31. BINOMIAL THEOREM

1. Assuming $|x|$ to be so small, that x^2 and higher powers of x can be neglected, then $\frac{\sqrt{1+x}+(1-x)^{3/2}}{(1+x)+\sqrt{1+x}}=$

[AP EAMCET 17-09-20_Shift-1]

1. $1+\frac{5x}{4}$
2. $1-\frac{5x}{4}$
3. $1+\frac{4x}{5}$
4. $1-\frac{4x}{5}$

2. If $[x]$ denotes the greatest integer function on x, then the number of positive integral divisors of $\left[\left(2+\sqrt{3}\right)^5\right]$ is **[AP EAMCET 17-09-20_Shift-1]**

1. 6
2 4
3. 2
4. 8

3. The coefficient of x^{50} in the expansion of $(1+x)^{1000}+x(1+x)^{999}+x^2(1+x)^{998}+.....+x^{1000}$ is

[AP EAMCET 17-09-20_Shift-2]

1. $^{1000}C_{50}$
2. $^{999}C_{50}$
3. $^{1000}C_{51}$
4. $^{1001}C_{50}$

4. If $1\times1!+2\times2!+3\times3!+.....+n\times n!=11!-1$, then the maximum value of ^nC_r is

[AP EAMCET 17-09-20_Shift-2]

1. 462
2. 252
3. 162
4. 512

5. If the m^{th} term is the middle term in expansion of $\left(x^2-\frac{1}{2x}\right)^{20}$, then the co-efficient of T_{m+3}

[AP EAMCET 17-09-20_Shift-2]

1. $^{20}C_{13}2^{-13}$
2. $-^{20}C_{13}2^{13}$
3. $-^{20}C_{13}2^{-13}$
4. $^{20}C_{13}2^{13}$

6. If $x=\frac{1}{5}+\frac{1\times3}{5\times10}+\frac{1\times3\times5}{5\times10\times15}+......$ then $3x^2+6x=$

[AP EAMCET 17-09-20_Shift-2]

1. 1
2. 2
3. -1
4. -2

7. In the expansion of $\left(\sqrt[5]{3}+\sqrt[3]{2}\right)^{15}$

[AP EAMCET 18-09-20_Shift-1]

1. Number of rational terms is 3
2. Sum of all rational terms is 58
3. Sum of all rational terms is greater than the sum of all irrational terms
4. Sum of all irrational terms is greater than the sum of all rational terms

8 Choose the correct option regarding the following statements

(i) $C_0+C_2+C_4+.....+C_n=2^{n-1}$ if n is even

(ii) $C_1+C_3+C_5+.....+C_{n-1}=2^{n-1}$ if n is even

[AP EAMCET 18-09-20_Shift-1]

1. (i) is true,(ii) is false
2. (i) is false,(ii) is true
3. Both (i) and (ii) are false
4. Both (i) and (ii) are true

9. Find the coefficient of x^5 in $\left(1+x+x^2\right)^8$

[AP EAMCET 18-09-20_Shift-2]

1. 405
2. 508
3. 404
4. 504

10. In the expansion of $\left(a+1+\frac{1}{a}\right)^n$, where $n\in N$ there are 2029 terms. Then n=

[AP EAMCET 18-09-20_Shift-2]

1. 1015
2. 1013
3. 1014
4. 1012

11. If the coefficients of x^9 and x^{10} in the binomial expansion of $\left(3+\frac{x}{2}\right)^n$ are equal, then n=

[AP EAMCET 21-09-20_Shift-1]

1. 69
2. 96
3. 66
4. 99

12. By neglecting x^4 and higher powers of x, find approximate value of $\sqrt[3]{x^2+64}-\sqrt[3]{x^2+27}$

[AP EAMCET 21-09-20_Shift-1]

1. $1-\frac{7}{234}x^2$ 2. $1-\frac{7}{432}x^2$
3. $1-\frac{7}{32}x^2$ 4. $1-\frac{7}{42}x^2$

13. The coefficients of x^{50} in $(1+x)^{101}(1-x+x^2)^{100}$ is____ **[AP EAMCET 21-09-20_Shift-1]**

1. 1 2. -1
3. 0 4. 2

14 If $\frac{^{n+1}C_{r+1}}{^{n+1}C_r}=\frac{n-r+1}{m}$, then m=

[AP EAMCET 21-09-20_Shift-1]

1. r 2. r-1
3. r+1 4. 1-r

15. What is the coefficient of $\frac{y^3}{x^8}$ in $(x+y)^{-5}, when\left|\frac{y}{x}\right|<1$?

[AP EAMCET 21-09-20_Shift-2]

1. -35 2. -30
3. -25 4. 10

16. If 'n' is a positive interger then

$\sum_{r=1}^{n} r^2.c_r=(\_\_\_\_\_)2^{n-2}$

[AP EAMCET 21-09-20_Shift-2]

1. $n(n-1)$ 2. n
3. $n(n+1)$ 4. $(n+1)$

17. Assuming x to be so small that x^2 and higher powers of x can be neglected, the coefficient of x in $\frac{(1-x)^{1/3}+(1-5x)^2}{(16-x)^{1/4}}$ is equal to

[AP EAMCET 22-09-20_Shift-1]

1. $\frac{989}{96}$ 2. $\frac{989}{192}$
3. $-\frac{989}{96}$ 4. $-\frac{989}{192}$

18. If the sum of all the coefficients of $\left(\alpha x^2-2x+1\right)^{2019}$ is equal to the sum of all the coefficients of $(x-\alpha y)^{2019}$ then $\alpha=$

[AP EAMCET 22-09-20_Shift-1]

1. -1 2. 0
3. 1 4. 2

19. If the 6th term in $\left(\frac{2p}{3}+\frac{3q}{2}\right)^9$ is "ap^bq^c", then a,b and c respectively are

[AP EAMCET 22-09-20_Shift-1]

1. 189, 5, 4 2. 189, 4, 5
3. 212, 4, 5 4. 212, 5, 4

20. What is the constant term in the binomial expansion of $(1+3x)^n\left(1+\frac{1}{3x}\right)^n$?

[AP EAMCET 22-09-20_Shift-2]

1. $\binom{2n}{n}$ 2. $\binom{2n}{n-1}$
3. $\binom{2n}{n+1}$ 4. No such term exists

21. Let R=$(5\sqrt{5}+11)^{2n+1}$ and f=R- [R] , where [x] denotes the greatest integer less than or equal to x , then Rf=

[AP EAMCET 23-09-20_Shift-1]

1. 2^{n+1} 2. 2^{2n+1}
3. 4^{n+1} 4. 4^{2n+1}

22. If the term independent of x in the expansion of $(\sqrt{x}-\frac{k}{x^2})^{10}$ is 405 , then k=

[AP EAMCET 23-09-20_Shift-1]

1. 3 only 2. -3
3. ±3 4. 0

23. The index of the power of x occurring in the 5th term from the end in the expansion of $(\frac{x^3}{2}-\frac{2}{x^2})^{12}$ is **[AP EAMCET 23-09-20_Shift-1]**

1. 3 2. -3
3. 4 4. -4

24. A bag contains 5 blue and an unknown number x of red balls. Two balls are drawn at random. If the probability of both of them being blue is $\frac{5}{14}$, then the value of x is equal to

[AP EAMCET 23-09-20_Shift-1]

1. 8
2. 5
3. 3
4. 6

25. If [x] represents the greatest integer not greater than x, then $\left[\left(1+\frac{1}{100000}\right)^{100000}\right]=$

[AP EAMCET 23-09-20_Shift-1]

1. 1
2. 3
3. 2
4. 4

26. Let n be positive integer. If the coefficient of $2^{nd}, 3^{rd}$ *and* 4^{th} terms in the expansion of $(1+x)^n$ are in A.P, then the value of n=

1. 6
2. 7
3. 8
4. 9

27. In the binomial expansion of $(a-b)^n, n \geq 5$, the sum of the 5^{th} *and* 6^{th} terms is zero. Then the value of $\frac{a}{b}$ is____

1. $\frac{n-2}{3}$
2. $\frac{n-4}{5}$
3. $\frac{n-5}{7}$
4. $\frac{n-1}{3}$

28. If $y=\frac{3}{4}+\frac{3.5}{4.8}+\frac{3.5.7}{4.8.12}+.....\infty$, then

[TS EAMCET 09-09-20_Shift-1]

1. $y^2-2y+5=0$
2. $y^2+2y-7=0$
3. $y^2-3y+4=0$
4. $y^2+4y-6=0$

29. If the coefficient of x^{13} in the expansion of $\frac{(1+x)^2}{(1-2x)^3}$ is $A\times 2^{10}$, then A=

[TS EAMCET 09-09-20_Shift-1]

1. 862
2. 1304
3. 1360
4. 1724

30. For $|x|<\frac{1}{2}$ if the coefficient of x^{10} and the constant term in the expansion of $\frac{2x^3+8x^2-2x-2}{(1-x)(1+x)(1-2x)}$ in powers of x are l *and* m respectively, then $lm=$

[TS EAMCET 09-09-20_Shift-2]

1. $6(1+2^9)$
2. $4(1+2^9)$
3. $6(1+2^{10})$
4. $4(1+2^{10})$

31. The number of irrational terms in the expansion of $\left(5^{\frac{1}{2}}+7^{\frac{1}{8}}\right)^{1024}+\left(5^{\frac{1}{2}}-7^{\frac{1}{8}}\right)^{1024}$ is

[TS EAMCET 09-09-20_Shift-2]

1. 896
2. 129
3. 513
4. 384

32. Let $x\in R$ be so small that the powers of x beyond two are insignificant and negligibly small. For such x, if $(1-x)^3(2+x)^6$ is approximated by $a+bx+cx^2$, then $a+b+c=$

[TS EAMCET 10-09-20_Shift-1]

1. -80
2. 144
3. 80
4. 127

33. For 0< x <1, the expansion of $\left(1+\frac{1}{x}\right)^{\frac{1}{2}}$ is

[TS EAMCET 10-09-20_Shift-1]

1. $1+\frac{1}{2x}-\frac{1}{2!}\left(\frac{1}{2x}\right)^2+\frac{1.3}{3!}\left(\frac{1}{2x}\right)^3-\frac{1.3.5}{4!}\left(\frac{1}{2x}\right)^4+....\infty$
2. $\frac{1}{\sqrt{x}}+\frac{1}{2}\sqrt{x}-\frac{1}{2!}\frac{x\sqrt{x}}{2^2}+\frac{1.3x^2\sqrt{x}}{3!2^3}-....\infty$
3. $1+\frac{1}{\sqrt{x}}+\frac{1}{2}x\sqrt{x}+\frac{1}{2!}\frac{x^2\sqrt{x}}{2^3}+\frac{1.3}{3!}\frac{x^3\sqrt{x}}{2^4}+.....\infty$
4. $\frac{1}{\sqrt{x}}+\frac{1}{2x\sqrt{x}}-\frac{1}{2!}\left(\frac{1}{2x}\right)^2\frac{1}{\sqrt{x}}+\frac{1.3}{3!}\left(\frac{1}{2x}\right)^3\frac{1}{\sqrt{x}}-....\infty$

34 If the 9th and 10th terms are the numerically greatest terms in the expansion of $(5x-6y)^n$ when $x=\frac{2}{5}$ and $y=\frac{1}{2}$, then the absolute value of the middle term of that expansion is

[TS EAMCET 10-09-20_Shift-2]

1. $14C_8 6^7$
2. $14C_7 6^7$
3. $15C_7 6^7$
4. $15C_8 6^8$

35. $1-\frac{3}{16}+\frac{1.4}{1.2}\left(\frac{3}{16}\right)^2-\frac{1.4.7}{1.2.3}\left(\frac{3}{16}\right)^3+...=$

[TS EAMCET 10-09-20_Shift-2]

1. $\left(\frac{15}{6}\right)^{\frac{3}{8}}$
2. $\left(\frac{4}{5}\right)^{\frac{2}{3}}$
3. $\left(\frac{7}{4}\right)^{\frac{1}{16}}$
4. $\left(\frac{4}{15}\right)^{\frac{-2}{5}}$

36. Suppose l, m, n respectively represents the coefficient of x^{10}, the constant and the coefficient of x^{-10} in the expansion of $\left(ax^2+\frac{b}{x^3}\right)^{15}$. If $\frac{l}{m}+\frac{m}{n}=\frac{26}{11}$, then $a^2:b^2=?$

[TS EAMCET 11-09-20_Shift-1]

1. 16:9
2. 9:4
3. 4:1
4. 1:25

37 For $z\in C$, if $(1+z)^n=1+nc_1z+nc_2z^2+...nc_nz^n$ and $\sum_{r=0}^{100}100c_r(\sin rx)=\left(2\cos\frac{x}{2}\right)^{100}\sin kx$, then $k=$

[TS EAMCET 11-09-20_Shift-1]

1. 25
2. 100
3. 50
4. 75

38. If sum of the coefficients of $x^r\ (r=0,1,2,...,2n)$ in the expansion of $\left(1+3x-2x^2\right)^n$, is 128, then $\sum_{r=1}^{2n} r\frac{(2n)_{c_r}}{^{(2n)}c_{r-1}}=$

[TS EAMCET 11-09-20_Shift-2]

1. 120
2. 135
3. 90
4. 105

39. If $^nc_0, ^nc_1, ^nc_2, ..., ^nc_n$ respectively are the binomial coefficients in the expansion of $(1+x)^n$, then when $n=10, \sum_{r=1}^{10} {^nc_r}.r(r-4)=$

[TS EAMCET 11-09-20_Shift-2]

1. 5120
2. 7680
3. 20480
4. 28160

40. If x is so small that all terms containing x^2 and higher powers of x can be neglected, then the approximate value of $\frac{\left(1+\frac{2x}{3}\right)^{-4}(4+5x)^{1/2}}{(9+x)^{\frac{3}{2}}}$, when $x=\frac{6}{371}$, is

[TS EAMCET 14-09-20_Shift-2]

1. $\frac{1}{27}$
2. $\frac{29}{378}$
3. $\frac{3}{27}$
4. $\frac{1}{14}$

41. The sum of the coefficients of $x^{-3/2}$ and x^3 in the expansion of $\sqrt{3+x}+\sqrt{5+x}$ when $3<x<5$, is

[TS EAMCET 14-09-20_Shift-2]

1. $\frac{-9+\sqrt{5}}{16}$
2. $\frac{5^{-5/2}-18}{16}$
3. $\frac{-6+\sqrt{5}}{6}$
4. $\frac{5-\sqrt{6}}{6}$

42. If $\left(2+\sqrt{3}\right)^{49}+\left(\sqrt{3}-2\right)^{49}=a+b\sqrt{3}$, then

[TS EAMCET 04-08-2021_Shift-2]

1. $a\neq 0, b\neq 0$
2. $b\neq 0, a=0$
3. $b=0, a\neq 0$
4. $a=b$

43. In the expansion of $\left(1+\frac{3x}{2}\right)^{-5}$, the coefficient of x^{10} s equal to the coefficient of x^{10} in $(1+\alpha x)^n, n\in N$, then $n\alpha=$

[TS EAMCET 04-08-2021_Shift-2]

1. 15
2. 18
3. 24
4. 21

44. For $x>0$, if p^{th} term is the first negative term in the expansion of $\left(1+\frac{3x}{5}\right)^{\frac{22}{3}}$ and in the expansion of $\left(1-\frac{3x}{5}\right)^{\frac{22}{3}}$ from r^{th} term onwards all the terms are positive, then the number of terms in the expansion of $\left(px+\frac{r}{x}\right)^{pr}$ is

[TS EAMCET 04-08-2021_Shift-2]

1. 101
2. 119
3. 200
4. 99

45. The value of the numerically greatest term in the expansion of $(2x+3y)^{11}$ when $x=\frac{1}{2}$ & $y=\frac{1}{3}$ is **[TS EAMCET 04-08-2021_Shift-1]**

1. 462
2. ${}^{11}C_5\left(\frac{2}{3}\right)^6$
3. ${}^{11}C_6\left(\frac{3}{2}\right)^5$
4. 576

46. The co-efficient of x^4 in the expansion of $\left(1-x-x^2+x^3\right)^6$ is :

[TS EAMCET 04-08-2021_Shift-1]

1. 120
2. 15
3. -75
4. -60

47. The sum of the co-efficient in the expansion of $\left(1+\frac{x}{2}\right)^{12}$ is :

[TS EAMCET 04-08-2021_Shift-1]

1. 0
2. 2^{11}
3. $\left(\frac{3}{2}\right)^{12}$
4. 2^{12}

48. If $f(n)$ is the coefficient of x^n in the expansion of $(1+x)(1-x)^n$, then $f(2021)=$

[TS EAMCET 05-08-2021_Shift-1]

1. -2019
2. 2020
3. 2021
4. -2022

49. If p and q are respectively the coefficient of x^{-3} and x^{-5} in the expansion of $\left(x^{1/3}+\frac{1}{2x^{1/3}}\right)^{21}$, $x>0$, then $\frac{5p}{4q}=$

[TS EAMCET 05-08-2021_Shift-1]

1. 102
2. 408
3. 182
4. 468

50. For $n, p\in N-\{1\}$, the coefficient of x^3 in $\frac{(1-x)^{-1/p}}{(1-x)^n}=$

[TS EAMCET 05-08-2021_Shift-1]

1. $\frac{(np+1)(np+p+1)(np+2p+1)}{p^3\times 3!}$
2. $\frac{(np+1)(np+p)(np+2p)}{p^3\times 3!}$
3. $\frac{(np+p)(np+2p)(np+3p)}{p^3\times 3!}$
4. $\frac{(np+1)(np+2)(np+3)}{p^3 3!}$

51. In the binomial expression of $(1+x)^{2k}$, if its middle term is the only numerically greatest term, then x lies in the interval

[TS EAMCET 05-08-2021_Shift-2]

1. $(-2k,2k)$
2. $\left(-\frac{k+1}{k},\frac{k+1}{k}\right)$
3. $(-k,k)$
4. $(-(k+1),(k+1))$

52. Sum of the coefficient of x^r $(r=0,12,3,...15)$ in the expansion of $(3x-1)^{15}$ is equal to the sum of the binomial coefficient of which of the following expansions?

$(a)(1+x)^{15}$ $(b)(1+x)^{16}+(1-x)^{16}$

$(c)(1+x)^{16}-(1-x)^{16}$

[TS EAMCET 05-08-2021_Shift-2]

1. (a),(b) and (c)
2. (a) and (c) only
3. (b) and (c) only
4. (a) and (b) only

44. For $x>0$, if p^{th} term is the first negative term in the expansion of $\left(1+\frac{3x}{5}\right)^{\frac{22}{3}}$ and in the expansion of $\left(1-\frac{3x}{5}\right)^{\frac{22}{3}}$ from r^{th} term onwards all the terms are positive, then the number of terms in the expansion of $\left(px+\frac{r}{x}\right)^{pr}$ is

[TS EAMCET 04-08-2021_Shift-2]

1. 101
2. 119
3. 200
4. 99

45. The value of the numerically greatest term in the expansion of $(2x+3y)^{11}$ when $x=\frac{1}{2}$ & $y=\frac{1}{3}$ is **[TS EAMCET 04-08-2021_Shift-1]**

1. 462
2. ${}^{11}C_5\left(\frac{2}{3}\right)^6$
3. ${}^{11}C_6\left(\frac{3}{2}\right)^5$
4. 576

46. The co-efficient of x^4 in the expansion of $\left(1-x-x^2+x^3\right)^6$ is :

[TS EAMCET 04-08-2021_Shift-1]

1. 120
2. 15
3. -75
4. -60

47. The sum of the co-efficient in the expansion of $\left(1+\frac{x}{2}\right)^{12}$ is :

[TS EAMCET 04-08-2021_Shift-1]

1. 0
2. 2^{11}
3. $\left(\frac{3}{2}\right)^{12}$
4. 2^{12}

48. If $f(n)$ is the coefficient of x^n in the expansion of $(1+x)(1-x)^n$, then $f(2021)=$

[TS EAMCET 05-08-2021_Shift-1]

1. -2019
2. 2020
3. 2021
4. -2022

49. If p and q are respectively the coefficient of x^{-3} and x^{-5} in the expansion of $\left(x^{1/3}+\frac{1}{2x^{1/3}}\right)^{21}$, $x>0$, then $\frac{5p}{4q}=$

[TS EAMCET 05-08-2021_Shift-1]

1. 102
2. 408
3. 182
4. 468

50. For $n, p\in N-\{1\}$, the coefficient of x^3 in $\frac{(1-x)^{-1/p}}{(1-x)^n}=$

[TS EAMCET 05-08-2021_Shift-1]

1. $\frac{(np+1)(np+p+1)(np+2p+1)}{p^3\times 3!}$
2. $\frac{(np+1)(np+p)(np+2p)}{p^3\times 3!}$
3. $\frac{(np+p)(np+2p)(np+3p)}{p^3\times 3!}$
4. $\frac{(np+1)(np+2)(np+3)}{p^3 3!}$

51. In the binomial expression of $(1+x)^{2k}$, if its middle term is the only numerically greatest term, then x lies in the interval

[TS EAMCET 05-08-2021_Shift-2]

1. $(-2k,2k)$
2. $\left(-\frac{k+1}{k},\frac{k+1}{k}\right)$
3. $(-k,k)$
4. $\left(-(k+1),(k+1)\right)$

52. Sum of the coefficient of x^r $(r=0,12,3,\ldots 15)$ in the expansion of $(3x-1)^{15}$ is equal to the sum of the binomial coefficient of which of the following expansions?

$(a)(1+x)^{15}$ $(b)(1+x)^{16}+(1-x)^{16}$

$(c)(1+x)^{16}-(1-x)^{16}$

[TS EAMCET 05-08-2021_Shift-2]

1. $(a),(b)$ *and* (c)
2. (a) *and* (c) *only*
3. (b) *and* (c) *only*
4. (a) *and* (b) *only*

53. The 13^{th} term in the expansion of $(1-4x)^{-4}$ is

[TS EAMCET 05-08-2021_Shift-2]

1. $^{15}C_4 4^{12}x^{12}$
2. $728x^{12}$
3. $^{15}C_3 4^{12}x^{12}$
4. $1092x^{12}$

54. If the coefficient of x^3 in the binomial expansion of $x^3\left(2\sqrt{3}x^2+\frac{1}{kx}\right)^{12}$ is 880, then $k=$

[TS EAMCET 06-08-2021_Shift-2]

1. $2\sqrt{2}$
2. $4\sqrt{3}$
3. $2\sqrt{3}$
4. $\sqrt{3}$

55. The middle term in the expansion of $\left(4x^3-\frac{15}{4x}\right)^8$ is

[TS EAMCET 06-08-2021_Shift-2]

1. $70(15x)^4$
2. $1820x^8$
3. $70(15x^2)^4$
4. $2560x^4$

56 If $5|b|<2|a|$, then the 4th term in the expansion of $(2a+5b)^{-4}$ is

[TS EAMCET 06-08-2021_Shift-2]

1. $^4C_3 2^5 5^3 a^5 b^3$
2. $-^6C_3 \frac{5^3}{2^7}\frac{b^3}{a^7}$
3. $-^6C_3 \frac{5^4}{2^8}\frac{b^4}{a^8}$
4. $^4C_3 2^5 5^4 a^4 b^4$

57. The term independent of x in the expansion of $\left(x-\frac{2}{\sqrt{x}}\right)^{21}$ is

[TS EAMCET 06-08-2021_Shift-1]

1. $^{21}C_{15}(-2)^{15}$
2. $^{21}C_{14}2^{14}$
3. $-^{21}C_7(2)^7$
4. $-^{21}C_7 2^{14}$

58. If the set of all values of x for which the expansion of $(7-5x)^{-2/3}$ valid is equal to $(-a,a)$, then $5a+7=$

[TS EAMCET 06-08-2021_Shift-1]

1. 14
2. 21
3. 0
4. 12

59. The coefficient of x^3 in the expansion of $\left(1-\frac{3}{4}x\right)^{\frac{1}{2}}$ is

[TS EAMCET 06-08-2021_Shift-1]

1. $\frac{27}{1024}$
2. $\frac{-27}{1024}$
3. $\frac{81}{1024}$
4. $\frac{-81}{1024}$

60 If L and M are respectively the coefficient of x^{-7} in $\left(ax+\frac{b}{x^2}\right)^{11}$ and the coefficient of x^7 in $\left(bx^2+\frac{a}{x}\right)^{11}$ then L + M =

[TS EAMCET 18-07-2022_Shift-1]

1. $\frac{1}{b}\left[\text{coefficient of } x^{-6} \text{ in}\left(ax+\frac{b}{x^2}\right)^{12}\right]$
2. $\frac{1}{a}\left[\text{coefficient of } x^{6} \text{ in}\left(ax^2+\frac{b}{x}\right)^{12}\right]$
3. $a\left[\text{coefficient of } x^{-10} \text{in}\left(ax+\frac{b}{x^2}\right)^{11}\right]$
4. $b\left[\text{coefficient of } x^{4} \text{ in}\left(ax^2+\frac{b}{x}\right)^{11}\right]$

61. If the 4th term in the expansion of $\left(\frac{x}{2}-\frac{2y}{3}\right)^6$ is – 20, then xy =

[TS EAMCET 18-07-2022_Shift-2]

1. 2
2. 3
3. 8
4. 27

62. If k is the coefficient of x^5 in the expansion of $\left(2x^2-\frac{1}{3x^3}\right)^5$ then $\frac{3k}{2}=$

[TS EAMCET 19-07-2022_Shift-I]

1. -20
2. -40
3. 20
4. 40

63. The expansion of $(a+x)^n$ contains 15 terms. When x=1 the ratio of the neighbouring terms to the middle term in this expansion is 16.Then the positive integral value of 'a' is

[TS EAMCET 19-07-2022_Shift-2]

1. 1
2. 3
3. 4
4. 2

64. $\frac{1}{8}-\frac{7}{8.12}+\frac{7.10}{8.12.16}-........=$

[TS EAMCET 20-07-2022_Shift-1]

1. $\sqrt[3]{\frac{4}{7}}$
2. $\sqrt[3]{\frac{4}{7}}-\frac{3}{4}$
3. $\sqrt[3]{\frac{4}{7}}+\frac{3}{4}$
4. $\sqrt[3]{\frac{7}{4}}-\frac{3}{4}$

65. Numerically greatest term in the expansion of $(2x-3y)^{11}$ when $x=\frac{1}{3}$ and $y=\frac{1}{2}$ is

[TS EAMCET 20-07-2022_Shift-2]

1. ${}^{11}C_8\left(\frac{2}{3}\right)^5$
2. ${}^{11}C_3\left(\frac{3}{2}\right)^5$
3. ${}^{11}C_2\left(\frac{3}{2}\right)^7$
4. ${}^{11}C_2\left(\frac{2}{3}\right)^7$

66 C_j stands for ${}^nC_j$, then

$$\frac{C_1}{C_0}+\frac{2\times C_2}{C_1}+\frac{3\times c_3}{C_2}+....+\frac{n\times C_n}{C_{n-1}}=$$

[15th May 2023 Shift 1]

1. $\sum_{k=1}^{n}k^2$
2. $\sum_{k=1}^{n}\frac{k}{2}$
3. $\sum_{k=1}^{n}2k$
4. $\sum_{k=1}^{n}k$

67 If $(2-5x)^{\frac{-1}{5}}=a_0+a_1x+a_2x^2+....$, then $\frac{a_1}{a_2}=$

[15th May 2023 Shift 1]

1. $\frac{1}{3}$
2. $-\frac{2}{3}$
3. $-\frac{1}{3}$
4. $\frac{2}{3}$

68 If $C_i={}^nc_i$, then $C_0C_r+C_1C_{r+1}+C_2C_{r+2}+...+C_{n-r}C_n=$

[15th May 2023 Shift 2]

1. $\frac{(2n)!}{(n-2r)!(n+2r)!}$
2. $\frac{(2n)!}{(n-r)!(n+r)!}$
3. ${}^{2n}C_r$
4. ${}^{2n}C_{r+1}$

69 The coefficient of the highest power of x in the expansion of

$\left(x+\sqrt{x^2-1}\right)^8+\left(x-\sqrt{x^2-1}\right)^8$ *is*

[15th May 2023 Shift 2]

1. 64
2. 128
3. 256
4. 512

70 The coefficient of x^r in the expansion of $\frac{1}{\sqrt[3]{(1-2x)^2}}$ is **[16th May 2023 Shift 1]**

1. $\frac{2.5.8...(3r-1)}{r!}(-1)^r\left(\frac{2}{3}\right)^r$
2. $\frac{2.5.8...(3r-1)}{r!}(-1)^r\left(\frac{3}{2}\right)^r$
3. $\frac{2.5.8...(3r-1)}{r!}\left(\frac{2}{3}\right)^r$
4. $\frac{2.5.8...(3r-1)}{r!}\left(\frac{3}{2}\right)^r$

71 *If* $3\times{}^5C_0+8\times{}^5C_1+13\times{}^5C_2+18\times{}^5C_3+23\times{}^5C_4$ $+28\,{}^5C_5=k\times 2^4$, *then* $k=$

[16th May 2023 Shift 1]

1. 33
2. 37
3. 31
4. 30

72 The coefficient of x^3 in the expansion of $(1-2x)^{\frac{1}{2}}(1+3x)^{\frac{1}{3}}$ is

[16th May 2023 Shift 2]

1. $-\frac{20}{3}$
2. $\frac{20}{3}$
3. $\frac{17}{3}$
4. $-\frac{17}{3}$

73 If the numerically greatest term in the expansion of $(2\text{-}3x)^9$ when $x = 1$ is $P_1^{\alpha} P_2^{\beta} P_3^{\gamma} P_4^{\delta}$ ($P_1 < P_2 < P_3 < P_4$ are the first four prime numbers), then $\alpha + \beta + \gamma + \delta =$

[17th May 2023 Shift 1]

1. 13 2. 12
3. 14 4. 11

74 The degree of the polynomial $\left(x+\sqrt{x^4-1}\right)^9+\left(x-\sqrt{x^4-1}\right)^9$ is

[17th May 2023 Shift 1]

1. 14 2. 15
3. 16 4. 17

75 If $\lim\limits_{x\to 1}\dfrac{\sqrt[4]{x^{-3}}+a\sqrt[4]{x^5}}{x-1}=-2$, then the coefficient of x in the expansion of $\left(\sqrt[4]{x^{-3}}+a\sqrt[4]{x^5}\right)^4$ is **[17th May 2023 Shift 1]**

1. 6 2. -1
3. 5 4. 4

76. The coefficient of x^2 in the expansion of $(1-3x)^{\frac{1}{3}}(1+2x)^{\frac{-1}{2}}$ is

[17th May 2023 Shift 2]

1. $-\dfrac{3}{2}$ 2. $\dfrac{3}{2}$
3. $\dfrac{1}{2}$ 4. $-\dfrac{1}{2}$

77. $(3+\sqrt{8})^5+(3-\sqrt{8})^5$

[17th May 2023 Shift 2]

1. 6926 2. 6826
3. 6726 4. 6626

78 If C_r stands for ${}^nC_r$ then

$$\frac{C_0}{2}+\frac{C_1}{2.2^2}+\frac{C_2}{3.2^3}+\dots+\frac{C_n}{(n+1)2^{n+1}}=$$

[17th May 2023 Shift 2]

1. $\dfrac{3^n}{2^{n+1}(n+1)}$ 2. $\dfrac{3^{n+1}-2^{n+1}}{2^{n+1}(n+1)}$
3. $\dfrac{3^n}{2^n(n+1)}$ 4. $\dfrac{3^{n+1}}{2^n(n+1)}$

79 If $\left(3+\sqrt{2}\right)^6-\left(3-\sqrt{2}\right)^6=a+b\sqrt{2}$, then a+b=

[18th May2023 shift -1]

1. 5220 2. 5230
3. 5240 4. 5250

80 If

$$A=\left\{\begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix} : a_i, b_i, c_i,\ i=(1,2,3) \text{ are the binomial coeffcients in the expansion of } (1+x)^{11}\right\}$$

Then the number of elements in set A is

[18th May2023 shift -1]

1. 9^9 2. 6^9
3. 11^9 4. 12^9

81 The coefficient of x^3 in the expansion of $(1-x)^{\frac{3}{2}},(|x|<1)$ is **[18th May 2023 Shift 2]**

1. $-\frac{3}{16}$
2. $\frac{1}{16}$
3. $\frac{1}{8}$
4. $\frac{3}{16}$

82 If the sum of the coefficients of even powers of x in the expansion of $(1-x+x^2)^{2n}$ is 3281, then n= **[18th May 2023 Shift 2]**

1. 4
2. 5
3. 6
4. 3

83 When x is so small that its square and its higher powers may be neglected, then value of $\frac{\left(1+\frac{3}{4}x\right)^{-4}\sqrt{(3+x)}}{\sqrt{(3-x)^3}}$ is approximately equal to

[19th May 2023 Shift 1]

1. $\frac{1}{3}-\frac{7x}{9}$
2. $\frac{1}{3}+\frac{7x}{9}$
3. $\frac{1}{3}+\frac{11x}{18}$
4. $\frac{1}{3}-\frac{11x}{18}$

84. The coefficient of x^2 in the expansion of $(1-3x)^{\frac{-1}{4}}$ is **[19th May 2023 Shift 1]**

1. $\frac{45}{64}$
2. $\frac{45}{8}$
3. $\frac{45}{16}$
4. $\frac{45}{32}$

85 The greatest term in the expansion of $(1+x)^{15}$, $when\ x=\frac{1}{2}$ is

[19th May 2023 Shift 1]

1. $\frac{1}{32}\,{}^{15}C_5$
2. $\frac{1}{64}\,{}^{15}C_6$
3. $\frac{1}{32}\,{}^{15}C_6$
4. $\frac{1}{64}\,{}^{15}C_5$

86 If the term independent of x in the expansion of $\left(\sqrt{x}-\frac{k}{x^2}\right)^{10}$ is 405, then k =

[12TH MAY 2023 SHIFT-1]

1. ±1
2. 0
3. ±3
4. ±5

87 The number of rational terms in the binomial expansion of $\left(\sqrt[4]{5}+\sqrt[5]{4}\right)^{100}$ is

[12TH MAY 2023 SHIFT-1]

1. 10
2. 20
3. 6
4. 5

88 The coefficient of x^{50} in the expansion of $(1+x)^{101}(1-x+x^2)^{100}$ is

[12TH MAY 2023 SHIFT-1]

1. 0
2. -1
3. 50
4. 100

86 The number of integral terms in the expansion of $\left(\sqrt{3}+\sqrt[8]{5}\right)^{256}$ is

[12TH MAY 2023 SHIFT -2]

1. 32
2. 33
3. 34
4. 35

90 The expansion of $(1+x+x^2)^{\frac{-3}{2}}$ in powers of x is valid if **[12TH MAY 2023 SHIFT -2]**

1. $|x|<1$
2. $|x|<\frac{1}{2}$
3. $\left|x+\frac{1}{2}\right|<\frac{\sqrt{5}}{2}$
4. $-\frac{1}{2}-\frac{\sqrt{5}}{2}<x<1$

91 If $(1+x)^n=C_0+C_1x+C_2x^2+...+C_nx^n$ for $n\in N$, then

$C_0+\frac{C_1}{2}+\frac{C_2}{3}+...+\frac{C_n}{n+1}=$

[12TH MAY 2023 SHIFT -2]

1. $\frac{2^n-1}{n+1}$
2. $\frac{2^n-1}{n}$
3. $\frac{2^{n+1}-1}{n+1}$
4. $\frac{2^{n+1}-1}{n}$

92 In the expansion of $(x-2y+3z)^5$, if the total number of terms is p and the coefficient of x^2yz^2 is q , then $\frac{q}{p}=$

[13TH MAY 2023 SHIFT-1]

1. 60 2. $-\frac{180}{7}$
3. 72 4. $-\frac{1080}{7}$

93 Let $c_0, c_1, c_2, ..., c_n$ be the binomial coefficients in the expansion of $(1+x)^n$ .if $S_{n+1} = 5.c_0 + 8.c_1 + 11.c_2 + ...(n+1)$ terms ,then S_{11}=**[13TH MAY 2023 SHIFT-1]**

1. 18944 2. 17920
3. 20480 4. 40960

94 *if* $|x|$ is so small that x^3 and higher powers of x can be neglected , then an approximate value of $\frac{1}{\sqrt{4-x}(2+x)^3}$ is

[13TH MAY 2023 SHIFT-1]

1. $\frac{1}{16}\left(1+\frac{13x}{8}+\frac{219}{128}x^2\right)$ 2. $\frac{1}{8}\left(1+\frac{11x}{8}-\frac{165}{128}x^2\right)$
3. $\frac{1}{32}\left(1-\frac{11x}{8}+\frac{219}{128}x^2\right)$ 4. $\frac{1}{16}\left(1-\frac{11x}{8}+\frac{171}{128}x^2\right)$

95 If (-c,c)is the set of all values of x for which the expansion of $(7-5x)^{-2/3}$ is valid, then 5c+7= **[EAPCET 14-05-23 SHIFT-1]**

1. 0 2. 12
3. 41 4. 14

96. If n is a positive integer and f (n) is the coefficient of x^n in the expansion of $(1+x)(1-x)^n$, then f(2023)= **[EAPCET 14-05-23 SHIFT-1]**

1. -2021 2. 2022
3. 2023 4. -2023

97 If y= $\frac{3}{4}+\frac{3.5}{4.8}+\frac{3.5.7}{4.8.12}+...to\infty$, then **[EAPCET 14-05-23 SHIFT-1]**

1. $y^2-2y+5=0$ 2. $y^2+2y-7=0$
3. $y^2-3y+4=0$ 4. $y^2+4y-6=0$

98 The numerically greatest term in the binomial expansion of $(2x-3y)^5$ when $x=\frac{3}{2}$ *and* $y=\frac{2}{3}$ is

[EAPCET 13-05-23 SHIFT-2]

1. 360 2. 1080
3. 720 4. 2160

99 When 3^{2023} is divided by 16, the remainder obtained is **[EAPCET 13-05-23 SHIFT-2]**

1. 15 2. 11
3. 9 4. 7

KEY

1)	2	2)	2	3)	4	4)	2	5)	3
6)	2	7)	4	8)	4	9)	4	10)	3
11)	1	12)	2	13)	3	14)	3	15)	1
16)	3	17)	4	18)	3	19)	2	20)	1
21)	4	22)	3	23)	4	24)	3	25)	3
26)	2	27)	2	28)	2	29)	4	30)	2
31)	4	32)	1	33)	2	34)	2	35)	2
36)	2	37)	3	38)	4	39)	2	40)	4
41)	2	42)	2	43)	4	44)	1	45)	1
46)	4	47)	3	48)	2	49)	2	50)	1
51)	2	52)	1	53)	3	54)	4	55)	3
56)	2	57)	2	58)	1	59)	2	60)	2
61)	2	62)	2	63)	3	64)	2	65)	2
66)	4	67)	4	68)	2	69)	3	70)	3
71)	3	72)	1	73)	1	74)	4	75)	1
76)	2	77)	3	78)	2	79)	1	80)	2
81)	2	82)	1	83)	1	84)	4	85)	1
86)	3	87)	3	88)	1	89)	2	90)	3
91)	3	92)	2	93)	3	94)	4	95)	4
96)	2	97)	2	98)	2	99)	2	100)	

SOLUTIONS

1. $\dfrac{(1+x)^{1/2}+(1-x)^{3/2}}{(1+x)^{1/2}\left(\sqrt{1+x}+1\right)}$

$$\Rightarrow \frac{\left((1+x)^{1/2}+(1-x)^{3/2}\right)\left(\sqrt{1+x}-1\right)}{x(1+x)^{1/2}}$$

$$=\frac{\left(1+\frac{x}{2}+1-\frac{3}{2}x\right)(1+x)^{-1/2}\left((1+x)^{1/2}-1\right)}{x}$$

$$=\frac{(2-x)\left(1-\frac{x}{2}\right)\left(\frac{x}{2}-\frac{x^2}{8}\right)}{x}$$

$$=\frac{(2-2x)\left(\frac{1}{2}x-\frac{1}{8}x^2\right)}{x}$$

$$\Rightarrow (2-2x)\left(\frac{1}{2}-\frac{1}{8}x\right)=1-\frac{5x}{4}$$

2. Let

$$I+F=\left(2+\sqrt{3}\right)^5$$

$$0<F<1 \quad .....(1)$$

$$f=\left(2-\sqrt{3}\right)^5$$

$$0<f<1 \quad ....(2)$$

From (1) and(2)

$$\Rightarrow F+f=1$$

$$I+F+f=2\left[{}^5C_0 2^5+{}^5C_2 2^3\ 3+{}^5C_4 2\ 3^2\right]$$

$$I+1=724$$

$$I=\left[\left(2+\sqrt{3}\right)^5\right]=723=3^1\times 241^1$$

Divisors$=(1+1)(1+1)=4$

3. $(1+x)^{1000}+x(1+x)^{999}+x^2(1+x)^{998}+........+x^{1000}$

$$=(1+x)^{1000}\left[1+\frac{x}{1+x}+\left(\frac{x}{1+x}\right)^2+.......+\frac{x^{1000}}{(1+x)^{1000}}\right]$$

$$=(1+x)^{1000}\left[\frac{1\left(1-\left(\frac{x}{1+x}\right)^{1001}\right)}{1-\frac{x}{1+x}}\right]$$

$$=(1+x)^{1000}\left[\frac{(1+x)^{1001}-x^{1001}}{(1+x)^{1001}}\times\frac{1+x}{1}\right]$$

$$=(1+x)^{1000}\left[\frac{(1+x)^{1001}-x^{1001}}{(1+x)^{1000}}\right]$$

$$=(1+x)^{1001}-x^{1001}$$

Coeff of $x^{50}={}^{1001}c_{50}$

4. $1\times 1!+2\times 2!+3\times 3!+.....+n\times n!$

$$=(2-1)1!+(3-1)2!+(4-1)3!+...+(n+1-1)n!$$

$$=2!-1!+3!-2!+4!-3!+....+(n+1)!-n!$$

$$\Rightarrow (n+1)!-1=11!-1$$

$$\Rightarrow n+1=11$$

$$\Rightarrow n=10$$

$$\max imum\ \ value={}^nC_{\frac{n}{2}}={}^{10}c_5=252$$

5. Given expansion

$$\left(x^2-\frac{1}{2x}\right)^{20}$$

$$T_m=T_{\frac{n}{2}+1}=T_{10+1}=T_{11}$$

$$\therefore m=11$$

$$T_{m+3}=T_{14}=T_{13+1}={}^{20}c_{13}\left(x^2\right)^7\left(\frac{-1}{2x}\right)^7$$

$$coeff.\ of\ \ T_{m+3}=-{}^{20}c_{13}2^{-13}$$

SOLUTIONS

1. $\dfrac{(1+x)^{1/2}+(1-x)^{3/2}}{(1+x)^{1/2}\left(\sqrt{1+x}+1\right)}$

$$\Rightarrow \frac{\left((1+x)^{1/2}+(1-x)^{3/2}\right)\left(\sqrt{1+x}-1\right)}{x(1+x)^{1/2}}$$

$$=\frac{\left(1+\frac{x}{2}+1-\frac{3}{2}x\right)(1+x)^{-1/2}\left((1+x)^{1/2}-1\right)}{x}$$

$$=\frac{(2-x)\left(1-\frac{x}{2}\right)\left(\frac{x}{2}-\frac{x^2}{8}\right)}{x}$$

$$=\frac{(2-2x)\left(\frac{1}{2}x-\frac{1}{8}x^2\right)}{x}$$

$$\Rightarrow (2-2x)\left(\frac{1}{2}-\frac{1}{8}x\right)=1-\frac{5x}{4}$$

2. Let

$$I+F=\left(2+\sqrt{3}\right)^5$$

$0<F<1$ .....(1)

$$f=\left(2-\sqrt{3}\right)^5$$

$0<f<1$ ....(2)

From (1) and(2)

$\Rightarrow F+f=1$

$I+F+f=2\left[{}^5C_0 2^5+{}^5C_2 2^3\ 3+{}^5C_4 2\ 3^2\right]$

$I+1=724$

$$I=\left[\left(2+\sqrt{3}\right)^5\right]=723=3^1\times 241^1$$

Divisors$=(1+1)(1+1)=4$

3. $(1+x)^{1000}+x(1+x)^{999}+x^2(1+x)^{998}+........+x^{1000}$

$$=(1+x)^{1000}\left[1+\frac{x}{1+x}+\left(\frac{x}{1+x}\right)^2+.......+\frac{x^{1000}}{(1+x)^{1000}}\right]$$

$$=(1+x)^{1000}\left[\frac{1\left(1-\left(\frac{x}{1+x}\right)^{1001}\right)}{1-\frac{x}{1+x}}\right]$$

$$=(1+x)^{1000}\left[\frac{(1+x)^{1001}-x^{1001}}{(1+x)^{1001}}\times\frac{1+x}{1}\right]$$

$$=(1+x)^{1000}\left[\frac{(1+x)^{1001}-x^{1001}}{(1+x)^{1000}}\right]$$

$$=(1+x)^{1001}-x^{1001}$$

Coeff of $x^{50}={}^{1001}c_{50}$

4. $1\times 1!+2\times 2!+3\times 3!+.....+n\times n!$

$=(2-1)1!+(3-1)2!+(4-1)3!+...$
$+(n+1-1)n!$

$=2!-1!+3!-2!+4!-3!+....$
$+(n+1)!-n!$

$\Rightarrow (n+1)!-1=11!-1$

$\Rightarrow n+1=11$

$\Rightarrow n=10$

$\max imum\ value={}^nC_{\frac{n}{2}}={}^{10}c_5=252$

5. Given expansion

$$\left(x^2-\frac{1}{2x}\right)^{20}$$

$T_m=T_{\frac{n}{2}+1}=T_{10+1}=T_{11}$

$\therefore m=11$

$$T_{m+3}=T_{14}=T_{13+1}={}^{20}c_{13}\left(x^2\right)^7\left(\frac{-1}{2x}\right)^7$$

$coeff.\ of\ T_{m+3}=-{}^{20}c_{13}2^{-13}$

6. Given

$$x=\frac{1}{5}+\frac{1\times3}{5\times10}+\frac{1\times3\times5}{5\times10\times15}+......$$

$$1+x=1+\frac{1}{5}+\frac{1\times3}{5\times10}+\frac{1\times3\times5}{5\times10\times15}+......$$

$$1+x=1+\frac{1}{2}.\frac{2}{5}+....$$

$$1+x=\left(1-\frac{2}{5}\right)^{\frac{-1}{2}}=\left(\frac{3}{5}\right)^{\frac{-1}{2}}$$

$$1+x=\sqrt{\frac{5}{3}}$$

$$\Rightarrow(1+x)^2=\frac{5}{3}$$

$$\therefore 3x^2+6x=2$$

7. Conceptual

8. Conceptual

9. $\left((1+x)+x^2\right)^8$

$$=8_{C_0}(1+x)^8+8_{C_1}(1+x)^7x^2+8_{C_2}(1+x)^6(x^2)^2+.....$$

coeff of $x^5=8_{C_0}.8_{C_5}+8_{C_1}.7_{C_3}+8_{C_2}.6_{C_1}$

$=504$

10. $2n+1=2029$

$\Rightarrow n=1014$

11. Given expansion $\left(3+\frac{x}{2}\right)^n$

coeff of "x^9" = *coeff of* "x^{10}"

$$n_{C_9}(3)^{n-9}\left(\frac{1}{2}\right)^9=n_{C_{10}}(3)^{n-10}\left(\frac{1}{2}\right)^{10}$$

$$\frac{n_{C_{10}}}{n_{C_9}}=6$$

$$\Rightarrow\frac{n-9}{10}=6$$

$n=69$

12. $4\left(1+\frac{x^2}{64}\right)^{\frac{1}{3}}-3\left(1+\frac{x^2}{27}\right)^{\frac{1}{3}}$

$$=4\left(1+\frac{x^2}{64.3}\right)-3\left(1+\frac{x^2}{27.3}\right)$$

($\because$ by neglecting x^4 & higher powers of 'x')

$$=1+\frac{x^2}{48}-\frac{x^2}{27}$$

$$=1-\frac{7x^2}{432}$$

13. $(1+x)^{101}(1-x+x^2)^{100}$

$$=(1+x)\left[(1+x)(1-x+x^2)\right]^{100}$$

$$=(1+x)\left(1+x^3\right)^{100}$$

The coefficients of $x^{50}=0$

($\because$ term not exist)

14. $\frac{{}^{n+1}C_{r+1}}{{}^{n+1}C_r}=\frac{(n+1)-r}{r+1}$

$$=\frac{n-r+1}{r+1}$$

$m=r+1$

15. w.k.t $(1+x)^{-n}=1-nx+\frac{n(n+1)}{2!}x^2-\frac{n(n+1)(n+2)}{3!}x^3$ +......

given $(x+y)^{-5}=x^{-5}\left(1+\frac{y}{x}\right)^{-5}$

now coeff of $\frac{y^3}{x^8}=-\frac{n(n+1)(n+2)}{3!}$,put n=5

=-35

16. By verification method

$$\sum_{r=1}^{n}r^2.c_r=n(n+1)2^{n-2}$$

Take n=2

$$L.H.S=\sum_{r=1}^{2}r^2\,{}^2c_r={}^2c_1+4\,{}^2c_2=6$$

$$R.H.S=n(n+1)2^{n-2}=2(3)=6$$

17. Conceptual

18. $\alpha - 2 + 1 = 1 - \alpha$

$2\alpha = 2$

$\alpha = 1$

19 $T_{5+1} = 9_{C_5}\left(\frac{2p}{3}\right)^4\left(\frac{3q}{2}\right)^5$

$= \frac{9\times8\times7\times6}{1\times2\times3\times4}\times\frac{3}{2}p^4q^5$

$= 189p^4q^5$

20. $(1+3x)^n\left(1+\frac{1}{3x}\right)^n$

$= \frac{(1+3x)^{2n}}{(3x)^n} = \frac{(1+3x)^{2n}}{3^n x^n}$

$cons\tan t\ \ term = {}^{2n}c_n$

21. $R = (5\sqrt{5}+11)^{2n+1},\quad f = (5\sqrt{5}-11)^{2n+1}$

$Rf = 4^{2n+1}$

22.

$T_{r+1} = {}^{10}C_r\left(x^{1/2}\right)^{10-r}\frac{(-k)^r}{x^{2r}}$

$r = 2,\quad {}^{10}C_2k^2 = 405$

$k^2 = \frac{405}{45} = 9,\quad k = \pm3$

23. 5^{th} term from end $= T_9 = T_{8+1}$

$= {}^{12}C_8\left(\frac{x^3}{2}\right)^4\left(\frac{-2}{x^2}\right)^8$

Index of power of x is -4

24. $n(S) = {}^{(x+5)}C_2$

Given that $\frac{{}^5C_2}{{}^{(x+5)}C_2} = \frac{5}{14}$

$x^2 + 9x - 36 = 0 \Rightarrow x = 3$

25. $\left(1+\frac{1}{100000}\right)^{100000} = 1+1+\frac{100000\times99999}{2}\left(\frac{1}{100000}\right)^2+.........$

=2.---

$\left[\left(1+\frac{1}{100000}\right)\right]^{100000} = [2.-\!-\!-] = 2$

26. Given expansion, $(1+x)^n$

$T_{r+1} = {}^nC_rx^r$

The coefficients of terms 2^{nd}, 3^{rd} and 4^{th}

${}^nC_1, {}^nC_2, {}^nC_3 \rightarrow A.P$

$2\,{}^nC_2 = {}^nC_1 + {}^nC_3$

$2 = \frac{{}^nC_1}{{}^nC_2} + \frac{{}^nC_3}{{}^nC_2} \Rightarrow n = 7$

27. Given expansion $(a-b)^n$

$T_5 = T_{4+1} = {}^nC_4a^{n-4}(-b)^4$

$T_6 = T_{5+1} = {}^nC_5a^{n-5}(-b)^5$

Sum=0

$\Rightarrow {}^nC_4a^{n-4}b^4 - {}^nC_5a^{n-5}b^5 = 0$

$\Rightarrow {}^nC_4a^{n-4}b^4 = {}^nC_5a^{n-5}b^5$

$\Rightarrow \frac{{}^nC_4}{{}^nC_5}\frac{1}{a^4} = \frac{b}{a^5}$

$\Rightarrow \frac{5}{n-4} = \frac{b}{a} \Rightarrow \frac{a}{b} = \frac{n-4}{5}$

28

$y+1 = 1+\frac{3}{1!}\left(\frac{1}{4}\right)+\frac{3.5}{2!}\left(\frac{1}{4}\right)^2+....+\infty$

$y+1 = \left(1-\frac{1}{2}\right)^{-3/2} = \left(\frac{1}{2}\right)^{-3/2}$

$y+1 = (2)^{3/2}$

$y^2 + 2y - 7 = 0$

29. $(1+x)^2(1-2x)^{-3}$

$= (1+2x+x^2)(1-2x)^{-3}$

Coefficient of $x^{13} = 105\times2^{13}+182\times2^{12}+78\times2^{11}$

$= (840+728+156)2^{10}$

$= 1724\times2^{10}$

30. $\left(2x^3+8x^2-2x-2\right)(1-x)^{-1}(1+x)^{-1}(1-2x)^{-1}$

$l = 2(2^7+2^5+2^3+2)+8(2^8+2^6+2^4+2^2+1)$

$-2(2^9+2^7+2^5+2^3+2)-2(2^{10}+2^8+....+1)$

$= -2^{10} - 2$

$m =$ constant term = -2

$lm = \left(-2^{10}-2\right)(-2) = 4\left(2^9+1\right)$

31. Conceptual

No. of terms $=\frac{n}{2}+1$

No. of rational terms $=\left[\frac{n}{lcm(l,m)}\right]+1$, l, m is

atleast one factor of n

$=\left[\frac{n}{lcm(l,m)}\right]$, l, m is not a factor of n

32. $(1-x)^3(2+x)^6$

$=(1-x^3+3x^2-3x)$

$\left(x^6+12x^5+4.{}^6C_2x^4+8.{}^6C_3x^3+16.{}^6C_4x^2+32.{}^6C_5x+64.{}^6C_6\right)$

Here

$a=64.{}^6C_6$

$b=32.{}^6C_5$

$c=-96.{}^6C_5+16.{}^6C_4$

$a+b+c=-80$

33 $\frac{1}{\sqrt{x}}(1+x)^{\frac{1}{2}}$

$$=\frac{1}{\sqrt{x}}\left(1+\frac{x}{2}-\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}x^2+\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}x^3+\ldots\infty\right)$$

$$=\frac{1}{\sqrt{x}}+\frac{1}{2}\sqrt{x}-\frac{1}{2!}\frac{x\sqrt{x}}{2^2}+\frac{1.3x^2\sqrt{x}}{3!2^3}-\ldots\infty$$

34. $\frac{(n+1)1\times1}{1\times1+1}=9$

$$\frac{(n+1)\left(\frac{3}{2}\right)}{\left(\frac{3}{2}\right)+1}=9$$

n=14

$\therefore$ *given* $(5x-6y)^{14}$

Middle term $=\left(\frac{14}{2}+1\right)=8$

$T_8=T_{7+1}$

$={}^{14}C_7(5x)^7(-6y)^7$

$={}^{14}C_7 5^7(-6)^7\frac{2^7}{5^7}\left(\frac{1}{2^7}\right)$

Absolute value $= {}^{14}C_7 6^7$

35 Here $nx=\frac{3}{16}$ ....(1)

$$\frac{nx(nx+x)}{3!}=\frac{1.4}{1.2}\left(\frac{3}{16}\right)^2$$

$x=\frac{9}{16}$

$(1)\Rightarrow n=\frac{1}{3}$

$\therefore$ *given* $=\left(1+\frac{9}{16}\right)^{\frac{-1}{3}}=\left(\frac{4}{5}\right)^{\frac{2}{3}}$

36 Given expansion $\left[ax^2+\frac{b}{x^3}\right]^{15}$

l=coeff of $x^{10}\Rightarrow{}^{15}C_4.a^{11}.b^4$

m=constant term= ${}^{15}C_6.a^9.b^6$

n= coeff of $x^{-10}\Rightarrow{}^{15}C_8a^7b^8$

now,

$$\frac{l}{m}+\frac{m}{n}=\frac{26}{11}\Rightarrow\frac{a^2}{b^2}\left[\frac{{}^{15}c_4}{{}^{15}c_6}+\frac{{}^{15}c_6}{{}^{15}c_8}\right]=\frac{26}{11}$$

on simplifying weget $a^2:b^2=9:4$

37. Given $(1+z)^n = 1 + nc_1 z + nc_2 z^2 + \ldots nc_n z^n$

$Let\ z = \cos x + i\sin x$

$now, 2^n \cos^n \frac{x}{2}\left(\cos\frac{nx}{2} + i\sin\frac{nx}{2}\right)$

$= \left(1 + {}^nc_1\cos x + {}^nc_2\cos 2x + \ldots + {}^nc_n\cos nx\right) +$

$i\left({}^nc_1\sin x + {}^nc_2\sin 2x + \ldots + {}^nc_n\sin nx\right)..(1)$

$now, \sum_{r=0}^{100} {}^{100}c_r(\sin rx) = \left(2\cos\frac{x}{2}\right)^{100}.\sin kx$

comparing imaginary part of eqn. (1) on B.S

$we\,get, \left(2\cos\frac{x}{2}\right)^n.\sin\frac{nx}{2}\ on\ R.H.S$

$put\ n = 100, then \sin kx = \sin 50x$

$\Rightarrow k = 50$

---

38. Sum of the coefficients

$= 128 = f(1)$

$\Rightarrow (1+3-2)^n = 128 \Rightarrow 2^n = 2^7 \Rightarrow n = 7$

$$\sum_{r=1}^{14} r.\frac{{}^{14}C_r}{{}^{14}C_{r-1}} = \sum_{r=1}^{14} r.\frac{14-r+1}{r} = \sum_{r=1}^{14}(15-r)$$

$= 14 + 13 + 12 + .. + 3 + 2.1 = \frac{14.15}{2} = 105$

---

39 $n = 10, \sum_{r=1}^{10} {}^nC_r.r(r-4)$

$$= \sum_{r=1}^{10} r^2.C_r - 4.\sum_{r=1}^{10} r.C_r$$

$$= \sum_{r=1}^{10} r^2.C_r = n(n+1).2^{n-2} = 10.11.2^8$$

$$= \sum_{r=1}^{10} r.c_r = n.2^{n-1} = 10.2^9$$

$= 10.11.2^6 - 4.10.2^9 = 7680$

---

40. $$\frac{\left(1+\frac{2x}{3}\right)^{-4}(4+5x)^{1/2}}{(9+x)^{\frac{3}{2}}} = \frac{\left(1+\frac{2x}{3}\right)^{-4} 4^{\frac{1}{2}}\left(1+\frac{5x}{4}\right)^{\frac{1}{2}}}{9^{\frac{3}{2}}\left(1+\frac{x}{9}\right)^{\frac{3}{2}}}$$

$$= \frac{2}{27}\left(1-\frac{8x}{3}\right)\left(1+\frac{5x}{3}\right)\left(1+\frac{x}{6}\right)$$

$\left[\because x^2\ and\ higher\ power\ of\ x\ neglected\right]$

$$= \frac{2}{27}\left[1 - \frac{8x}{3} + \frac{5x}{8} - \frac{x}{6}\right]$$

$\left[\because x^2\ and\ higher\ power\ of\ x\ neglected\right]$

$$= \frac{2}{27}\left[1 - \frac{53x}{24}\right] = \frac{1}{4}$$

$\left(\because x = \frac{6}{371}\right)$

---

41 $$\sqrt{3+x} + \sqrt{5+x} = x^{\frac{1}{2}}\left(1+\frac{3}{x}\right)^{\frac{1}{2}} + 5^{\frac{1}{2}}\left(1+\frac{x}{5}\right)^{\frac{1}{2}}$$

$$= \begin{cases} x^{\frac{1}{2}}\left[1 + \sum_{r=1}^{\infty} \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\ldots\left(\frac{1}{2}-r+1\right)}{r!}\left(\frac{3}{x}\right)^r\right] \\ +5^{\frac{1}{2}}\left[1 + \sum_{s=1}^{\infty} \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\ldots\left(\frac{1}{2}-s+1\right)}{s!}\left(\frac{3}{x}\right)^s\right] \end{cases}$$

$for\ coefficients\ of\ x^{-3/2}, \frac{1}{2} - r = -\frac{3}{2} \Rightarrow r = 2$

$for\ coefficients\ of\ x^3, s = 3$

$\therefore coefficients\ of\ x^{-3/2} + coefficients\ of\ x^3$

$$= \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2}3^2 + 5^{\frac{1}{2}}\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{5}\frac{1}{5^3}$$

$$= \frac{5^{-5/2} - 18}{16}$$

42. $\left(2+\sqrt{3}\right)^{49}+\left(\sqrt{3}-2\right)^{49}=a+b\sqrt{3}$

$$2^{49}\left(1+\frac{\sqrt{3}}{2}\right)^{49}+(-2)^{49}\left(1-\frac{\sqrt{3}}{2}\right)^{49}$$

$$=2^{49}\left[\left(1+\frac{\sqrt{3}}{2}\right)^{49}-\left(1-\frac{\sqrt{3}}{2}\right)^{49}\right]$$

$$=2^{49}\left[2\left({}^{49}C_1\left(\frac{\sqrt{3}}{2}\right)+{}^{49}C_3\left(\frac{\sqrt{3}}{2}\right)^3+....+{}^{49}C_{49}\left(\frac{\sqrt{3}}{2}\right)^{49}\right)\right]$$

$$=a+b\sqrt{3}$$

$$a=0, b\neq 0$$

43 $\left(1+\frac{3x}{2}\right)^{-5}=1-{}^5C_1\left(\frac{3x}{2}\right)+{}^6C_2\left(\frac{3x}{2}\right)^2+....+{}^{14}C_{10}\left(\frac{3x}{2}\right)^{10}+...$

Coefficient of $x^{10}={}^{14}C_{10}\left(\frac{3}{2}\right)^{10}$

Coefficient of x^{10} in $(1+ax)^n={}^nC_{10}a^{10}$

$${}^{14}C_{10}\left(\frac{3}{2}\right)^{10}={}^nC_{10}a^{10}$$

$n=14, a=3/2$

$na=21$

44. First negative term in $\left(1+\frac{3x}{5}\right)^{22/3}$ is

$P=T_{[22/3]+3}=T_{10}$

45. $(2x+3y)^{11}=(2x)^{11}\left(1+\frac{3y}{2x}\right)^{11}$

$n=11, X=\frac{3y}{2x}=\frac{3}{2}\times\frac{1}{3}\times\frac{2}{1}=1$

$\frac{(n+1)1\times 1}{1\times 1+1}=\frac{12\times 1}{1+1}$

$r=6$

$T_{r+1}=N.G.T$

$T_{6+1}={}^{11}C_6(2x)^5(3y)^6$

$at\ x=\frac{1}{2}, y=\frac{1}{3}$

$={}^{11}C_6=462$

46 $\left(1-x-x^2+x^3\right)^6$

$$=\left[(1-x)-x^2(1-x)\right]^6$$

$$=(1-x)^6\left(1-x^2\right)^6$$

$$\left\{\sum_{r=0}^{6}{}^6C_r(-x)^r\right\}.\left\{\sum_{s=0}^{6}{}^6C_s\left(-x^2\right)^s\right\}$$

Coefficient of x^4

$={}^6C_0\times{}^6C_2-{}^6C_2\times{}^6C_1+{}^6C_4\times{}^6C_0$

$=15-90+15=-60$

47. $\left(1+\frac{x}{2}\right)^{12}$ $put\ x=1$

Sum of the coefficient $=\left(1+\frac{1}{2}\right)^{12}=\left(\frac{3}{2}\right)^{12}$

48. Given expansion $(1+x)(1-x)^n$

Coefficient of x^n in the above expansion is,

$$f(n)=(-1)^{n-1}\cdot{}^nC_{n-1}+(-1)^n\,{}^nC_n$$

Now,

$$f(2021)=(-1)^{2020}\cdot{}^{2021}C_{2020}+(-1)^{2021}\,{}^{2021}C_{2021}$$

$$={}^{2021}C_1+(-1)=2020$$

49. Given binomial expansion,

$$\left(x^{1/3}+\frac{1}{2x^{1/3}}\right)^{21}, x>0\left(\because r=\frac{np-m}{p+q}\right)$$

P=Coefficient of x^{-3} is, ${}^{21}C_{15}\frac{1}{2^{15}}$

Q=Coefficient of x^{-5} is, ${}^{21}C_{18}\frac{1}{2^{18}}$

Now, $\frac{5p}{4q}=\frac{5\times{}^{21}C_{15}\frac{1}{2^{15}}}{4\times{}^{21}C_{18}\frac{1}{2^{18}}}=408$

50. Given

$$\frac{(1-x)^{-1/p}}{(1-x)^{n}}=(1-x)^{-1/p}(1-x)^{-n}$$

$$=\left[1+\frac{1}{1!}\left(\frac{x}{p}\right)+\frac{1(1+p)}{2!}\left(\frac{x}{p}\right)^2+\frac{1(1+p)(1+2p)}{3!}\left(\frac{x}{p}\right)^3+...\right]$$

$$\left[1+\frac{n}{1!}\left(\frac{x}{1}\right)+\frac{n(n+1)}{2!}\left(\frac{x}{1}\right)^2+\frac{n(n+1)(n+2)}{3!}\left(\frac{x}{1}\right)^3+...\right]$$

Now, coeff. of x^3 is given by,

$$\frac{n(1+p)}{2!\times p^2}+\frac{n(n+1)}{2!p}+\frac{(1+p)(1+2p)}{3!p^3}+\frac{n(n+1)(n+2)}{3!}$$

After simplifying, we get

$$\frac{(np+1)(np+p+1)(np+2p+1)}{p^3\times 3!}$$

51. Middle term $T_{\frac{2k}{2}+1}=T_{k+1}$

It is numerical greatest term

$$T_{k+1}>T_{k+2}$$

$$^{2k}C_k x^k>{}^{2k}C_{k+1}x^{k+1}$$

$$1>\frac{^{2k}C_{k+1}}{^{2k}C_k}.x$$

$$1>\frac{2k-k-1+1}{k+1}.x$$

Then $T_k<T_{k+1}>T_{k+2}$ $\quad \frac{k+1}{k}>x\Rightarrow x<\frac{k+1}{k}$

52. Sum of the coefficient of x^r $(r=0,1,2,3,..,15)$

In the expansion of $(3x-1)^{15}=f(x)=2^{15}$.

Sum of the binomial coefficient of the expansion $(1+x)^{15}=2^{15}$

Sum of the binomial coefficient of the expansion

$$(1+x)^{16}+(1-x)^{16}=2\left({}^{16}C_0+{}^{16}C_2x^2+{}^{16}C_4x^4+...+{}^{16}C_{16}x^{16}\right)$$

$$=(1+x)^{16}+(1-x)^{16}={}^{16}C_0+{}^{16}C_2+{}^{16}C_4+...+{}^{16}C_{16}=2^{16-1}=2^{15}$$

Similarly sum of the coefficient of the expansion $(1+x)^{16}-(1-x)^{16}=2^{16-1}=2^{15}$

53. $(1-4x)^{-4}=1+{}^4C_1(4x)+{}^{4+1}C_2(4x)^2+{}^{4+2}C_3(4x)^3+...+{}^{4+11}C_{12}(4x)^{12}+...$

13^{th} *term* $={}^{15}C_{12}4^{12}.x^{12}={}^{15}C_3 4^{12}.x^{12}$

54 $x^3\left(2\sqrt{3}x^2+\frac{1}{kx}\right)^{12}$

Coefficient of x^3 in $x^3\left(2\sqrt{3}x^2+\frac{1}{kx}\right)^{12}$

= coefficient of constants term in $\left(2\sqrt{3}x^2+\frac{1}{kx}\right)^{12}$

$$r=\frac{np}{p+q}=\frac{12}{3}=8$$

$$880={}^{12}C_8.\left(2\sqrt{3}x^2\right)^{12-8}.\left(\frac{1}{kx}\right)^8$$

$$k^8=9^2\Rightarrow k=\sqrt{3}$$

55. Middle term $=\left(\frac{n}{2}+1\right)^{th}$ term =5th term

$$T_5=T_{4+1}={}^8C_4\times\left(4x^3\right)^{8-4}\times\left(\frac{-15}{4x}\right)^4$$

$$=70\left(15x^2\right)^4$$

56. $\frac{|b|}{|a|}<\frac{2}{5}\Rightarrow\frac{-2}{5}<\frac{b}{a}<\frac{2}{5}$

$$T_{r+1}=(-1)^r.{}^{n+r-1}C_rx^r$$

T_4 in $(2a+5b)^{-4}=(2a)^{-4}\left(1+\frac{5b}{2a}\right)^{-4}$

$$T_4=(2a)^{-4}\frac{-4(4+1)(4+2)}{3!}\left(\frac{5b}{2a}\right)^3$$

$$=-{}^6C_3\frac{5^3}{2^7}.\frac{b^3}{a^7}$$

57. $\left(x-\frac{2}{\sqrt{x}}\right)^{21}$

General term $T_{r+1}={}^{21}C_r(x)^{21-r}\left(\frac{-2}{\sqrt{x}}\right)^r$

$$={}^{21}C_r(-2)^r x^{21-r-\frac{r}{2}}$$

$$21-r-\frac{r}{2}=0\Rightarrow r=14$$

Independent term ${}^{21}C_{14}(-2)^{14}$

58. $(7-5x)^{-2/3}=7^{-2/3}\left(1-\frac{5x}{7}\right)^{-2/3}$

$\left|\frac{5x}{7}\right|<1 \Rightarrow |x|<\frac{7}{5}$

$x\in\left(-\frac{7}{5},\frac{7}{5}\right)=(-a,a)$

$a=\frac{7}{5}\Rightarrow 5a+7=14$

59. $\left(1-\frac{3}{4}x\right)^{\frac{1}{2}}$

Above expansion is in the form of $(1-x)^{p/q}$

$$(1-x)^{p/q}=1-\frac{\frac{p}{q}(x)}{1!}+\frac{\frac{p}{q}\left(\frac{p}{q}-1\right)}{2!}x^2-\frac{\frac{p}{q}\left(\frac{p}{q}-1\right)\left(\frac{p}{q}-2\right)}{3!}x^3+....$$

Coefficient of x^3

$$=\frac{-\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}\left(\frac{3}{4}\right)^3=\frac{-27}{1024}$$

60.

Exp: $\left(ax+\frac{b}{x^2}\right)^{11}$

coeff of x^{-7}

$r=\frac{np-s}{p+q}=6$

$L=coeff\ of\ x^{-7}={}^{11}C_6a^5b^6$

$Exp:\left(bx^2+\frac{a}{n}\right)^{11}$

coeff of x^7

$r=5$

$M=coeff\ of\ x^7$

$={}^{11}c_5b^6a^5$

$L+M=2\left({}^{11}c_6a^5b^6\right)$

By verify: from opt '2'

$\frac{1}{a}\left[coefficient\ of\ x^6\ in\left(ax^2+\frac{b}{x}\right)^2\right]=L+M$

61.

$T_4=T_{3+1}=6c_3\left(\frac{x}{2}\right)^3\left(\frac{-2}{3}y\right)^3=-20$

$\Rightarrow\frac{x^3y^3}{3^3}=1$

$\Rightarrow(xy)^3=3^3$

$xy=1$

62. $r-\frac{np-s}{p+q}=\frac{5(2)-5}{5}=1$

$coeff\ of\ x^5=k=5c_1 2^4\left(\frac{-1}{3}\right)^1.$

$\Rightarrow k=-\frac{80}{3}$

$3\frac{k}{2}=-40$

63. No. of terms $=n+1=15$

$n=14$

$x=1\Rightarrow \exp:(a+1)^{14}$

$n-even$

$middle\ term=\frac{T_n}{2}+1=T_8$

$\Rightarrow T_7:T_9=14c_6a^8:14c_8a^6$

$\Rightarrow\frac{T_7}{T_9}=\frac{16}{1}\Rightarrow\frac{16}{1}=\frac{a^2}{1}\Rightarrow a=4$

64.

Let $S=\frac{1}{8}-\frac{7}{8.12}+......$

$\frac{3}{4}+S=1-\frac{1}{4}+\frac{1}{8}-\frac{7}{8.12}+......(1)$

$(1+x)^n=l+nx+\frac{n(n-1)x^2}{2!}+......(2)$

$nx=\frac{-1}{4}$

$\frac{n(n-1)x^2}{2}=\frac{1}{8}$

$\Rightarrow x=3/4$

$\Rightarrow n=-1/3$

From (1) & (2)

$\frac{3}{4}+s=\left(1+\frac{3}{4}\right)^{-1/3}=\left(\frac{4}{7}\right)^{1/3}$

$s=3\sqrt{\frac{4}{7}}-\frac{3}{4}$

65. Exp: $(2x-3y)^{11}, n=\frac{1}{3}, y=\frac{1}{2}$

$$\Rightarrow\left[2x\left(1-\frac{3y}{2x}\right)\right]^{11}$$

$$p=\frac{(n+1)|x|}{|x|+1}=\frac{12\left(\frac{9}{4}\right)}{\frac{9}{4}+1}=\frac{108}{13}$$

$$[p]=[8.3]=8$$

$$T_{[p]+1}=T_{8+1}=11c_8\left[2\left(\frac{1}{3}\right)\right]^3\left[-3\left(\frac{1}{2}\right)\right]^8$$

$$=11c_8\left(\frac{3}{2}\right)^5=11c_3\left(\frac{3}{2}\right)^5$$

66. $$n+\not{2}\left(\frac{n-2+1}{\not{2}}\right)+\not{3}\left(\frac{n-3+1}{\not{3}}\right)+........\not{n}\left(\frac{n-n+1}{\not{n}}\right)$$

$$n+n-1+n-2+......1=\frac{n(n+1)}{2}$$

67. Expand & Compare

68. Formula

69. The coefficient of the highest power of x in the expansion of $\left(x+\sqrt{x^2-1}\right)^8+\left(x-\sqrt{x^2-1}\right)^8$

$$=2\left[{}^8c_0x^8+{}^8c_2x^6\left(x^2-1\right)+{}^8c_4x^4\left(x^2-1\right)^2+{}^8c_6x^2\left(x^2-1\right)^3+{}^8c_8\left(x^2-1\right)^4\right]$$

$$=2\left[{}^8c_0+{}^8c_2+{}^8c_4+{}^8c_6+{}^8c_8\right]$$

$$=2\left(2^7\right)\Rightarrow 2^8\Rightarrow 256$$

70. $(1-2x)^{-2/3}$

P = 2, q = 3, X = 2x

$$T_{r+1}=\frac{p(p+q)(p+2q).....(p+(r-1)q)}{r!}\left(\frac{2x}{3}\right)^r$$

$$=\frac{2.5.8.......(3r-1)}{r!}\left(\frac{2}{3}\right)^r$$

71. $(2a+nd)2^{n-1}=(2.3+25)2^{5-1}$

a = 3 $\qquad = 31\times 2^4$

d = 5

n = 5

72. $(1-2x)^{\frac{1}{2}}(1+3x)^{\frac{-1}{3}}$

Coefficient of x^3

$$\left\{\frac{\left(\frac{-1}{3}\right)\left(\frac{-1}{3}-1\right)}{3!}3^3+\frac{1}{2}(-2)\frac{\left(\frac{-1}{3}\right)\left(\frac{-1}{3}-1\right)}{2!}.3^2+\frac{\left(\frac{-1}{2}\right)\left(\frac{-1}{3}-1\right)}{2!}(-2)^2\left(\frac{-1}{3}\right)(3)+\frac{\left(\frac{-1}{2}\right)\left(\frac{-1}{2}-1\right)}{3!}(-2)^2\right\}$$

$$=-\frac{1.4.7}{3^3.3!}3^3-\frac{1.4}{3^2.2!}3^2+\frac{1.1}{2^2.2!}2^2.(1)+\frac{1.1.3}{2^3.3!}(-2^3)$$

$$=\frac{28}{6}-2+\frac{1}{2}-\frac{1}{2}$$

$$=\frac{-40}{6}=\frac{-20}{3}$$

73. $(2-3x)^9$, when $x=1$

$$=2^9\left(1-\frac{3x}{2}\right)^9$$

$$N.G.T=\frac{(n+1)|x|}{|x|+1}=6$$

$$T_{6+1}=9_{c_6}(2)^{9-6}(-3x)^6$$

$$=9_{c_6}2^33^6$$

$$=2^5\times3^7\times5^0\times7^1$$

$$\alpha+\beta+\gamma+\delta=5|7+0+1=13$$

74. $\left(x+\sqrt{x^4-1}\right)^9+\left(x-\sqrt{x^4-1}\right)^9$

$$(x+a)^n+(x-a)^n=2\left({}^nc_0x^n+{}^nc_2x^{n-2}a^2+.....\right)$$

$$=2\{{}^9c_0x^9+9_{c_2}x^7\left(x^4-1\right)+9_{c_4}x^5\left(x^4-1\right)^2$$

$$+9_{c_6}x^3\left(x^4-1\right)^3+9_{c_8}x\left(x^4-1\right)^4\}$$

Degree =17

75. $\frac{x^{-3/4}+ax^{5/4}}{x-1}=-2$

$1+a=0$

$\boxed{a=-1}$

$T_{r+1}={}^4C_r\left(x^{-3/4}\right)^{4-r}\left(-x^{5/4}\right)^r$

$\Rightarrow \frac{-12+3r}{4}+\frac{5r}{4}=1$

$\Rightarrow -12+8r=4$

$\boxed{r=2}$

$\Rightarrow {}^4C_2(-1)^2=\frac{4\times 3}{2}=6$

76. $(1-3x)^{\frac{1}{3}}(1+2x)^{\frac{-1}{2}}$

$=\left[1-x+\frac{\frac{1}{3}\left(\frac{1}{3}-1\right)}{2}\times 9x^2....\right]$

$\left[1-x+\frac{\frac{1}{2}\left(\frac{1}{2}+1\right)}{2}(4x^2).......\right]$

$=\left(1-x-x^2.....\right)\left(1-x+\frac{3}{2}x^2....\right)$

coefficeint of $x^2=\frac{3}{2}+1-1$

$=\frac{3}{2}$

77. $\left(3+\sqrt{8}\right)^5+\left(3-\sqrt{8}\right)^5$

$=2\left[5c_0(3^5)+5c_2(3)^3(\sqrt{8})^2+5c_4(3)^1(\sqrt{8})^4\right]$

$=2[243+10\times 27\times 8+5\times 3\times 64]$

$=6726$

78.

$c_0x+\frac{c_1}{2}x^2+\frac{c_2}{3}x^3+.....+\frac{c_n}{n+1}x^{n+1}=\frac{(1+x)^{n+1}-1}{(n+1)}$

put $x=\frac{1}{2}$

$c_0\left(\frac{1}{2}\right)+\frac{c_1}{2}\left(\frac{1}{2}\right)^2+\frac{c_2}{3}\left(\frac{1}{2}\right)^3+.....+\frac{c_n}{n+1}\left(\frac{1}{2}\right)^{n+1}$

$=\frac{\left(1+\frac{1}{2}\right)^{n+1}-1}{n+1}$

$=\frac{3^{n+1}-2^{n+1}}{(n+1)\,2^{n+1}}$

79. $\left(3+\sqrt{2}\right)^6-\left(3-\sqrt{2}\right)^6$

$=3^6+6_{c_1}.3^5.\sqrt{2}+6_{c_2}.3^4.2+....+6_{c_6}.(2^3)-$

$\left(3^6-6_{c_1}.3^5.\sqrt{2}-6_{c_2}.3^4.2-....\right)$

$=2\left[6_{c_1}.3^5.\sqrt{2}+6_{c_3}.3^3.(2\sqrt{2})+6_{c_5}.3^1.(4\sqrt{2})\right]$

$=2\sqrt{2}\left[6\times 243+20\times 27\times 2+6.(3)4\right]$

$=5220\sqrt{2}=a+b\sqrt{2}$

$\therefore a=0, b=5220$

$\therefore a+b=5220$

80. Binomial coefficients in $(1+x)^{11}$ are

${}^{11}C_0, {}^{11}C_1, {}^{11}C_2, {}^{11}C_3, {}^{11}C_4, {}^{11}C_5$

(distinct)

Each matrix is formed by these 6 elements in 6^9 ways.

81. Coefficient of x^3 in the expansion of $(1-x)^{\frac{3}{2}}$ is :

$(1-x)^{3/2}=1+\frac{3/2}{1!}(-x)^1+\frac{\frac{3}{2}\left(\frac{3}{2}-1\right)}{2!}(-x)^2+$

$\frac{\frac{3}{2}\left(\frac{3}{2}-1\right)\left(\frac{3}{2}-2\right)}{3!}(-x)^3$

Coefficient of x^3 =

$\frac{\frac{3}{2}\left(\frac{1}{2}\right)\left(\frac{-1}{2}\right)}{3!}=\frac{3}{8(6)}=\frac{1}{16}$

82. conceptual

83.
$$3^{1/2}.3^{-3/2}(1-3x)\left(1+\frac{x}{3}\right)^{1/2}\left(1-\frac{x}{3}\right)^{-3/2}$$
$$\Rightarrow 3^{-1}(1-3x)\left(1+\frac{x}{6}\right)\left(1+\frac{x}{2}\right)$$
$$=\frac{1}{3}\left[1+\frac{x}{6}-3x-\frac{x^2}{2}\right]\left[1+\frac{x}{2}\right]$$
$$=\frac{1}{3}\left[1+\frac{x}{2}+\frac{x}{6}-3x\right]$$
$$=\frac{1}{3}-\frac{7x}{9}$$

84. $(1-3x)^{-1/4}$
$$=1+\frac{1}{4}(3x)+\frac{\frac{1}{4}\left(\frac{1}{4}+1\right)}{2!}(3x)^2+......$$
$$=1+\frac{3}{4}x+\frac{\frac{1}{4}\left(\frac{5}{4}\right)}{2!}9x^2+.......$$

Co–eff of x^2 is $\frac{1}{4}\times\frac{5}{4}\times\frac{9}{2}=\frac{45}{32}$

85. Conceptual

86. $p=\frac{1}{2}, q=2, n=10$
$$r=\frac{np}{p+q}=\frac{10\times\frac{1}{2}}{\frac{1}{2}+2}=\frac{5}{\frac{5}{2}}=2$$
$$T_{2+1}=405$$
$$10C_2\left(\sqrt{x}\right)^8\frac{(-k)^2}{x^4}=405$$
$$45.\cancel{x^4}.\frac{k^2}{\cancel{x^4}}=405$$
$$k^2=9$$
$$k=\pm3.$$

87. Given $\left(4\sqrt{5}+5\sqrt{4}\right)^{100}=\left(5^{\frac{1}{4}}+4^{\frac{1}{5}}\right)^{100}$

The no of rational terms $=\left[\frac{100}{L.C.M\,4,5}\right]+1$
$$=\left[\frac{100}{20}\right]+1$$
$$=5+1=6$$

88. *Given expansion is* $(1+x)^{101}\left(1-x+x^2\right)^{100}$
$$=(1+x)\ \left((1+x)^{100}\ \left(1-x+x^2\right)\right)^{100}$$
$$=(1+x)\left(1+x^3\right)^{100}$$
$$=(1+x)[100_{c_0}+100_{c_1}x^3+$$
$$100_{c_2}\left(x^3\right)^2+....+100_{c_{48}}\left(x^3\right)^{48}+........$$
Req coff $x^{50}=0$

89. $\left[\frac{256}{8}\right]+1$
$$=33$$
No of rataional *terms* $=33$

90. $\left(1+x+x^2\right)^{-3/2}$
$$\left|x^2+x\right|<1$$
$$\left|x^2+2(x)\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^2\right|<1+\frac{1}{4}$$
$$\left|x+\frac{1}{2}\right|^2<\frac{5}{4}$$
$$\left|x+\frac{1}{2}\right|<\frac{\sqrt{5}}{2}$$

91. Conceptual

92. Given $(x-2y+3z)^5$

No. of terms in above expansion
$${}^{n+r-1}C_{r-1}=7c_2=21=p$$
Coefficient of x^2yz^2 in the above expansion
$$((x-2y)+3z)^5=5c_0(x-2y)^5(3z)^0+5c_1(x-2y)^4(3z)^1$$
$$+5c_2(x-2y)^3(3z)^2+5c_3(x-2y)^2(3z)^3$$

$+5c_4(x-2y)^1(3z)^4+5c_5(x-2y)^0(3z)^5$

is $5c_2.9.(x-2y)^3z^2$

$=-540x^2yz^2$

$q=-540$

$\therefore q/p=\frac{-540}{21}=\frac{-180}{7}$

93. $S_{n+1}={}^5C_0+{}^8C_1+{}^{11}C_2+.......+(n+1)$ terms

$t_{n+1}=a+(n-1)d$

$=3n+5$

$S_{n+1}={}^5C_0+{}^8C_1+{}^{11}C_2+.......+(3n+5)c_n$

$S_{n+1}=(5+3n+5)2^{n-1}$

$=(3n+10)2^{n-1}$

$S_{11}=S_{10+1}=(3(10)+10)2^{10-1}$

$=40\times2^9=20480$

94. $\frac{1}{\sqrt{4-x}\,(2+x)^3}=\frac{1}{2\left(1-x/4\right)^{\frac{1}{2}}.2^3\left(1+x/2\right)^3}$

$=\frac{1}{16}\left(1-x/4\right)^{\frac{-1}{2}}\left(1+\frac{x}{2}\right)^{-3}$

$=\frac{1}{16}\left[1+\frac{1}{2}.\frac{x}{4}+\frac{\frac{1}{2}\left(\frac{1}{2}+1\right)}{2!}\frac{x^2}{4^2}+....\right]\ \left[1-3_{c_1}x/2+4_{c_2}\left(x/2\right)^2.....\right]$

By neglecting x^3 and higher powers

$=\frac{1}{16}\left[1+x/8+\frac{1.3}{8(16)}x^2\right]\ \left[1-3/2x+3/2x^2\right]$

$=\frac{1}{16}\left[1-\frac{11x}{8}+\frac{171}{128}x^2\right]$

95. $(7-5x)^{-\frac{2}{3}}=7^{-\frac{2}{3}}\left(1-\frac{5}{7}x\right)^{-\frac{2}{3}}$

If it is valid then $\left|\frac{5}{7}x\right|<1$

$|x|<\frac{7}{5}$

$-\frac{7}{5}<x<\frac{7}{5}$

$C=\frac{7}{5}$

Consider $5c+7=5\left(\frac{7}{5}\right)+7=7+7=14$

96. $(1+x)(1-x)^n$

$=(1+x)\,(n_{C_0}-n_{C_1}x+n_{C_2}x^2+.......n_{C_r}(-x)^r$

f(2023) = coefficient or x^{2023}

$=-n_{C_n}+n_{C_{n-1}}$

$=-2023_{C_{2023}}+2023_{C_{2022}}$

$=-1+2023$

$=2022$

97. IPE

98. $(2x-3y)^5=(2x)^5\left[1-\frac{3y}{2x}\right]^5$

$\alpha=\frac{-3}{2x}=\frac{-2}{3}$

$\frac{(n+1)|\alpha|}{|\alpha|+1}=\frac{12}{5}=2..........=T_3(N.G.T)$

$T_3=T_{2+1}=5c_2(2x)^3(-3y)^2$

$=1080$

99. $3^{2023}=(3^4)^{505}\,27$

$=(80+1)^{505}\,27$

$=27+16\,(Integer)$

Divisible by 16

$=\frac{27}{16}+(Integer)$

Remainder $=11$