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Aimstutorial

ONLINE CLASSES

1. Find the equation of the circle with centre (-7, -3) and radius '4'.

Sol: Given Centre, C (h, k) = (-7, -3),

radius, r = 4

$\therefore$ The equation of the circle is

$(x - h)^2 + (y - k)^2 = r^2$

$\Rightarrow (x + 7)^2 + (y + 3)^2 = 4^2$

$\Rightarrow x^2 + 49 + 14x + y^2 + 9 + 6y = 16$

$\therefore$ **$x^2 + y^2 + 14x + 6y + 42 = 0$**

(H/W) Find the equation of the circle with centre C = (-1, -8) and radius r = 5.

Ans: $x^2 + y^2 + 2x + 16y + 40 = 0$.

2. Find the equation of the circle passing through (3, 4) and having the centre at (-3, 4).

Sol: Given centre C (h, k) = (-3, 4)

Let the given point A = (3, 4)

Since 'A' is the point on the circle.

$\therefore$ Radius, r = CA

$= \sqrt{(-3-3)^2 + (4-4)^2}$

$= \sqrt{(-6)^2 + (0)^2}$

$= \sqrt{36+0} = \sqrt{36} = 6$

$\therefore$ The equation of the required circle is

$(x-h)^2 + (y-k)^2 = r^2$

$\Rightarrow (x + 3)^2 + (y - 4)^2 = 6^2$

$\Rightarrow x^2 + 9 + 6x + y^2 + 16 - 8y = 36$

$\therefore$ **$x^2 + y^2 + 6x - 8y - 11 = 0$.**

3. Find the equation of the circle passing through the origin and has centre at (-4, -3).

Sol: Given centre C = (-4, -3), origin O = (0, 0)

Radius = distance between (0, 0) and (-4, -3)

$= \sqrt{16+9} = \sqrt{25} = 5.$

Equation of the circle which has centre at (-4, -3) and radius 5 is

$(x - x_1)^2 + (y - y_1)^2 = r^2$

$\Rightarrow (x + 4)^2 + (y + 3)^2 = 5^2$

$\Rightarrow x^2 + 8x + 16 + y^2 + 6y + 9 = 25$

$\therefore$ **$x^2 + y^2 + 8x + 6y = 0.$**

4. Find the equation of the circle passing through (2, - 1) having the centre at (2, 3).

Sol: Given C = (2, 3), P = (2, -1)

radius = r = CP = $\sqrt{(2-2)^2 + (3+1)^2} = 4$

Equation of the required circle with centre

C =(2, 3) = (x_1, y_1) and the radius (r) = 4

Equation of circle is $(x - x_1)^2 + (y - y_1)^2 = r^2$

$\Rightarrow (x - 2)^2 + (y - 3)^2 = 4^2$

$\Rightarrow x^2 + 4 - 4x + y^2 + 9 - 6y = 16$

$\therefore$ **$x^2 + y^2 - 4x - 6y - 3 = 0$**

5. Find the equation of the circle passing through (-2,3) having the centre at (0,0).

Sol: Given centre C(h,k) = (0,0)

Let the given point A (- 2,3)

Since A(- 2,3) is a point on the circle.

Radius r = CA = $\sqrt{(-2-0)^2+(3-0)^2}$

$=\sqrt{(-2)^2+(3)^2}=\sqrt{4+9}=\sqrt{13}$

$\therefore$ The equation of the required circle is

$(x-h)^2+(y-k)^2=r^2$.

$\Rightarrow (x-0)^2+(y-0)^2=\sqrt{(13)^2}$ $\Rightarrow$ **$x^2+y^2=13$.**

(H/W)Find the equation of the circle whose centre is (-1,2) and which passes through (5,6).

Ans: $x^2+y^2+2x-4y-47=0$.

6. Find the values of a,b if $ax^2+bxy+3y^2-5x+2y-3=0$ represents a circle. Also find the radius and centre of the circle.

Sol: $ax^2+bxy+3y^2-5x+2y-3=0$

represents a circle

$\Rightarrow$ Coeff. x^2 = Coeff.y^2 and Coeff.of xy = 0

a = 3 and b = 0

Equation of the circle is

$\Rightarrow 3x^2+3y^2-5x+2y-3=0$ $\div 3$

$\Rightarrow x^2+y^2-\frac{5}{3}x+\frac{2}{3}y-1=0$

centre C = $\left(\frac{5}{6},-\frac{1}{3}\right)$, Radius= $\sqrt{g^2+f^2-c}$

$=\sqrt{\left(\frac{-5}{6}\right)^2+\left(\frac{2}{6}\right)^2-(-1)}=\sqrt{\frac{25}{36}+\frac{4}{36}+1}=\sqrt{\frac{25+4+36}{36}}=\frac{\sqrt{65}}{6}$.

7. Find the value of 'a' of $2x^2 + ay^2 - 3x + 2y - 1 = 0$ represents a circle and also find its radius.

Sol: $2x^2 + ay^2 - 3x + 2y - 1 = 0$ represents a circle

$\Rightarrow$ Coe. of x^2 = coe. of y^2 $\Rightarrow a = 2$

Equation of the circle is

$2x^2 + 2y^2 - 3x + 2y - 1 = 0 \quad \div 2$

$\Rightarrow x^2 + y^2 - \frac{3}{2}x + y - \frac{1}{2} = 0$

$2g = \frac{-3}{2} \Rightarrow g = -3/4,\ 2f = 1 \Rightarrow f = \frac{1}{2}, c = \frac{-1}{2}$

Radius = $\sqrt{g^2+f^2-c}$ **=** $\sqrt{\left(\frac{-3}{4}\right)^2+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)}$ **=** $\sqrt{\frac{9}{16}+\frac{1}{4}+\frac{1}{2}}=\sqrt{\frac{9+4+8}{16}}=\frac{\sqrt{21}}{4}$.

8. If $x^2 + y^2 + 2gx + 2fy - 12 = 0$ represents a circle with centre (2, 3) find g, f and its radius.

Sol: Centre of the circle

$x^2 + y^2 + 2gx + 2fy - 12 = 0$ is $(-g, -f) = (2, 3)$

$\Rightarrow g = -2, f = -3$ and $c = -12$

Radius = $\sqrt{g^2+f^2-c} = \sqrt{(-2)^2+(+3)^2-(-12)} = \sqrt{4+9+12} = 5$.

9. If the centre of the circle $x^2 + y^2 + ax + by - 12 = 0$ is (2, 3), find the values of a, b and the radius of the circle.

Sol: Comparing $x^2 + y^2 + ax + by - 12 = 0$ with

$x^2 + y^2 + 2gx + 2fy + c = 0$,

$2g = a \Rightarrow g = \frac{a}{2}$, $2f = b \Rightarrow f = \frac{b}{2}$, $c = -12$.

Now centre $(-g, -f) = \left(\frac{-a}{2}, \frac{-b}{2}\right) = (2, 3) \Rightarrow$ **a = - 4, b = - 6.**

Radius = $\sqrt{g^2+f^2-c} = \sqrt{4+9+12}$ **= 5.**

10. If $x^2 + y^2 - 4x + 6y + c = 0$ represent a circle with radius 6 then find the value of c.

Sol: Given equation of circle $x^2 + y^2 - 4x + 6y + c = 0$

center $c = (-g, -f) = (2, -3)$, Radius = 6

Radius = $\sqrt{g^2+f^2-c} = \sqrt{(2)^2+(-3)^2-c} = 6$

$\Rightarrow 4 + 9 - c = 36 \quad \Rightarrow c = 13 - 36 \quad \therefore c = -23.$

(H/W)If the cirlcle $x^2 + y^2 - 4x + 6y + a = 0$ has radius 4 then find 'a'. (Ans: $a = -3$)

11. Find the centre and radius of circle $x^2 + y^2 + 6x + 8y - 96 = 0$

sol: Given circle is $x^2 + y^2 + 6x + 8y - 96 = 0$

centre = (-3, -4)

Radius = $\sqrt{g^2+f^2-c} = \sqrt{3^2+4^2+96} = \sqrt{9+16+96} = \sqrt{121} = 11.$

12. Find the centre and radius of the circle $\sqrt{1+m^2}(x^2 + y^2) - 2cx - 2mcy = 0$ $(c > 0)$.

Sol: Given equation of the circle is

$\sqrt{1+m^2}(x^2 + y^2) - 2cx - 2mcy = 0 \qquad \div \sqrt{1+m^2}$

$x^2 + y^2 - \frac{2c}{\sqrt{1+m^2}}x - \frac{2mc}{\sqrt{1+m^2}}y = 0$

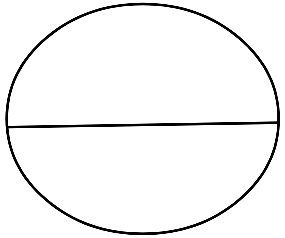
Coordinates of centre = $(-g, -f) = \left(\frac{c}{\sqrt{1+m^2}}, \frac{mc}{\sqrt{1+m^2}}\right)$

Radius = $\sqrt{g^2+f^2-c} = \sqrt{\frac{c^2}{1+m^2}+\frac{m^2c^2}{1+m^2}} = \sqrt{\frac{c^2(1+m^2)}{1+m^2}} = c.$

13. If one end of the diameter of the circle $x^2 + y^2 - 2x + 4y = 0$ is (3, -1), then find the other end of the diameter.

Sol: Centre of the circle $x^2 + y^2 - 2x + 4y = 0$ is C(1, -2)

Let B(a, b) be the other end of the diameter through A(3, -1).



Here C is the midpoint of $\overline{AB} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{3+a}{2}, \frac{-1+b}{2}\right) = (1, -2)$

$\frac{3+a}{2} = 1 \qquad \frac{-1+b}{2} = -2$

$3 + a = 2 \qquad -1 + b = -4$

$a = -1 \qquad b = -3.$

Required point B is (-1, -3).

14. Show that A (- 3, 0) lies on $x^2 + y^2 + 8x + 12y + 15 = 0$ and find the other end of diameter through A.

Sol : Given equation of circle

$x^2 + y^2 + 8x + 12y + 15 = 0$ .... (1)

given A(-3,0)

verify A on (1)

$(-3)^2 + 0^2 + 8(-3) + 12(0) + 15 = 0$

$\Rightarrow 9 + 0 - 24 + 0 + 15 = 0 \quad \Rightarrow 24 - 24 = 0 \quad \therefore$ A lies on (1)

Centre of circle (- 4, - 6) = C

Let B (a,b) be other end of diameter through A

$\Rightarrow$ C = mid point of AB=(- 4, - 6) $= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-3+a}{2}, \frac{0+b}{2}\right)$

$\frac{a-3}{2} = -4 \quad \Rightarrow a - 3 = -8 \quad \Rightarrow a = -8 + 3 = -5$

$\Rightarrow \frac{b}{2} = -6 \Rightarrow b = -12 \therefore$ **B(- 5, - 12).**

15. Obtain the parametric equations of the circle. $x^2 + y^2 = 4$.

Sol: Given circle is $x^2 + y^2 = 4$ (Standard form)

Centre = $(-g_1 - f) = (0, 0)$ radius = r = 2

Parametric equations of the circle

$x = -g + r\cos\theta$, $y = -f + r\sin\theta$

$x = 0 + 2\cos\theta$, $y = 0 + 2\sin\theta$

$x = 2\cos\theta, y = 2\sin\theta \ (0 \le \theta < 2\Pi)$

(H/W) Obtain the parametric equations of the circle $4(x^2 + y^2) = 9$.

Ans: $x = \frac{3}{2}\cos\theta, y = \frac{3}{2}\sin\theta, 0 \le \theta < 2\Pi$.

16. Obtain the parametric equations of the circle $(x - 3)^2 + (y - 4)^2 = 8^2$.

Sol: Given circle is $(x - 3)^2 + (y - 4)^2 = 8^2$ (Central form)

Centre (h,k) = (3, 4), r = 8

Parametric equations is

$x = h + r\cos\theta, y = k + r\sin\theta \ (0 \le \theta < 2\Pi)$

$\Rightarrow x = 3 + 8\cos\theta, y = 4 + 8\sin\theta \ (0 \le \theta < 2\Pi)$

17. Obtain the parametric equations of the circle $x^2 + y^2 - 6x + 4y - 12 = 0$.

Sol: For the circle $x^2 + y^2 - 6x + 4y - 12 = 0$ (genral equation)

Centre (-g, -f) = (3, -2)

Radius = $\sqrt{g^2 + f^2 - c} = \sqrt{(-3)^2 + 2^2 + 12} = 5 = r$

Parametric equations of the circle are

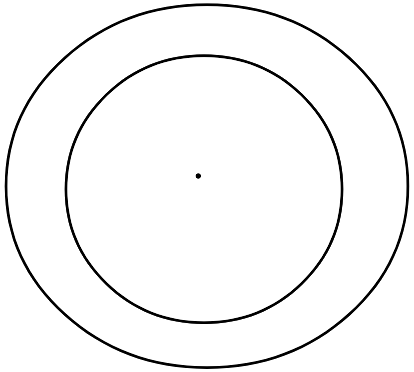
$x = -g + r\cos\theta, \ y = -f + r\sin\theta.$ $\quad 0 \le \theta < 2\Pi$

$x = 3 + 5\cos\theta, y = -2 + 5\sin\theta \quad 0 \le \theta < 2\Pi$.

18. Find equation of the circle which is concentric with $x^2 + y^2 - 6x - 4y - 12 = 0$ and passing through (-2, 14).

Sol: Equation of a circle concentric with $x^2 + y^2 - 6x - 4y - 12 = 0$. is in the form of

$$x^2 + y^2 - 6x - 4y + k = 0$$



Since, it is passing through (-2, 14).

$\Rightarrow 4 + 196 + 12 - 56 + k = 0.$

$\Rightarrow 156 + k = 0 \quad \Rightarrow k = -156$

Required equation of a circle

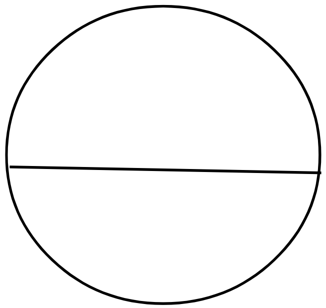
$x^2 + y^2 - 6x - 4y - 156 = 0.$

(H/W) Find the equation of the circle passing through (2, 3) and concentric with the circle $x^2 + y^2 + 8x + 12y + 15 = 0$. Ans : $x^2 + y^2 + 8x + 12y - 65 = 0$.

19. Find the equation of the circle having line joining (-4, 3) (3, - 4) as a diameter.

Sol: Equation of the circle with A (- 4, 3), B (3, - 4) as ends of diameter is

$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$$



$\Rightarrow (x+4)(x - 3) + (y - 3)(y + 4) = 0$

$\Rightarrow x^2 - 3x + 4x - 12 + y^2 + 4y - 3y - 12 = 0$

$\therefore$ **$x^2 + y^2 + x + y - 24 = 0.$**

20. Find the power of the point P(2, 3) with respect to the circle $x^2 + y^2 - 2x + 8y - 23 = 0$.

Sol:Given P(2,3) and $S \equiv x^2 + y^2 - 2x + 8y - 23 = 0$.(power=PA.PB)

$S_{11} = (2)^2 + (3)^2 - 2(2) + 8(3) - 23 = 4 + 9 - 4 + 24 - 23 = 10$

Power of P w.r.t S = 0 is 10.

(H/W)Find the power of the point (2, 4) with respect to $x^2 + y^2 - 4x - 6y - 12 = 0$ Ans: -24

7. Find the position of the point (3, 2) with respect to the circle $x^2 + y^2 - 4x - 6y - 12 = 0$.

Sol: $S = x^2 + y^2 - 4x - 6y - 12 = 0$, $P(3, 2) = (x_1, y_1)$

$S_{11} = 3^2 + 2^2 - 4(3) - 6(2) - 12$

$= 9 + 4 - 12 - 12 - 12 = -23$.

p lies inside the circle

$S_{11} = 0$, point lies on the circle.
$S_{11} < 0$, point lies inside the circle.
$S_{11} > 0$, point lies outside the circle

21. Find the length of the tangent from (1,3) to the circle $x^2 + y^2 - 2x + 4y - 11 = 0$.

Sol: Here $(x_1, y_1) = (1, 3)$ and

$S = x^2 + y^2 - 2x + 4y - 11 = 0$

The length of the tangent is $\sqrt{S_{11}}$

$$= \sqrt{(1)^2 + (3)^2 - 2(1) + 4(3) - 11} = \sqrt{1 + 9 - 2 + 12 - 11} = \sqrt{9} = 3.$$

22. If the length of the tangent from (2, 5) to the circle $x^2 + y^2 - 5x + 4y + k = 0$ is $\sqrt{37}$, then find k.

Sol: $x^2 + y^2 - 5x + 4y + k = 0$, P(2, 5)

Given $\sqrt{S_{11}} = \sqrt{37}$

$\Rightarrow S_{11} = 37$

$\Rightarrow 2^2 + 5^2 - 5(2) + 4(5) + k = 37$

$\Rightarrow k = 37 - 39$

$\Rightarrow$ **k = -2.** (H/W) If the L.T from (5, 4) to the circle $x^2 + y^2 + 2ky = 0$ is 1 then find k. (Ans: - 5)

23. Find the length of the chord formed on thecircle $x^2 + y^2 = a^2$ on the line $x \cos a + y \sin a = p$.

Sol: Given equation of circle $x^2 + y^2 = a^2$ .... (1)

Centre = C (0,0), radius (r) = a

given equation of chord $x \cos a + y \sin a - p = 0$ .... (2)

d = perpendicular distance from C (0,0) to (2)

$$= \left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$$

$$= \left|\frac{0 + 0 - p}{\sqrt{\cos^2\alpha + \sin^2\alpha}}\right| = \left|\frac{-p}{1}\right| = p$$

Length of the chord = $2\sqrt{r^2 - d^2} = 2\sqrt{a^2 - p^2}$ units.

24. Find the equation of tangent and normal of $x^2 + y^2 - 6x + 4y - 12 = 0$ at (-1, 1).

Sol: Eqation of tangent to $x^2 + y^2 - 6x + 4y - 12 = 0$ at (-1, 1) is

$xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0.$

$\Rightarrow x(-1) + y(1) - 3(x - 1) + 2(y + 1) - 12 = 0$

$\Rightarrow -4x + 3y - 7 = 0$

$\Rightarrow$ **$4x - 3y + 7 = 0.$**

Equation of normal at (-1, 1) is in the form

$b(x - x_1) - a(y - y_1) = 0$

$\Rightarrow -3(x+1)-4(y-1) = 0.$

$\Rightarrow -3x-3-4y+4=0$

Equation of normal at (-1, 1) is $3x + 4y - 1 = 0$.

25. Find the equation of the normal at P(3,5) of the circle $S = x^2 + y^2 - 10x - 2y + 6 = 0$.

Sol: Given circle is $x^2 + y^2 - 10x - 2y + 6 = 0$

centre $C = (-g, -f) = (5,1)$

Given point $P = (3,5)$

equation of the normal passing through P is equation of CP.

$\Rightarrow y - 5 = \frac{1-5}{5-3}(x - 3)$

$\Rightarrow y - 5 = \frac{-4}{2}(x - 3)$

$\Rightarrow y - 5 = -2(x - 3)$

$\Rightarrow y - 5 = -2x + 6 \quad \Rightarrow$ **$2x + y - 11 = 0.$**

26. Find the area of the triangle formed with the coordinate axes and the tangent drawn at the point (x_1, y_1) on the circle $x^2 + y^2 = a^2$.

Sol: Equation of tangent to $x^2 + y^2 = a^2$ at (x_1, y_1) is

$$xx_1 + yy_1 = a^2$$

$$\Rightarrow \frac{x}{a^2/x_1} + \frac{y}{a^2/y_1} = 1.$$

Area of the triangle formed by this tangent with coordinate axes.

$$= \frac{1}{2}|(x - \text{intercept})\ (y - \text{intercept})|$$

$$= \frac{1}{2}\left|\left(\frac{a^2}{x_1}\right)\left(\frac{a^2}{y_1}\right)\right| = \frac{a^4}{2|x_1y_1|}$$ **square units.**

27. Find the area of the triangle formed by the normal at (3, - 4) to the circle $x^2 + y^2 - 22x - 4y + 25 = 0$ with the coordinate axis.

Sol: Given circle is $x^2 + y^2 - 22x - 4y + 25 = 0$

centre $(-g, -f) = C(11,2)$

Equation of normal passing through $(3, -4)$ $(11,2)$ is

$$y + 4 = \frac{2+4}{11-3}(x - 3)$$

$$y + 4 = \frac{6}{8}(x - 3)$$

$$\Rightarrow 4y + 16 = 3x - 9$$

$$\Rightarrow 3x - 4y - 25 = 0$$

Area of the triangle formed by the normal with coordinate axes is

$$\frac{c^2}{2|ab|} = \frac{(-25)^2}{2|3(-4)|} = \frac{625}{24} \text{ sq.units}.$$

28. State the necessary and sufficient condition for $lx + my + n = 0$ to be a normal to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$.

Sol: The straight line $lx + my + n = 0$ is normal to the circle $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$

the centre (-g, -f) of the circle lies

on $lx + my + n = 0$

$\Rightarrow l(-g) + m(-f) + n = 0$

$\Rightarrow$ **$lg + mf = n.$**

29. Find the polar of (1,2) w.r.t to $x^2 + y^2 = 7$.

Sol: Given circle $S \equiv x^2 + y^2 - 7 = 0$.
Given point is P(1,2)
Equation of polar of P with respect to S = 0 is
$S_1 = 0 \Rightarrow x(1) + y(2) - 7 = 0$
$\Rightarrow$ **$x + 2y - 7 = 0$.**

(H/W)Find the polar of (3, - 1) with respect to $2x^2 + 2y^2 = 11$. (Ans:$6x - 2y - 11 = 0$.)

30. Find the polar of (1, -2) with respect to $x^2 + y^2 - 10x - 10y + 25 = 0$.

Sol. Given $x^2 + y^2 - 10x - 10y + 25 = 0$......... (1)
point = (1, -2)
Equation of polar of P(1,-2) w.r.t. (1) is $S_1 = 0$
$xx_1 + yy_1 - 5(x + x_1) - 5(y + y_1) + 25$
$x(1) + y(-2) - 5(x + 1) - 5(y - 2) + 25 = 0$
$x - 2y - 5x - 5 - 5y + 10 + 25 = 0$
$-4x - 7y + 30 = 0$
$4x + 7y - 30 = 0$.

(H/W)Find the equation of the polar of (2,3) with respect to the circle $x^2 + y^2 + 6x + 8y - 96 = 0$.
Ans: $5x + 7y - 78 = 0$

31. Find the angle between the tangents drawn from (3, 2) to the circle $x^2 + y^2 - 6x + 4y - 2 = 0$.

Sol: Given circle $x^2 + y^2 - 6x + 4y - 2 = 0$

center $C = (3, -2)$

$r = \sqrt{(-3)^2 + (2)^2 + 2} = \sqrt{9+4+2} = \sqrt{15}$

$\sqrt{S_{11}} = \sqrt{(3)^2 + 2^2 - 6(3) + 4(2) - 2} = \sqrt{9+4-18+8-2} = 1$

$\tan\frac{\theta}{2} = \frac{r}{\sqrt{S_{11}}} = \frac{\sqrt{15}}{1}$

$\cos\theta = \left|\frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}\right|$

$\Rightarrow \cos\theta = \left|\frac{1-15}{1+15}\right| = \left|\frac{-14}{16}\right| = \frac{7}{8} \quad \Rightarrow \theta = \cos^{-1}\left(\frac{7}{8}\right)$.

32. Find the pole of $ax + by + c = 0$ with respect to $x^2 + y^2 = r^2$.

Sol: Given equation of circle

$S \equiv x^2 + y^2 - r^2 = 0$ .... (1)

Given equation of line $ax + by + c = 0$ .... (2)

Let $P(x_1, y_1)$ be the pole.

Equation of polar P with respect to $S = 0$ is $S_1 = 0$

$xx_1 + yy_1 - r^2 = 0$ .... (3)

(2), (3) represents same line

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{x_1}{a} = \frac{y_1}{b} = \frac{-r^2}{c}$

$\Rightarrow \frac{x_1}{a} = \frac{-r^2}{c} \Rightarrow x_1 = -\frac{ar^2}{c}$ and $\frac{y_1}{b} = \frac{-r^2}{c} \Rightarrow y_1 = -\frac{br^2}{c}$

$\therefore$ Coordinates of pole $= \left(-\frac{ar^2}{c}, -\frac{br^2}{c}\right)$.

34. Find the value of k if the points (1,3) and (2, k) are conjugate with respect to the circle $x^2 + y^2 = 35$.

Sol: Here $(x_1, y_1) = (1, 3)$, $(x_2, y_2) = (2, k)$ and

$x^2 + y^2 - 35 = 0$ since the given points are conjugate we have $S_{12} = 0$.

$x_1 x_2 + y_1 y_2 - 35 = 0$

$\Rightarrow (1)(2) + 3(k) - 35 = 0$

$\Rightarrow 2 + 3k - 35 = 0$

$\Rightarrow 3k = 33 \Rightarrow$ **$k = 11$.**

35. Show that (4, -2) and (3, -6) are conjugate with respect to the circle $x^2 + y^2 - 24 = 0$.

Sol: Given $(x_1, y_1) = (4, -2)$, $(x_2, y_2) = (3, -6)$

$s = x^2 + y^2 - 24 = 0$

For conjugate points, $S_{12} = 0 \Rightarrow x_1 x_2 + y_1 y_2 - 24 = 0$

$S_{12} = 4(3) + (-2)(-6) - 24 = 0$

The given points are conjugate with respect to the given circle.

(H/W) Show that the points (- 6,1) and (2,3) are conjugate points with respect to the circle $x^2 + y^2 - 2x + 2y + 1 = 0$. Ans: Verify $S_{12} = 0$.

36. Find the value of k, if the points (4,2) and (k, - 3) are conjugate points with respect to the circle $x^2 + y^2 - 5x + 8y + 6 = 0$.

Sol: Given equation of circle

$S \equiv x^2 + y^2 - 5x + 8y + 6 = 0$ .... (1)

given P(4,2) Q (k,- 3) are conjugate points with respect to the circle S = 0

$\Rightarrow S_{12} = 0 \Rightarrow x_1x_2 + y_1y_2 - \frac{5}{2}(x_1 + x_2) + 4(y_1 + y_2) + 6 = 0$

$\Rightarrow 4(k) + 2(-3) - \frac{5}{2}(4 + k) + 4(2 - 3) + 6 = 0$

$\Rightarrow 4k - 6 - \frac{5}{2}(4) - \frac{5}{2}k - 4 + 6 = 0 \Rightarrow 4k - \frac{5}{2}k - 14 = 0 \Rightarrow 8k - 5k - 28 = 0$

$\Rightarrow 3k = 28 \Rightarrow$ **$k = \frac{28}{3}$.**

37. Find the chord of contact of (1,2) w.r.t to $x^2 + y^2 = 7$.

Sol: Given circle $S \equiv x^2 + y^2 - 7 = 0$.

Given point is P(1,2)

Equation of chord of contact of P with respect to S = 0 is

$S_1 = 0 \Rightarrow x(1) + y(2) - 7 = 0$

$\Rightarrow$ **$x + 2y - 7 = 0$.**

38. Find the number of common tangents that exist for the pair of circles $x^2 + y^2 = 4$, $x^2 + y^2 - 6x - 8y + 16 = 0$.

Sol: Given equations of the circles are

$x^2 + y^2 = 4$ .... (1), $x^2 + y^2 - 6x - 8y - 16 = 0$ .... (2)

For the circle (1), centre, $C_1 = (0,0)$, Radius $r_1 = 2$

For the circle (2), centre, $C_2 = (3,4)$

Radius $r_2 = \sqrt{(-3)^2 + (-4)^2 - 16} = \sqrt{9 + 16 - 16} = \sqrt{9} = 3$.

$|\overline{C_1C_2}| = \sqrt{3^2 + 4^2} = 5$ $|\overline{C_1C_2}|$ $= r_1 + r_2$

The given circles touch each other externally, No. of common tangents = 3.

39. Find the number of common tangents that exist for the pair of circles $x^2 + y^2 + 6x + 6y + 14 = 0$, $x^2 + y^2 - 2x - 4y - 4 = 0$.

Sol: Given equations of the circles are

$x^2 + y^2 + 6x + 6y + 14 = 0$ .... (1), $x^2 + y^2 - 2x - 4y - 4 = 0$ .... (2)

For the circle (1), centre, $C_1 = (-3, -3)$, Radius $r_1 = \sqrt{(-3)^2 + (-3)^2 - 14} = \sqrt{9 + 9 - 14} = \sqrt{4} = 2$

For the circle (2), centre, $C_2 = (1, 2)$

Radius $r_2 = \sqrt{(1)^2 + (2)^2 + 4} = = \sqrt{9} = 3$.

$|\overline{C_1C_2}| = \sqrt{(-3-1)^2 + (-3-2)^2} = \sqrt{16 + 25} = \sqrt{41}$ $|\overline{C_1C_2}| > r_1 + r_2$

Thus one circle lies outside the other, Hence No. of common tangents = 4.

40.Find the internal centre and external centres of similitude for the circles
$x^2 + y^2 +6x+6y+14=0$, $x^2 + y^2 - 2x- 4y - 4 = 0$.

ANS:

41.Find the equation of the circle with centre (2, 3) and touching the line 3x - 4y + 1 = 0.

Sol: Radius = perpendicular distance from (2, 3) to the line 3x - 4y + 1 = 0

$$r = \frac{|3(2)-4(3)+1|}{5} = \frac{6-12+1}{5} = 1$$

Equation of the required circle

$(x - 2)^2 + (y - 3)^2 = 1^2 \Rightarrow x^2 + 4 - 4x + y^2 + 9 - 6y = 1 \Rightarrow$ **$x^2 + y^2 - 4x - 6y + 12 = 0$.**

42.Find the equation of the circle with centre (-3,4) and touching y - axis.

Sol: Centre = (h, k) = (-3, 4), Radius = |h| = | - 3| = 3

Equation of the required circle is

$(x +3)^2 + (y - 4)^2 = 3^2 \Rightarrow x^2 + 9 + 6x + y^2 + 16 - 8y = 9 \Rightarrow$ **$x^2 + y^2 + 6x - 8y + 16 = 0$.**

(H/W)Find the equation of the circle with centre (-3,4) and touching X - axis.

43. Find the equation of the tangent at the point 30^0 (Parametric value of θ) of the circle $x^2 + y^2 + 4x + 6y - 39 = 0$.

Sol: Given circle $x^2 + y^2 + 4x + 6y - 39 = 0$

Here g = 2, f = 3, $r = \sqrt{4+9+39} = \sqrt{52} = 2\sqrt{13}$

The required equation of tangent at P(θ) is $(x + g)\cos\theta + (y + f)\sin\theta = r$

$(x + 2)\cos 30^0 + (y + 3)\sin 30^0 = 2\sqrt{13}$

$$\Rightarrow (x+2)\left(\frac{\sqrt{3}}{2}\right) + (y+3)\left(\frac{1}{2}\right) = 2\sqrt{13}$$

$$\Rightarrow \sqrt{3}x + 2\sqrt{3} + y + 3 = 4\sqrt{13} \Rightarrow \sqrt{3}x + y + 3 + 2(\sqrt{3} - 2\sqrt{13}) = 0.$$

1. If a point P is moving such that the lengths of tangents drawn from P to the circles $x^2+y^2-4x-6y-12=0$ and $x^2+y^2+6x+18y+26=0$ are in the ratio 2 : 3, then find the equation of the locus of P.

Sol: Given equations of circles are

$S = x^2 + y^2 - 4x - 6y - 12 = 0, \quad S' = x^2 + y^2 + 6x + 18y + 26 = 0$

Let $P(x_1, y_1)$ be any point on the locus.

Given $\sqrt{S_{11}} : \sqrt{S'_{11}} = 2 : 3 \Rightarrow \frac{\sqrt{S_{11}}}{\sqrt{S'_{11}}} = \frac{2}{3}$

$\Rightarrow 3\sqrt{S_{11}} = 2\sqrt{S'_{11}}$ squaring on both sides

$\Rightarrow 9(S_{11}) = 4(S'_{11})$

$\Rightarrow 9(x_1^2+y_1^2-4x_1-6y_1-12) = 4(x_1^2 + y_1^2 + 6x_1 + 18y_1+26)$

$\Rightarrow 9x_1^2 + 9y_1^2 - 36x_1-54y_1-108 = 4x_1^2 + 4y_1^2 + 24x_1 + 72y_1 + 104$

$\Rightarrow 5x_1^2 + 5y_1^2 - 60x_1 - 126y_1 - 212 = 0$

Hence the required equation of locus of $P(x_1, y_1)$ is $5x^2 + 5y^2 - 60x - 126y - 212 = 0.$

2. If a point P is moving such that the lengths of tangents drawn from 'P' to the circles $x^2 + y^2 + 8x + 12y + 15 = 0$ and $x^2 + y^2 - 4x - 6y - 12 = 0$ are equal, then find the equation of the locus of P.

Sol: Given equations of the circles are $S \equiv x^2 + y^2 + 8x + 12y + 15 = 0, \quad S' \equiv x^2 + y^2 - 4x - 6y - 12 = 0$

Let $P(x_1, y_1)$ be any point on the locus.

PA, PB be the lengths of the tangent from P to the circles (1) & (2) respectively.

Given condition is $\sqrt{S_{11}} = \sqrt{S'_{11}} \Rightarrow S_{11} = S'_{11}$

$\Rightarrow x_1^2 + y_1^2 + 8x_1 + 12y_1 + 15 = x_1^2 + y_1^2 - 4x_1 - 6y_1 - 12$

$\Rightarrow 8x_1 + 12y_1 + 15 + 4x_1 + 6y_1 + 12 = 0.$

$\Rightarrow 12x_1 + 18y_1 + 27 = 0.$

The equation of the locus of 'P' is $12x + 18y + 27 = 0 \Rightarrow 4x + 6y + 9 = 0.$

3. Find the length of the chord intercepted by the circle $x^2 + y^2 - 8x - 2y - 8 = 0$ on the line $x + y + 1 = 0$.

Sol: Given circle $x^2 + y^2 - 8x - 2y - 8 = 0$ ------ (1)

Centre (C) = (4, 1)

Radius (r) = $\sqrt{4^2+1^2+8} = \sqrt{16+1+8} = \sqrt{25} = 5$

Given chord, $x + y + 1 = 0$ --------- (2)

d = perpendicular distance from C to (2)

$$\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}} = \left|\frac{4+1+1}{\sqrt{1^2+1^2}}\right| = \frac{6}{\sqrt{2}}$$

Length of chord = $|\overline{AB}|$ = $2\sqrt{r^2-d^2}$ $= 2\sqrt{25-\left(\frac{6}{\sqrt{2}}\right)^2} = 2\sqrt{25-\frac{36}{2}} = 2\sqrt{\frac{50-36}{2}} = 2\sqrt{\frac{14}{2}} = 2\sqrt{7}$ **units.**

4. Find the length of the chord intercepted by the circle $x^2 + y^2 - x + 3y - 22 = 0$ on the line $y = x - 3$.

Sol: Given equation of the circle is

$x^2 + y^2 - x + 3y - 22 = 0$

Centre C = (- g, - f) = $\left(\frac{1}{2}, \frac{-3}{2}\right)$.

Radius r = $\sqrt{g^2+f^2-c} = \sqrt{\left(\frac{-1}{2}\right)^2+\left(\frac{3}{2}\right)^2+22} = \sqrt{\frac{49}{2}}$

Now, d = The perpendicular distance from the centre $C\left(\frac{1}{2}, \frac{-3}{2}\right)$ to the line $x - y - 3 = 0$

$$d = \frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}} = \frac{\left|1\left(\frac{1}{2}\right)-1\left(\frac{-3}{2}\right)-3\right|}{\sqrt{(1)^2+(-1)^2}} = \frac{1}{\sqrt{2}}$$

Length of the chord = $2\sqrt{r^2-d^2} = 2\sqrt{\left(\frac{49}{2}\right)^2-\left(\frac{1}{\sqrt{2}}\right)^2} = 2\sqrt{\frac{49}{2}-\frac{1}{2}} = 2\sqrt{\frac{48}{2}} = 2\sqrt{24} = 4\sqrt{6}$ units.

5. The line $y = mx + c$ and the circle $x^2 + y^2 = a^2$ intersect at A and B. If $AB = 2l$, then show that $c^2 = (1 + m^2)(a^2 - \lambda^2)$

Sol: Given equation of circle $x^2 + y^2 = a^2$ .... (1)

Centre C (0,0) radius, $r = a$

given equation of line $y = mx + c$, $mx - y + c = 0$ ....(2) and (1), (2) intersect at A and B.

d = perpendicular distance from centre C to (2). $= \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}} = \left|\frac{0+0-c}{\sqrt{m^2+1}}\right| = \left|\frac{c}{\sqrt{m^2+1}}\right|$.

Given $|AB| = 2\lambda \Rightarrow 2\sqrt{r^2 - d} = 2\lambda \Rightarrow \sqrt{r^2 - d^2} = \lambda$

squaring on both sides $\Rightarrow r^2 - d^2 = \lambda^2$

$\Rightarrow a^2 - \frac{c^2}{1+m^2} = \lambda^2 \Rightarrow a^2 - \lambda^2 = \frac{c^2}{1+m^2} \Rightarrow$ **$(a^2 - \lambda^2)(1 + m^2) = c^2$.**

6. Find the equation of the circle with centre (-2, 3) cutting a chord of length 2 units on $3x + 4y + 4 = 0$.

Sol: Given centre C(-2, 3)

Given equation of chord, $3x + 4y + 4 = 0$ ------------- (1)

d = perpendicular distance from C (-2, 3) to (1)

$$= \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}} = \left|\frac{3(-2)+4(3)+4}{\sqrt{3^2+4^2}}\right| = \left|\frac{-6+12+4}{\sqrt{9+16}}\right| = \left|\frac{10}{\sqrt{25}}\right| = \frac{10}{5} = 2$$

Given length of chord $= 2 \Rightarrow 2\sqrt{r^2 - d^2} = 2 \Rightarrow \sqrt{r^2 - d^2} = 1$

Squaring on both sides $\Rightarrow r^2 - d^2 = 1$

substituting $d = 2 \Rightarrow r^2 - 2^2 = 1$

$r^2 = 1 + 4 = 5$

Required equation of the circle is $(x - h)^2 + (y - k)^2 = r^2$

$(x + 2)^2 + (y - 3)^2 = 5 \Rightarrow x^2 + 4x + 4 + y^2 - 6y + 9 - 5 = 0$

$x^2 + y^2 + 4x - 6y + 8 = 0$.

7. Find the pole of $3x + 4y - 45 = 0$ with respect to $x^2 + y^2 - 6x - 8y + 5 = 0$.

Sol: Given circle $x^2 + y^2 - 6x - 8y + 5 = 0$

$2g = -6 \Rightarrow g = -3, \quad 2f = -8 \Rightarrow f = -4$

$r = \sqrt{(-3)^2 + (-4)^2 - 5} = \sqrt{9 + 16 - 5} = \sqrt{20}$

Given line $3x + 4y - 45 = 0$

$l = 3, m = 4, n = -45$ and $lg+mf-n = 3(-3) + 4(-4) + 45 = -9 - 16 + 45 = 20$

$$\text{pole} = \left(-g + \frac{\ell r^2}{\ell g + mf - n}, -f + \frac{mr^2}{\ell g + mf - n}\right) = \left(3 + \frac{3(20)}{20}, \quad 4 + \frac{4(20)}{20}\right)$$

$= (3 + 3, 4 + 4) =$ **(6, 8).**

(H/W)Find the pole of $x + y + 2 = 0$ with respect to the circle $x^2 + y^2 - 4x + 6y - 12 = 0$. Ans: (- 23, - 28).

8. Find the value of k, if $kx + 3y - 1 = 0$, $2x + y + 5 = 0$ are conjugate lines with respect to the circle $x^2 + y^2 - 2x - 4y - 4 = 0$.

Sol: Given equation of circle

$x^2 + y^2 - 2x - 4y - 4 = 0$ ------------- (1) $\quad 2g = -2 \Rightarrow g = -1, \quad 2f = -4 \Rightarrow f = -2$ and $\quad c = -4$

$r = \sqrt{(-1)^2 + (-2)^2 - (-4)} = \sqrt{1 + 4 + 4} = \sqrt{9} = 3$

given lines $kx + 3y - 1 = 0$, $2x + y + 5 = 0$

$l_1 = k, m_1 = 3, n_1 = -1, \quad l_2 = 2, m_2 = 1, n_2 = 5$

Conjugate lines condition : $r^2 (l_1 l_2 + m_1 m_2) = (l_1 g + m_1 f - n_1)(l_2 g + m_2 f - n_2)$

$9(k.2 + 3.1) = [k(-1) + 3(-2) + 1]\,[2(-1) + 1(-2) - 5] \cancel{9} \Rightarrow (2k + 3) = (-k - 5)(-\cancel{9})$

$2k + 3 = k + 5$

$2k - k = 5 - 3$

$k = 2$.

(H/W)Find the value of k, if $x + y - 5 = 0$, $2x + ky - 8 = 0$ are conjugate lines with respect to the circle $x^2 + y^2 - 2x - 2y - 1 = 0$. Ans: k = 2

9. Find the equations of the tangents to the circle $x^2 + y^2 - 4x + 6y - 12 = 0$ which are parallel to $x + y - 8 = 0$.

Sol: Given equation of circle $x^2 + y^2 - 4x + 6y - 12 = 0$ ---------- (1)

Centre (2, -3), radius = $\sqrt{(2)^2 + (-3)^2 - (-12)} = \sqrt{4+9+12} = \sqrt{25} = 5$.

Given line $x + y - 8 = 0$ ---------- (2)

Equation of a line parallel to (2) is,

$x + y + k = 0$ ---------- (3)

Given (3) is tangent to (1)

Condition : r = d [perpendicular distance from C to (3)]

$$\Rightarrow 5 = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}} = \left|\frac{2 + (-3) + k}{\sqrt{1^2 + 1^2}}\right| \Rightarrow 5 = \left|\frac{-1 + k}{\sqrt{2}}\right|$$

$\Rightarrow 5\sqrt{2} = |k - 1| \quad \Rightarrow k - 1 = \pm 5\sqrt{2}$

$\Rightarrow k = 1 \pm 5\sqrt{2}$

Hence required equations of tangents are $x + y + 1 \pm 5\sqrt{2} = 0$.

10. Find the equations of the tangents to the circle $x^2 + y^2 + 2x - 2y - 3 = 0$ which are perpendicular to $3x - y + 4 = 0$.

Sol: Let the equation of the tangent perpendicular to $3x - y + 4 = 0$ is $x + 3y + k = 0$, —— (1)

centre of $x^2 + y^2 + 2x - 2y - 3 = 0$ is (-1, 1)

Radius = $\sqrt{(-1)^2 + (1)^2 + 3} = \sqrt{5}$

Given (1) is tangent of a given cirlce

Now r = d

Radius = length of the perpendicular from (-1, 1) to $x + 3y + k = 0$

$$\Rightarrow r = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}} \Rightarrow \sqrt{5} = \frac{|(-1) + 3(1) + k|}{\sqrt{10}} \Rightarrow 5\sqrt{2} = |k + 2| \qquad \Rightarrow k = -2 \pm 5\sqrt{2}.$$

Required tangents are $x + 3y - 2 \pm 5\sqrt{2} = 0$.

11. $x^2 + y^2 - 2x + 4y = 0$ at (3, - 1). Also find the equation of tangent parallel to it.

Sol: Given equation of the circle is $x^2 + y^2 - 2x + 4y = 0$ .... (1)

The given point $P(x_1, y_1) = (3, -1)$

$\therefore$ The equation of tangent at P is $S_1 = 0$.

$x(3) + y(-1) - 1(x + 3) + 2(y - 1) = 0$

$\Rightarrow 3x - y - x - 3 + 2y - 2 = 0 \quad \Rightarrow 2x + y - 5 = 0$

The equation of the straight line parallel to the tangent $2x + y - 5 = 0$ is $2x + y + k = 0$ .... (2)

If (2) is a tangent to the given circle

where r = d (perpendicular distance from centre (1, - 2) to the line (2)).

$$\Rightarrow r = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}} \Rightarrow \sqrt{(-1)^2 + 2^2} = \frac{|2(1) + 1(-2) + k|}{\sqrt{2^2 + 1^2}} \quad \Rightarrow \sqrt{5} = \frac{|k|}{\sqrt{5}}$$

$\Rightarrow |k| = 5 \Rightarrow k = \pm 5$

The equations of the tangents to the circle are $2x + y \pm 5 = 0$

One of these tangnets namely $2x + y - 5 = 0$ is the tangent at (3, - 1).

The other tangent parallel to it is $2x + y + 5 = 0$.

12. Find the area of the triangle formed by the normal at (3, - 4) to the circle $x^2 + y^2 - 22x - 4y + 25 = 0$ with the coordinate axis.

Sol: Given circle is $x^2 + y^2 - 22x - 4y + 25 = 0$

centre $(-g, -f) = C(11,2)$

Equation of normal passing through (3, - 4) (11,2) is

$$y + 4 = \frac{2+4}{11-3}(x - 3) \Rightarrow y + 4 = \frac{6}{8}(x - 3) \Rightarrow 4y + 16 = 3x - 9$$

$\Rightarrow 3x - 4y - 25 = 0$

Area of the triangle formed by the normal with coordinate axes is $\frac{c^2}{2|ab|} = \frac{(-25)^2}{2|3(-4)|} = \frac{625}{24}$ sq.units.

13. Show that $x + y + 1 = 0$ touches the circle $x^2 + y^2 - 3x + 7y + 14 = 0$ and find its point of contact.

Sol: Given equation of circle

$x^2 + y^2 - 3x + 7y + 14 = 0$ ---------- (1)

Centre (C) $= \left(\frac{3}{2}, \frac{-7}{2}\right)$

Radius (r) $= \sqrt{\left(\frac{-3}{2}\right)^2 + \left(\frac{7}{2}\right)^2 - 14} = \sqrt{\frac{9}{4} + \frac{49}{4} - 14} = \sqrt{\frac{9+49-56}{4}} = \sqrt{\frac{2}{4}} = \frac{1}{\sqrt{2}}$

Given line $x + y + 1 = 0$ ---------- (2)

d = perpendicular distance from C to (2)

$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}} = \left|\frac{\frac{3}{2} - \frac{7}{2} + 1}{\sqrt{1^2 + 1^2}}\right| = \left|\frac{\left(\frac{3-7+2}{2}\right)}{\sqrt{2}}\right| = \left|\frac{-1}{\sqrt{2}}\right| = \frac{1}{\sqrt{2}} = r.$$

The line (2) touches the circle (1)

The point of contact is the foot of the perpendicular from C to (2)

$$\frac{h - x_1}{a} = \frac{k - y_1}{b} = \frac{-(ax_1 + by_1 + c)}{a^2 + b^2} \Rightarrow \frac{h - \frac{3}{2}}{1} = \frac{k + \frac{7}{2}}{1} = \frac{-\left(\frac{3}{2} - \frac{7}{2} + 1\right)}{1^2 + 1^2}$$

$$h - \frac{3}{2} = k + \frac{7}{2} = \frac{-\left(\frac{3-7+2}{2}\right)}{1+1}$$

$$h - \frac{3}{2} = k + \frac{7}{2} = \frac{1}{2}$$

$\Rightarrow h - \frac{3}{2} = \frac{1}{2} \quad \Rightarrow h = \frac{3}{2} + \frac{1}{2} \quad \Rightarrow h = 2$

$\Rightarrow k + \frac{7}{2} = \frac{1}{2} \quad \Rightarrow k = \frac{-7}{2} + \frac{1}{2} \quad \Rightarrow k = -3$

Coordinates of point of contact (h, k) = (2, - 3).

14. Show that the tangent at (-1, 2) of the circle $x^2 + y^2 - 4x - 8y + 7 = 0$ touches the circle $x^2 + y^2 + 4x + 6y = 0$ and also find its point of contact.

Sol: Equation of tangent at (-1, 2) to the circle $x^2 + y^2 - 4x - 8y + 7 = 0$ is

$S_1 = xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0.$

$\Rightarrow x(-1) + y(2) - 2(x - 1) - 4(y + 2) + 7 = 0$

$\Rightarrow -3x - 2y + 1 = 0 \quad \Rightarrow 3x + 2y - 1 = 0$ ---------- (1)

For the circle $x^2 + y^2 + 4x + 6y = 0$

Centre = (-2, -3), radius = $\sqrt{2^2 + 3^2 - 0} = \sqrt{13}$

Perpendicular distance from (-2, -3) to the line (1) $d = \dfrac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}} = \dfrac{|3(-2) + 2(-3) - 1|}{\sqrt{3^2 + 2^2}} = \dfrac{|-13|}{\sqrt{13}} = \sqrt{13} = r$

So line (1) also touches the 2nd circle.

Let (h, k) be the required point of contact.
So it is the foot of the perpendicular from the centre (-2, -3)

$$\frac{h - x_1}{a} = \frac{k - y_1}{b} = \frac{-(ax_1 + by_1 + c)}{a^2 + b^2}$$

$$\Rightarrow \frac{h + 2}{3} = \frac{k + 3}{2} = \frac{-[3(-2) + 2(-3) - 1]}{3^2 + 2^2}$$

$\dfrac{h + 2}{3} = 1 \qquad \dfrac{k + 3}{2} = 1$

$h + 2 = 3 \qquad k + 3 = 2$

$h = 1 \qquad k = -1$

Coordinates of point of contact = (1, -1).

15. Find the mid point of the chord intercepted by $x^2 + y^2 - 2x - 10y + 1 = 0$ on the line $x - 2y + 7 = 0$.

Sol: Given equation of circle $x^2 + y^2 - 2x - 10y + 1 = 0$ -------- (1)

Centre = C(1, 5)

Given equation of chord, $x - 2y + 7 = 0$ ------ (2)

Midpoint of chord is foot of the perpendicular from centre to (2)

$$\frac{h-x_1}{a} = \frac{k-y_1}{b} = \frac{-(ax_1+by_1+c)}{a^2+b^2} \quad \Rightarrow \frac{h-1}{1} = \frac{k-5}{-2} = \frac{-(1-2(5)+7)}{1^2+(-2)^2}$$

$$\frac{h-1}{1} = \frac{k-5}{-2} = \frac{-(-2)}{1+4} \quad \Rightarrow \frac{h-1}{1} = \frac{k-5}{-2} = \frac{2}{5}$$

$$\frac{h-1}{1} = \frac{2}{5} \qquad \frac{k-5}{-2} = \frac{2}{5}$$

$5h - 5 = 2$ $\qquad$ $5k - 25 = -4$

$5h = 2 + 5 = 7$ $\qquad$ $5k = -4 + 25 = 21$

$h = \frac{7}{5}$ $\qquad$ $k = \frac{21}{5}$

Coordinates of mid point (h, k) = $\left(\frac{7}{5}, \frac{21}{5}\right)$.

16. Find the inverse point of (-2, 3) with respect to the circle $x^2 + y^2 - 4x - 6y + 9 = 0$.

Sol: Given equation of circle $x^2 + y^2 - 4x - 6y + 9 = 0$--------- (1)

Centre C(2, 3) $\quad$ P(-2, 3)

Equation of CP, $y = 3$ ------ (2)

Equation of polar of P is $S_1 = 0$.

$x(-2) + y(3) - 2(x - 2) - 3(y + 3) + 9 = 0$

$-2x + \cancel{3y} - 2x + 4 - \cancel{3y} - \cancel{9} + \cancel{9} = 0$

$-4x + 4 = 0 \Rightarrow -4x = -4$

$x = 1$ ------------- (3)

P.I. of (2), (3) is (1, 3) . **The inverse point of P is (1, 3).**

17. Find the condition that the tangents drawn from (0,0) to $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$, be perpendicular to each other.

Sol: Given equation of the circle is

$S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$

Centre $C = (-g, -f)$, Radius, $r = \sqrt{g^2 + f^2 - c}$

Let, the given point $P(x_1, y_1) = (0,0)$

Angle between the tangents $\theta = 90^0$

The length of the tangent $= \sqrt{S_{11}} = \sqrt{0^2 + 0^2 + 2(0) + 2f(0) + c} = \sqrt{c}$.

If 'θ' is the angle between the tangents then

$$\tan\frac{\theta}{2} = \frac{r}{\sqrt{S_{11}}}$$

$$\tan 45^0 = \frac{\sqrt{g^2 + f^2 - c}}{\sqrt{c}}$$

$$1 = \frac{\sqrt{g^2 + f^2 - c}}{\sqrt{c}}$$

$$\sqrt{c} = \sqrt{g^2 + f^2 - c}$$

Squaring on both sides, we get

$c = g^2 + f^2 - c$

$\therefore$ **$g^2 + f^2 - 2c = 0$.**

18. Find the locus of 'P' where the tangent drawn from 'P' to $x^2 + y^2 = a^2$ are perpendicular to each other.

Sol: Given equation of the circle $x^2 + y^2 = a^2$

Radius $r = a$

Let $P(x_1 y_1)$ be any point on the locus.

Length of the tangent $= \sqrt{S_{11}}$

$$= \sqrt{x^2{}_1 + y^2{}_1 - a^2}$$

Given that angle between the tangents $q = 90^0$.

If 'q' is the angle between the tangents through 'P' to the given circle then

$$\tan\left(\frac{\theta}{2}\right) = \frac{r}{\sqrt{S_{11}}}$$

$$\Rightarrow \tan 45^0 = \frac{a}{\sqrt{x_1^2 + y_1^2 - a^2}}$$

$$\Rightarrow 1 = \frac{a}{\sqrt{x_1^2 + y_1^2 - a^2}}$$

$$\Rightarrow \sqrt{x_1^2 + y_1^2 - a^2} = a$$

Squaring on both sides

$\Rightarrow x^2{}_1 + y^2{}_1 - a^2 = a^2$

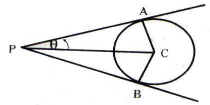
$\Rightarrow x_1{}^2 + y_1{}^2 = a^2 + a^2 = 2a^2$

$\therefore$ **The locus of $P(x_1, y_1)$ is $x^2 + y^2 = 2a^2$.**

19. Find the condition that the tangents drawn from the exterior point (g, f) to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ are perpendicular to each other.

Sol: Given equation of the circle is

$x^2 + y^2 + 2gx + 2fy + c = 0$, Radius $r = \sqrt{g^2 + f^2 - c}$



Let the given point $P(x_1, y_1) = (g, f)$

given $\theta = 90^0$

Length of the tangent $= \sqrt{S_{11}}$

$= \sqrt{x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c}$

$= \sqrt{g^2 + f^2 + 2g^2 + 2f^2 + c}$

$= \sqrt{3g^2 + 3f^2 + c}$

If 'θ' is the angle between the tangents then

$\tan\left(\frac{\theta}{2}\right) = \frac{r}{\sqrt{S_{11}}}$ $\Rightarrow \tan 45^0 = \frac{\sqrt{g^2 + f^2 - c}}{\sqrt{3g^2 + 3f^2 + c}}$

$\Rightarrow 1 = \frac{\sqrt{g^2 + f^2 - c}}{\sqrt{3g^2 + 3f^2 + c}}$

$\Rightarrow \sqrt{3g^2 + 3f^2 + c} = \sqrt{g^2 + f^2 - c}$

squaring on both sides

$\Rightarrow 3g^2 + 3f^2 + c = g^2 + f^2 - c$

$\Rightarrow 2g^2 + 2f^2 + 2c = 0$

$\Rightarrow$ **$g^2 + f^2 + c = 0$.**

20. Show that the area of the triangle formed by the two tangents through $P(x_1, y_1)$ to the circle $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ and the chord of contact of P with respect to S = 0 is $\frac{r(S_{11})^{3/2}}{S_{11}+r^2}$, where r is radius of the circle.

Sol: Given equation of circle

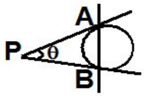
$S = x^2 + y^2 + 2gx + 2fy + c = 0$ ----- (1)

$P(x_1, y_1)$ be an external point

PA, PB be the tangents $|PA| = |PB| = \sqrt{S_{11}}$

$\overline{AB}$ is chord of contact of P w.r.to S = 0

If 'θ' be the angle between the tangents



$\tan(\theta/2) = \frac{r}{\sqrt{S_{11}}}$ (where r is radius of circle)

Area of $\Delta PAB = \frac{1}{2}|PA|.|PB| \sin\theta$

$$= \frac{1}{2}\sqrt{S_{11}}\sqrt{S_{11}}.\frac{2\tan(\theta/2)}{1+\tan^2(\theta/2)} = S_{11}\left[\frac{\frac{r}{\sqrt{S_{11}}}}{1+\left(\frac{r}{\sqrt{S_{11}}}\right)^2}\right] = S_{11}\left[\frac{\frac{r}{\sqrt{S_{11}}}}{1+\frac{r^2}{S_{11}}}\right] = S_{11}\left[\frac{\frac{r}{\sqrt{S_{11}}}}{\frac{S_{11}+r^2}{S_{11}}}\right]$$

$$= S_{11}\frac{r.S_{11}}{\sqrt{S_{11}}(S_{11}+r^2)} = \frac{r(S_{11})\sqrt{S_{11}}}{S_{11}+r^2} = \frac{r(S_{11})^{3/2}}{S_{11}+r^2}.$$

21. Find the pair of tangents from the origin to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ and hence deduce a condition for these tangents to be perpendicular.

Sol: Given equation of the circle is

$x^2 + y^2 + 2gx + 2fy + c = 0$

Let the given point $P(x_1, y_1) = (0,0)$

The equation of the pair of tangents drawn from (0,0) to the given circle is $S_1^2 = SS_{11}$

$\Rightarrow [x(0) + y(0) + g(x + 0) + f(y + 0) + c]^2 = (x^2 + y^2 + 2gx + 2fy + c)(0 + 0 + 0 + 0 + c)$

$\Rightarrow (gx + fy + c)^2 = (x^2 + y^2 + 2gx + 2fy + c)c$

$\Rightarrow g^2x^2 + f^2y^2 + c^2 + 2fgxy + 2fcy + 2gcx = cx^2 + cy^2 + 2gcx + 2fcy + c^2$

$\Rightarrow (g^2 - c)x^2 + 2gf\,xy + (f^2 - c)y^2 = 0$

Given that these tangents to be perpendicular then coefficient of x^2 + coefficient of $y^2 = 0$

$\Rightarrow g^2 - c + f^2 - c = 0 \Rightarrow$ **$g^2 + f^2 = 2c$.**

22. Find the equation of pair of tangents drawn from (0, 0) to $x^2 + y^2 + 10x + 10y + 40 = 0$.

Sol: Given equation of circle $S = x^2 + y^2 + 10x + 10y + 40 = 0$

$P(0, 0) = (x_1, y_1)$

$S_1 = x(0) + y(0) + 5(x + 0) + 5(y + 0) + 40$

$S_1 = 5x + 5y + 40$

$S_{11} = 0^2 + 0^2 + 10(0) + 10(0) + 40 = 40$

Equation of pair of tangents, $S_1^2 = SS_{11}$

$(5x + 5y + 40)^2 = (x^2 + y^2 + 10x + 10y + 40)\,40$

$5^2(x + y + 8)^2 = (x^2 + y^2 + 10x + 10y + 40)\,40$

${}^5\cancel{25}\,[x^2 + y^2 + 8^2 + 2xy + 2.y.8 + 2.8.x] = (x^2 + y^2 + 10x + 10y + 40)\,\cancel{40}_8$

$5x^2 + 5y^2 + \cancel{320} + 10xy + \cancel{80y} + \cancel{80x} = 8x^2 + 8y^2 + \cancel{80x} + \cancel{80y} + \cancel{320}$

$\Rightarrow$ **$3x^2 - 10xy + 3y^2 = 0$.**

23. If the abscissae of points A, B are the roots of the equation $x^2 + 2ax - b^2 = 0$ and ordinates of A, B are the roots of $y^2 + 2py - q^2 = 0$, then find the equation of a circle for which $\overline{AB}$ as a diameter.

Sol: Let A be (x_1, y_1) and B be (x_2, y_2).

Given x_1, x_2 be the roots of $x^2 + 2ax - b^2 = 0$

$\Rightarrow (x - x_1)(x - x_2) = x^2 + 2ax - b^2$

Given y_1, y_2 be the roots of $y^2 + 2py - q^2 = 0$

$\Rightarrow (y - y_1)(y - y_2) = y^2 + 2py - q^2$

Equation of circle with $\overline{AB}$ as a diameter is $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

$\Rightarrow (x^2 + 2ax - b^2) + (y^2 + 2py - q^2) = 0$

$\Rightarrow$ **$x^2 + y^2 + 2ax + 2py - b^2 - q^2 = 0$**

is the equation of required circle.