1. The probability distribution of a random variable is given by

$X = x$	1	2	3	4	5
P(X=x)	1k	2k	3k	4k	5k

Find the value of k, mean & variance of X.

Sol:

Given that X is a random variable

Sum of the probabilities =1⇨ $\sum \boldsymbol{P(X = x_i) = 1}$

⇨ $1k + 2k + 3k + 4k + 5k = 1$

⇨ $15k = 1$

⇨ $k = \frac{1}{15}$

Let

μ *be the mean and* σ^2 *be the variance of* X

$\underline{\boldsymbol{\mu = \sum x_i . P(X = x_i)}}$

$= 1(k) + 2(2k) + 3(3k) + 4(4k) + 5(5k)$

$= k + 4k + 9k + 16k + 25k$

$= 55k$

$= \frac{55}{15}$

$\mu = \frac{11}{3}$

$\underline{\boldsymbol{\sigma^2 = \sum x_i^2 . P(X = x_i) - \mu^2}}$

$= 1(k) + 4(2k) + 9(3k) + 16(4k) + 25(5k) - \boldsymbol{\mu^2}$

$= k + 8k + 27k + 64k + 125k - \boldsymbol{\mu^2}$

$= 225k - (\frac{11}{3})^2$

$= \frac{225}{15} - \frac{121}{9}$

$= 15 - \frac{121}{9}$

$= \frac{135-121}{9}$

$= \frac{14}{9}$

2. The probability distribution of a random variable is given by

$X = x$	-2	-1	0	1	2	3
P(X=x)	0.1	k	0.2	2k	0.3	k

Find the value of k, mean & variance of X.

Sol: Given that X is a random variable

Sum of the probabilities =1⇨ $\sum \boldsymbol{P(X = x_i) = 1}$

⇨ $0.1 + k + 0.2 + 2k + 0.3 + k = 1$

⇨ $4k + 0.6 = 1$

⇨ $4k = 1 - 0.6$

⇨ $4k = 0.4$ ⇨ $k = 0.1$ *or* $\frac{1}{10}$

Let μ *be the mean and* σ^2 *be the variance of* X

$\underline{\boldsymbol{\mu = \sum x_i . P(X = x_i)}}$

$= (-2)(0.1) + (-1)(k) + (0)(0.2) + (1)(2k) + (2)(0.3) + (3)(k)$

$= -0.2 - k + 0 + 2k + 0.6 + 3k$

$= 4k + 0.4$

$= 0.4 + 0.4$

$\mu = 0.8$

$\underline{\boldsymbol{\sigma^2 = \sum x_i^2 . P(X = x_i) - \mu^2}}$

$= (-2)^2(0.1) + (-1)^2(k) + (0)^2(0.2) + (1)^2(2k) + (2)^2(0.3) + (3)^2(k) - \boldsymbol{\mu^2}$

$= 4(0.1) + 1(k) + 0 + 1(2k) + 4(0.3) + 9(k) - \boldsymbol{\mu^2}$

$= 0.4 + 1.2 + 12k - \boldsymbol{\mu^2}$

$= 0.4 + 1.2 + 1.2 - (0.8)^2$

$= 2.8 - (0.8)^2$

$= 2.8 - 0.64$

$= 2.16$

3. **The probability distribution of a random variable is given by**

$X = x$			[illegible]	3	[illegible]		6	7
P(X =x)			[illegible]	2 k	[illegible]	k^2	$2k^2$	$7k^2 + k$

Find the value of k, mean of X. and P(0 < X < 5).

Sol: Given that X is a random variable

Sum of the probabilities =1⇨ $\sum \boldsymbol{P(X = x_i) = 1}$

⇨ $0 + k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$

⇨ $10k^2 + 9k - 1 = 0$

⇨ $10k^2 + 10k - 1k - 1 = 0$

⇨ $10k(k + 1) - 1(k + 1) = 0$

⇨ $(k + 1)(10k - 1) = 0$

⇨ $(k + 1) = 0, (10k - 1) = 0$

⇨ $k = -1,\ k = \frac{1}{10}$

Let

μ *be the mean and* σ^2 *be the variance of* X

$\boldsymbol{\mu = \sum x_i . P(X = x_i)}$

$= (0)(0) + (1)(k) + (2)(2k) + (3)(2k) + (4)(3k) + (5)(k^2) + (6)(2k^2) + 7(7k^2 + k) = 1$

Aims

$= 0 + k + 4k + 6k + 12k + 5k^2 + 12k^2 + 49k^2 + 7k$

$= 30k + 66k^2 \{ k = \frac{1}{10}, k^2 = \frac{1}{100} \}$

$= \frac{30}{10} + \frac{66}{100}$

$= \frac{300}{100} + \frac{66}{100}$

$= \frac{366}{100} = 3.66$

P (0 < X < 5) $= P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4)$

$= k + 2k + 2k + 3k$

=8k

$= 8(0.1) \{ k = \frac{1}{10} \}$

$= 0.8$

4. **A cubical die is thrown. Find the mean and variance of X, given number on the face that shows up.**

Sol:

$X = x$	1	2	3	4	5	6
P(X=x)	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

Let μ *be the mean and* σ^2 *be the variance of* X

$\boldsymbol{\mu = \sum x_i . P(X = x_i)}$

$= 1\left(\frac{1}{6}\right) + 2\left(\frac{1}{6}\right) + 3\left(\frac{1}{6}\right) + 4\left(\frac{1}{6}\right) + 5\left(\frac{1}{6}\right) + 6\left(\frac{1}{6}\right)$

$= \frac{1}{6}(1 + 2 + 3 + 4 + 5 + 6)$

$= \frac{1}{6}(21)$

$\mu = \frac{7}{2}$

$\boldsymbol{\sigma^2 = \sum x_i^2 . P(X = x_i) - \mu^2}$

$= 1\left(\frac{1}{6}\right) + 4\left(\frac{1}{6}\right) + 9\left(\frac{1}{6}\right) + 16\left(\frac{1}{6}\right) + 25\left(\frac{1}{6}\right) + 36\left(\frac{1}{6}\right) - \boldsymbol{\mu^2}$

$= \frac{1}{6}(1 + 4 + 16 + 25 + 36) - \boldsymbol{\mu^2}$

$= \frac{1}{6}(92) - \left(\frac{7}{2}\right)^2$

$= \frac{46}{3} - \frac{49}{4}$

$= \frac{184 - 147}{12}$

$= \frac{35}{12}$

5. The range of a random variable X is {0, 1, 2}. Given that P(X=0)=3c^3, P(x=1)4c-10c^2, P(X=2)=5c-1 where c is constant Find (i) the value of c (ii)P(X<1) (iii) P(1<X≤ 2) (iv) P(0<X<3)

Sol: Given

P(X=0) =3c^3, P(x=1)=4c-10c^2,

P(X=2) =5c-1.

Given that X is a random variable

Sum of the probabilities =1⇨ $\sum \boldsymbol{P(X = x_i) = 1}$

⇨ $3c^3 + 4c - 10c^2 + 5c - 1 = 1$

⇨ $3c^3 - 10c^2 + 9c - 2 = 0$

By observation $c = 1$ *satisfy the the eq''n*

By synthetic division

	3	-10	9	-2
1	0	3	-7	2
	3	-7	2	0

Now the above eq''n becomes

⇨$(c-1)(3c^2 - 7c + 2) = 0$

⇨$(c = 1), (3c^2 - 7c + 2) = 0$

⇨ $3c^2 - 7c + 2 = 0$

⇨ $3c^2 - 6c - c + 2 = 0$

⇨ $3c(c-2) - 1(c-2) = 0$

⇨ $(c-2)(3c-1) = 0$

⇨ $(c-2) = 0, (3c-1) = 0$

⇨ $c = 2,\ c = \frac{1}{3}$

$\therefore c = 1, 2, \frac{1}{3}$

$c = 1, 2$ *are not possible* So, the value of c is $\frac{1}{3}$

(ii) $P(X < 1) = P(X = 0) = 3c^3$

$= 3\left(\frac{1}{3}\right)^3 = \frac{3}{27} = \frac{1}{9}$

$(iii) P(1 < X \leq 2) = P(X = 2) = 5c - 1$

$= 5\left(\frac{1}{3}\right) - 1 = \frac{5-3}{3} = \frac{2}{3}$

$(iv) P(0 < X < 3) = P(X = 1) = P(X = 2)$

$= 4c - 10c^2 + 5c - 1$

$= 9c - 10c^2 - 1 = \frac{9}{3} - \frac{10}{9} - 1$

$= \frac{27-10-9}{9}$

$= \frac{8}{9}$

6. A random variable X has the range {1, 2, 3....}. If P(X= K) = $\frac{c^K}{K!}$ *for* $K = 1, 2, \ldots$..then find C and P (0 < X < 3).

Sol: $P(X = K) = \frac{c^K}{K!}$ $for\ K = 1, 2, \ldots$..

Sum of the probabilities =1⇨ $\sum \boldsymbol{P(X = x_i) = 1}$

⇨ $\frac{c^1}{1!} + \frac{c^2}{2!} + \frac{c^3}{3!} + \cdots = 1$

Adding '1'on both sides

⇨ $1 + \frac{c^1}{1!} + \frac{c^2}{2!} + \frac{c^3}{3!} + \cdots = 1+1$

$$\left\{ \begin{array}{c} \therefore e^x = 1 + \frac{x^1}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \\ a^x = N ⇨ x = \log_a N \end{array} \right\}$$

⇨$e^c = 2$

⇨$c = \log_e 2$

$P(0 < X < 3) = P(X = 1) + P(X = 2)$

$= \frac{c^1}{1!} + \frac{c^2}{2!}$

$= \frac{(\log_e 2)^1}{1!} + \frac{(\log_e 2)^2}{2!}$

Aims

7. A random variable X has the range {0, 1, 2, 3....}. If P(X= K) = $\frac{c(k+1)}{2^k}$ *for* $K = 0, 1, 2, \ldots$..find c.

Sol:

$P(X = K) = \frac{c(k+1)}{2^k}$ $for\ K = 0, 1, 2, \ldots$..

$= \frac{c(0+1)}{2^0} + \frac{c(1+1)}{2^1} + \frac{c(2+1)}{2^2} + \frac{c(3+1)}{2^3} + \cdots. = 1$

⇨ $\frac{c(1)}{1} + \frac{c(2)}{2} + \frac{c(3)}{2^2} + \frac{c(4)}{2^3} + \cdots. = 1$

⇨ $c\left[1 + 2\left(\frac{1}{2}\right) + 3\left(\frac{1}{2}\right)^2 + 4\left(\frac{1}{2}\right)^3 + \cdots\right] = 1$

$\{\therefore\ (1-x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + \cdots\}$

⇨ $c\left[\left(1 - \frac{1}{2}\right)^{-2}\right] = 1$

⇨ $c\left[\left(\frac{1}{2}\right)^{-2}\right] = 1$

⇨ $c\left[\left(\frac{2}{1}\right)^2\right] = 1$

⇨ $c[4] = 1$ ⇨$c = \frac{1}{4}$

8. **If the mean and variance of binomial variants X are 2.4 and 1.44 respectively, find $P(1 < X \le 4)$.**

Sol:

$Given\ mean(np) = 2.4 \quad \dots.(1)$

$variance(npq) = 1.44 \quad \dots..(2)$

$\frac{(1)}{(2)} \Rightarrow \frac{npq}{np} = \frac{1.44}{2.4}$

$\Rightarrow q = \frac{144}{240} = \frac{3}{5}$

$\therefore p + q = 1 \Rightarrow p = 1 - q$

$p = 1 - \frac{3}{5} \quad = \frac{2}{5}$

$from\ (1) \Rightarrow np = 2.4$

$\Rightarrow n = \frac{2.4}{p} = \frac{2.4}{\frac{2}{5}} = 6$

$required\ probability\ \boldsymbol{P}(\mathbf{1} < X \le 4)$

$= P(X = 2) = P(X = 3) + P(X = 4)$

$\{\ P(X = r) = n_{c_r} p^r q^{n-r}\ \}$

$=6_{c_2}\left(\frac{2}{5}\right)^2\left(\frac{3}{5}\right)^{6-2} + 6_{c_3}\left(\frac{2}{5}\right)^3\left(\frac{3}{5}\right)^{6-3} + 6_{c_4}\left(\frac{2}{5}\right)^4\left(\frac{3}{5}\right)^{6-4}$

$= 6_{c_2}\left(\frac{2}{5}\right)^2\left(\frac{3}{5}\right)^4 + 6_{c_3}\left(\frac{2}{5}\right)^3\left(\frac{3}{5}\right)^3 + 6_{c_4}\left(\frac{2}{5}\right)^4\left(\frac{3}{5}\right)^2$

$= 15\left(\frac{4\times81}{5^6}\right) + 20\left(\frac{8\times27}{5^6}\right) + 15\left(\frac{16\times9}{5^6}\right)$

$= \frac{12\times81+32\times27+48\times9}{5^5}$

$= \frac{972+864+432}{5^5}$

$= \frac{2268}{3125}$

9. **If the difference between the mean and the variance of a binomial variance is 5/9, then find the probability for the event of 2 successes, when the experiment is conducted 5 times.**

Sol:

$let\ n, p\ be\ the\ parameters\ of\ the$

$Binomial$ Distribution

$given\ that\ n = 5,$

$np - npq = \frac{5}{9} \dots \dots..(1)$

$\Rightarrow np(1 - q) = \frac{5}{9}$

$\Rightarrow np(p) = \frac{5}{9}$

$\Rightarrow 5p^2 = \frac{5}{9}$

$\Rightarrow p^2 = \frac{1}{9} \Rightarrow p = \frac{1}{3}, q = 1 - p = 1 - \frac{1}{3} = \frac{2}{3}$

$required\ probability\ \boldsymbol{P}(X = 2)$

$\{\ P(X = r) = n_{c_r} p^r q^{n-r}\ \}$

$=5_{c_2}\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^{5-2}$

$=\frac{5\times4}{2\times1}\left(\frac{1}{9}\right)\left(\frac{8}{27}\right)$

$= \frac{80}{243}$

10. **One in nine ships is likely to be wrecked when they are set on sail. When 6 ships are set on sail, find the probability (i)at least one will arrive safely.(ii)exactly three will arrive safely.**

Sol:$let\ n, p\ be\ the\ parameters$

$Given\ q = \frac{1}{9}\{failure\}$

$p = 1 - q = 1 - \frac{1}{9} = \frac{8}{9}(successes)$, n=6

$(i) probability\ of$ at least one will

arrive safely $= total - non\ of\ them\ arrive\ safely.$

$= 1 - P(X = 0)$

$=1 - 6_{c_0}\left(\frac{8}{9}\right)^0\left(\frac{1}{9}\right)^{6-0} = 1 - \frac{1}{9^6}$

(ii) $probability\ of$ Exactly three will arrive safely

$(X = 3) = 6_{c_3}\left(\frac{8}{9}\right)^3\left(\frac{1}{9}\right)^{6-3}$

$= \frac{6\times5\times4}{3\times2\times1}\left(\frac{8^3}{9^6}\right) = 20\left(\frac{8^3}{9^6}\right)$

11. In the experiment to tossing a coin n times, if the variable X denotes the number of heads and P(X=4), P(X=5), P(X=6) are in A.P, and then find n.

Sol:

let n, p be the parameters of the Binomial Distribution

$p = \frac{1}{2}$ *getting a head(successes)*

$q = \frac{1}{2}$ *getting a head(failure)*

Given that

$P(X = 4), P(X = 5), P(X = 6)$ *are in A.P*

$$\left\{\begin{array}{c} P(X = r) = n_{c_r} p^r q^{n-r} \\ P(X = 4) = n_{c_4} \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^{n-4} \\ = n_{c_4} \left(\frac{1}{2}\right)^n \end{array}\right\}$$

$P(X = 4) = n_{c_4} \left(\frac{1}{2}\right)^n$

$P(X = 5) = n_{c_5} \left(\frac{1}{2}\right)^n$

$P(X = 6) = n_{c_6} \left(\frac{1}{2}\right)^n$

if a, b, c are in A.P

$\Rightarrow 2 = \frac{(a+c)}{b} = \frac{a}{b} + \frac{c}{b}$

$$\frac{n_{c_r}}{n_{c_{r+1}}} = \frac{r+1}{n-r}$$

Aims

$\Rightarrow 2 = \frac{n_{c_4}\left(\frac{1}{2}\right)^n}{n_{c_5}\left(\frac{1}{2}\right)^n} + \frac{n_{c_6}\left(\frac{1}{2}\right)^n}{n_{c_5}\left(\frac{1}{2}\right)^n}$

$\Rightarrow 2 = \frac{n_{c_4}}{n_{c_5}} + \frac{n_{c_6}}{n_{c_5}}$ $\Rightarrow 2 = \frac{5}{n-4} + \frac{n-5}{6}$

$\Rightarrow 2(n-4)(6) = 30 + (n-5)(n-4)$

$\Rightarrow 12n - 48 = 30 + n^2 - 4n - 5n + 20$

$\Rightarrow 12n - 48 = n^2 - 9n + 50$

$\Rightarrow n^2 - 9n - 12n + 50 + 48 = 0$

$\Rightarrow n^2 - 21n + 98 = 0$

$\Rightarrow n^2 - 7n - 14n + 98 = 0$

$\Rightarrow n(n-7) - 14(n-7) = 0$

$\Rightarrow (n-7)(n-14) = 0$

$\Rightarrow (n-7) = 0, (n-14) = 0$

$\Rightarrow n = 7, n = 14$