MATHEMATICS – IIB MODEL QUESTION PAPER -1

TIME: 3Hrs **MAX.MARKS: 75 Marks**

SECTION –A (aimstutorial.in)

ANSWER ALL THE QUESTIONS **10 X 2 = 20 Marks**

1. If $ax^2 + bxy + 3y^2 - 5x + 2y - 3 = 0$ represents a circle, then find the values of a and b and also find its radius and centre.
2. If the length of the tangent from (2, 5) to the circle $x^2 + y^2 - 5x + 4y + k = 0$ is $\sqrt{13}$ then find k.
3. Find the angle between the circles $x^2 + y^2 - 12x - 6y + 41 = 0$ and $x^2 + y^2 + 4x + 6y - 59 = 0$
4. Find the coordinates of the points on the parabola $y^2 = 8x$ whose focal distance is 10.
5. Find the equations of tangents to the hyperbola $3x^2 - 4y^2 = 12$ which are parallel to the line $y = x - 7$.
6. Evaluate $\int \frac{1+\cos^2 x}{1-\cos 2x}\,dx$
7. Evaluate $\int \frac{e^x(1+x)}{\cos^2(xe^x)}\,dx$
8. Evaluate $\int_0^{\pi} \sqrt{2+2\cos\theta}\;d\theta$
9. Find $\int_0^{2\pi} \sin^2 x \cos^4 x\,dx$
10. Find the order and degree of the differential equation $\left[\frac{d^2y}{dx^2} - \left(\frac{dy}{dx}\right)^3\right]^{\frac{6}{5}} = 6y$.

SECTION – B

ANSWER ANY FIVE QUESTIONS **5 X 4 = 20 Marks**

11. Find the equations of the tangents to the circle $x^2 + y^2 - 4x + 6y - 12 = 0$ which the parallel to $x + y - 8 = 0$.
12. If the angle between the circles $x^2 + y^2 - 12x - 6y + 41 = 0$ and $x^2 + y^2 + kx + 6y - 59 = 0$ is 45^0, find k.
13. Find the equations of tangents to the ellipse $2x^2 + y^2 = 8$ which are (i) parallel to $x - 2y - 4 = 0$. (ii) Perpendicular to $x + y + 2 = 0$ (iii) which are making angle 45 with x-axis.
14. If the line $y = mx + c$ touches the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, prove that $c^2 = a^2m^2 + b^2$
15. Show that the angle between the two asymptotes of a hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $2\text{Tan}^{-1}\left(\frac{b}{a}\right)$ or $2\,\text{Sec}^{-1}(e)$.
16. Evaluate $\lim_{n\to\infty} \frac{\sqrt{n+1}+\sqrt{n+2}+\ldots\ldots+\sqrt{n+n}}{n\sqrt{n}}$
17. Solve: $\frac{dy}{dx} + y\tan x = \sin x$.

SECTION – C

ANSWER ANY FIVE QUESTIONS **5 X 7 = 35 Marks**

18. Find the equation of the circle passing through the points (3, 4), (3, 2), (1, 4)
19. Show that the circles $x^2 + y^2 - 6x - 2y + 1 = 0$ and $x^2 + y^2 + 2x - 8y + 13 = 0$ touch each other. Also find the point of contact and common tangent at this point of contact.
20. Derive the equation of the parabola $y^2 = 4ax$ in the standard form.
21. Evaluate $\int \frac{2\sin x + 3\cos x + 4}{3\sin x + 4\cos x + 5}\,dx$
22. If $I_n = \int \sec^n x\,dx$, then prove that $I_n = \frac{1}{n-1}\sec^{n-2} x \cdot \tan x + \frac{n-2}{n-1} I_{n-2}$, and deduce value of $\int \sec^5 dx$.
23. Evaluate $\int_0^{\pi/4} \frac{\sin x + \cos x}{9 + 16\sin 2x}\,dx$
24. Solve the differential equation $\frac{dy}{dx} + \frac{10x + 8y - 12}{7x + 5y - 9} = 0$.

MATHEMATICS – IIB MODEL QUESTION PAPER -2

TIME: 3Hrs **MAX.MARKS: 75 Marks**

SECTION –A (aimstutorial.in)

ANSWER ALL THE QUESTIONS **10 X 2 = 20 Marks**

1. Find the centre and radius of the circle $\sqrt{1+m^2}\;(x^2 + y^2) - 2cx - 2mcy = 0$ $(c > 0)$.
2. Obtain the parametric equations of the circle $(x - 3)^2 + (y - 4)^2 = 8^2$
3. Show that the circles $x^2 + y^2 + 4x - 2y - 11 = 0$ and $x^2 + y^2 - 4x - 8y + 11 = 0$ intersect each other orthogonally.
4. Show that the line $2x - y + 2 = 0$ is a tangent to the parabola $y^2 = 16x$. Also find the point of contact
5. If $3x – 4y + k = 0$ is a tangent to $x^2 + y^2 = 5$ then find the value of 'k'.
6. Find $\int \sqrt{1-\cos 2x}\,dx$
7. Evaluate $\int \frac{1}{(x+3)\sqrt{x+2}}\,dx$
8. Evaluate $\int_0^4 |\,2 - x\,|\,dx$.
9. Evaluate $\int_0^{\pi/2} \cos^{11} x\,dx$
10. Form the differential equation corresponding to $y = ae^{3x} + be^{4x}$.

SECTION – B

ANSWER ANY FIVE QUESTIONS **5 X 4 = 20 Marks**

11. If a point P is moving such that the lengths of tangents drawn from P to the circles $x^2 + y^2 - 4x - 6y - 12 = 0$ and $x^2 + y^2 + 6x + 18y + 26 = 0$ are in the ratio 2: 3, then find the equation of the locus of P.
12. Show that the circles $x^2 + y^2 - 8x - 2y + 8 = 0$ and $x^2 + y^2 - 2x + 6y + 6 = 0$ touch each other and find point of contact.
13. Find the equation of tangent and normal to the ellipse $2x^2 + 3y^2 = 11$ at the point whose ordinate is one.
14. Show that the locus of feet of perpendicular drawn from foci to an of the ellipse is the auxiliary circle.
15. Find the equations of tangents to the hyperbola $3x^2 - 4y^2 = 12$ which are i) parallel ii) perpendicular to the line $y = x – 7$
16. Evaluate $\int_0^{\pi/2} \frac{a\sin x + b\cos x}{\sin x + \cos x}\,dx$
17. Solve: $(1 + x^2)\frac{dy}{dx} + 2xy - 4x^2 = 0$.

SECTION – C

ANSWER ANY FIVE QUESTIONS **5 X 7 = 35 Marks**

18. Show that the four points (1, 2), (3, -4), (5, -6) and (19, 8) are concyclic and find the equation of the circle.
19. Find the transverse common tangents of the circles $x^2 + y^2 - 4x - 10y + 28 = 0$ and $x^2 + y^2 + 4x - 6y + 4 = 0$.
20. Find the coordinates of vertex, focus, and the equation of directrix and axis of the parabola $x^2 - 2x + 4y - 3 = 0$.
21. Evaluate $\int \sqrt{\frac{5-x}{x-2}}$
22. If $I_n = \int \sin^n x\,dx$, then show that $I_n = \frac{-\sin^{n-1} x\cos x}{n} + \frac{n-1}{n} I_{n-2}$ and hence find I_5, I_4.
23. Evaluate $\int_0^{\pi} \frac{x\sin x}{1+\sin x}\,dx$.
24. Solve the differential equation $\frac{dy}{dx} = \frac{3y - 7x + 7}{3x - 7y - 3}$

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MATHEMATICS – IIB MODEL QUESTION PAPER -3

TIME: 3Hrs **MAX.MARKS: 75 MARKS**

SECTION –A (aimstutorial.in)

ANSWER ALL THE QUESTIONS **10 X 2 = 20 MARKS**

1. If the circle $x^2 + y^2 + ax + by - 12 = 0$ has centre at (2, 3) find a, b and also the radius of the circle.
2. Show that (4, 2) and (3, -5) are conjugate with respect to the circle $x^2 + y^2 - 3x - 5y + 1 = 0$
3. Find k if the pair of circles $x^2 + y^2 - 6x - 8y + 12 = 0$, $x^2 + y^2 - 4x + 6y + k = 0$ are orthogonal.
4. Find the coordinates of the points on the parabola $y^2 = 2x$ whose focal distance is 5/2.
5. If e, e_1 are the eccentricities of a hyperbola and its conjugate hyperbola, prove that $\frac{1}{e^2} + \frac{1}{e_1^2} = 1$.
6. Evaluate $\int \frac{x^8}{1+x^{18}} dx$.
7. Evaluate $\int \frac{\cot(\log x)}{x} dx$.
8. Evaluate $\int_0^2 \sqrt{4 - x^2}\ dx$
9. Evaluate $\int_0^{\pi/2} \frac{\sin^2 x - \cos^2 x}{\sin^3 x + \cos^3 x} dx$.
10. Find the order and degree of the differential equation $\frac{d^2y}{dx^2} = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{5}{3}}$.

SECTION – B

ANSWER ANY FIVE QUESTIONS **5 X 4 = 20 MARKS**

11. Find the value of k if $kx + 3y - 1 = 0$, $2x + y + 5 = 0$ are conjugate lines with respect to the circle $x^2 + y^2 - 2x - 4y - 4 = 0$.
12. Find the equation of the circle passing through the points of the intersection of circles $x^2 + y^2 - 8x - 6y + 21 = 0$ and $x^2 + y^2 - 2x - 15 = 0$ and the point (1, 2).
13. Find the value of k if $4x + y + k = 0$ is a tangent to the ellipse $x^2 + 3y^2 = 3$.
14. Find the equation of the ellipse with focus at (1, -1), e = 2/3 and directrix as $x + y + 2 = 0$.
15. Find eccentricity, foci, equations of directrix, length of latus rectum of the hyperbola $5x^2 - 4y^2 + 20x + 8y = 4$.
16. Evaluate $\int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$
17. Solve: $(xy^2 + x)\,dx + (yx^2 + y)\,dy = 0$.

SECTION – C

ANSWER ANY FIVE QUESTIONS **5 X 7 = 35 MARKS**

18. Find the equation of circle which passes through the points (4, 1), (6, 5) and whose centre lies on $4x + 3y - 24 = 0$.
19. Show that the circles $x^2 + y^2 - 4x - 6y - 12 = 0$ and $5(x^2 + y^2) - 8x - 14y - 32 = 0$ touch each other. Also find the point of contact and common tangent at this point of contact.
20. Find the equation of the parabola whose axis is parallel to the Y-axis and which passes through the points (4, 5), (-2, 11) and (-4, 21).
21. Evaluate $\int \frac{x+1}{x^2 + 3x + 12} dx$
22. If $I_n = \int \operatorname{cosec}^n x\, dx$, then prove that $I_n = -\frac{1}{n-1} \operatorname{cosec}^{n-2} x \cot x + \frac{n-2}{n-1} I_{n-2}$, and deduce value of $\int \operatorname{cosec}^5 dx$.
23. Find the area enclosed between $y^2 = 4x$, $y^2 = 4(4 - x)$
24. Solve the differential equation $\frac{dy}{dx} + x\sin 2y = x^3 \cos^2 y$.

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MATHEMATICS – IIB MODEL QUESTION PAPER -4

TIME: 3Hrs **MAX.MARKS: 75 MARKS**

SECTION –A (aimstutorial.in)

VERY SHORT ANSWER TYPE QUESTIONS

ANSWER ALL THE QUESTIONS **10 X 2 = 20 MARKS**

1. Find the equation of a circle passing through (2, -1) and having centre at (2, 3).
2. Find the equation of the polar of (1, -2) with respect to write $x^2 + y^2 - 10x - 10y + 25 = 0$
3. Show that the angle between the circles $x^2 + y^2 = a^2$, $x^2 + y^2 = ax + ay$ is $\frac{3\pi}{4}$.
4. If $\left(\frac{1}{2}, 2\right)$ is one extremity of a focal chord of the parabola $y^2 = 8x$, find the coordinates of the other extremity.
5. Find the equation of the normal at $\theta = \frac{\pi}{3}$ to the hyperbola $3x^2 - 4y^2 = 12$.
6. Evaluate $\int \frac{\log(1+x)}{1+x} dx$
7. **Evaluate** $\int e^x(\sec x + \sec x \tan x)\,dx$.
8. **Evaluate** $\int_0^{\pi/2} \frac{\sin^5 x}{\sin^5 x + \cos^5 x} dx$.
9. Find the area under the curve f(x) = sinx in $[0, 2\pi]$.
10. Form the differential equation corresponding to y = A cos 3x + B sin 3x, where A and B are parameters.

SECTION – B

ANSWER ANY FIVE QUESTIONS **5 X 4 = 20 MARKS**

11. Show that x + y + 1 = 0 touches the circle $x^2 + y^2 - 3x + 7y + 14 = 0$ and find its point of contact.
12. Find the equation of the circle which passes through the point (0, - 3) and intersects the circles given by the equations $x^2 + y^2 - 6x + 3y + 5 = 0$ and $x^2 + y^2 - x - 7y = 0$ orthogonally.
13. Find the length of major axis, minor axis, length of latus rectum, eccentricity, foci, equations of directrix of the ellipse $9x^2 + 16y^2 = 144$.
14. If P(x, y) is any point on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with foci **S and S′** then prove that **SP + S′P** is a constant.
15. Find the eccentricity, foci, equations of directrix, length of latus rectum of the hyperbola $16y^2 - 9x^2 = 144$.
16. Evaluate $\int_0^1 x\ \mathrm{Tan}^{-1}x\ dx$.
17. Solve: $(1+y^2)\,dx = (\mathrm{Tan}^{-1}\,y - x)\,dx$.

SECTION – C

LONG ANSWER TYPE QUESTIONS

ANSWER ANY FIVE QUESTIONS **5 X 7 = 35 MARKS**

18. Find the equations of the circles which touch $2x - 3y + 1 = 0$ at (1, 1) and having radius $\sqrt{13}$.
19. Find the pair of tangents drawn from (1, 3) to the circle $x^2 + y^2 - 2x + 4y - 11 = 0$ and also find the angle between them.
20. If y_1, y_2, y_3 are the y- coordinates vertices of the triangle inscribed in the parabola $y^2 = 4ax$, then show that the area of the triangle is $\frac{1}{8a}\left|(y_1 - y_2)(y_2 - y_3)(y_3 - y_1)\right|$.
21. Evaluate $\int (6x+5)\sqrt{6 - 2x^2 + x}\ dx$.
22. Evaluate $\int \frac{2x+3}{(x+3)(x^2+4)} dx$.
23. Find the value of $\int_0^{\pi} x \cdot \sin^7 x \cos^6 x\, dx$.
24. Solve the differential equation $\frac{dy}{dx}(x^2 y^3 + xy) = 1$.

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MATHEMATICS – IIB MODEL QUESTION PAPER -5

TIME: 3Hrs **MAX.MARKS: 75 MARKS**

SECTION –A (aimstutorial.in)

ANSWER ALL THE QUESTIONS **10 X 2 = 20 MARKS**

1. If $x^2 + y^2 + 2gx + 2fy - 12 = 0$ represents a circle with centre (2, 3); find g, f and its radius.
2. Find the value of k if the points (1, 3) & (2, k) are conjugate with respect to circle $x^2 + y^2 = 35$.
3. Find the equation of radical axis of the two circles $2x^2 + 2y^2 + 3x + 6y - 5 = 0$, $3x^2 + 3y^2 - 7x + 8y - 11 = 0$.
4. Find the equation of tangent to the parabola $y^2 = 16x$ inclined at an angle 60^0 with its axis and also find the point of contact.
5. Find the equation of the hyperbola whose foci are (+ 5,0), the transverse axis is of length 8.
6. Evaluate $\int \frac{(a^x - b^x)^2}{a^x b^x} dx$.
7. Evaluate $\int e^x \left(\frac{1 + x\log x}{x}\right) dx$.
8. Evaluate $\int_0^1 \frac{x^2}{x^2+1} dx$.
9. Find the area bounded between the curves $y^2 - 1 = 2x$ and $x = 0$.
10. Find the order and degree of $\left(\frac{d^3y}{dx^3}\right)^2 - 3\left(\frac{dy}{dx}\right)^2 - e^x = 4$.

SECTION – B

ANSWER ANY FIVE QUESTIONS **5 X 4 = 20 MARKS**

11. Find the equation to the pair of tangents drawn from (0,0) to $x^2 + y^2 + 10x + 10y + 40 = 0$.
12. Show that the circles $x^2 + y^2 + 2ax + c = 0$ and $x^2 + y^2 + 2by + c = 0$ touch each other if $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c}$
13. Find the equations of the tangent and normal to the ellipse $9x^2 + 16y^2 = 144$ at the end of latus rectum in the first quadrant.
14. The tangent and normal to the ellipse $x^2 + y^2 = 4$ at a point $p(\theta)$ on it meets the major axis in Q and R respectively if $0 < \theta < \frac{\pi}{2}$ and QR = 2 then show that $\theta = \text{Cos}^{-1} = \left(\frac{2}{3}\right)$.
15. Find the equation of the hyperbola whose foci are (4, 2), (8, 2) and eccentricity is 2.
16. Evaluate $\int_0^{\pi/2} \frac{dx}{4 + 5\cos x}$.
17. Solve $\sqrt{1+x^2}\, dx + \sqrt{1+y^2}\, dy = 0$.

SECTION – C

ANSWER ANY FIVE QUESTIONS **5 X 7 = 35 MARKS**

18. If (2, 0), (0, 1), (4, 5) and (0, c) are concyclic, then find the value of c.
19. Prove that the combined equation of pair of tangents drawn from an external point P (x_1, y_1) to the circle S = 0 is $S_1^2 = SS_{11}$
20. Show that equation of common tangent to the parabola $y^2 = 4ax$ and $x^2 = 4by$ is $xa^{\frac{1}{3}} + yb^{\frac{1}{3}} + a^{\frac{2}{3}}b^{\frac{2}{3}} = 0$.
21. Evaluate $\int \frac{1}{3\cos x + 4\sin x + 6}\, dx$.
22. If $I_n = \cos^n x\, dx$, then show that $I_n = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n} I_{n-2}$ and hence find I_4
23. Evaluate $\int_3^7 \sqrt{\frac{7-x}{x-3}}$
24. Solve $(x^3 - 3xy^2)\, dx + (3x^2y - y^3)\, dy = 0$.

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MATHEMATICS – IIB MODEL QUESTION PAPER -6

TIME: 3Hrs **MAX.MARKS: 75 MARKS**

SECTION –A (aimstutorial.in)

ANSWER ALL THE QUESTIONS **10 X 2 = 20 MARKS**

25. Find the centre and radius of the circle $x^2 + y^2 + 6x + 8y - 96 = 0$.
26. Find the equation of the circle which is concentric with $x^2 + y^2 - 6x - 4y - 12 = 0$ and passing through (-2, 14)
27. Find the equation of common chord of circles $x^2 + y^2 - 4x - 4y + 3 = 0$, $x^2 + y^2 - 5x - 6y + 4 = 0$.
28. Find the equation of the parabola whose focus is S (1, -7) and vertex is A (1, -2).
29. Find the equations of tangents to the hyperbola $3x^2 - 4y^2 = 12$ which is parallel to the line $y = x - 7$.
30. Evaluate $\int \frac{\sin\left(\text{Tan}^{-1}x\right)}{1+x^2} dx$.
31. Evaluate $\int \frac{\cos x + \sin x}{\sqrt{1+\sin 2x}} dx$.
32. Evaluate $\int_0^2 |1 - x|\, dx$.
33. Evaluate $\int_0^{\frac{\pi}{2}} \sin^4 x \cos^5 x dx$.
34. Find the order and degree of differential Equation $\left[\left(\frac{dy}{dx}\right)^{\frac{1}{2}} + \left(\frac{d^2y}{dx^2}\right)^{\frac{1}{3}}\right]^{\frac{1}{4}}$.

SECTION – B

ANSWER ANY FIVE QUESTIONS **5 X 4 = 20 MARKS**

35. Find the equation of circle with centre (-2, 3) cutting chord of length 2 units on $3x + 4y + 4 = 0$.
36. Find the equation of the circle whose diameter is the common chord of the circle $x^2 + y^2 + 2x + 3y + 1 = 0$, $x^2 + y^2 + 4x + 3y + 2 = 0$.
37. Find the length of major axis, minor axis, length of latus rectum, eccentricity, foci, equations of directrix of the ellipse $4x^2 + y^2 - 8x + 2y + 1 = 0$.
38. The distance of a point on the ellipse $x^2 + 3y^2 = 6$ from its centre is equal to 2. Find the eccentric angles.
39. Show that the condition for the line $\ell x + my + n = 0$ to be a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $a^2\ell^2 - b^2m^2 = n^2$.
40. Evaluate $\lim_{n\to\infty}\left[\frac{1}{n+1} + \frac{1}{n+2} + \ldots + \frac{1}{6n}\right]$.
41. Solve: $\frac{dy}{dx} = \frac{2x - y + 1}{x + 2y - 3}$.

SECTION – C

ANSWER ANY FIVE QUESTIONS **5 X 7 = 35 MARKS**

42. Find the equation of circle whose centre lies on x-axis and passing through (-2, 3) and (4, 5).
43. Find the equations of direct common tangents of the circles $x^2 + y^2 + 22x - 4y - 100 = 0$ and $x^2 + y^2 - 22x + 4y + 100 = 0$.
44. The normal at a point t_1 on $y^2 = 4ax$ meets the parabola again in the point t_2. prove that $t_1t_2 + t_1^2 + 2 = 0$.
45. Evaluate $\int (3x - 2)\sqrt{2x^2 - x + 1}\, dx$.
46. Find the reduction formula for $\int \cot^n x\, dx$ and hence find $\int \cot^4 x\, dx$.
47. Evaluate $\int_0^1 \frac{\log(1+x)}{1+x^2} dx$.
48. Solve differential equation $\frac{dy}{dx} = \frac{2x + y + 3}{2y + x + 1}$.

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MATHEMATICS – IIB MODEL QUESTION PAPER -7

TIME: 3Hrs **MAX.MARKS: 75 MARKS**

SECTION –A (aimstutorial.in)

ANSWER ALL THE QUESTIONS **10 X 2 = 20 MARKS**

49. Find the equation of the circle with (-4, 3), (3, -4) as ends of a diameter.
50. State the necessary and sufficient condition for $\ell x + my + n = 0$ to be normal to circle $x^2 + y^2 + 2gx + 2fy + c = 0$.
51. If angle between the circles $x^2 + y^2 - 12x - 6y + 41 = 0$ & $x^2 + y^2 + kx + 6y - 59 = 0$ is 45^0 find k.
52. Find the value of k if the line $2y = 5x + k$ is a tangent to the parabola $y^2 = 6x$.
53. Find the product of length of perpendiculars from any point on the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$ to its asymptotes.
54. Evaluate $\int \sec^2 x \operatorname{cosec}^2 x \, dx$.
55. Evaluate $\int \frac{dx}{\sqrt{x^2 + 2x + 10}}$.
56. Evaluate $\int_0^{\pi/2} \sec^4\theta \, d\theta$
57. Evaluate $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \sin^2 x \cos^4 x \, dx$
58. Find the general solution of $\frac{dy}{dx} = e^{x+y}$.

SECTION – B

ANSWER ANY FIVE QUESTIONS **5 X 4 = 20 MARKS**

59. Find the equations of tangents to the circle $x^2 + y^2 + 2x - 2y - 3 = 0$ which are perpendicular to $3x - y + 4 = 0$.
60. Find the equation of the circle which cuts the circles $x^2 + y^2 - 4x - 6y + 11 = 0$ and $x^2 + y^2 - 10x - 4y + 21 = 0$ orthogonally and has the diameter along the straight line $2x + 3y = 7$.
61. Find the equations of tangents to $9x^2 + 16y^2 = 144$, which makes equal intercepts on the coordinate axes.
62. If the normal at one end of a latus rectum of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ passes through one end of the minor axis, then show that $e^4 + e^2 = 1$.
63. Show that the equation of normal at $P(\theta)$ to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $\frac{ax}{\sec\theta} + \frac{by}{\tan\theta} = a^2 + b^2$.
64. Find the area bounded between the curves $y = 3x$ and $y = 6x - x^2$.
65. Solve $(1 + x^2)\frac{dy}{dx} + y = \tan^{-1} x$.

SECTION – C
LONG ANSWER TYPE QUESTIONS

ANSWER ANY FIVE QUESTIONS **5 X 7 = 35 MARKS**

66. Find the equation of the circle passing through the points (1, 2), (3, -4), (5, -6).
67. Find the equation of all possible common tangents of the circles $x^2 + y^2 - 2x - 6y + 6 = 0$ and $x^2 + y^2 - 1 = 0$.
68. Prove that the two parabolas $y^2 = 4ax$ and $x^2 = 4by$ intersect (other than the origin) at an angle of $\mathbf{Tan^{-1}\left[\frac{3a^{1/3}b^{1/3}}{2(a^{2/3}+b^{2/3})}\right]}$.
69. Evaluate $\mathbf{\int \frac{1}{(1+x)\sqrt{2x^2+3x+1}}\,dx}$
70. Find the reduction formula for $\int \tan^n x \, dx$ and hence find $\int \tan^6 x \, dx$.
71. Evaluate $\int_0^{\pi/2} \frac{x}{\sin x + \cos x} dx$.
72. Solve the differential equation $(2x + y + 1)\,dx + (4x + 2y - 1)\,dy = 0$.

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MATHEMATICS – IIB MODEL QUESTION PAPER -8

TIME: 3Hrs **MAX.MARKS: 75 MARKS**

SECTION –A (aimstutorial.in)

ANSWER ALL THE QUESTIONS **10 X 2 = 20 MARKS**

1. If $x^2 + y^2 - 4x + 6y + c = 0$ represents a circle with radius 6, then find the value of c.
2. Obtain the parametric equations of the circle represented by $x^2 + y^2 = 4$.
3. Find the equation of common chord of circles $(x - a)^2 + (y - b)^2 = c^2$, $(x - b)^2 + (y - a)^2 = c^2$.
4. Find the coordinates of the points on the parabola $y^2 = 8x$ whose focal distance is 10.
5. If the eccentricity of a hyperbola is $\frac{5}{4}$, then find the eccentricity of its conjugate hyperbola.
6. Evaluate $\int \left(1 - \frac{1}{x^2}\right) e^{x+\frac{1}{x}} dx$.
7. Evaluate $\int e^x \frac{1+x}{(2+x)^2} dx$.
8. Evaluate $\int_1^5 \frac{dx}{\sqrt{2x-1}}$.
9. Find the area of the region enclosed by the curves $y = x^3 + 3$, $y = 0$, $x = -1$, $x = 2$.
10. Find the order degree of differential equation $\mathbf{x^{\frac{1}{2}}\left(\frac{d^2y}{dx^2}\right)^{(1/3)} + x\frac{dy}{dx} + y = 0}$

SECTION – B

ANSWER ANY FIVE QUESTIONS **5 X 4 = 20 MARKS**

11. Show that the tangent at (-1, 2) of the circle $x^2 + y^2 - 4x - 8y + 7 = 0$ touches the circle $x^2 + y^2 + 4x + 6y = 0$, also find its point of contact.
12. Find the equation of the circle which is orthogonal to $x^2 + y^2 + 2x + 17y + 4 = 0$, $x^2 + y^2 + 7x + 6y + 11 = 0$ and $x^2 + y^2 - x + 22y + 3 = 0$.
13. Find the equation of tangent and normal to the ellipse $x^2 + y^2 - 4x + 12y + 14 = 0$ at the point (2, -1).
14. Find the equation of the ellipse in the standard form whose distance between foci is 2 and the length of latus rectum is 15/2.
15. Find the eccentricity, foci, equation of directrix, and length of latus rectum of hyperbola $5x^2 - 4y^2 + 20x + 8y = 4$.
16. Evaluate $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{i^3}{i^4 + n^4}$.
17. Solve: $\frac{dy}{dx} = \frac{x(2\log x + 1)}{\sin y + y\cos y}$.

SECTION – C

ANSWER ANY FIVE QUESTIONS **5 X 7 = 35 MARKS**

18. If θ_1, θ_2 are the angles of inclination of tangents through a point P to the circle $x^2 + y^2 = a^2$, then find the locus of P when $\cot\theta_1 + \cot\theta_2 = k$.
19. find the equation of the circle which touches the circle $x^2 + y^2 - 2x - 4y - 20 = 0$ externally at (5, 5) with radius 5.
20. If the normal of a chord at a point 't' on the parabola $y^2 = 4ax$ subtends a right angle at its vertex, then prove that $\mathbf{t = \pm\sqrt{2}}$.
21. Evaluate $\int \frac{\cos x + 3\sin x + 7}{\cos x + \sin x + 1} dx$.
22. Evaluate $\int \frac{\sin x \cos x}{\cos^2 x + 3\cos x + 2} dx$.
23. Evaluate $\int_0^{\pi} \frac{x \sin^3 x}{1 + \cos^2 x} dx$.
24. Solve the differential equation $\frac{dy}{dx}(x^2 y^3 + xy) = 1$.

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