ ,\quad (3p - 5) = 0$

$\Rightarrow 2p = 5 \quad and \qquad p = \frac{5}{3}$

Case (1)

$x + \frac{1}{x} = \frac{5}{3}$

$\Rightarrow \frac{x^2+1}{x} = \frac{5}{3}$

$\Rightarrow 3x^2 + 3 = 5x$

$\Rightarrow 3x^2 - 5x + 3 = 0$

case (2)

$x + \frac{1}{x} = \frac{5}{2}$

$\frac{x^2+1}{x^2} = \frac{5}{2}$

$2x^2 + 2 = 5x$

$2x^2 + 2 = 5x$

$2x^2 - 5x + 2 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Case (1):

$= \frac{5 \pm \sqrt{5^2 - 4.3.3}}{2.3}$

$= \frac{5 \pm \sqrt{25 - 36}}{6}$

$= \frac{5 \pm \sqrt{11i^2}}{6}$

$= \frac{5 \pm i\sqrt{11}}{6}$

Case (2):

$= \frac{5 \pm \sqrt{5^2 - 4.2.2}}{2.2}$

$= \frac{5 \pm \sqrt{25 - 16}}{4}$

$= \frac{5 \pm \sqrt{9}}{4}$

$= \frac{5 \pm 3}{4}$

$= \frac{5+3}{4} \ or \ \frac{5-3}{4}$

$= 2 \ or \ \frac{1}{2}$

$\therefore req\ roots\ are\ -1, 1, 2, \frac{1}{2}, = \frac{5 \pm \sqrt{11}i}{6}$

## When condition is given

8. **Solve the eq'n $x^4 + 4x^3 - 2x^2 - 12x + 9 = 0$, Given that it has two pairs of equal roots.**

Sol: f(x) $= x^4 + 4x^3 - 2x^2 - 12x + 9 = 0$

Given that above eq''n has two pairs of equal roots.

let the roots $\alpha, \alpha, \beta, \beta$

$S_1 = -\frac{b}{a}$ $S_4 = \frac{e}{a}$

$\alpha + \alpha + \beta + \beta = -\frac{4}{1}$ $\alpha.\alpha.\beta.\beta = \frac{9}{1}$

⇨ $2(\alpha + \beta)$ =-4 ⇨ $(\alpha\beta)^2$ =9

⇨ $(\alpha + \beta)$ = -2..... (1) ⇨ $(\alpha\beta) = \pm 3$....(2)

Quadratic eq''n with roots $\alpha, \beta$ is

$x^2 - (\alpha + \beta)x + (\alpha\beta) = 0$

$x^2 + 2x - 3 = 0$

$x^2 + 3x - 1x - 3 = 0$

$x(x+3) - 1(x+3) = 0$

$(x+3)(x-1) = 0$

$(x+3) = 0, (x-1) = 0$

⇨ $x = -3, x = 1$

Hence the required roots of the given biquadratic eq''n are -3, -3, 1, 1.

9. **Solve $x^3 - 9x^2 + 14x + 24 = 0$, given that two of the roots are in the ratio 3:2.**

**Sol:** f(x) $= x^3 - 9x^2 + 14x + 24 = 0$

Given that above eq''n has two of the roots are in the ratio 3:2

let the roots $2\alpha, 3\alpha, \beta$

$S_1 = -\frac{b}{a}$ $S_3 = -\frac{d}{a}$

$2\alpha + 3\alpha + \beta = -(-\frac{9}{1})$ $2\alpha.3\alpha.\beta. = -24$

⇨ $(5\alpha + \beta) = 9$ ⇨ $\alpha^2(\beta) = -4$...(2)

$\beta = 9 - 5\alpha$ ..... (1)

$S_2 = \frac{c}{a}$

$= (2\alpha)(3\alpha) + (3\alpha)(\beta) + (2\alpha)(\beta) = 14$

$6\alpha^2 + 5\alpha\beta = 14$ ... (3)

Solving eq''n(1) in (2)

⇨ $\alpha^2(\beta) = -4$

⇨ $\alpha^2(9 - 5\alpha) = -4$

⇨ $5\alpha^3 - 9\alpha^2 - 4 = 0$ ... (4)

*by observation*

if $\alpha = 1$ ⇨ $5 - 9 - 4 \neq 0$

if $\alpha = -1$ ⇨ $-5 - 9 - 4 \neq 0$

if $\alpha = 2$ ⇨ $5(8) - 9(4) - 4 = 40 - 40 = 0$

∴ $\alpha = 2$ *is a root*

Sub $\alpha = 2$ in (1) ⇨ $\beta = 9 - 5(2) = 1$

$\alpha = 2$ *and* $\beta = -1$ *satisfy the* (3) *also*

∴roots are $2\alpha = 4, 3\alpha = 6, \beta = -1\{-1, 4, 6\}$

10. **Solve $x^4 + x^3 - 16x^2 - 4x + 48 = 0$, given that the product of two of the roots is 6.**

**Sol:**

$f(x) = x^4 + x^3 - 16x^2 - 4x + 48 = 0$ ....(1)

*let* $\alpha, \beta, \gamma, \delta$ *be the root of* (1)

Given $\alpha\beta = 6$, *let* $\alpha + \beta = p$

$S_4 = \frac{e}{a} = \alpha\beta\gamma\delta = 48$

⇨ $(6)\gamma\delta = 48$ ⇨ $\gamma\delta = 8$, *let* $(\gamma + \delta) = q$

| | |
|---|---|
| *now, Q.E with roots* $\alpha, \beta$ *is*<br>$x^2 - (\alpha + \beta)x + \alpha\beta = 0$<br>$x^2 - px + 6 = 0$ ... ... (2) | *Q.E with roots* $\gamma, \delta$ *is*<br>$x^2 - (\gamma + \delta)x + \gamma\delta = 0$<br>$x^2 - qx + 8 = 0$ ... ... (3) |

From (1), (2), (3)

⇨ $x^4 + x^3 - 16x^2 - 4x + 48$

$= (x^2 - px + 6)(x^2 - qx + 8)$

⇨ $x^4 + x^3 - 16x^2 - 4x + 48$

$= \begin{bmatrix} x^4 - qx^3 + 8x^2 - px^3 \\ +pqx^2 - 8px + 6x^2 - 6qx + 48 \end{bmatrix}$

⇨ $x^4 + x^3 - 16x^2 - 4x + 48$

$= [x^4 - (p+q)x^3 + (pq+14)x^2 - (8p+6q)x + 48]$

Comparing the coefficient of $x^3$, *we get*

$P + q + 1 = 0$ ... (a)

Comparing the coefficient of $x$, *we get*

$8p + 6q - 4 = 0$

$4p + 3q - 2 = 0$ ... (b)

Solving (a), (b)

| | | | |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 4 | 3 | -2 | 4 |

$(p, q) = \left[\frac{-2-3}{3-4}, \frac{4+2}{3-4}\right] = (5, -6)$

| | |
|---|---|
| $x^2 - px + 6 = 0$<br>⇨ $x^2 - 5x + 6 = 0$<br>⇨ $x^2 - 2x - 3x + 6 = 0$<br>⇨ $x(x-2) - 3(x-2) = 0$<br>⇨ $(x-2)(x-3) = 0$<br>$x = 2, 3$ | $x^2 - qx + 8 = 0$<br>⇨ $x^2 + 6x + 8 = 0$<br>⇨ $x^2 + 4x + 2x + 8 = 0$<br>⇨ $x(x+4) + 2(x+4) = 0$<br>⇨ $(x+4)(x+2) = 0$<br>$x = -4, -2$ |

∴ the roots are 2, 3, -2, -4

**Solve $x^4 - 5x^3 + 5x^2 + 5x - 6 = 0$, given that the product of two of the roots is 3.(H\W :Ans: 1, 3, -1, 2)**

**11. Solve $x^4 - 4x^2 + 8x + 35 = 0$, given that $2 + i\sqrt{3}$ is a root of the equation.**

Sol: $f(x) = x^4 - 4x^2 + 8x + 35 = 0 \ldots (1)$

Given $2 + i\sqrt{3}$ *is one of the root,*

*then* $2 - i\sqrt{3}$ *also a root of* $(1)$

$Q.E$ *with roots* $\alpha = 2 + i\sqrt{3}, \beta = 2 - i\sqrt{3}$ *is*

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$x^2 - \left(2 + i\sqrt{3} + 2 - i\sqrt{3}\right)x + (2 + i\sqrt{3})(2 + i\sqrt{3}) = 0$

⇨$x^2 - (4)x + (4 - 3i^2) = 0\ [i^2 = -1]$

⇨$x^2 - 4x + (4 + 3) = 0$

⇨$x^2 - 4x + 7 = 0$ is a factor of (1)

| | | | | |
|---|---|---|---|---|
| 1 | 0 | -4 | 8 | 35 |
| - | 4 | 16 | 20 | 0 |
| - | - | -7 | -28 | -35 |
| 1 | 4 | 5 | 0 | 0 |

*now,* $x^2 + 4x + 5 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

X$= \frac{-4 \pm \sqrt{4^2 - 4(1)(5)}}{2(1)} = \frac{-4 \pm \sqrt{-4}}{2} = \frac{-4 \pm \sqrt{i^2 4}}{2}$

$= \frac{-4 \pm 2i}{2} = -2 \pm i$

The roots are $\mathbf{2 \pm i\sqrt{3}, -2 \pm i}$

Solve $x^4 + 2x^3 - 5x^2 + 6x + 2 = 0$, given that $1 + i$ is a root of the equation.(H\W $[Ans: 1 \pm i, -2 \pm \sqrt{3}]$)

**11 Find the root of**

$$x^4 - 16x^3 + 86x^2 - 176x + 105 = 0.$$

***SOl*:**

$f(x) = x^4 - 16x^3 + 86x^2 - 176x + 105 = 0.$

***now*** $f(1) = 1 - 16 + 86 - 176 + 105 = 0$

***hence* 1 *is a root of*** $f(x)$

$f(2) \neq 0$ 2 *is not a root of* $f(x)$

$f(3) = 0$, 3 *is also root of* $f(x)$

*By S.D*

| | | | | | |
|---|---|---|---|---|---|
| | 1 | -16 | 86 | -176 | 105 |
| 1 | 0 | 1 | -15 | 71 | -105 |
| | 1 | -15 | 71 | -105 | 0 |
| 3 | 0 | 3 | -36 | 105 | |
| | 1 | -12 | 35 | 0 | |

$x^2 - 12x + 35 = 0$

$x^2 - 5x - 7x + 35 = 0$

$x(x - 5) - 7(x - 5) = 0$

$(x - 5)(x - 7) = 0$

$\boldsymbol{x = 5, 7}$

$\therefore$ *roots are* $1, 3, 5, 7$

**12. find the polynomial equation whose roots are the translates of those of the equation**

$$\boldsymbol{x^4 - 5x^3 + 7x^2 - 17x + 11 = 0\ by - 2.}$$

***Sol*:**

***Given equation is***

$f(x) = \boldsymbol{x^4 - 5x^3 + 7x^2 - 17x + 11 = 0}$

| | | | | | |
|---|---|---|---|---|---|
| 2 | 1 | -5 | 7 | -17 | 11 |
| | **0** | 2 | -6 | 2 | -30 |
| **2** | **1** | -3 | 1 | -15 | $-19A_4$ |
| | 0 | 2 | -2 | -2 | |
| 2 | 0 | 1 | -1 | $-17A_3$ | |
| | 0 | 2 | 2 | | |
| 2 | 1 | 1 | $1A_2$ | | |
| | 0 | 2 | | | |
| 2 | $1A_0$ | $3A_1$ | | | |
| | 0 | $A0$ | | | |

$\therefore$ *requried transformed eq''n is*

$x^4 + 3x^3 + x^2 - 17x - 19 = 0$

Aims

13 find the polynomial equation whose roots are the translates of those of the equation

$$x^5 + 4x^3 - x^2 + 11 = 0 \quad by - 3.$$

***Sol:Given equation is***

$$f(x) = x^5 + 4x^3 - x^2 + 11 = 0$$

| | | | | | | |
|---|---|---|---|---|---|---|
| 3 | 1 | 0 | 4 | -1 | 0 | 11 |
| | **0** | 3 | 9 | 39 | 114 | 342 |
| **3** | **1** | 3 | 13 | 38 | 114 | $353A_5$ |
| | 0 | 3 | 18 | 93 | 393 | |
| 3 | 1 | 6 | 31 | 131 | $507A_4$ | |
| | 0 | 3 | 27 | 174 | | |
| 3 | 1 | 9 | 58 | $305A_3$ | | |
| | 0 | 3 | 36 | | | |
| 3 | 1 | 12 | $94A_2$ | | | |
| | 0 | 3 | | | | |
| | $1A_0$ | $15A_1$ | | | | |
| | | 0 | $A0$ | | | |

$\therefore$ *requried transformed eq''n is*

$$x^5 + 15x^4 + 94x^3 + 305x^2 + 507x + 353 = 0.$$