1. If $f: A \rightarrow B, g: B \rightarrow C$ are two bijections, then prove that gof : $A \rightarrow C$ is also a bijection.
2. If $f: A \rightarrow B, g: B \rightarrow C$ are two bijections, then prove tht (gof) ${ }^{-1}: f^{-1} \circ g^{-1}$.
3. If $f: A \rightarrow B, I_{A}$ and $I_{B}$ are identity functions on $A$ and $B$ respectively, then prove that foI $I_{A}=I_{B}$ of $=f$.
4. If $f: A \rightarrow B$ is a bijection then show that $f$ of $f^{-1}=I_{B}$ and $f-1$ of $=I_{A}$.
5. Prove that composition of mappings is associative. OR

If $f: A \rightarrow B, g: B \rightarrow C, h: C \rightarrow D$ are functions, then prove that ho $(g \circ f)=(h \circ g)$ of.
6. If $f: A \rightarrow B, g: B \rightarrow A$ are two functions such that $g o f=I_{A}$ and fog $=I_{B}$, then prove that $g=f^{-1}$.
7. Let $f=\{(1, a),(2, c),(3, b),(4, d)\}$ and $g^{-1}=\{(2, a),(4, b),(1, c),(3, d)\}$, then show that $(g \circ f)^{-1}=f^{-1} \circ g^{-1}$.
8. If $f: Q \rightarrow Q$ is defined by $f(x)=5 x+4$ for all $x \in Q$, show that $f$ is a bijection and find $f^{-1}$.
9. If $f: A \rightarrow B$ is a bijection, then prove that $f^{-1}: B \rightarrow A$ is a bijection

## MATHEMATICAL INDUCTION

1. Using induction, prove that $1^{2}+\left(1^{2}+2^{2}\right)+\left(1^{2}+2^{2}+3^{2}\right)+\ldots \ldots$. upto $n$ terms $=\frac{n(n+1)^{2}(n+2)}{12}$.
2. Using induction, prove that $1.2 .3+2.3 .4+3.4 .5+\ldots \ldots .$. upto $n$ terms $=\frac{n(n+1)(n+2)(n+3)}{4}$.
3. Prove that $a+(a+d)+(a+2 d)+$ $\qquad$ upto $n$ terms $=\frac{n}{2}[2 a+(n-1) d]$ by induction.
4. Prove that $a+a r+a r^{2}+$ upto $n$ terms $=\frac{a\left(r^{n}-1\right)}{r-1}, r \neq 1$ by induction.
5. Show that $\frac{1}{1.4}+\frac{1}{4.7}+\frac{1}{7.10}+\ldots \ldots .$. upto $n$ terms $=\frac{n}{3 n+1}$ for all $n \in N$.
6. Using mathematical induction, prove that $2.4^{2 n+1}+3^{3 n+1}$ is divisible by $11, \forall n \in N$.
7. Use mathematical induction to prove that $3 \cdot 5^{2 n+1}+2^{3 n+1}$ is divisible by 17 .
8. Show that $49^{n}+16 n-1$ is divisible by 64 for all positive integers $n$.
9. Prove that $\frac{1^{3}}{1}+\frac{1^{3}+2^{3}}{1+3}+\frac{1^{3}+2^{3}+3^{3}}{1+3+5}+\ldots \ldots \ldots$. upto $n$ terms $=\frac{n}{24}\left(2 n^{2}+9 n+13\right)$.
10. Prove that $2.3+3.4+4.5+\ldots \ldots \ldots$ upto $n$ terms $=\frac{n\left(n^{2}+6 n+11\right)}{3}$.
11. Use mathematical induction to prove the statement $2+3.2+4.2^{2}+\ldots \ldots$. upto $n$ terms $=n .2^{n} n \in N$.
12. Use induction to prove that $(1+x)^{n}>1+n x$ for $n \geq 2, x>-1, x \neq 0$.
13. By induction, prove that $4^{3}+8^{3}+12^{3}+\ldots \ldots$. upto $n$ terms $=16 n^{2}(n+1)^{2}$.
14. Using mathematical induction prove that $\left(1+\frac{3}{1}\right)\left(1+\frac{5}{4}\right)\left(1+\frac{7}{9}\right) \cdots \cdots\left(1+\frac{2 n+1}{n^{2}}\right)=(n+1)^{2}$

MATRICES

1. Show that $\left|\begin{array}{lll}a & b & c \\ b & c & a \\ c & a & b\end{array}\right|^{2}=\left|\begin{array}{ccc}2 b c-a^{2} & c^{2} & b^{2} \\ c^{2} & 2 c a-b^{2} & a^{2} \\ b^{2} & a^{2} & 2 a b-c^{2}\end{array}\right|=\left(a^{3}+b^{3}+c^{3}-3 a b c\right)^{2}$.
2. Show that $\left|\begin{array}{ccc}a+b+2 c & a & b \\ c & b+c+2 a & b \\ c & a & c+a+2 b\end{array}\right|=2(a+b+c)^{3}$.
3. Show that $\left|\begin{array}{ccc}a-b-c & 2 a & 2 a \\ 2 b & b-c-a & 2 b \\ 2 c & 2 c & c-a-b\end{array}\right|=(a+b+c)^{3}$.
4. Without expanding the determinant, show that $\left|\begin{array}{lll}b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a\end{array}\right|=2\left|\begin{array}{lll}a & b & c \\ b & c & a \\ c & a & b\end{array}\right|$.
5. Find the value of $x$ if $\left|\begin{array}{ccc}x-2 & 2 x-3 & 3 x-4 \\ x-4 & 2 x-9 & 3 x-16 \\ x-8 & 2 x-27 & 3 x-64\end{array}\right|=0$.
6. Show that $\left|\begin{array}{ccc}b+c & c+a & a+b \\ a+b & b+c & c+a \\ a & b & c\end{array}\right|=a^{3}+b^{3}+c^{3}-3 a b c$.
7. If $\left|\begin{array}{lll}a & a^{2} & 1+a^{3} \\ b & b^{2} & 1+b^{3} \\ c & c^{2} & 1+c^{3}\end{array}\right|=0$ and $\left|\begin{array}{lll}a & a^{2} & 1 \\ b & b^{2} & 1 \\ c & c^{2} & 1\end{array}\right| \neq 0$ then show that $a b c=-1$.
8. Show that $\left|\begin{array}{ccc}a & b & c \\ a^{2} & b^{2} & c^{2} \\ a^{3} & b^{3} & c^{3}\end{array}\right|=a b c(a-b)(b-c)(c-a)$.
9. Show that $\left|\begin{array}{lll}1 & a^{2} & a^{3} \\ 1 & b^{2} & b^{3} \\ 1 & c^{2} & c^{3}\end{array}\right|=(a-b)(b-c)(c-a)(a b+b c+c a)$.
10. If $A=\left[\begin{array}{lll}a_{1} & b_{1} & c_{1} \\ a_{2} & b_{2} & c_{2} \\ a_{3} & b_{3} & c_{3}\end{array}\right]$ is a non-singular matrix, then prove that (i) $A$ is invertible and (ii) $A^{-1}=\frac{\operatorname{AdjA}}{\operatorname{det} A}$.
11. If $A=$ then find $\left[\begin{array}{ccc}1 & -2 & 3 \\ 0 & -1 & 4 \\ -2 & 2 & 1\end{array}\right]$, then find $\left(A^{\prime}\right)^{-1}$.
12. If $3 A=\left[\begin{array}{ccc}1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1\end{array}\right]$ then show that $A^{-1}=A^{\prime}$.
13. If $A=\left[\begin{array}{ccc}-1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1\end{array}\right]$ then show that $\operatorname{Adj} A=3 A^{\prime}$, find $A^{-1}$.
14. Solve the following simultaneous linear equations by using Cramer's rule $3 x+4 y+5 z=18,2 x-y+8 z=13,5 x-2 y+7 z=20$.
15. Solve the following equations by using Matrix inversion method.
i) $3 x+4 y+5 z=18,2 x-y+8 z=13,5 x-2 y+7 z=20$
ii) $2 x-y+3 z=9, x+y+z=6, x-y+z=2$.
16. Solve the following linear equations by Gauss - Jordan method.
i) $x+y+z=1,2 x+2 y+3 z=6, x+4 y+9 z=3$.
ii) $x+y+z=9,2 x+5 y+7 z=52,2 x+y-z=0$
iii) $2 x-y+3 z=9, x+y+z=6, x-y+z=2$.
iv) $2 x-y+3 z=8,-x+2 y+z=4,3 x+y-4 z=0$.
v) $3 x+4 y+5 z=18,2 x-y+8 z=13,5 x-2 y+7 z=20$
17. Apply the test of rank to examine whether the following equations are consistent $2 x-y+3 z=8,-x+2 y+z=4,3 x+y-4 z=0$.
18. Show that the following system of equations is consistent and solve it completely: $x+y+z=3,2 x+2 y-z=3, x+y-z=1$.

## MULTIPLICATION OF VECTORS

1. If $\bar{a}, \bar{b}, \bar{c}$ are three vectors, prove that i) $(\bar{a} \times \bar{b}) \times \bar{c}=(\bar{a} \cdot \bar{c}) \bar{b}-(\bar{b} \cdot \bar{c}) \bar{a} . \quad$ ii) $\overline{\mathrm{a}} \times(\overline{\mathrm{b}} \times \overline{\mathrm{c}})=(\overline{\mathrm{a}} \cdot \overline{\mathrm{c}}) \overline{\mathrm{b}}-(\overline{\mathrm{a}} \cdot \overline{\mathrm{b}}) \overline{\mathrm{c}}$
2. Find the shortest distance between the lines $\bar{r}=6 \bar{i}+2 \bar{j}+2 \bar{k}+t(\bar{i}-2 \bar{j}+2 \bar{k})$ and $\bar{r}=-4 \bar{i}-\bar{k}+s(3 \bar{i}-2 \bar{j}-2 \bar{k})$. 3.S If $A=(1,-2,-1), B=(4,0,-3), C=(1,2,-1)$ and $D=(2,-4,-5)$, find the distance between $A B$ and CD.
3. Let $\overline{\mathrm{a}}=\overline{\mathrm{i}}+\overline{\mathrm{j}}+\overline{\mathrm{k}}, \overline{\mathrm{b}}=2 \overline{\mathrm{i}}-\overline{\mathrm{j}}+3 \overline{\mathrm{k}}, \overline{\mathrm{c}}=\overline{\mathrm{i}}-\overline{\mathrm{j}}$ and $\overline{\mathrm{d}}=6 \overline{\mathrm{i}}+2 \overline{\mathrm{j}}+3 \overline{\mathrm{k}}$. Express $\overline{\mathrm{d}}$ interms of $\overline{\mathrm{b}} \times \overline{\mathrm{c}}$, $\overline{\mathrm{c}} \times \overline{\mathrm{a}}$, and $\overline{\mathrm{a}} \times \overline{\mathrm{b}}$.
4. Let $\overline{\mathrm{a}}=\overline{\mathrm{i}}-2 \overline{\mathrm{j}}+\overline{\mathrm{k}}, \overline{\mathrm{b}}=2 \overline{\mathrm{i}}+\overline{\mathrm{j}}+\overline{\mathrm{k}}, \overline{\mathrm{c}}=\overline{\mathrm{i}}+2 \overline{\mathrm{j}}-\overline{\mathrm{k}}$ then find $(\overline{\mathrm{a}} \times \overline{\mathrm{b}}) \times \overline{\mathrm{c}}$ and $|\overline{\mathrm{a}} \times(\overline{\mathrm{b}} \times \overline{\mathrm{c}})|$.
5. If $\overline{\mathrm{a}}=\overline{\mathrm{i}}-2 \overline{\mathrm{j}}-3 \overline{\mathrm{k}}, \overline{\mathrm{b}}=2 \overline{\mathrm{i}}+\overline{\mathrm{j}}-\overline{\mathrm{k}}, \overline{\mathrm{c}}=\overline{\mathrm{i}}+3 \overline{\mathrm{j}}-2 \overline{\mathrm{k}}$, verify that $\overline{\mathrm{a}} \times(\overline{\mathrm{b}} \times \overline{\mathrm{c}}) \neq(\overline{\mathrm{a}} \times \overline{\mathrm{b}}) \times \overline{\mathrm{c}}$.
6. Find the equation of the plane passing throwgh the points $A(2,3,-1) B(4,5,2)$ and $C(3,6,5)$.
7. Find the equation of the plane passing through the point $A(3,-2,-1)$ and parallel to the vectors $\overline{\mathrm{b}}=\overline{\mathrm{i}}-2 \overline{\mathrm{j}}+4 \overline{\mathrm{k}}$, and $\overline{\mathrm{c}}=3 \overline{\mathrm{i}}+2 \overline{\mathrm{j}}-5 \overline{\mathrm{k}}$ in cartésian form.
8. Find $\lambda$ in order that the four points $A(3,2,1), B(4, \lambda, 5), C(4,2,-2)$, and $D(6,5,-1)$ be coplanar.
9. Show that the points $(5,-1,1)(7,-4,7)(1,-6,10)$ and $(-1,-3,4)$ are vertices of a rhombus.
10. Prove that the angle $\theta$ between any two diagonals of a cube is given by $\cos \theta=\frac{1}{3}$.
11. In any triangle, prove that the altitudes are concurrent.
12. In any triangle, prove that the perpendicular bisectors of the sides are concurrent.
13. If $\bar{a}=2 \bar{i}+\bar{j}-3 \bar{k}, \bar{b}=\bar{i}-2 \bar{j}+\bar{k}, \bar{c}=-\bar{i}+\bar{j}-4 \bar{k}$, and $\bar{d}=\bar{i}+\bar{j}+\bar{k}$ then compute $|(\bar{a} \times \bar{b}) \times(\bar{c} \times \bar{d})|$.
14. A line makes angles $\theta_{1}, \theta_{2}, \theta_{3}$ and $\theta_{4}$ with the diagonals of a cube, show that $\cos ^{2} \theta_{1}+\cos ^{2} \theta_{2}+\cos ^{2} \theta_{3}+\cos ^{2} \theta_{4}=\frac{4}{3}$.

## TRANSFORMATIONS

1. If $A+B+C=180^{\circ}$, then show that $\sin 2 A-\sin 2 B+\sin 2 C=4 \cos A \sin B \cos C$.
2. $A+B+C=180^{\circ}$ Then show that $\cos A+\cos B+\cos C=1+4 \sin A / 2 \sin A / 2 \sin C / 2$.
3. If $A, B, C$ are angles of a triangle, prove that $\cos 2 A+\cos 2 B+\cos 2 C=-4 \cos A \cos B \cos C-1$.
4. If $A, B, C$ are angles of a triangle, prove that $\sin A+\sin B-\sin C=4 \sin \frac{A}{2} \sin \frac{B}{2} \cos \frac{C}{2}$.
5. If $A, B, C$ are angles of a triangle, then prove that $\sin ^{2} A+\sin ^{2} B-\sin ^{2} C=2 \sin A \sin B \cos C$.
6. If $A, B, C$ are angles of a triangle, then prove that $\sin ^{2} \frac{A}{2}+\sin ^{2} \frac{B}{2}-\sin ^{2} \frac{C}{2}=1-2 \cos \frac{A}{2} \cos \frac{B}{2} \sin \frac{C}{2}$.
7. If $A+B+C=\pi$, then prove that $\cos ^{2} \frac{A}{2}+\cos ^{2} \frac{B}{2}+\cos ^{2} \frac{C}{2}=2\left(1+\sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}\right)$.
8. In triangle $A B C$, prove that $\cos \frac{A}{2}+\cos \frac{B}{2}+\cos \frac{C}{2}=4 \cos \left(\frac{\pi-A}{4}\right) \cos \left(\frac{\pi-B}{4}\right) \cos \left(\frac{\pi-C}{4}\right)$.
9. If $A, B, C$ are the angles in a triangle then prove that $\sin \frac{A}{2}+\sin \frac{B}{2}+\sin \frac{C}{2}=1+4 \sin \left(\frac{\pi-A}{4}\right) \sin \left(\frac{\pi-B}{4}\right) \sin \left(\frac{\pi-C}{4}\right)$
10. In $\triangle A B C$, prove that $\sin \frac{A}{2}+\sin \frac{B}{2}-\sin \frac{C}{2}=-1+4 \cos \left(\frac{\pi-A}{4}\right) \cos \left(\frac{\pi-B}{4}\right) \sin \left(\frac{\pi-C}{4}\right)$.
11. If $\mathrm{A}+\mathrm{B}+\mathrm{C}=0$, then prove that $\cos ^{2} \mathrm{~A}+\cos ^{2} \mathrm{~B}+\cos ^{2} \mathrm{C}=1+2 \cos \mathrm{~A} \cos \mathrm{~B} \cos \mathrm{C}$.
12. If $A+B+C=\frac{\pi}{2}$, then prove that $\cos 2 A+\cos 2 B+\cos 2 C=1+4 \sin A \sin B \sin C$.
13. If $A+B+C=\frac{3 \pi}{2}$, then prove that $\sin 2 A+\sin 2 B-\sin 2 C=-4 \sin A \sin B \cos C$.
14. If $A+B+C=2 S$, then prove that $\cos (S-A)+\cos (S-B)+\cos (S-C)+\cos S=4 \cos \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2}$.

## PROPERTIES OF TRIANGLES

1. Show that $\frac{r_{1}}{b c}+\frac{r_{2}}{c a}+\frac{r_{3}}{a b}=\frac{1}{r}-\frac{1}{2 R}$.
2. If $a=13, b=14, c=15$, show that $R=\frac{65}{8}, r=4, r_{1}=\frac{21}{2}, r_{2}=12, r_{3}=14$.
3. If $\triangle A B C$, if $r_{1}=8, r_{2}=12, r_{3}=24$, find $a, b, c$.
4. If $\triangle A B C$, if $r_{1}=2, r_{2}=3, r_{3}=6$, and $r=1$, prove that $a=3, b=4, c=5$.
5. If $p_{1}, p_{2}, p_{3}$ are the altitudes drawn from vertices $A, B, C$ to the opposite sides of a triangle respectively, then show that
i) $\frac{1}{p_{1}}+\frac{1}{p_{2}}+\frac{1}{p_{3}}=\frac{1}{r}$
ii) $\frac{1}{p_{1}}+\frac{1}{p_{2}}-\frac{1}{p_{3}}=\frac{1}{r_{3}}$
iii) $p_{1} p_{2} p_{2}=\frac{(a b c)^{2}}{8 R^{3}}=\frac{8 \Delta^{3}}{a b c} \quad$ iv) $\frac{1}{p_{1}^{2}}+\frac{1}{p_{2}^{2}}+\frac{1}{p_{3}^{2}}=\frac{\cot \mathrm{A}+\cot \mathrm{B}+\cot \mathrm{C}}{\Delta}$.
6. Show that $r+r_{3}+r_{1}-r_{2}=4 R \cos B$.
7. Show that $\frac{a b-r_{1} r_{2}}{r_{3}}=\frac{b c-r_{2} r_{3}}{r_{1}}=\frac{c a-r_{3} r_{1}}{r_{2}}$.
8. In $\triangle A B C$, prove that $\frac{\cot \frac{A}{2}+\cot \frac{B}{2} \cot \frac{C}{2}}{\cot A+\cot B+\cot C}=\frac{(a+b+c)^{2}}{a^{2}+b^{2}+c^{2}}$.
9. Show that $\operatorname{acos}^{2} \frac{A}{2}+b \cos ^{2} \frac{B}{2}+\cos ^{2} \frac{C}{2}=S+\frac{\Delta}{R}$.
10. Prove that $a^{3} \cos (B-C)+b^{3} \cos (C-A)+c^{3} \cos (A-B)=3 a b c$.
11. Show that $\sin ^{2} \frac{A}{2}+\sin ^{2} \frac{B}{2}+\sin ^{2} \frac{C}{2}=1-\frac{r}{2 R}$.
12. Show that $\cos ^{2} \frac{A}{2}+\cos ^{2} \frac{B}{2}+\cos ^{2} \frac{C}{2}=2+\frac{r C}{2 R}$
13. In triangle $\triangle A B C$, prove that $\frac{r_{1} r+r_{2} r_{3} r_{2} r+r_{1} r_{3}}{c a}=\frac{r_{3} r+r_{1} r_{2}}{a b}$.
14. The upper $\frac{3}{4}$ th portion ofla vertical pole subtends an angle $\operatorname{Tan}^{-1} \frac{3}{5}$ at a point in the horizontal plane through its foot and at a distance 40 m from the foot. Given that the vertical pole is at a height less than 100 m from the ground, find its height.

## SHORT ANSWER QUESTIONS (4 MARKS)

MATRICES

1. If $\theta-\phi=\pi / 2$, then show that $\left[\begin{array}{cc}\cos ^{2} \theta & \cos \theta \sin \theta \\ \cos \theta \sin \theta & \sin ^{2} \theta\end{array}\right]\left[\begin{array}{cc}\cos ^{2} \phi & \cos \phi \sin \phi \\ \cos \phi \sin \phi & \sin ^{2} \phi\end{array}\right]=0$.
2. If $A=\left[\begin{array}{lll}1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1\end{array}\right]$ then show that $A^{2}-4 A-5 I=0$.
3. If $\mathrm{I}=\left[\begin{array}{ll}1 & 0 \\ 0 & 1\end{array}\right], \mathrm{E}=\left[\begin{array}{ll}0 & 1 \\ 0 & 0\end{array}\right]$, S.T. $(\mathrm{aI}+\mathrm{bE})^{3}=a^{3} \mathrm{I}+3 \mathrm{a}^{2} \mathrm{bE}$.
4. If $A=\left[\begin{array}{ll}3 & -4 \\ 1 & -1\end{array}\right]$ then show that $A^{n}=\left[\begin{array}{cc}1+2 n & -4 n \\ n & 1-2 n\end{array}\right]$ for all positive integers $n$.
5. If $A=\left[\begin{array}{cc}\cos \theta & \sin \theta \\ -\sin \theta & \cos \theta\end{array}\right]$ then show that $A^{n}=\left[\begin{array}{cc}\cos n \theta & \sin n \theta \\ -\sin n \theta & \cos n \theta\end{array}\right]$ for all positive integers.
6. Show that $\left|\begin{array}{lll}b c & b+c & 1 \\ c a & c+a & 1 \\ a b & a+b & 1\end{array}\right|=(a-b)(b-c)(c-a)$.
7. Show that $\left|\begin{array}{ccc}a^{2}+2 a & 2 a+1 & 1 \\ 2 a+1 & a+2 & 1 \\ 3 & 3 & 1\end{array}\right|=(a-1)^{3}$.
8. If $A=\left[\begin{array}{lll}2 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 2 & 1\end{array}\right]$, then find the inverse of $A$.
9. Find the adjoint matrix and inverse matrix of the matrix $\left[\begin{array}{lll}1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4\end{array}\right]$
10. Find the inverse matrix of $\left[\begin{array}{lll}\mathbf{a} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{b} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{c}\end{array}\right]$.
11. If $A=\left[\begin{array}{lll}3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1\end{array}\right]$, then show that $A^{-1}=A^{3}$.
12. If $3 A=\left[\begin{array}{ccc}1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1\end{array}\right]$. Show that $A^{-1}=A^{1}$.
13. If $A=\left[\begin{array}{ccc}-1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1\end{array}\right]$ then show that $\operatorname{adj} A=3 A^{\top}$. Also find $A^{-1}$.
14. Show that $\left|\begin{array}{lll}x & \mathbf{a} & \mathbf{a} \\ \mathbf{a} & \mathbf{x} & \mathbf{a} \\ \mathbf{a} & \mathbf{a} & \mathbf{x}\end{array}\right|=(\mathbf{x}+\mathbf{2 a})(\mathbf{x}-\mathbf{a})^{2}$.
15. Examine whether the following system of equations are consistent or inconsistent and if consistent, find the complete solution: $x+y+z=1,2 x+y+z=2, x+2 y+2 z=1$.

## ADDITION OF VECTORS

1. If $O$ is centre of a regular hexagon $A B C D E F$, show that $\overline{A B}+\overline{A C}+\overline{\mathrm{AD}}+\overline{\mathrm{AE}}+\overline{\mathrm{AF}}=3 \overline{\mathrm{AD}}=6 \overline{\mathrm{AO}}$.
2. In $\triangle A B C$, if $O$ is the circumcentre and $H$ is the orthocentre, then show that
(i) $\overline{\mathrm{OA}}+\overline{\mathrm{OB}}+\overline{\mathrm{OC}}=\overline{\mathrm{OH}}$
(ii) $\overline{\mathrm{HA}}+\overline{\mathrm{HB}}+\overline{\mathrm{HC}}=2 \overline{\mathrm{HO}}$
3. Prove that the triangle formed by the vectors $3 \bar{i}+5 \bar{j}+2 \bar{k}, 2 \bar{i}-3 \bar{j}-5 \bar{k}$ and $-5 \bar{i}-2 \bar{j}+3 \bar{k}$ is equilateral.
4. Show that the points $A=(2 \bar{i}-\bar{j}+\bar{k}), B=(\bar{i}-3 \bar{j}-5 \bar{k}), C=(3 \bar{i}-4 \bar{j}-4 \bar{k})$ are the vertices of right angled triangle.
5. If $\bar{a}$ and $\bar{b}$ are the position vectors of the points $A$ and $B$ with respect to the origin and $P$ divides $A B$ in the ratio $m: n$, then prove that the position vector of $P$ is $\frac{n b+n a}{m+n}$.
6. If $\bar{a}, \bar{b}, \bar{c}$ are non coplanar vectors, then test for collinearity of the points whose position vectors are give as $3 \bar{a}-4 \bar{b}+3 \bar{c},-4 \bar{a}+5 \bar{b}-6 \bar{c}, 4 \bar{a}-7 \bar{b}+6 \bar{c}$.
7. If $\bar{a}, \bar{b}, \bar{c}$ are non-coplanar vectors, then prove that $3 \bar{a}+2 \bar{b}-5 \bar{c},-3 \bar{a}+8 \bar{b}-5 \bar{c},-3 \bar{a}+2 \bar{b}+\bar{c},-\bar{a}+4 \bar{b}-3 \bar{c}$ are coplanar points
8. If $\bar{a}, \bar{b}, \bar{c}$ are non-coplanar vectors, then prove that the following four points are coplanar.

$$
-\bar{a}+4 \bar{b}-3 \bar{c}, 3 \bar{a}+2 \bar{b}-5 \bar{c},-3 \bar{a}+8 b-5 \bar{c} \text { and }-3 \bar{a}+2 \bar{b}+\bar{c}
$$

9. If the points whose position vectors are $3 \bar{i}-2 \bar{j}-\bar{k}, 2 \bar{i}+3 \bar{j}-4 \bar{k},-\bar{i}+\bar{j}+2 \bar{k}$ and $4 \bar{i}+5 \bar{j}+\lambda \bar{k}$ are coplanar, then show that $\lambda=\frac{-146}{17}$.
10. If $\bar{a}+\bar{b}+\bar{c}=\alpha \bar{d}, \bar{b}+\bar{c}+\bar{d}=\beta \overline{\mathrm{a}}$ and $\overline{\mathrm{a}}, \overline{\mathrm{b}}, \overline{\mathrm{c}}$ are noncoplanar vectors, then show that $\overline{\mathrm{a}}+\overline{\mathrm{b}}+\overline{\mathrm{c}}+\overline{\mathrm{d}}=\overline{0}$.
11. In the two dimensional plane, prove by using vector methods, the equation of the line whose intercepts on the axes are ' $a$ ' and ' $b$ ' is $\frac{x}{a}+\frac{y}{b}=1$.
12. Find the vector equation of the plane passing through the points $4 \bar{i}-3 \bar{j}-\bar{k}, 3 \bar{i}+7 \bar{j}-10 \bar{k}$, and $2 \bar{i}+5 \bar{j}-7 \bar{k}$ and show that the point $\bar{i}+2 \bar{j}-3 \bar{k}$ lies in the plane.
13. Find the point of intersection of the line $\bar{r}=2 \bar{a}+\bar{b}+t(\bar{b}-\bar{c})$ and the plane $\bar{r}=\bar{a}+x(\bar{b}+\bar{c})+y(\bar{a}+2 \bar{b}-\bar{c})$, where are $\overline{\mathrm{a}}, \overline{\mathrm{b}}$ and $\overline{\mathrm{c}}$ are noncoplanar vectors.
14. $A B C D$ is a parallelogram. If $L$ and $M$ are the middle points of $B C$ and $C D$ respectively, then find
(i) $\overline{\mathbf{A L}}$ and $\overline{\mathbf{A M}}$ in terms of $\overline{\mathbf{A B}}$ and $\overline{\mathbf{A D}}$.
(ii) $\lambda$, if $\overline{\mathrm{AM}}=\lambda \overline{\mathrm{AD}}-\overline{\mathrm{LM}}$
15. In $\triangle A B C, P, Q$ and $R$ are the midpoints of the sides $A B, B C$ and $C A$ respectively. If $D$ is any point
(i) then express $\overline{\mathrm{DA}}+\overline{\mathrm{DB}}+\overline{\mathrm{DC}}$ in terms of $\overline{\mathrm{DP}}, \overline{\mathrm{DQ}}$ and $\overline{\mathrm{DR}}$.
(ii) If $\overline{\mathrm{PA}}+\overline{\mathrm{QB}}+\overline{\mathrm{RC}}=\bar{\alpha}$, then find $\overline{\boldsymbol{\alpha}}$.

# INTERMEDIATE FIRST YEAR MATHEMATICS IMPORTANT QUESTIONS AIMSTUTORIAL.IN <br> MULTIPLICATION OF VECTORS 

1. By using vector method, prove that angle in a semicircle is a right angle.
2. In $\triangle A B C$, if $\overline{\mathrm{BC}}=\overline{\mathrm{a}}, \overline{\mathrm{CA}}=\overline{\mathrm{b}}, \overline{\mathrm{AB}}=\overline{\mathrm{c}}$ then show that $\overline{\mathrm{a}} \times \overline{\mathrm{b}}=\overline{\mathrm{b}} \times \overline{\mathrm{c}}=\overline{\mathrm{c}} \times \overline{\mathrm{a}}$.
3. If $\bar{a}=2 \bar{i}-\bar{j}+\bar{k}$ and $\bar{b}=3 \bar{i}+4 \bar{j}-\bar{k}$ and $\theta$ is the angle between $\bar{a}$ and $\bar{b}$, then find $\sin \theta$.
4. Find a unit vector perpendicular to the plane determined by the points $P(1,-1,2), Q(2,0,-1)$ and $R(0,2,1)$.
5. Find the area of the triangle whose vertices are $\mathrm{A}(1,2,3), \mathrm{B}(2,3,1)$ and $\mathrm{C}(3,1,2)$.
6. Find the volume of the parallelepiped with coterminous edges $2 \bar{i}-3 \bar{j}+\bar{k}, \bar{i}-\bar{j}+2 \bar{k}$ and $2 \bar{i}+\bar{j}-\bar{k}$.
7. Find the volume of the tetrahedron having the coterminous edges $\bar{i}+\bar{j}+\bar{k}, \bar{i}-\bar{j}$ and $\bar{i}+2 \bar{j}+\bar{k}$.
8. Find the volume of the tetrahedron whose vertices are (1, 2, 1), (3, 2, 5), (2, -1, 0), (-1, 0, 1).
9. $\bar{a}, \bar{b}, \bar{c}$ are non zero vectors and $\bar{a}$ is perpendicular to both $\bar{b}$ and $\bar{c}$. If $|\bar{a}|=2,|\bar{b}|=3,|\bar{c}|=4$ and $(\bar{b}, \bar{c})=2 \pi / 3$, then find $|[\bar{a} \bar{b} \bar{c}]|$.
10. If $\bar{a}=\bar{i}-2 \bar{j}-3 \bar{k}, \bar{b}=2 \bar{i}+\bar{j}-\bar{k}$, and $\bar{c}=\bar{i}+3 \bar{j}-2 \bar{k}$, verify $\bar{a} \times(\bar{b} \times \bar{c}) \neq(\bar{a} \times \bar{b}) \times \bar{c}$.
11. Let $\overline{\mathrm{a}}=2 \overline{\mathrm{i}}+\overline{\mathrm{j}}-2 \overline{\mathrm{k}}, \overline{\mathrm{b}}=\overline{\mathrm{i}}+\overline{\mathrm{j}}$. If $\overline{\mathrm{c}}$ is a vector such that $\overline{\mathrm{a}} \cdot \overline{\mathrm{c}}=|\overline{\mathrm{c}}| .|\overline{\mathrm{c}}-\overline{\mathrm{a}}|=2 \sqrt{2}$ and the angle between
$\overline{\mathrm{a}} \times \overline{\mathrm{b}}$ and $\overline{\mathrm{c}}$ is $30^{\circ}$, than find the value of $|(\overline{\mathrm{a}} \times \overline{\mathrm{b}}) \times \overline{\mathrm{c}}|$.
If $\bar{a}=2 \bar{i}+\bar{j}-\bar{k}, \bar{b}=-i+2 \bar{j}-4 \bar{k}$ and $\bar{c}=\bar{i}+\bar{j}+\bar{k}$, then find $(\bar{a} \times \bar{b}) .(\bar{b} \times \bar{c})$.
For any vectors $\bar{a}, \bar{b}, \bar{c}, \bar{d}$ prove that $(\bar{a} \times \bar{b}) x(\bar{c} \times \bar{d})=[\bar{a} \bar{c} \bar{d}] \bar{b}-[\bar{b} \bar{c} \bar{d}] \bar{a}$
For any three vectors $\overline{\mathrm{a}}, \overline{\mathrm{b}}, \overline{\mathrm{c}}$ prove that $\left[\begin{array}{llll}\overline{\mathrm{b}} \times \overline{\mathrm{c}} & \overline{\mathrm{c}} \times \overline{\mathrm{a}} & \overline{\mathrm{a}} \times \overline{\mathrm{b}}\end{array}\right]=\left[\begin{array}{lll}\overline{\mathrm{a}} \overline{\mathrm{b}} & \overline{\mathrm{c}}\end{array}\right]^{2}$.
12. If $[\bar{b} \bar{c} \bar{d}]+[\bar{c} \bar{a} \bar{d}]+[\bar{a} \bar{b} \bar{d}]=[\bar{a} \bar{b} \bar{c}]$, then show that the points with position vectors $\overline{\mathbf{a}}, \overline{\bar{b}}, \overline{\mathbf{c}}, \overline{\mathbf{d}}$ are coplanar.
13. Let $\overline{\mathbf{a}}$ and $\overline{\mathbf{b}}$ be vectors, satisfying $|\overline{\mathbf{a}}||\overline{\mathbf{b}}|=\mathbf{5}$ and $(\overline{\mathbf{a}}, \overline{\mathbf{b}})=45^{\circ}$. Find the area of the triangle having $\overline{\mathbf{a}}-\mathbf{2} \overline{\mathbf{b}}$ and $\mathbf{3} \bar{a}+\mathbf{2} \bar{b}$ as two of its sides.
14. If $|\bar{a}|=\mathbf{2},|\overline{\mathbf{b}}|=\mathbf{3},|\overline{\mathbf{c}}|=\mathbf{4}$ and each of $\overline{\mathbf{a}}, \overline{\mathrm{b}}, \overline{\mathrm{c}}$ is perpendicular to the sum of other two vectors, then find the magnitude $\overline{\mathrm{a}}+\overline{\mathrm{b}}+\overline{\mathrm{c}}$.

## COMPOUND ANGLES \& MULTIPLES AND SUB MULTIPLES

1. Prove that $\frac{\boldsymbol{\operatorname { t a n }} \theta+\boldsymbol{\operatorname { s e c }} \theta-\mathbf{1}}{\boldsymbol{\operatorname { t a n }} \theta-\boldsymbol{\operatorname { s e c }} \theta+\mathbf{1}}=\frac{\mathbf{1}+\boldsymbol{\operatorname { s i n }} \theta}{\boldsymbol{\operatorname { c o s }} \theta}$.
2. If $A+B=45^{\circ}$, then prove that (i) $(1+\tan A)(1+\tan B)=2 \quad$ (ii) $(\cot A-1)(\cot B-1)=2$.
3. If $0<A<B<\pi / 4, \sin (A+B)=24 / 25, \cos (A-B)=4 / 5$, then find the value of $\tan 2 A$.
4. If $A+B, A$ are acute angles such that $\sin (A+B)=24 / 25$ and $\tan A=3 / 4$, find the value of $\cos B$.
5. Prove that $\tan 70^{\circ}-\tan 20^{\circ}=2 \tan 50^{\circ}$.
6. Prove that $\cos A \cos \left(60^{\circ}+A\right) \cos \left(60^{\circ}-A\right)=1 / 4 \cos 3 A$.
7. If $A+B+C=\frac{\pi}{2}$, then show that $\cot A+\cot B+\cot C=\cot A \cot B \cot C$.
8. Prove that $\sin 18^{\circ}=\frac{\sqrt{5}-1}{4}$.
9. Prove that (i) $\tan A+\cot A=2 \operatorname{cosec} 2 A$.
(ii) $\cot \mathrm{A}-\tan \mathrm{A}=2 \cot 2 \mathrm{~A}$.
10. 
11. 
12. 
13. Prove that $\sin ^{4} \frac{\pi}{8}+\sin ^{4} \frac{3 \pi}{8}+\sin ^{4} \frac{5 \pi}{8}+\sin ^{4} \frac{7 \pi}{8}=\frac{3}{2}$.
14. If $A$ is not an integral multiple of $\pi$, prove that $\cos A \cos 2 A \cos 4 A \cos 8 A=\frac{\sin 16 A}{16 \sin A}$.

If $\cos x+\cos y=\frac{4}{5}$ and $\cos x-\cos y=\frac{2}{7}$, find the value of $14 \tan \left(\frac{x-y}{2}\right)+5 \cot \left(\frac{x+y}{2}\right)$.
16. If $a, b, c$ are non zero real numbers and $\alpha, \beta$ are the solutions of the equation $a \cos \theta+b \sin \theta=c$ then show that
$\begin{array}{ll}\text { i) } \sin \alpha+\sin \beta=\frac{\mathbf{2 b c}}{\mathbf{a}^{2}+\mathbf{b}^{2}} & \text { ii) } \sin \alpha \cdot \sin \beta=\frac{\mathbf{c}^{2}-\mathbf{a}^{\mathbf{2}}}{\mathbf{a}^{2}+\mathbf{b}^{2}} \text {. }\end{array}$
17.
12. $\left(\frac{\cos A+\cos B}{\sin A-\sin B}\right)^{n}+\left(\frac{\sin A+\sin B}{\cos A-\cos B}\right)^{n}=\left\{\begin{array}{l}2 \cot ^{n}\left(\frac{A-B}{2}\right), \text { if } n \text { is even. } \\ 0, \text { if } n \text { is odd }\end{array}\right.$

## TRIGONOMETRIC EQUATIONS

1. Solve : $2 \cos ^{2} \theta-\sqrt{3} \sin \theta+1=0$.
2. Solve : $\cot ^{2} x-(\sqrt{3}+1) \cot x+\sqrt{3}=0$ in $0<x<\pi / 2$.
3. Find the solution set for $\tan \theta+3 \cot \theta=5 \sec \theta$.
4. Solve tan $\theta+\sec \theta=\sqrt{3}, 0 \leq \theta \leq 2 \pi$.
5. Solve : $1+\sin ^{2} \theta=3 \sin \theta \cos \theta$.
6. Solve : $\sqrt{2}(\sin x+\cos x)=\sqrt{3}$
7. Solve the equation $\sqrt{3} \sin \theta-\cos \theta=\sqrt{2}$.
8. Find the general solution for the trigonometric equation $\sin 7 \theta+\sin 4 \theta+\sin \theta=0$.
9. Find the solution set for $\sin 2 x-\cos 2 x=\sin x-\cos x$.
10. Solve : $4 \sin x \sin 2 x \sin 4 x=\sin 3 x$.
11. If $0<\theta<\pi$, solve $\cos \theta \cos 2 \theta \cos 3 \theta=1 / 4$.
12. Find the values of $x$ in $(-\pi, \pi)$ satisfying the equation $8^{1+\cos x+\cos ^{2} x+\ldots . . . . . . . . . . . ~} \infty=4^{3}$
13. If $\tan (\pi \cos \theta)=\cot (\pi \sin \theta)$, then prove that $\cos (\theta-\pi / 4)= \pm 1 / 2 \sqrt{ } 2$.
14. If $\theta_{1}, \theta_{2}$ are the solutions of the equation $a \cos 2 \theta+b \sin 2 \theta=c$, then find the values of (i) $\tan \theta_{1}+\tan \theta_{2}$
(ii) $\tan \theta_{1} \tan \theta_{2}$

INVERSE TRIGONOMETRIC FUNCTIONS

1. Prove that $\operatorname{Sn}^{-1} \frac{4}{5}+\operatorname{Sin}^{-1} \frac{7}{25}=\operatorname{Sin}^{-1} \frac{117}{125}$.
2. Prove that $\operatorname{Sin}^{-1} \frac{3}{5}+\operatorname{Sin}^{-1} \frac{8}{17}=\operatorname{Cos}^{-1} \frac{36}{85}$
3. $\quad$ Show that $\operatorname{Cos}^{-1} \frac{4}{5}+\operatorname{Sin}^{-1} \frac{3}{\sqrt{34}}=\operatorname{Tan}^{-1} \frac{27}{11}$.
4. $\quad$ Prove that $2 \operatorname{Sin}^{-1}\left(\frac{3}{5}\right)-\operatorname{Cos}^{-1}\left(\frac{5}{13}\right)=\operatorname{Cos}^{-1}\left(\frac{323}{325}\right)$.
5. Show that $\operatorname{Sin}^{-1} \frac{4}{5}+2 \operatorname{Tan}^{-1} \frac{1}{3}=\pi / 2$.
6. Prove that $\cos \left(2 \operatorname{Tan}^{-1} 1 / 7\right)=\sin \left(4 \operatorname{Tan}^{-1} 1 / 3\right)$.
7. Prove that $\operatorname{Tan}^{-1} 1 / 2+\operatorname{Tan}^{-1} 1 / 5+\operatorname{Tan}^{-1} 1 / 8=\pi / 4$.
8. Find the value of $\tan \left[\operatorname{Sin}^{-1} \frac{3}{5}+\operatorname{Cos}^{-1} \frac{5}{\sqrt{34}}\right]$.
9. If $\operatorname{Cos}^{-1} p+\operatorname{Cos}^{-1} q+\operatorname{Cos}^{-1} r=\pi$, then prove that $p^{2}+q^{2}+r^{2}+2 p q r=1$
10. If $\operatorname{Tan}^{-1} x+\operatorname{Tan}^{-1} y+\operatorname{Tan}^{-1} z=\pi / 2$, then prove that $x y+y z+z x=1$.
11. If $\operatorname{Sin}^{-1} x+\operatorname{Sin}^{-1} y+\operatorname{Sin}^{-1} z=\pi$, then prove that $x \sqrt{1-x^{2}}+y \sqrt{1-y^{2}}+z \sqrt{1-z^{2}}=2 x y z$.
12. Solve : $\operatorname{Tan}^{-1}\left(\frac{x-1}{x-2}\right)+\operatorname{Tan}^{-1}\left(\frac{x+1}{x+2}\right)=\pi / 4$.
13. Solve : $\operatorname{Tan}^{-1}\left(\frac{1}{2 x+1}\right)+\operatorname{Tan}^{-1}\left(\frac{1}{4 x+1}\right)=\operatorname{Tan}^{-1}\left(\frac{2}{x^{2}}\right)$.
14. Solve : $3 \operatorname{Sin}^{-1}\left(\frac{2 x}{1+x^{2}}\right)-4 \operatorname{Cos}^{-1}\left(\frac{1-x^{2}}{1+x^{2}}\right)+2 \operatorname{Tan}^{-1}\left(\frac{2 x}{1-x^{2}}\right)=\pi / 3$.
15. If $\operatorname{Cos}^{-1}\left(\frac{p}{a}\right)+\operatorname{Cos}^{-1}\left(\frac{q}{b}\right)=\alpha$, then prove that $\frac{p^{2}}{a^{2}}-\frac{2 p q}{a b} \cos \alpha+\frac{q^{2}}{b^{2}}=\sin ^{2} \alpha$.

## PROPERTIES OF TRIANGLES

1. In $\triangle A B C$, prove that $b^{2}=c^{2}+a^{2}-2 c a \cos B$.
2. In $\triangle A B C$, prove that $\tan \left(\frac{B-C}{2}\right)=\frac{b-c}{b+c} \cot \frac{A}{2}$.
3. In $\triangle A B C$, show that $a=b \cos C+c \cos B$.
4. In $\triangle A B C$, prove that $\frac{a+b}{c}=\frac{\cos \left(\frac{A-B}{2}\right)}{\sin \frac{C}{2}}$.
5. In $\triangle A B C$, show that $(b-c)^{2} \cos ^{2} \frac{A}{2}+(b+c)^{2} \sin ^{2} \frac{A}{2}=a^{2}$.
6. Show that $\frac{\cos A}{a}+\frac{\cos B}{b}+\frac{\cos C}{c}=\frac{a^{2}+b^{2}+c^{2}}{2 a b c}$.
7. $\quad$ In $\triangle A B C$, show that $a^{2} \cot A+b^{2} \cot B+c^{2} \cot C=\frac{a b c}{R}$.
8. If $b+c=3 a$ then find the value of $\cot \frac{B}{2} \cot \frac{C}{2}$.
9. In $\triangle A B C$, if $a: b: c=7: 8: 9$, then find $\cos A: \cos B: \cos C$.
10. $\operatorname{In} \triangle A B C$, if $\frac{1}{a+c}+\frac{1}{b+c}=\frac{3}{a+b+c}$, show that $C=60^{\circ}$.
11. If $\sin \theta=\frac{a}{b+c}$ then show that $\cos \theta=\frac{2 \sqrt{b c}}{b+c} \cos A / 2$.
12. If $a=(b-c) \sec \theta$, prove that $\tan \theta=\frac{2 \sqrt{b c}}{b-c} \sin \frac{A}{2}$.
13. Prove that $\cot \frac{A}{2}+\cot \frac{B}{2}+\cot \frac{C}{2}=\frac{s^{2}}{\Delta}$.
14. Prove that $\cot A+\cot B+\cot C=\frac{a^{2}+b^{2}+c^{2}}{4 \Delta}$.
15. Prove that $r_{1}+r_{2}+r_{3}-r=4 R$.
16. Prove that $\left(\frac{1}{r}-\frac{1}{r_{1}}\right)\left(\frac{1}{r}-\frac{1}{r_{2}}\right)\left(\frac{1}{r}-\frac{1}{r_{3}}\right)=\frac{4 R}{r^{2} s^{2}}$.
17. 

If $A_{1}, A_{2}, A_{3}$ are the area of incircle and excircles of a triangle respectively, then prove that $\frac{1}{\sqrt{A_{1}}}+\frac{1}{\sqrt{A_{2}}}+\frac{1}{\sqrt{A_{3}}}=\frac{1}{\sqrt{A}}$.
18. In a triangle $\triangle A B C$, if $a^{2}+b^{2}+c^{2}=8 R^{2}$, then show that it is right angled.

## 1ST YEAR - MATHEMATICS IA

VERY SHORT ANSWER QUESTIONS (2 MARKS)
FUNCTIONS (Q1)

1. Define one - one function. Give an example.
2. Define onto function. Give an example.
3. $f: N \rightarrow N$ is defined as $f(x)=2 x+3$. Is $f$ onto ? Explain with reason.
4. If a function is defined as $f(x)=\left\{\begin{array}{l}x+2, x>-1 \\ 2,-1 \leq x \leq 1 \\ x-1,-3<x<-1\end{array}\right.$. Find the values of (i) $f(0)(i i) f(2)+f(-2)$
5. If the function $f$ defined by $f(x)=\left\{\begin{array}{cc}3 x-2, & x>3 \\ x^{2}-2, & -2 \leq x \leq 2 \\ 2 x+1, & x<-3\end{array}\right.$ then find the values of $f(4)$ and $f(2.5)$.

Ans. 10, not defined
6. If $A=\left\{0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}\right\}$ and $f: A \rightarrow B$ is a surjection defined by $f(x)=\cos x$, then find $B$.

If $f$ and $g$ are real valued functions defined by
$f(x)=2 x-1$ and $g(x)=x^{2}$, then find (i) $(\mathrm{fg})(\mathrm{x})$
8. If $A=\{-2,-1,0,1,2\}$ and $f: A \rightarrow B$ is a surjection defined $b y f(x)=x^{2}+x+1$ then find $B$. Ans: $B=f(A)=\{3,1,7\}$ 9. If $f=\{(1,2),(2,-3),(3,-1)\}$, then find
(i) $2 f$
(ii) $f^{2}$

Ans: $\{(1,4),(2,-6),(3,-2)\},\{(1,4),(2,9),(3,1)\}$
10. If $f: R \rightarrow R, g: R \rightarrow R$ are defined by $f(x)=4 x-1, g(x)=x^{2}+2$,
then find (i) (gof) $\left(\frac{a+1}{4}\right)$
(ii) $\mathrm{go}(\mathrm{fof})(0)$.

Ans: $\mathrm{a}^{2}+2,27$.
$f: R \rightarrow R, g: R \rightarrow R$ are defined by $f(x)=3 x-1, g(x)=x^{2}+1$, then find (fog) (2).
12. If $f(x)=2, g(x)=x^{2}, h(x)=2 x$ then find (fogoh) ( $x$ ).
13. If $f(x)=\frac{x+1}{x-1}$, then find (fofof)(x).
14. If $f: R \rightarrow R, g: R \rightarrow R$ are defined by $f(x)=3 x-2, g(x)=x^{2}+1$, then find ( $\left.\mathrm{gof}^{-1}\right)(2)$.
15. Find the inverse function of $f(x)=5^{x}$.
16. If $f: Q \rightarrow Q$ defined by $f(x)=5 x+4$, then find $f^{-1}$.
17. Find the inverse of the function $f: R \rightarrow R$ defined $b y f(x)=a x+b(a \neq 0) ; a, b \in R$.
18. $f: R \rightarrow R$ define by $f(x)=\frac{2 x+1}{3}$, then this function is injection or not? Justify.

Ans: 14.
Ans: 2

Ans: $\mathrm{f}(\mathrm{x})$.
Ans: $\frac{25}{9}$
Ans: $\mathrm{f}^{-1}(\mathbf{x})=\log _{5} \mathrm{x}$
Ans: $f^{-1}(x)=\frac{x-4}{5}$
Ans: $f^{-1}(x)=\frac{x-b}{a}$

Ans: $f$ is a injection
19. Determine whether the following functions are even or odd.
(i) $f(x)=a^{x}-a^{-x}+\sin x(i i) f(x)=x\left(\frac{e^{x}-1}{e^{x}+1}\right)$ (iii) $f(x)=\log \left(x+\sqrt{x^{2}+1}\right)$.

## FUNCTIONS (Q2)

1. Find the domain of the real valued function $f(x)=\frac{1}{\left(x^{2}-1\right)(x+3)}$.

Ans. R-\{-3, -1, 1\}
2. Find the domain of the real valued function $f(x)=\frac{1}{\log (2-x)}$.

Ans: $(-\infty, 2)-\{1\}$ or $(-\infty, 1) \cup(1,2)$
3. Find the domain of the real function $f(x)=\sqrt{4 x-x^{2}}$.

Ans: [0, 4]
4. Find the domain of the real valued function $f(x)=\sqrt{x^{2}-25}$.

Ans : $(-\infty,-5] \cup[5, \infty)$
5. Find the domain of the function $f(x)=\frac{1}{\sqrt{1-x^{2}}}$, where $f$ is a real valued function.

Ans: (-1, 1)
6. Find the domain of the real function $f(x)=\log \left(x^{2}-4 x+3\right)$.

Ans : $(-\infty, 1) \cup(3, \infty)$
7. Find the domain of the real valued function $f(x)=\frac{\sqrt{2+x}+\sqrt{2-x}}{x}$.

Ans : $[-2,0) \cup(0,2)$
8. Find the domain of $f(x)=\frac{2 x^{2}-5 x+7}{(x-1)(x-2)(x-3)}$.

Ans: $\mathbf{R - \{ 1 , 2 , 3 \}}$
9. Find the range of
(i) $f(x)=\frac{x^{2}-4}{x-2}$
(ii) $f(x)=\sqrt{9+x^{2}}$
10. Find the domain and range of (i) $f(x)=\frac{2+x}{2-x} \quad$ (ii) $f(x)=\frac{x}{1+x^{2}} \quad$ (iii) $f(x)=\sqrt{9-x^{2}} \quad$ (iv) $f(x)=\frac{x}{2-3 x}$

## MATRICES(Q3)

1. Define "Triangular matrix".
2. Construct a $3 \times 2$ matrix, whose elements are defined by $a_{i j}=\frac{1}{2}|i-3 j|$.

Ans: $\left[\begin{array}{cc}1 & 5 / 2 \\ 1 / 2 & 2 \\ 0 & 3 / 2\end{array}\right]$
3. If $\left[\begin{array}{cc}x-3 & 2 y-8 \\ z+2 & 6\end{array}\right]=\left[\begin{array}{cc}5 & 2 \\ -2 & a-4\end{array}\right]$ then find the values of $x, y, z$ and $a$.

Ans. $\mathrm{x}=8, \mathrm{y}=5, \mathrm{z}=-4$ and $\mathrm{a}=10$
4. If $A=\left[\begin{array}{lll}1 & 2 & 3 \\ 3 & 2 & 1\end{array}\right]$ and $B=\left[\begin{array}{lll}3 & 2 & 1 \\ 1 & 2 & 3\end{array}\right]$, find $3 B-2 A$.

Ans: $\left[\begin{array}{ccc}7 & 2 & -3 \\ -3 & 2 & 7\end{array}\right]$
5. If $\mathrm{A}=\left[\begin{array}{ll}1 & 2 \\ 3 & 4\end{array}\right], \mathrm{B}=\left[\begin{array}{ll}3 & 8 \\ 7 & 2\end{array}\right]$ and $2 \mathrm{X}+\mathrm{A}=\mathrm{B}$, the finf Cin .

Ans: $X=\left[\begin{array}{cc}1 & 3 \\ 2 & -1\end{array}\right]$
6. Define 'Trace of the matrix' and find the trace of the matrix $A=\left[\begin{array}{ccc}1 & 3 & -5 \\ 2 & -1 & 5 \\ 2 & 0 & 1\end{array}\right]$.

Ans: The trace of a square matrix is the sum of the elements in principal diagonal. Trace of $A=1$.
7. If $A=\left[\begin{array}{ll}i & 0 \\ 0 & i\end{array}\right]$, find $A^{2}$.

Ans: $\left[\begin{array}{cc}-1 & 0 \\ 0 & -1\end{array}\right]$
8. A certain bookshop has 10 dozen chemistry books, 8 dozen physics books, 10 dozen economics books. Their selling prices are Rs. 80 , Rs. 60 and Rs. 40 each respectively. Find the total amount the bookshop will receive by selling all the books using matrix algebra.

Ans: Rs.20,160/-
9. If $A=\left[\begin{array}{cc}2 & 4 \\ -1 & k\end{array}\right]$ and $A^{2}=\left[\begin{array}{ll}0 & 0 \\ 0 & 0\end{array}\right]$, find the value of $k$.

Ans: $\mathrm{k}=\mathbf{- 2}$.
10. If $A=\left[\begin{array}{ccc}-2 & 1 & 0 \\ 3 & 4 & -5\end{array}\right], B=\left[\begin{array}{cc}1 & 2 \\ 4 & 3 \\ -1 & 5\end{array}\right]$, then find $A+B^{\prime}$.

Ans: $\left[\begin{array}{ccc}-1 & 5 & -1 \\ 5 & 7 & 0\end{array}\right]$
11. If $A=\left[\begin{array}{cc}\cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha\end{array}\right]$, show that $A A^{\prime}=A^{\prime} A$.

Ans : $\left[\begin{array}{ll}1 & 0 \\ 0 & 1\end{array}\right]$
12. If $\mathrm{A}=\left[\begin{array}{ccc}2 & 0 & -1 \\ -1 & 1 & 5\end{array}\right]$ and $\mathrm{B}=\left[\begin{array}{ccc}-1 & 1 & 0 \\ 0 & 1 & -2\end{array}\right]$, then find $\left(A B^{\prime}\right)^{\prime}$.

Ans: $\left[\begin{array}{cc}-2 & 2 \\ 2 & -9\end{array}\right]$
13. If $A=\left[\begin{array}{cc}-2 & 1 \\ 5 & 0 \\ -1 & 4\end{array}\right]$ and $B=\left[\begin{array}{ccc}-2 & 3 & 1 \\ 4 & 0 & 2\end{array}\right]$, then find $2 A+B^{\prime}$ and $3 B^{\prime}-A$.

Ans: $\left[\begin{array}{cc}-6 & 6 \\ 13 & 0 \\ -1 & 10\end{array}\right],\left[\begin{array}{cc}-4 & 11 \\ 4 & 0 \\ 4 & 2\end{array}\right]$
14. Find the product $\left[\begin{array}{ccc}0 & c & -b \\ -c & 0 & a \\ b & -a & 0\end{array}\right]\left[\begin{array}{ccc}a^{2} & a b & a c \\ a b & b^{2} & b c \\ a c & b c & c^{2}\end{array}\right]$

## MATRICES(Q4)

1. Define symmetric matrix and give an example.

Ans : A square matrix $A$ is said to be a symmetric matrix if $A^{\prime}=A . E g\left[\begin{array}{lll}1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6\end{array}\right]$
2. If $A=\left[\begin{array}{cc}2 & -4 \\ -5 & 3\end{array}\right]$, then find $A+A^{\prime}$ and $A A^{\prime}$.

Ans : $\left[\begin{array}{cc}4 & -9 \\ -9 & 6\end{array}\right],\left[\begin{array}{cc}20 & -22 \\ -22 & 34\end{array}\right]$
3. If $A=\left[\begin{array}{ccc}-1 & 2 & 3 \\ 2 & 5 & 6 \\ 3 & x & 7\end{array}\right]$ is a symmetric matrix then find $x$.

Ans: 6.
4. If $A=\left[\begin{array}{ccc}0 & 2 & 1 \\ -2 & 0 & -2 \\ -1 & x & 0\end{array}\right]$ is a skew symmetric matrix, then find $x$.

Ans: $x=2$
5. If $A=\left[\begin{array}{ccc}1 & 0 & 0 \\ 2 & 3 & 4 \\ 5 & -6 & x\end{array}\right]$, $\operatorname{det} A=45$, then find $x$.

Ans: $x=-7$
6. Show that $\left|\begin{array}{lll}a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c\end{array}\right|=0$.
7. If $\omega$ is a complex cube root of unity, then show that $\left|\begin{array}{ccc}\omega & \omega^{2} & 1 \\ \omega^{2} & 1 & \omega\end{array}\right|=0$.
8. Find the determinant of $\left[\begin{array}{lll}2^{2} & 3^{2} & 4^{2} \\ 3^{2} & 4^{2} & 5^{2}\end{array}\right]$

Ans: - 8
9. Find the adjoint and inverse of the matrix $\left[\begin{array}{cc}1 & 2 \\ 3 & -5\end{array}\right]$.

Ans: $\left[\begin{array}{cc}-5 & -2 \\ -3 & 1\end{array}\right], \frac{-1}{11}\left[\begin{array}{cc}-5 & -2 \\ -3 & 1\end{array}\right]$
10. Find the adjoint of the matrix $\left[\begin{array}{cc}\cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha\end{array}\right]$.

Ans : $\left[\begin{array}{cc}\cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha\end{array}\right],\left[\begin{array}{cc}\cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha\end{array}\right]$
11. Find the rank of the matrix $\left[\begin{array}{lll}1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1\end{array}\right]$

Ans: 1.
12. Find the rank of $A=\left[\begin{array}{lll}0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 2 & 1\end{array}\right]$ using elementary transformations.

Ans: A: 2

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13. If $A=\left[\begin{array}{ccc}1 & 2 & 1 \\ -1 & 0 & 2 \\ 0 & 1 & -1\end{array}\right]$, find the rank of $A$.

Ans: A: 3

Solve the following system of homogeneous equations $x-y+z=0, x+2 y-z=0,2 x+y+3 z=0$.
Solve the following system of homogeneous equations $x+y-2 z=0,2 x+y-3 z=0,5 x+4 y-9 z=0$.

## ADDITION OF VECTORS (Q-5)

1. Let $\bar{a}=2 \bar{i}+4 \bar{j}-5 \bar{k}, \bar{b}=\bar{i}+\bar{j}+\bar{k}, \bar{c}=\bar{j}+2 \bar{k}$. Find the unit vector in the opposite direction of $\bar{a}+\bar{b}+\bar{c}$.

$$
\text { Ans. } \frac{-(3 \overline{\mathrm{i}}+6 \overline{\mathrm{j}}-2 \overline{\mathrm{k}})}{7} \text {. }
$$

2. Find the unit vector in the direction of sum of the vectors $\bar{a}=2 \bar{i}+2 \bar{j}-5 \bar{k}$ and $2 \bar{i}+\bar{j}+3 \bar{k}$.

$$
\text { Ans. } \frac{4 \overline{\mathrm{i}}+3 \overline{\mathrm{j}}-2 \overline{\mathrm{k}}}{\sqrt{29}} \text {. }
$$

3. If the position vectors of $A, B, C$ are $-2 \bar{i}+\bar{j}-\bar{k},-4 \bar{i}+2 \bar{j}+2 \bar{k}, 6 \bar{i}-3 \bar{j}-13 \bar{k}$ and $\overline{A B}=\lambda \overline{A C}$, find $\lambda$.

$$
\text { Ans. } \lambda=\frac{-1}{4} \text {. }
$$

4. Show that the points whose position vectors are $-2 \bar{a}+3 \bar{b}+5 \bar{c}, \bar{a}+2 \bar{b}+3 \bar{c}, 7 \bar{a}-\bar{c}$ are collinear, where $\bar{a}, \bar{b}, \bar{c}$ are non-coplanar vectors.
5. If the vectors $-3 \bar{i}+4 \bar{j}+\lambda \bar{k}$ and $\mu \bar{i}+8 \overline{\mathrm{j}}+6 \overline{\mathrm{k}}$ are collinear vectors, then find $\lambda$ and $\mu$.

Ans. $\mu=3, \lambda=6$.
6. If $\overline{\mathrm{a}}=2 \overline{\mathrm{i}}+5 \overline{\mathrm{j}}+\overline{\mathrm{k}}, \overline{\mathrm{b}}=4 \overline{\mathrm{i}}+\mathrm{m} \overline{\mathrm{j}}+\mathrm{n} \overline{\mathrm{k}}$ and $\overline{\mathrm{a}}, \overline{\mathrm{b}}$ are collinear, find $\mathrm{m}, \mathrm{n}$.

Ans: $\mathrm{m}=10, \mathrm{n}=2$.
7. If $\overline{\mathrm{OA}}=\overline{\mathrm{i}}+\overline{\mathrm{j}}+\overline{\mathrm{k}}, \overline{\mathrm{AB}}=3 \overline{\mathrm{i}}-2 \overline{\mathrm{j}}+\overline{\mathrm{k}}, \overline{\mathrm{BC}}=\overline{\mathrm{i}}+2 \overline{\mathrm{j}}-2 \overline{\mathrm{k}}, \overline{\mathrm{CD}}=2 \overline{\mathrm{i}}+\overline{\mathrm{j}}+3 \overline{\mathrm{k}}$ then find the vector $\overline{\mathrm{OD}}$.

Ans. $7 \overline{\mathrm{i}}+2 \overline{\mathrm{j}}+3 \overline{\mathrm{k}}$.
8. Define 'linear combination of vectors'.
9. $A B C D E$ is a pentagon. If the sum of the vectors $\overline{A B}, \overline{A E}, \overline{B C}, \overline{D C}, \overline{E D}$ and $\overline{A C}$ is $\lambda \overline{A C}$, then find the value of $\lambda$.

Ans. $\lambda=3$.
10. If $\alpha, \beta, \gamma$ are the angles made by the vector $3 \bar{i}-6 \bar{j}+2 \bar{k}$ with the positive directions of the coordinate axes then find $\cos \alpha, \cos \beta$ and $\cos \gamma$.

Ans. $\cos \alpha=\frac{3}{7}, \cos \beta=\frac{-6}{7}, \cos \gamma=\frac{2}{7}$.
11. Find the angles made by the straight line passing through the points $(T,-3,2)$ and $(3,-5,1)$ with the coordinate axes.

$$
\text { Ans. } \cos ^{-1} \frac{2}{3}, \cos ^{-1}\left(\frac{-2}{3}\right), \cos ^{-1}\left(\frac{-1}{3}\right) .
$$

12. Define coplanar vectors, non-coplanar vectors'.
13. Define linear combination of vectors?

## ADDITION OF VECTORS (Q-6)

1. Find the vector equation of the line passing through the point $2 \bar{i}+3 \bar{j}+\bar{k}$ and parallel to the vector $4 \bar{i}-2 \bar{j}+3 \bar{k}$.

$$
\text { Ans. } \bar{r}=2 \bar{i}+3 \bar{j}+\bar{k}+\mathbf{t}(4 \overline{\mathbf{i}}-2 \overline{\mathbf{j}}+3 \overline{\mathrm{k}}) \text { where } \mathbf{t} \in \mathbf{R}
$$

2. OABC is a parallelogram. If $\overline{\mathrm{OA}}=\overline{\mathrm{a}}$ and $\overline{\mathrm{OC}}=\overline{\mathrm{c}}$, find the vector equation of the side BC .

Ans. $\overline{\mathbf{r}}=\mathbf{t} \overline{\mathbf{a}}+\overline{\mathbf{c}}, \mathbf{t} \in \mathbf{R}$.
3. Find the vector equation of the line passing through the points. $\mathbf{2} \overline{\mathbf{i}}+\overline{\mathbf{j}}+\mathbf{3} \overline{\mathbf{k}},-\mathbf{4} \overline{\mathbf{i}}+\mathbf{3} \overline{\mathbf{j}}-\overline{\mathbf{k}}$

$$
\text { Ans. } \bar{r}=(1-t)(2 \overline{\mathrm{i}}+\overline{\mathrm{j}}+3 \overline{\mathrm{k}})+\mathbf{t}(-4 \overline{\mathrm{i}}+3 \overline{\mathrm{j}}-\overline{\mathrm{k}}) \text { where } \mathrm{t} \in \mathrm{R} \text {. }
$$

4. If $\bar{a}, \bar{b}, \bar{c}$ are the position vectors of the vertices $A, B$ and $C$ respectively of $\triangle A B C$, then find the vector equation of the median through the vertex $A$.

$$
\text { Ans. } \overline{\mathrm{r}}=(1-\mathrm{t}) \overline{\mathrm{a}}+\mathrm{t}\left(\frac{\overline{\mathrm{~b}}+\overline{\mathrm{c}}}{2}\right) \text { where } \mathrm{t} \in \mathrm{R} .
$$

5. Find the vector equation of the plane passing through the points $\overline{\mathrm{i}}-2 \overline{\mathrm{j}}+5 \overline{\mathrm{k}},-5 \overline{\mathrm{j}}-\overline{\mathrm{k}}$ and $-3 \overline{\mathrm{i}}+5 \overline{\mathrm{j}}$.

$$
\text { Ans. } \overline{\mathbf{r}}=(1-\mathbf{t - s})(\overline{\mathrm{i}}-2 \overline{\mathrm{j}}+5 \overline{\mathrm{k}})+\mathbf{t}(-5 \overline{\mathrm{j}}-\overline{\mathrm{k}})+\mathbf{s}(-3 \overline{\mathrm{i}}+5 \overline{\mathrm{j}}) \text { where } \mathbf{t}, \mathbf{s} \in \mathbf{R} \text {. }
$$

6. Find the vector equation of the plane passing through the points $(0,0,0),(0,5,0)$ and $(2,0,1)$.

$$
\text { Ans. } \bar{r}=t(5 \bar{j})+s(2 \bar{i}+\bar{k}) \text { where } t, s \in R .
$$

7. Find the vector equation of the plane which passes through the points $2 \bar{i}+4 \bar{j}+2 \bar{k}, 2 \bar{i}+3 \bar{j}-5 \bar{k}$ and parallel to the vector $3 \bar{i}-2 \bar{j}+\bar{k} . \quad$ Ans. $\overline{\mathbf{r}}=(1-\mathbf{t})(2 \bar{i}+4 \bar{j}+2 \bar{k})+\mathbf{t}(2 \bar{i}+3 \bar{j}-5 \bar{k})+5(3 \bar{i}-2 \bar{j}+\bar{k})$ where $\mathbf{t}, \mathbf{s} \in \mathbf{R}$.
8. Find the vector equation of the plane passing through the point $(1,2,3)$ and parallel to the vectors $(-2,3,1),(2,-3,4)$

$$
\text { Ans : } \bar{r}=\bar{i}+2 \bar{j}+3 \bar{k}+t(-2 \bar{i}+3 \bar{j}+\bar{k})+s(2 \bar{i}-3 \bar{j}+4 \bar{k}) \text { where } \mathbf{t}, \mathbf{s} \in \mathbf{R} \text {. }
$$

## PRODUCT OF VECTORS (Q-7)

1. If $\bar{a}=\bar{i}+2 \bar{j}-3 \bar{k}$ and $\bar{b}=3 \bar{i}-\bar{j}+2 \bar{k}$, then show that $\bar{a}+\bar{b}$ and $\bar{a}-\bar{b}$ are perpendicular to each other.

$$
\text { Ans. }(\bar{a}+\bar{b}) \cdot(\bar{a}-\bar{b})=0
$$

2. If the vectors $\mathbf{2} \overline{\mathbf{i}}+\lambda \overline{\mathbf{j}}-\overline{\mathbf{k}}, \mathbf{4} \overline{\mathbf{i}}-\mathbf{2} \overline{\mathbf{j}}+\mathbf{2} \overline{\mathbf{k}}$ are perpendicular to each other, find $\lambda$.

Ans. $\lambda=3$.
3. If the vectors $\lambda \bar{i}-3 \bar{j}+5 \bar{k}$ and $2 \lambda \bar{i}-\lambda \bar{j}-\bar{k}$ are perpendicular each other. Find the values of $\lambda$.

Ans. $\lambda=\frac{-5}{2}$ or 1 .
4. If $4 \bar{i}+\frac{2 p}{3} \bar{j}+p \bar{k}$ is parallel to the vector $\bar{i}+2 \bar{j}+3 \bar{k}$, find $p$.

Ans. 12.
5. If $|\bar{a}+\bar{b}|=|\bar{a}-\bar{b}|$, then find the angle between $\bar{a}$ and $\bar{b}$.

Ans. $90^{0}$
6. Let $\bar{a}=\bar{i}+\bar{j}+\bar{k}$ and $\bar{b}=2 \bar{i}+3 \bar{j}+\bar{k}$, find projection vector of $\bar{b}$ on $\bar{a}$ and its magnitude.

Ans. $2(\bar{i}+\bar{j}+\bar{k}), 2 \sqrt{3}$.
7. Find the angle between the vectors $\bar{i}+2 \bar{j}+3 \bar{k}$ and $3 \bar{i}-\bar{j}+2 \bar{k}$.

Ans. $\theta=60^{\circ}$
8. If $\bar{a}=2 \bar{i}-\bar{j}+\bar{k}$ and $\bar{b}=\bar{i}-3 \bar{j}-5 \bar{k}$, then find $|\overline{\mathrm{a}} \times \overline{\mathrm{b}}|$.

Ans. $\sqrt{210}$.
Ans. $\sqrt{\frac{155}{156}}$.
9. If $\bar{a}=2 \bar{i}-\bar{j}+\bar{k}$ and $\bar{b}=3 \bar{i}+4 \bar{j}-\bar{k}$ and $\theta$ is the angle between $\bar{a}$ and $\bar{b}$, then find $\sin \theta$.

Ans. 25.
10. If $|\overline{\mathrm{a}}|=13,|\overline{\mathrm{~b}}|=5$ and $\overline{\mathrm{a}} \cdot \overline{\mathrm{b}}=60$, then find $|\overline{\mathrm{a}} \times \overline{\mathrm{b}}|$.
11. Find the area of the parallelogram with $2 \bar{i}-3 \bar{j}$ and $3 \bar{i}-\bar{k}$ as adjacent sides.

Ans. $\sqrt{94}$ sq. units.
12. Find the area of parallelogram whose diagonals are $3 \bar{i}+\bar{j}-2 \bar{k}, \bar{i}-3 \bar{j}+4 \bar{k}$.

Ans. $5 \sqrt{3}$ sq. units.
13. Find the unit vector perpendicular to the plane containing the vectors $\bar{a}=4 \bar{i}+3 \bar{j}-\bar{k}, \bar{b}=2 \bar{i}-6 \bar{j}-3 \bar{k}$.

Ans. $\pm \frac{(3 \overline{\mathrm{i}}-2 \overline{\mathrm{j}}+6 \overline{\mathrm{k}})}{7}$
14. Find the angle between the planes $\overline{\mathbf{r}} .(\mathbf{2} \overline{\mathbf{i}}-\overline{\mathbf{j}}+\mathbf{2} \overline{\mathbf{k}})=\mathbf{3}$ and $\overline{\mathbf{r}} .(\mathbf{3} \overline{\mathbf{i}}+\mathbf{6} \overline{\mathbf{j}}+\overline{\mathbf{k}})=\mathbf{4} . \quad$ Ans: $\cos ^{-1}(2 / 21)$.
15. Find the vector equation of the plane through the point $(3,-2,1)$ and perpendicular to the vector $(4,7,-4)$.
16. Find the angle between the planes $\overline{\mathbf{r}} .(2 \overline{\mathbf{i}}-\overline{\mathbf{j}}+2 \overline{\mathbf{k}})=\mathbf{3}$ and $\overline{\mathbf{r}} .(3 \overline{\mathbf{i}}+6 \overline{\mathbf{j}}+\overline{\mathbf{k}})=\mathbf{4}$.

## TRIGONOMETRY UPTO TRANSFORMATIONS (Q-8)

1. If $\sin \theta=\frac{-1}{3}$ and $\theta$ does not lie in the $3^{\text {rd }}$ quadrant, find the value of cos $\theta$.

Ans: $\frac{2 \sqrt{2}}{3}$
2. If $\cos \theta+\sin \theta=\sqrt{2} \cos \theta$, then prove that $\cos \theta-\sin \theta-\sqrt{2} \sin \theta$.
3. If $3 \sin \theta+4 \cos \theta=5$, then find the value of $4 \sin \theta-3 \cos \theta$.
4. Show that $\cot \frac{\pi}{20} \cdot \cot \frac{3 \pi}{20} \cdot \cot \frac{5 \pi}{20} \cdot \cot \frac{7 \pi}{20} \cdot \cot \frac{9 \pi}{20}=1$.
5. If $\sin \alpha+\operatorname{cosec} \alpha=2$, find the value of $\sin \theta+\operatorname{cosec}^{n} \alpha, n \in Z$.

Ans. 2
6. Find the period of the function defined by $f(x)=\tan \left(x+4 x+9 x+\ldots \ldots . .+n^{2} x\right)$.
7. Find the period of $f(x)=\cos \left(\frac{4 x+9}{5}\right)$.
8. Find a sine function, whose period is $2 / 3$. Ans. $\begin{array}{r}\frac{6 \pi}{n(n+1)(2 n+1)} . \\ \text { Ans. } \frac{5 \pi}{2} \\ \text { Ans. } \pm \sin (3 \pi x) . \\ \text { Ans }: \cos \frac{2 \pi x}{7}\end{array}$ Ans. $\begin{array}{r}\frac{6 \pi}{n(n+1)(2 n+1)} . \\ \text { Ans. } \frac{5 \pi}{2} \\ \text { Ans. } \pm \sin (3 \pi x) . \\ \text { Ans }: \cos \frac{2 \pi x}{7}\end{array}$
9. Find a cosine function whose period is 7.
10. Eliminate ' $\theta$ ' from $x=a \cos ^{3} \theta, y=b \sin ^{3} \theta$.
11. Draw the graph of $y=\sin x$ between $-\pi$ and $\pi$ taking 4 values on $x$-axis.
12. Draw the graph of $y=\tan x$ in $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$.
13. Draw the graph of $y=\sin 2 x$ in $[-\pi, \pi]$.
14. Draw the graph of $y=\cos ^{2} x$ in $(0, \pi)$.

1. If $\mathrm{A}+\mathrm{B}=\frac{\pi}{4}$, then prove that $(1+\tan \mathrm{A})(1+\tan \mathrm{B})=2$.
2. Show that $\cos 100^{\circ} \cos 40^{\circ}+\sin 100^{\circ} \sin 40^{\circ}=\frac{1}{2}$.
3. Show that $\cos 42^{\circ}+\cos 78^{\circ}+\cos 162^{\circ}=0$.
4. Find the value of $\sin 34^{\circ}+\cos 64^{\circ}-\cos 4^{\circ}$.

Ans. 0
5. What is the value of $\tan 20^{\circ}+\tan 40^{\circ}+\sqrt{3} \tan 20^{\circ} \tan 40^{\circ}$.
6. Prove that $\frac{\cos 9^{\circ}+\sin 9^{\circ}}{\cos 9^{\circ}-\sin 9^{\circ}}=\cot 36^{\circ}$.
7. Show that $\frac{1}{\sin 10^{\circ}}-\frac{\sqrt{3}}{\cos 10^{\circ}}=4$.
8. If $\frac{\sin \alpha}{\mathrm{a}}=\frac{\cos \alpha}{\mathrm{b}}$, then prove that $\operatorname{asin} 2 \alpha+\mathrm{b} \cos 2 \alpha=\mathrm{b}$.
9. If $180^{\circ}<\theta<270^{\circ}$ and $\sin \theta=\frac{-4}{5}$, calculate $\sin \frac{\theta}{2}$ and $\cos \frac{\theta}{2}$.
10. Find the maximum and minimum values of $f(x)=7 \cos x-24 \sin x+5$.
11. Find the maximum and minimum values of $\cos \left(x+\frac{\pi}{3}\right)+2 \sqrt{2} \sin \left(x+\frac{\pi}{3}\right)-3$.
12. If $\sin \alpha=3 / 5$, where $\pi / 2<\alpha<\pi$, evaluate $\cos 3 \alpha$.

## HYPERBOLIC FUNOCTIONS (Q-10)

1. If $\sinh x=\frac{3}{4}$, find $\cosh 2 x$ and $\sinh 2 x$.

Ans. $\frac{2}{\sqrt{5}}, \frac{-1}{\sqrt{5}}$
Ans. - 20, 30.
Ans. 0, -6.
Ans. -117/125.

Ans. $\frac{17}{8}, \frac{15}{8}$.
2. If $\cosh x=\frac{5}{2}$, find the values of $\cosh 2 x$ and $\sinh 2 x$.
3. If $\sinh x=\frac{1}{2}$, find the value of $\cosh 2 x+\sinh 2 x$.

Ans. $\frac{23}{2}, \frac{5 \sqrt{21}}{2}$.
Ans. $\frac{3+\sqrt{5}}{2}$.
4. Prove that $(\cosh x+\sinh x)^{n}=\cosh n x+\sinh n x$.
5. Prove that $(\cosh x-\sinh x)^{n}=\cosh n x-\sinh n x$.
6. For $x, y \in R$, prove that $\sinh (x+y)=\sinh x \cosh y+\cosh x \sinh y$.
7. If $\cosh x=\sec \theta$, then prove that $\tanh ^{2} \frac{x}{2}=\tan ^{2} \frac{\theta}{2}$.
8. If $\sinh x=3$, then show that $x=\log _{e}(3+\sqrt{10})$.
9. Show that $\operatorname{Tanh}^{-1}\left(\frac{1}{2}\right)=\frac{1}{2} \log _{\mathrm{e}} 3$.
10. For any $x \in R$, prove that $\sinh ^{-1} x \mid=\log _{e}\left(x+\sqrt{x^{2}+1}\right)$
11. For any $x \in[1, \infty)$, prove that $\cosh ^{-1} x=\log _{e}\left(x+\sqrt{x^{2}-1}\right)$.
12. For $x \in(-1,1)$, prove that $\operatorname{Tanh}^{-1} x=\frac{1}{2} \log _{e}\left(\frac{1+x}{1-x}\right)$.

## LONG ANSWER QUESTIONS (7 MARKS)

## STRAIGHT LINE

1. If $Q(h, k)$ is the foot of perpendicular from $P\left(x_{1}, y_{1}\right)$ on the line $a x+b y+c=0$, prove that $\left(h-x_{1}\right): a=\left(k-y_{1}\right): b=-\left(a x_{1}+b y_{1}+c\right)$ $:\left(a^{2}+b^{2}\right)$.
Find the foot of perpendicular from $(4,1)$ upon the straight line $3 x-4 y+12=0$.
2. If $Q(h, k)$ is the image of $P\left(x_{1}, y_{1}\right) w . r$.t the straight line $a x+b y+c=0$, then prove that $\left(h-x_{1}\right): a=\left(k-y_{1}\right): b=-2\left(a x_{1}+b y_{1}+c\right):\left(a^{2}+b^{2}\right)$ find the image of $(1,2)$ w.r.t. the straight line $3 x+4 y-1=0$.
3. If $p$ and $q$ are lengths of perpendiculars from the origin to the striaght lines $x \sec \alpha+y \operatorname{cosec} \alpha=a$ and $x \cos \alpha-y \sin \alpha=\operatorname{acos} 2 \alpha$, prove that $4 p^{2}+q^{2}=a^{2}$.
4. Find the circumcentre of the triangle whose vertices are $(-2,3),(2,-1)$ and $(4,0)$.
5. Find the circumcentre of the triangle whose vertices are $(1,0),(-1,2)$ and $(3,2)$.
6. Find the circumcentre of the triangle whose sides are given by $x+y+2=0,5 x-y-2=0$ and $x-2 y+5=0$.
7. Find the circumcentre of the triangle formed by the straight lines $x+y=0,2 x+y+5=0$ and $x-y=2$.
8. Find the orthocentre of the triangle whose vertices are $(-5,-7),(13,2),(-5,6)$.
9. Find the orthocentre of the triangle with the vertices $(-2,-1),(6,-1)$ and $(2,5)$.
10. If the equation of the sides a triangle are $7 x+y-10=0, x-2 y+5=0$ and $x+y+2=0$, find the orthocentre of the triangle.
11. Find the orthocentre of the triangle whose sides are given by $x+y+10=0, x-y-2=0$ and $2 x+y-7=0$.
12. Find the orthocentre of the triangle formed by the lines $x+2 y=0,4 x+3 y-5=0$ and $3 x+y=0$.
13. Find the equations of the straight line passing through the point $(1,2)$ and making an angle of $60^{\circ}$ with the line $\sqrt{3} x+y+2=0$
14. Find the equations of the straight lines passing the point of intersection of the lines $3 x+2 y+4=0,2 x+5 y=1$ and whose distance from $(2,-1)$ is 2 .
15. The base of an equilateral triangle is $x+y-2=0$ and the opposite vertex is $(2,-1)$. Find the equations of the remaining sides.
16. Two sides of a parallelogram are given by $4 x+5 y=0$ and $7 x+2 y=0$ and one diagonal is $11 x+7 y=9$. Find the equations of the remaining sides and the other diagonal.

## PAIR OF STRAIGHT LINES

1. If the equation $a x^{2}+2 h x y+b y^{2}=0$ represents a pair of lines, prove that combined equation of the pair of lines bisecting the angle between those lines is $h\left(x^{2}-y^{2}\right)=(a-b) x y$.
2. Show that the area of the triangle formed by the lines $a x^{2}+2 h x y+b y^{2}=0$ and $\ell x+m y+n=0$ is $\frac{n^{2} \sqrt{h^{2}-a b}}{\left|a m^{2}-2 h \ell m+b \ell^{2}\right|} s q$. units.
3. Prove that the product of perpendiculars from a point $(\alpha, \beta)$ to the pair of striaght lines $a x^{2}+2 h x y+b y^{2}=0$ is $\frac{\left|a \alpha^{2}+2 h \alpha \beta+b \beta^{2}\right|}{\sqrt{(a-b)^{2}+4 h^{2}}}$.
4. If $\theta$ is the angle between the pair of lines $a x^{2}+2 h x y+b y^{2}=0$, then prove that $\cos \theta=\frac{a+b}{\sqrt{(a-b)^{2}+4 h^{2}}}$.
5. Show that the lines represented by $(\ell x+m y)^{2}-3(m x-\ell y)^{2}=0$ and $\ell x+m y+n=0$ form an
6. Show that the lines represented by $(\ell x+m y)^{2}-3(m x-\ell y)^{2}=0$ and $(x+m y+n=0$ form an equilateral triangle with area $\frac{n^{2}}{\sqrt{3}\left(\ell^{2}+\mathrm{m}^{2}\right)}$.
7. If $(\alpha, \beta)$ is the centroid of the triangle formed by lines $a x^{2}+2 h x y+b y^{2}=0$ and $\ell x+m y+n=0$, prove that $\frac{\alpha}{\mathrm{b} \ell-\mathrm{hm}}=\frac{\beta}{\mathrm{am}-\mathrm{h} \ell}=\frac{2}{3\left(\mathrm{~b} \ell^{2}-2 \mathrm{~h} \ell \mathrm{~m}+\mathrm{am}^{2}\right)}$
8. Find the centroid and area of a triangle formed by the lines $3 x^{2}-4 x y+y^{2}=0,2 x-y=6$.
9. Prove that the lines represented by the equation $3 x^{2}+48 x y+23 y^{2}=0$ and $3 x-2 y+13=0$ form an equilateral triangle with area $\frac{13}{\sqrt{3}}$ sq.units.
10. If $a x^{2}+2 h x y+b g^{2}+2 g x+2 f y+c=0$ represents a pair of striaght lines, then prove that (i) $a b c+2 f g h-a f^{2}-b g^{2}-\mathrm{ch}^{2}=0$. (ii) $h^{2} \geq a b, g^{2} \geq a c$ and $f^{2} \geq b c$.
11. If the equation $a x^{2}+2 h x y+b y^{2}+2 g x+2 f y+c=0$ represents two parallel striaght lines then prove that
12. Show that the pair of striaght lines $6 x^{2}-5 x y-6 y^{2}=0$ and $6 x^{2}-5 x y-6 y^{2}+x+5 y-1=0$ form a square.
13. Show that the equation $2 x^{2}-13 x y-7 y^{2}+x+23 y-6=0$ represents a pair of striaght lines. Also find the angle between them and coordinates of point of intersection of the lines.
14. Find the angle between the lines joining the origin to the point of intersection of the curve $x^{2}+2 x y+y^{2}+2 x+2 y-5=0$ and the line $3 x-y+1=0$.
15. Show that the lines joining the origin to the points of intersection of the curve $x^{2}-x y+y^{2}+3 x+3 y-2=0$ and the straight line $x-y-\sqrt{2}=0$ are mutually perpendicular.
16. Find the value of ' $k$ ' if the lines joining the origin of the points of intersection of the curve $2 x^{2}-2 x y+3 y^{2}+2 x-y-1=0$ and the line $x+2 y=k$ are mutually perpendicular.
17. Write down the equation of pair of straight lines joining the origin to the points of intersection of the line $6 x-y+8=0$ with the pair of striaght lines $3 x^{2}+4 x y-4 y^{2}-11 x+2 y+6=0$, show that the lines so obtained make equal angles with the coordinate axes.
18. Find the condition for the chord $\ell x+m y=1$, of the circle $x^{2}+y^{2}=a^{2}$ (whose centre is the origin) to subtend a right angle at the origin.
19. If a ray makes angles $\alpha, \beta, \gamma, \delta$ with the four diagonals of the cube, find $\cos ^{2} \alpha+\cos ^{2} \beta+\cos ^{2} \gamma+\cos ^{2} \delta$.
20. Find the angle between two diagonals of a cube.
21. Find the direction cosines of two lines which are connected by the relations $\ell-5 m+3 n=0$ and $7 \ell^{2}+5 m^{2}-3 n^{2}=0$.
22. Find the angle between the lines whose direction cosines satisfy the equations $\ell+m+n=0, \ell^{2}+m^{2}-n^{2}=0$.
23. Find the angle between the lines whose direction cosines are given by the equation $3 \ell+m+5 n=0$ and $6 m n-2 n \ell+5 \ell m=0$.
24. Show that the lines whose d.c's are given by $\ell+m+n=0,2 m n+3 n \ell-5 \ell m=0$ are perpendicular to each other.
25. Find the direction cosines of two lines which are connected by the relations $\ell+m+n=0, m n-2 n \ell-2 \ell m=0$.
26. Show that the points $(4,7,8),(2,3,4),(-1,-2,1),(1,2,5)$ are vertices of a parallelogram.
27. If $(6,10,10),(1,0,-5)(6,-10,0)$ are vertices of a triangle find the direction ratios of its sides. Dertermine whether it is right angled or isosceles.
28. If $A(4,8,12), B(2,4,6), C(3,5,4)$ and $D(5,8,5)$ are four points, show that the lines $A B$ and $C D$ intersect.

## Differentiation

1. If $\sqrt{1-x^{2}}+\sqrt{1-y^{2}}=a(x-y)$, then prove that $\frac{d y}{d x}=\sqrt{\frac{1-y^{2}}{1-x^{2}}}$.
2. If $y=\operatorname{Tan}^{-1}\left(\frac{\sqrt{1+x^{2}}+\sqrt{1-x^{2}}}{\sqrt{1+x^{2}}-\sqrt{1-x^{2}}}\right)$ for $0<x<1$, find $\frac{d y}{d x}$.
3. If $y=x^{\tan x}+(\sin x)^{\cos x}$, find $\frac{d y}{d x}$.
4. If $y=(\sin x)^{\log x}+x^{\sin x}$, find $\frac{d y}{d x}$.
5. If $x^{y}+y^{x}=a^{b}$, show that $\frac{d y}{d x}=-\left[\frac{y x^{y-1}+y^{x} \log y}{x^{y} \log x+x y^{x-1}}\right]$.
6. If $y=x \sqrt{a^{2}+x^{2}}+a^{2} \log \left(x+\sqrt{a^{2}+x^{2}}\right)$, then show that $\frac{d y}{d x}=2 \sqrt{a^{2}+x^{2}}$.
7. If $f(x)=\operatorname{Sin}^{-1} \sqrt{\frac{x-\beta}{\alpha-\beta}}$ and $g(x)=\operatorname{Tan}^{-1} \sqrt{\frac{x-\beta}{\alpha-x}}$, then prove that $f^{\prime}(x)=g^{\prime}(x),(\beta<x<\alpha)$.
8. If $a>b>0$ and $0<x<\pi, f(x)=\left(a^{2}-b^{2}\right)^{-1 / 2} \cos ^{-1}\left(\frac{a \cos x+b}{a+b \cos x}\right)$ then $f^{\prime}(x)=(a+b \cos x)^{-1}$.
9. If $x=\frac{3 a t}{1+t^{3}}, y=\frac{3 a t^{2}}{1+t^{3}}$ then find $\frac{d y}{d x}$.
10. If $y=\frac{(1-2 x)^{2 / 3}(1+3 x)^{3 / 4}}{(1-6 x)^{5 / 6}(1+7 x)^{-6 / 7}}$, find $\frac{d y}{d x}$.

## Tangents and Normals

1. If the tangent at any point on the curve $x^{2 / 3}+y^{2 / 3}=a^{2 / 3}$ intersects the coordinate axes in $A$ and $B$, then show that the length $A B$ is constant.
2. If the tangent at any point $P$ on the ccurve $x^{m} \cdot y^{n}=a^{m+n}(m n \neq 0)$ meets the coordinate axes in $A, B$ then show that $A P$ : PB is a constant.
3. Show that the curves $y^{2}=4(x+1), y^{2}=36(9-x)$ intersect orthogonally.
4. Find the angle between the curves given by $x+y+2=0, x^{2}+y^{2}-10 y=0$.
5. Find the angle between the curves $y^{2}=8 x$ and $4 x^{2}+y^{2}=8$.
6. Find the angle between the curves $2 y^{2}-9 x=0,3 x^{2}+4 y=0$ (in the $4^{\text {th }}$ quadrant)
7. Find the angle between the curves $x y=2$ and $x^{2}+4 y=0$.
8. Find the angle between the curves $y^{2}=4 x$ and $x^{2}+y^{2}=5$.
9. Find the length of subtangent, subnormal at a point ' $t$ ' on the curve $x=a(\cos t+t \sin t) y=a(\operatorname{sint}-t \cos t)$.
10. At any point ' $t$ ' on the curve $x=a(t+\sin t) y=a(1-\cos t)$, find the lengths of tangent, normal, subtangent, and subnormal.
11. Show that the square of the length subtangent at any point on the curve by ${ }^{2}=(x+a)^{3}(b \neq 0)$ varies with the length of the sub normal and that point.
12. Show that the condition for the orthogonality of the curves $a x^{2}+b y^{2}=1$, and $a_{1} x^{2}+b_{1} y^{2}=1$ is $\frac{1}{a}-\frac{1}{b}=\frac{1}{a_{1}}-\frac{1}{b_{1}}$.

## Maxima and Minima

1. If the curved surface at right circular cylinder inscribed in a sphere of radius $r$ is maximum, show that the height of the cylinder is $\sqrt{2} \mathrm{r}$.
2. Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.
3. A window is in the shape of a rectangle surmounted by a semicircle. If the perimeter of the window is 20 ft , find the maximum area.
4. From a rectangular sheet of diminsions $30 \mathrm{~cm} \times 80 \mathrm{~cm}$, four equal squares of side $\times \mathrm{cm}$ are removed at the corners and the sides are then turned up so as to form an open rectangular box. Find the value of $x$, so that volume of the box is the greatest.
5. A wire of length ' $\ell$ ' is cut into two parts which are bent respectively in the form of a square and a circle. What are the lengths of the pieces of the wire respectively so that the sum of the areas is the least.
6. Find the maximum area of the rectangle that can be formed with fixed perimeter 20.
7. The profit function $p(x)$ of a company selling $x$ items per day is given by $p(x)=(150-x) x-1000$. Find the number of items that the company should manufacture to get maximum profit. Also find the maximum profit.
8. Find local maximum or local minimum of $f(x)=-\sin 2 x-x$ defined on $[-\pi / 2, \pi / 2]$.
9. Find the absolute maximum and absolute minimum of $f(x)=2 x^{3}-3 x^{2}-36 x+2$ on the interval [0,5].
10. Find the positive integers $x$ and $y$ such that $x+y=60$ and $x y^{3}$ is maximum.

## SHORT ANSWER QUESTIONS (4 MARKS)

## LOCUS

1. If the distances from $P$ to the points $(2,3)$ and $(2,-3)$ are in the ratio $2: 3$, then find the equation of locus of $P$.
2. $A(1,2), B(2,-3)$ and $C(-2,3)$ are three points. A point ' $P$ ' moves such that $P A^{2}+P B^{2}=2 P^{2}$. Show that the equation to the locus of ' $P$ ' is $7 x-7 y+4=0$
3. $\quad A(5,3)$ and $B(3,-2)$ are two fixed points. Find the equation of locus of $P$, so that the area of triangle $P A B$ is 9 .
4. Find the equation of locus of $P$, if the line segment joining $(2,3),(-1,5)$ subtends a right angle at $P$.
5. The ends of the hypotenuse of a right angled triangle are $(0,6)$ and $(6,0)$. Find the equation of locus of its third vertex.
6. Find the equation of the locus of point ' $P$ ' such that the distance of $P$ from the origin is twice the distance of $P$ from $A(1$, 2)
7. Find the equation of locus of the point, the sum of whose distances from $(0,2)$ and $(0,-2)$ is 6 units.
8. Find the equation of locus of a point, the difference of whose distances from $(-5,0)$ and $(5,0)$ is 8 units.
9. Find the equation of locus of $P$, if $A=(4,0), B-(-4,0)$ and $|P A-P B|=4$.

## TRANSFORMATION OF AXES

1. Find the point to which the origin is to be shifed by translation of axes so as to remove the first degree terms from the equation $a x^{2}+2 h x y+b y^{2}+2 g x+2 f y+c=0$, where $h^{2} \neq a b$.
2. Find the transformed equation of $2 x^{2}+4 x y+5 y^{2}=0$, when the origin is shifted to $(3,4)$ by the translation of axes.
3. If the transformed equation of a curve is $X^{2}+3 X Y-2 Y^{2}+17 X-7 Y-11=0$. when the origin is shifted to (2, 3$)$. Find the original equation of the curve.
4. Show that the axes are to be rotated through an angle of $\frac{1}{2} \operatorname{Tan}^{-1}\left(\frac{2 h}{a-b}\right)$ so as to remove xy term from the equation $a x^{2}+2 h x y+b y^{2}=0$ if $a \neq b$ and through an angle $\pi / 4$ if $a=b$.
5. Find the transformed equation of $3 x^{2}+10 x y+3 y^{2}=9$ when the axes are rotated through an angle $\pi / 4$.
6. Find the transformed equation of $x^{2}+2 \sqrt{3} x y-y^{2}=2 a^{2}$ when the axes are rotated through an angle $\pi / 6$.
7. Find the transformed equation of $x \cos \alpha+y \sin \alpha=p$ when the axes are rotated through an angle $\alpha$.
8. When the axes are rotated through an angle $45^{\circ}$, the transformed equation of a curve is $17 X^{2}-16 X Y+17 Y^{2}=225$. Find the original equation of the curve.

## STRAIGHT LINE

1. Find the points on the line $3 x-4 y-1=0$ which are at distance of 5 units from the point $(3,2)$.
2. Transform the equation $\frac{x}{a}+\frac{y}{b}=1$ into the normal form wheres $>0, b>0$. If the perpendicular distance of the straight line from the origin is $p$, deduce that
3. Find the equation of the straight line passing through the point $(1,3)$ and i) parallel to
ii) perpendicular to the line passing through the points $(3,-5)$ and $(-6,1)$.
4. Find the equation of the straight line passing through the points $(-1,2)$ and $(5,-1)$ and also find the area of the triangle formed by it with the coordinate axes.
5. A straight line through $Q(\sqrt{3}, 2)$ makes an angle $\frac{\pi}{6}$ with the positive direction of $x$-axis. If the straight line intersects
the line $\sqrt{3} x-4 y+8=0$ at $P$, find the distance $P Q$.
6. A straight line with slope 1 passes through $Q(-3,5)$ and meets the straight line $x+y-6=0$ at $P$. Find the distance $P Q$.
7. A straight line through $Q(2,3)$ makes an angle $3 \pi / 4$ with negative direction of the $x$-axis. If the straight line intersects the line $x+y-7=0$ at $P$. Find the distance $P Q$.
8. Prove that the ratio in which the straight line $L \equiv a x+b y+c=0$ divides the line segment joining the points $A\left(x_{1}, y_{1}\right)$ and $B\left(x_{2}, y_{2}\right)$ is $-L_{11}: L_{22}$
9. Find the value of ' $k$ ' if the lines $2 x-3 y+k=0,3 x-4 y-13=0$ and $8 x-11 y-33=0$ are concurrent.
10. If the straight lines $a x+b y+c=0, b x+c y+a=0$ and $c x+a y+b=0$ are concurrent, then prove that $a^{3}+b^{3}+c^{3}=3 a b c$.
11. Find the equation of the straight line perpendicular to the line $2 x+3 y=0$ and passing through the point of intersection of the lines $x+3 y-1=0$ and $x-2 y+4=0$.
12. Show that the lines $x-7 y-22=0,3 x+4 y+9=0$ and $7 x+y-54=0$ form a right angled isosceles triangle.
13. Find the values of $k$, if the angle between the straight lines $4 x-y+7=0$ and $k x-5 y-9=0$ is $45^{\circ}$.
14. Find the equations of the lines passing through the points $(-3,2)$ and making an angle of $45^{\circ}$ with the line $3 x-y+4=0$.
15. Find the equation of the straight line making non-zero equal interepts on the coordinate axes and passing through the point of intersection of the lines $2 x-5 y+1=0$ and $x-3 y-4=0$.
16. If $3 a+2 b+4 c=0$, then show that the equation $a x+b y+c=0$ represents a family of concurrent lines and find the point of concurrency.
17. Find the equation of the line passing through the point of intersection of $2 x+3 y=1,3 x+4 y=6$ and perpendicular to the line $5 x-2 y=7$.
18. Find the equation of the line perpendicular to the line $3 x+4 y+6=0$ and making an intercept ' -4 ' on the $x$-axis.
19. $x-3 y-5=0$ is the perpendicular bisector of the line segment joining the points $A, B$. If $A=(-1,-3)$, find the coordinates of $B$.

## Limits \& Continuity (SAQ)

1. Show that $f(x)=\left\{\begin{array}{lll}\frac{\cos a x-\cos b x}{x^{2}} & \text { if } & x \neq 0 \\ \frac{1}{2}\left(b^{2}-a^{2}\right) & \text { if } & x=0\end{array}\right.$, where a and $b$ are real constants is continuous at 0 .
2. Check the continuity of $f$ given by $f(x)=\left\{\begin{array}{ccc}\frac{x^{2}-9}{x^{2}-2 x-3} & \text { if } & 0<x<5 \\ 1.5 & \text { if } & x=3\end{array}\right.$ and $x \neq 3$ at the point 3 .
3. Check the continuity of the following function at 2: $f(x)=\left\{\begin{array}{c}\frac{1}{2}\left(x^{2}-4\right) \text { if } 0<x<2 \\ 0 \text { if } x=2 \\ 2-8 x^{-3} \text { if } x>2\end{array}\right.$
4. Find real constants $a$, $b$ so that the function $f$ given by $f(x)=\left\{\begin{array}{ccc}\sin x & \text { if } & x \leq 0 \\ x^{2}+a & \text { if } & 0<x<1 \\ b x+3 & \text { if } & 1 \leq x \leq 3 \\ -3 & \text { if } & x>3\end{array}\right.$ is continuous on $R$.
5. If $f$ given by $f(x)=\left\{\begin{array}{cll}k^{2} x-k & \text { if } & x \geq 1 \\ 2 & \text { if } & x<1\end{array}\right.$ is a continuous function on $R$, then find the values of $k$.
6. Check the continunity of the function $f$ at 1 and 2. $f(x)=\left\{\begin{array}{clc}x+1 & \text { if } & x \leq 1 \\ 2 x & \text { if } & 1<x<2 . \\ 1+x^{2} & \text { if } & x \geq 2\end{array}\right.$.
7. Check the continuity of ' $f$ ' given by $f(x)=\left\{\begin{array}{ccc}4-x^{2} & \text { if } & x \leq 0 \\ x-5 & \text { if } & 0<x \leq 1 \\ 4 x^{2}-9 & \text { if } & 1<x<2 \\ 3 x+4 & \text { if } & x \geq 2\end{array}\right.$ at the points 0,1 and 2.
8. Compute $\lim _{x \rightarrow \infty} \frac{x^{2}-\sin x}{x^{2}-2}$.
9. Compute $\lim _{x \rightarrow 0} \frac{\cos x-\cos b x}{x^{2}}$.
10. Evaluate $\lim _{x \rightarrow a} \frac{x \sin a-a \sin x}{x-a}$
11. Compute $\lim _{x \rightarrow 2} \frac{2 x^{2}-7 x-4}{(2 x-1)(\sqrt{x}-2)}$.
12. Compute $\lim _{x \rightarrow a} \frac{\sqrt{a+2 x}-\sqrt{3 x}}{\sqrt{3 a+x}-2 \sqrt{x}}$.
13. Compute $\lim _{x \rightarrow 0} \frac{x\left(e^{x}-1\right)}{\sqrt{1-\cos x}}$.
14. Find $\operatorname{Lt}_{x \rightarrow 0} \frac{\sqrt[3]{1+x}-\sqrt[3]{1-x}}{x}$.

## DIFFERENTIATION

1. Find the derivatives of the following functions from the first principles
1) $\sin 2 x$
2) $\tan 2 x$
3) $\sec 3 x$
4) $\cos a x$
5) $x^{3}$
2. Find the derivatives of the following functions using the definition
1) $\cos ^{2} x$
2) $\sqrt{x+1}$
3) $\log x$
4) $x \sin x$
3. If $u, v$ are two differentiable functions at $x$, then prove that $u v$ is also differnetiable at $x$ and $\frac{d}{d x}(u v)=u \frac{d v}{d x}+v \frac{d u}{d x}$ (Product rule).
4. If $u$, $v$ are two differentiable functions in $x$, then prove that $\frac{d}{d x}\left(\frac{u}{v}\right)=\frac{v \frac{d u}{d x}-u \frac{d v}{d x}}{v^{2}}$ (Quotient rule).
5. If $\sin y=x \cdot \sin (a+y)$ prove that $\frac{d y}{d x}=\frac{\sin ^{2}(a+y)}{\sin a}$
6. If $x^{y}=e^{x-y}$ prove that $\frac{d y}{d x}=\frac{\log x}{(1+\log x)^{2}}$
7. Find the derivative of the function $y=x^{y}$.
8. If $y=\operatorname{Tan}^{-1}\left(\frac{2 x}{1-x^{2}}\right)+\operatorname{Tan}^{-1}\left(\frac{3 x-x^{3}}{1-3 x^{2}}\right)-\operatorname{Tan}^{-1}\left(\frac{4 x-4 x^{3}}{1-6 x^{2}+x^{4}}\right)$ then prove that $\frac{d y}{d x}=\frac{1}{1+x^{2}}$.
9. Differentiable $f(x)=\operatorname{Tan}^{-1}\left(\frac{2 x}{1-x^{2}}\right)$ w.r.t. $g(x)=\operatorname{Sin}^{-1}\left(\frac{2 x}{1+x^{2}}\right)$
10. Differentiate $f(x)$ w.r.t. $g(x)$ if $f(x)=\operatorname{Sec}^{-1}\left(\frac{1}{2 x^{2}-1}\right), g(x)=\sqrt{1-x^{2}}$.
11. Find the derivative of $f(x)=\operatorname{Tan}^{-1}\left(\frac{\sqrt{1+x^{2}}-1}{x}\right)$ w.r.t. $g(x)=\operatorname{Tan}^{-1} x$.
12. Find $\frac{d y}{d x}$ if $x=a(\cos t+t \sin t), y=a(\sin t-t \cos t)$.
13. If $x=a\left[\cos t+\log \left(\tan \frac{t}{2}\right)\right], y=\operatorname{asin} t$, find $\frac{d y}{d x}$.
14. If $x=a(t-\sin t), y=a(1+\cos t)$, find $\frac{d^{2} y}{d x^{2}}$.
15. If $y=a x^{n+1}+b x^{-n}$, then show that $x^{2} y^{\prime \prime}=n(n+1) y$.
16. If $a x^{2}+2 h x y+b y^{2}=1$, then prove that $\frac{d^{2} y}{d x^{2}}=\frac{h^{2}-a b}{(h x+b y)^{3}}$

## Rate measure

1. The distance-time formula for the motion of a particle along a striaght line is $s=t^{3}-9 t^{2}+24 t-18$. Find when and where the velocity is zero.
2. A particle is moving in a straight line so that after $t$ seconds, its distances(incms) from a fixed point on the line is given by $s=f(t)=8 t+t^{3}$. find
i) velocity at time $t=2 \mathrm{sec}$
ii) the initial velocity
iii) acceleration at $t=2$ sec.
3. The displacement $s$ of a particle travelling in a straight line in seconds is given by $s=45 t+11 t^{2}-t^{3}$. Find the time when the particle comes to rest.
4. A particle is moving along a line according to $s=f(t)=4 t^{3}-3 t^{2}+5 t-1$, where $s$ is measured in meters and $t$ is measured in seconds. Find the velocity and acceleration at time $t$. At what time acceleration is zero?
5. A point $p$ is moving on the curve $y=2 x^{2}$. The $x$ coordinate of $P$ is incresing at the rate of 4 units per second. Find the rate at which the $y$ coordinate is increasing when the point is at $(2,8)$.
6. The volume of a cube is increasing at a rate of 9 cubic centimeters per second. How fast is the surface area increasing when the length gf the edge is 10 centimeters?
7. The radius of a circle is increasing at the rate of $0.7 \mathrm{~cm} / \mathrm{sec}$. What is the rate of increase of its circumference?
8. A stone is dropped into a quiet lake and ripples moves in circles at the speed of $5 \mathrm{~cm} / \mathrm{sec}$. At the instant when the radius of circular ripple is 8 cm. , how fast is the enclosed area increases?
9. The radius of an air bubble is increasing at the rate of $\frac{1}{2} \mathrm{~cm} / \mathrm{sec}$. At what rate is the volume of the bubble increasing when the radius is 1 cm ?
10. A balloon, which always remains spherical on inflation, is being inflated by pumping in 900 cubic centimeters of gas per second. Find the rate at which the radius of balloon increases when the radius is 15 cm .
11. A container in the shape of an inverted cone has height 12 cm and radius 6 cm at the top. If it is filled with water at the rate of $12 \mathrm{~cm}^{3} / \mathrm{sec}$., what is the rate of change in the height of water level when the tank is filled 8 cm .?
12. Suppose we have a rectangular aquarium with dimensions of length 8 m , width 4 m and height 3 m . Suppose we are filling the tank with water at the rate of $0.4 \mathrm{~m}^{3} / \mathrm{sec}$. How fast is the height of water changing when the water level is 2.5 m ?

## Tangents and Normals

1. Find the equations of tangent and normal to the curve $y^{4}=a x^{3}$ at $(a, a)$.
2. Find the length of normal and subnormal at a point $m$ the curve $y=\frac{a}{2}\left(e^{\frac{x}{a}}+e^{\frac{-x}{2}}\right)$.
3. Find the equations of tangent and normal to the curve $y=x^{3}+4 x^{2}$ at $(-1,3)$.
4. Show that the tangent at $P\left(x_{1}, y_{1}\right)$ on the curve $\sqrt{x}+\sqrt{y}=\sqrt{a}$ is $y_{y_{1}}{ }^{-1 / 2}+x x_{1}^{-1 / 2}=a^{1 / 2}$.
5. At a point $\left(x_{1}, y_{1}\right)$ on the curve $x^{3}+y^{3}=3 a x y$, show that the equation of tangent is $\left(x_{1}{ }^{2}-a y_{1}\right) x+\left(y_{1}{ }^{2}-a x_{1}\right) y=a x_{1}, y_{1}$.
6. Show that the tangent at any point $\theta$ on the curve $x=c \sec \theta, y=c \tan \theta$ is $y \sin \theta=x-c \cos \theta$.
7. Show that the curves $6 x^{2}-5 x+2 y=0$ and $4 x^{2}+8 y^{2}=3$ touch each other at $\left(\frac{1}{2}, \frac{1}{2}\right)$.
8. Show that the area of the triangle formed by the tangent at any point on the curve $x y=c(c \neq 0)$, with the coordinate axes is constant.
9. Find the value of $k$ so that the length of the subnormal at any point on the curve $y=a^{1-k} . x^{k}$ is constant.
10. Show that at any point $(x, y)$ on the curve $y=b e^{x / a}$, the length of the subtangent is a constant and the length of the subnormal is $\frac{y^{2}}{a}$.
11. Determine the intervals in which $f(x)=\frac{2}{x-1}+18 x$ for all $x \in R-\{0\}$ is strictly increasing and decreasing.
12. Find the tangent and normal to the curve $y=2 e^{-x / 3}$ at the point where the curve meets the $y-$ axis.

## MATHS IB

## VERY SHORT ANSWER QUESTIONS (2 MARKS)

## STRAIGHT LINE (Q1)

1. Find the value of $x$, of the slope of the line passing through $(2,5)$ and $(x, 3)$ is 2 .

Ans) $x=1$
2. $A(10,4), B(-4,9)$ and $C(-2,-1)$ are the vertices of a triangle $A B C$. Find the equation of the altitude through $B$.

Ans) $12 x+5 y+3=0$
3. Find the equation of the straight line passing through $(-2,4)$ and making non zero intercepts whose sum is zero.

Ans) $\mathbf{x - y + 6 = 0}$
4. Find the equation of the straight line passing through $(-4,5)$ and cutting off equal non-zero intercepts on the coordinate axes.

Ans) $\mathbf{x + y - 1 = 0}$
5. Find the equation of straight line passing through the point $(2,3)$ and making intercepts on the axes of coordinates, whose sum is zero.
6. Find the condition for the points $(a, 0),(h, k)$ and $(0, b)$ when $a b \neq 0$ to be collinear.

$$
\text { Ans) } \frac{\mathrm{h}}{\mathrm{a}}+\frac{\mathrm{k}}{\mathrm{~b}}=1
$$

7. Find the equation of the straight line passing through the points $\left(a t_{1}^{2}, 2 a t_{1}\right)$ and $\left(a t_{2}^{2}, 2 a t_{2}\right)$.

Find the area of the triangle formed by the coordinate axes and the line $\quad$ Ans) $\mathbf{y}\left(\mathbf{t}_{1}+\mathbf{t}_{\mathbf{2}}\right)=\mathbf{2 x}+\mathbf{2} \mathbf{a t}_{1} \mathbf{t}_{\mathbf{2}}$
8. Find the area of the triangle formed by the coordinate axes and the line $x \cos \alpha+y \sin \alpha=p$.

$$
\text { Ans) } p^{2}|\operatorname{cosec} 2 \alpha| s q \text { unts }
$$

9. If the area of the triangle formed by the straight lines $x=0, y=0$ and $3 x+4 y=a(a>0)$ is 6 , find the value of $a$. Ans) $\mathbf{a}=12$
10. Reduce the equation $4 x-3 y+12=0$ into (i) slope-intercept form (ii) intercept form. Ans) $y=\frac{4}{3} x+4, \frac{x}{-3}+\frac{y}{4}=1$

## STRAIGHT LINE (Q2)

1. Transform the equation $x+y+1=0$ into normal form.
2. Transform the equation $x+y-2=0$ intonormal form.

> Ans) $x \cos \frac{5 \pi}{4}+y \sin \frac{5 \pi}{4}=\frac{1}{\sqrt{2}}$
> Ans) $x \cos \frac{\pi}{4}+y \sin \frac{\pi}{4}=\sqrt{2}$
3. Find the value of $p$, if the lines $4 x-3 y-7=0,2 x+p y+2=0$ and $6 x+5 y-1=0$ are concurrent.

Ans. $p=4$
4. Find the value of $p$, if the straight lines $x+p=0, y+2=0,3 x+2 y+5=0$ are concurrent. $\quad$ Ans) $p=\frac{\mathbf{1}}{\mathbf{3}}$
5. Find the equations of the straight line passing through the origin and making equal angles with the coordinate axes.

Ans) $\mathbf{x - y}=\mathbf{0}, \mathbf{x + y}=\mathbf{0}$
6. Find the ratio in which the straight line $2 x+3 y=5$ divide the line joining the points $(0,0)$ and $(-2,1)$ Ans. 5:6 externally.
7. If $\theta$ is the angle between the lines $\frac{x}{a}+\frac{y}{b}=1, \frac{x}{b}+\frac{y}{a}=1$, find the value of $\sin \theta(a>b)$.

Ans) $\sin \theta=\frac{a^{2}-b^{2}}{a^{2}+b^{2}}$
8. Find the value of $k$, if the straight lines $6 x-10 y+3=0, k x-5 y+8=0$ are parallel.

Ans) $\mathbf{k}=3$
9. Find the values of $k$ if $y-3 k x+4=0,(2 k-1) x-(8 k-1) y-6=0$ are perpendicular.

Ans) $k=-1, \frac{1}{6}$
10. Find the equation of the straight line passing through the point $(5,4)$ and parallel to the line $2 x+3 y+7=0$.

Ans) $2 \mathrm{x}+3 \mathrm{y}-\mathbf{2 2}=\mathbf{0}$
11. Find the equation of the straight line perpendicular to the line $5 x-3 y+1=0$ and passing through the point (4,-3).

Ans. $3 x+5 y+3=0$.
12. Find the distance between the parallel lines $3 x+4 y-3=0$ and $6 x+8 y-1=0$.

Ans) $\frac{1}{2}$ Unit
13. Find the distance between the parallel lines $5 x-3 y-4=0,10 x-6 y-9=0$.
14. Find the foot of the perpendicular from $(-1,3)$ on the straight line $5 x-y-18=0$.

Ans) $\frac{1}{2 \sqrt{34}}$ Units
Ans) $(4,2)$

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## THREE DIMENSIONAL COORDINATES (Q3)

1. Show that the points $(1,2,3),(2,3,1)$ and $(3,1,2)$ form an equilateral triangle. $\quad$ Ans $) \mathbf{A B}=\mathbf{B C}=\mathbf{C A}=\mathbf{6}$
2. Show that the points $(2,3,5),(-1,5,-1)$ and $(4,-3,2)$ form a right angled isosceles triangle.
3. Find the distance between the midpoint of the line segment $\overline{A B}$ and the point $(3,-1,2)$ where $A=(6,3,-4), B=(-2,-1,2)$.

Ans) $\sqrt{14}$ Units
4. Show that the points $A=(1,2,3), B=(7,0,1), C=(-2,3,4)$ are collinear.
5. Find the ratio in which $Y Z$ plane divides the line joining $A(2,4,5)$ and $B(3,5,-4)$. Also find the point of intersection. Ans) -2 : 3, (0, 2, 23)
6. Find the coordinates of the vertex $C$ of $\triangle A B C$, it its centroid is the origin and the vertices $A, B$ are $(1,1,1)$ and $(-2,4,1)$ respectively.

Ans) (1, -5, -2)
7. Find the centroid of the tetrahedron whose vertices are $(2,3,-4),(-3,3,-2),(-1,4,2),(3,5,1) \quad$ Ans) $\left(\frac{1}{4}, \frac{15}{4}, \frac{-3}{4}\right)$
8. If $(3,2,-1),(4,1,1)$ and $(6,2,5)$ are three vertices and $(4,2,2)$ is the centroid of a tetrahedron, find the fourth vertex.

Ans) (3, 3, 3)
9. Find the fourth vertex of the parallelogram whose consecutive vertices are $(2,4,-1),(3,6,-1)$ and $(4,5,1)$

Ans) (3, 3, 1)
10. Find the ratio in which the $X Z$-plane divides the line joining $A(-2,3,4)$ and $B(1,2,3)$.

Ans. 3 : 2 externally

## THE PLANE (Q4)

1. Write the equation of the plane $4 x-4 y+2 z+5=0$ in intercepts form.
2. Find the equation of the plane whose intercepts on $X, Y, Z$ axes are 1, 2, 4 respectively.

$$
\text { Ans) } \frac{x}{1}+\frac{y}{2}+\frac{z}{4}=1 \Rightarrow 4 x+2 y+z-4=0
$$

Reduce the equation $x+2 y-3 z-6=0$ of the plane to the normal form.
4. Find the direction cosines of the normal to the plane $x+2 y+2 z-4=0$.
5. Find the equation of the plane passing through $(-2,1,3)$ and having $(3,-5,4)$ as direction ratios of its normal.

Ans) $3 \mathrm{x}-5 \mathrm{y}+4 \mathrm{z}-1=0$
6. Find the equation of the plane if the foot of the perpendicular from origin to the plane is $(2,3,-5)$.

Ans) $2 x+3 y-5 z-38=0$
7. Find the equation of the plane passing through the point $(1,1,1)$ and parallel to the plane $x+2 y+3 z-7=0$.

Ans) $x+2 y+3 z-6=0$
8. Find the equation of the plane bisecting the line segment joining $(2,0,6)$ and $(-6,2,4)$ and perpendicular to it.

Ans) $4 \mathrm{x}-\mathrm{y}+\mathrm{z}+4=0$
9. Find the angle between the planes $2 x-y+z=6$ and $x+y+2 z=7$
10. Find the angle between the planes $x+2 y+2 z-5=0$ and $3 x+3 y+2 z-8=0$.
11. Find the constant $k$ so that the planes $x-2 y \pm k z=0$ and $2 x+5 y-z=0$ are at right angles.

Ans. $\cos ^{-1}\left(\frac{13}{3 \sqrt{22}}\right)$

LIMITS (Q5)

1. Show that $\operatorname{Lt}_{x \rightarrow 0^{+}}\left(\frac{2|x|}{x}+x+1\right)=3$.
2. Evaluate $\operatorname{Lim}_{x \rightarrow 0} \frac{\sin a x}{x \cos x}$.

Ans) a
3. Evaluate $\operatorname{Lim}_{x \rightarrow 0} \frac{e^{x}-1}{\sqrt{1+x}-1}$.

Ans) 2
4. Compute $\operatorname{Lim}_{x \rightarrow 0} \frac{3^{x}-1}{\sqrt{1+x}-1}$.

Ans) $2 \log 3$
5. Show that $\operatorname{Lim}_{x \rightarrow 0} \frac{\sqrt{x+1}-1}{x}=\frac{1}{2}$.
6. Compute $\operatorname{Lt}_{x \rightarrow 0} \frac{a^{x}-1}{b^{x}-1}(a>b>0, b \neq 1)$.

Ans) $\log _{b} a$
7. Compute $\operatorname{Lim}_{x \rightarrow 0} \frac{e^{7 x}-1}{x}$.
8. Evaluate $\operatorname{Lt}_{x \rightarrow \frac{\pi}{2}} \frac{\cos x}{x-\frac{\pi}{2}}$.
9. Evaluate $\operatorname{Lim}_{x \rightarrow 0} \frac{\sin (a+b x)-\sin (a-b x)}{x}$.
10. Compute $\operatorname{Lim}_{x \rightarrow 0} \frac{1-\cos 2 m x}{\sin ^{2}(n x)}$.

## LIMITS (Q6)

1. Evaluate $\operatorname{Lt}_{x \rightarrow 0} \frac{e^{3+x}-e^{3}}{x}$.

Ans) $e^{3}$
2. Find $\operatorname{Lt}_{x \rightarrow 0}\left(\frac{e^{x}-\sin x-1}{x}\right)$.

Ans) 0
3. Compute $\lim _{x \rightarrow 1} \frac{\sin (x-1)}{x^{2}-1}$.
4. Compute $\lim _{x \rightarrow a} \frac{\tan (x-a)}{x^{2}-a^{2}}$. Ans) $\frac{1}{2}$

Ans) $\frac{1}{2 a}$
5. Compute $\lim _{x \rightarrow a} \frac{\sin (x-a) \tan ^{2}(x-a)}{\left(x^{2}-a^{2}\right)^{2}}$.

Ans) 0
6. Evaluate $\lim _{x \rightarrow 1} \frac{\log _{e} x}{x-1}$.

Ans) 1
7. Evaluate $\lim _{x \rightarrow \infty} \frac{11 x^{3}-3 x+4}{13 x^{3}-5 x^{2}-7}$.
8. Evaluate $\operatorname{Lt}_{x \rightarrow \infty}\left(\sqrt{x^{2}+x}-x\right)$.

Ans) $\frac{11}{13}$
9. Show that $\operatorname{Lt}_{x \rightarrow \infty}(\sqrt{x+1}-\sqrt{x})=0$.
10. Find the value of $\operatorname{Lt}_{x \rightarrow \infty} \frac{8|x|+3 x}{3|x|-2 x}$.

Ans) 11
11. Compute $\underset{x \rightarrow \infty}{\operatorname{Lt}} \frac{x^{2}+5 x+2}{2 x^{2}-5 x+1}$

Ans) $\frac{1}{2}$

## DIFFERENTIATION(Q7)

1. Find the derivative of $y=\sqrt{2 x-3}+\sqrt{7-3 x}$

Ans. $\frac{1}{\sqrt{2 x-3}}-\frac{3}{2 \sqrt{7-3 x}}$
2. If $f(x)=a^{x}$. $e^{x^{2}}$ then find $f^{\prime}(x)$.

Ans) $a^{x} e^{x^{2}}(2 x+\log a)$
3. If $y=e^{2 x} \cdot \log (3 x+4)$ then find $\frac{d y}{d x}\left(x>\frac{-4}{3}\right)$.

Ans) $\mathrm{e}^{2 x}\left[\frac{3}{3 x+4}+2 \log (3 x+4)\right]$
4. If $y=x^{2} e^{x} \sin x$, then find $\frac{d y}{d x}$.
5. If $y=\frac{a x+b}{c x+d}$, find $\frac{d y}{d x}$.
6. If $f(x)=7^{x^{3}+3 x}(x>0)$, then find $f^{\prime}(x)$.
7. If $y=\operatorname{Sin}^{-1} \sqrt{x}$, find $\frac{d y}{d x}$.

Ans) $x^{2} e^{x} \cos +x^{2} \sin x e^{x}+e^{x} \sin x .2 x=x^{2} e^{x} \sin x\left(\cot x+1+\frac{2}{x}\right)$

$$
\begin{gathered}
\text { Ans) } \frac{a d-b c}{(c x+d)^{2}} \\
\text { Ans) }(\log 7) 7^{x^{3}+3 x\left(3 x^{2}+3\right)} \\
\text { Ans) } \frac{1}{2 \sqrt{x-x^{2}}} \\
\text { Ans) } \frac{3}{\sqrt{9 x^{2}+16}}
\end{gathered}
$$

8. If $y=\operatorname{Sinh}^{-1}\left(\frac{3 x}{4}\right)$, then find $\frac{d y}{d x}$.
9. If $y=\log (\cosh 2 x)$, find $\frac{d y}{d x}$.
10. If $y=\log (\tan 5 x)$, find $\frac{d y}{d x}$.

Ans) $\mathbf{1 0}$ cose $10 x$
11. If $y=\log \left[\operatorname{Sin}^{-1}\left(e^{x}\right)\right]$, find $\frac{d y}{d x}$.

Ans) $\frac{\mathrm{e}^{\mathrm{x}}}{\sin ^{-1}\left(\mathrm{e}^{\mathrm{x}}\right) \cdot \sqrt{1-\mathrm{e}^{2 x}}}$

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12. If $y=\sec (\sqrt{\tan x})$, then find $\frac{d y}{d x}$.
13. Find the derivative of $\log [\sin (\log x)]$.


Ans) $\frac{\cot (\log x)}{x}$
14. Find the derivative of $\operatorname{Tan}^{-1}(\log x)$
15. If $f(x)=1+x+x^{2}+\ldots \ldots . .+x^{100}$, then find $f^{\prime}(1)$.
$\qquad$

## DIFFERENTIATION (Q8)

1. If $y=\cos [\log (\cot x)]$, find $\frac{d y}{d x}$.

Ans) $\boldsymbol{\operatorname { s i n }}[\log (\cot x)] \sec x \operatorname{cosec} x$

$$
\text { Ans) } \frac{-6 x^{2} \cot ^{-1}\left(x^{3}\right)}{1+x^{6}}
$$

Ans) $x^{x}(1+\log x)$
Ans) $\frac{a^{2} y-4 x^{3}}{4 y^{3}-a^{2}}$
Ans) $\frac{2}{1+\mathrm{x}^{2}}$
Ans) $\frac{1}{2}$ if $0<x<\pi$ and $\frac{-1}{2}$ if $-\pi<x<0$
Ans) $\frac{-2}{\sqrt{1-x^{2}}}$
Ans) $\frac{1}{1+x^{2}}$
Ans) $\frac{-3}{\sqrt{1-x^{2}}}$
10. If $y=\operatorname{Sin}^{-1}\left(3 x-4 x^{3}\right)$, find $\frac{d y}{d x}$.
11. If $\mathrm{x}=\tan \left(\mathrm{e}^{-y}\right)$, then show that $\frac{\mathrm{dy}}{\mathrm{dx}}=\frac{-\mathrm{e}^{y}}{1+\mathrm{x}^{2}}$.
12. If $x=\operatorname{acos}^{3} t, y=\operatorname{asin} 3 t$, find $\frac{d y}{d x}$.
13. Find the derivative of $e^{x}$ with respect to $\sqrt{x}$. Ans) $\frac{3}{\sqrt{1-x^{2}}}$
14. If $y=a e^{n x}+b e^{-n x}$, then prove that $y^{\prime \prime}=n^{2} y$.

## ERRORS AND APPROXIMATIONS (Q9)

1. If $y=x^{2}+x, x=10, \Delta x=0.1$, then find $\Delta y$ and $d y$.
2. If $y=x^{2}+3 x+6$, when $x=10, \Delta x=0.01$, then find $\Delta y$, $d y$.
3. If $y=e^{x}+x, x=5, \Delta x=0.02$, then find $\Delta y$ and $d y$.
4. Find an approximate value of $\sqrt{82}$.
5. Find the approximate value of $\sqrt[3]{65}$.
6. If the increase in the side of a square is $4 \%$, find the percentage of change in the area of the square. Ans : 8
7. The diameter of a sphere is measured to be 40 cm . If an error of 0.02 cm is made in it, then find approximate errors in volume and surface area of the sphere.

## MEAN VALUE THEOREMS (Q10)

1. Verify Rolle's theorem for the function $y=f(x)=x^{2}+4$ in $[-3,3]$.
2. Find the value of ' $c$ ' in Rolle's theorem for the function $f(x)=x^{2}-1$ on $[-1,1]$.
3. Let $f(x)=(x-1)(x-2)(x-3)$. Prove that there is more than one ' $c$ ' in $(1,3)$ such that $f^{\prime}(c)=0$. Ans : $c=2 \pm \frac{1}{\sqrt{3}}$.
4. Verify the conditions of Lagrange's mean value theorem for the function $f(x)=x^{2}-1$ on $[2,3]$. Ans: $\mathbf{c}=\mathbf{2 . 5}$.
5. Verify Lagrange's mean value theorem for the function $f(x)=\log x$ on $[1,2] . \quad$ Ans : $\mathbf{c}=\log _{2} \mathbf{e} \in(1,2)$

Ans : $16 \pi, 1.6 \pi$.
Ans: $\Delta \mathrm{y}=2.11, \mathrm{dy}=2.1$
Ans: $\Delta \mathrm{y}=0.2301, \mathrm{dy}=0.23$
Ans : $\Delta y=e^{5}\left(e^{0.02}-1\right)+0.02, d y=(0.02)\left(e^{5}+1\right)$
Ans: 9.055
Ans: 4.0208

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6. Find the value of $c$ so that $f^{\prime}(c)=\frac{f(b)-f(a)}{b-a}$ where $f(x)=e^{x}, a=0, b=1 . \quad$ Ans : $c=\log _{e}(e-1)$
7. Define the strictly increasing function and strictly decreasing function on an interval.

Ans. $f$ is strictly increasing in $[a, b]$ if $f^{\prime}(x)>0$ for each $x \in(a, b)$
$f$ is strictly decreasing in $[a, b]$ if $f^{\prime}(x)<0$ for each $x \in(a, b)$

## TANGENT AND NORMALS

1. Find the equation of tangent and normal to the curve $y=5 x^{4}$ at $P(1,5)$.
2. Find the equation of normal to the curve $y^{4}=a x^{3}$ at $(a, a)$.
3. Find the slope of the tangent to the curve $y=\frac{x-1}{x+1}$ at $x=0$.
4. Find the length of subtangent at a point on the curve $y=b \sin \left(\frac{\mathbf{x}}{\mathbf{a}}\right)$.
5. Show that at any point on the curve $y=b e^{x / a}$, the length of subnormal is $\frac{y^{2}}{a}$.
6. Show that at any point on the curve $y^{2}=4 a x$, the length of sub normal is constant.
7. Show that at any point $(x, y)$ on the curve $x y=a^{2}$, the subtangent varies as the abscissa.
