BINOMIAL THEOREM - 2

EXERCISE – 5(b)

1. Find the range of x for which the binomial expansions of the following are valid.

(i) $(2 + 3x)^{-2/3}$ (ii) $(5 + x)^{3/2}$

(iii) $(7 + 3x)^{-5}$ (iv) $\left(4-\frac{x}{3}\right)^{-1/2}$

Sol. (i) $(2 + 3x)^{-2/3}$ =

$$\left[2\left(1+\frac{3}{2}x\right)\right]^{-2/3} = 2^{-2/3}\left(1+\frac{3}{2}x\right)^{-2/3}$$

$\therefore$ The binomial expansion of $(2 + 3x)^{-2/3}$ is valid when $\left|\frac{3}{2}x\right|<1$.

i.e. $|x| < \frac{2}{3}$

i.e. $x\in\left(-\frac{2}{3},\frac{2}{3}\right)$

ii) $(5+x)^{3/2} = \left[5\left(1+\frac{x}{5}\right)\right]^{3/2} = 5^{3/2}\left(1+\frac{x}{5}\right)^{3/2}$

$\therefore$ The binomial expansion of $(5 + x)^{3/2}$ is valid when $\left|\frac{x}{5}\right|<1$.

i.e. $|x| < 5$

i.e. $x \in (-5, 5)$

iii) $(7+3x)^{-5} = 7\left[\left(1+\frac{3}{7}x\right)\right]^{-5} = 7^{-5}\left(1+\frac{3}{7}x\right)^{-5}$

$(7 + 3x)^{-5}$ is valid when $\left|\frac{3x}{7}\right|<1$

$\Rightarrow |x| < \frac{7}{3} \Rightarrow x\in\left(\frac{-7}{3},\frac{7}{3}\right)$

iv) $\left(4-\frac{x}{3}\right)^{-1/2} = \left[4\left(1-\frac{x}{12}\right)\right]^{-1/2}$

$\left(4-\frac{x}{3}\right)^{-1/2}$ is valid when $\left|\frac{-x}{12}\right|<1$

$\Rightarrow |x| < 12$

$\Rightarrow x \in (-12, 12)$

2. Find the (i) 6th term of $\left(1+\frac{x}{2}\right)^{-5}$ **.**

Sol. T_{r+1} in $(1 + x)^{-n}$

$$=(-1)^r \frac{(n)(n+1)(n+2)...(n+r-1)}{1\cdot 2\cdot 3\cdot ...r}\cdot x^r$$

Put r = 5, n = 5, x by x/2

$$T_6 = (-1)^5 \frac{(5)(5+1)(5+2)(5+3)(5+4)}{1\cdot 2\cdot 3\cdot 4\cdot 5}\cdot\left(\frac{x}{2}\right)^5$$

$$=\frac{-5\cdot 6\cdot 7\cdot 8\cdot 9}{1\cdot 2\cdot 3\cdot 4\cdot 5}\cdot\left(\frac{1}{2}\right)^5\cdot x^5 = \frac{-63}{16}\cdot x^5$$

ii) 7th term of $\left(1-\frac{x^2}{3}\right)^{-4}$

Sol. T_{r+1} in $(1 - x)^{-n}$ =

$$=\frac{(n)(n+1)(n+2)...(n+r-1)}{1\cdot 2\cdot 3\cdot ...r}\cdot x^r$$

Put r = 6, n = 4, x by $\frac{x^2}{3}$

Then 7th term in $\left(1-\frac{x^2}{3}\right)^{-4}$ is

$$=\frac{(4)(4+1)(4+2)(4+3)(4+4)(4+5)}{1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6}\left(\frac{-x^2}{3}\right)^6$$

$$=\frac{4\cdot 5\cdot 6\cdot 7\cdot 8\cdot 9}{1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6}\cdot\frac{x^{12}}{3^6}=\frac{28}{243}\cdot x^{12}$$

iii) 10th term of $(3 - 4x)^{-2/3}$.

Sol. $(3-4x)^{-2/3} = \left[3\left(1-\frac{4}{3}x\right)\right]^{-2/3}$

$$=(3)^{-2/3}\left(1-\frac{4}{3}x\right)^{-2/3} \quad ...(1)$$

First find 10th term of $\left(1-\frac{4}{3}x\right)^{-2/3}$

The general term of $(1 - x)^{-p/q}$ is

$$T_{r+1} = \frac{(p)(p+q)(p+2q)+...+[p+(r-1)q]}{(r)!}\left(\frac{x}{q}\right)^r$$

Here p = 2, q = 3, r = 9

$$\frac{x}{q} = \left(\frac{(4/3)x}{3}\right)\left(\frac{4}{9}x\right)$$