26. COMPLEX NUMBERS & DEMOIVRE'S THEOREM

1. The locus of a point on the Argand plane represented by the complex number z, which z satisfies the condition $\left|\frac{z-1+i}{z+1-i}\right| = \left|\operatorname{Re}\left(\frac{z-1+i}{z+1-i}\right)\right|$ is

[AP EAMCET 17-09-20_Shift-2]

1. A straight line that does not contain the point (-1+i)
2. A circle that does not contain the point (-1+i)
3. A parabola that does not contain the point (-1+i)
4. A hyperbola that does not contain the point (-1+i)

2. Let z_1, z_2 be two complex numbers such that $\bar{z}_1 - i\bar{z}_2 = \hat{0}$ and $\arg(z_1 z_2) = \frac{3\pi}{4}$, then $\arg(z_1) =$

[AP EAMCET 17-09-20_Shift-2]

1. $\frac{\pi}{4}$ 2. $\frac{-\pi}{8}$
3. $\frac{\pi}{8}$ 4. $\frac{\pi}{3}$

3. If $x+iy = \frac{(3+2i)(4-7i)(12+13i)}{(13-12i)(2-3i)(11+3i)}$, then $x^2+y^2 =$

[AP EAMCET 17-09-20_Shift-2]

1. 1 2. 2
3. $\frac{1}{2}$ 4. 3

4. What is the modulus of the complex number $(1+2i)(-2+i)$? **[AP EAMCET 17-09-20_Shift-2]**

1. $\sqrt{5}$ 2. 5
3. $5\sqrt{5}$ 4. $\sqrt{35}$

5. Let the complex numbers α and $\left(\frac{1}{\bar{\alpha}}\right)$ lie on circles $(x-x_0)^2+(y-y_0)^2=r^2$ and $(x-x_0)^2+(y-y_0)^2=4r^2$ respectively. If $z_0 = x_0 + iy_0$ satisfies the equation $2|z_0|^2 = r^2+2$ then $|\alpha| =$

[AP EAMCET 18-09-20_Shift-1]

1. $\frac{1}{\sqrt{2}}$ 2. $\frac{1}{2}$
3. $\frac{1}{\sqrt{7}}$ 4. $\frac{1}{3}$

6. Let $z = x+yi$, where x, y are integers and $i=\sqrt{-1}$ the area of the rectangle whose vertices are the roots of the equation $\bar{z}z^3 + z(\bar{z})^3 = 350$ is

[AP EAMCET 18-09-20_Shift-2]

1. 32 2. 40
3. 48 4. 80

7. Geometrically, the set $\{z \in C : |z-2-2i| \le 1\}$ represents

[AP EAMCET 21-09-20_Shift-1]

1. A closed circular disc with center at (-2,-2) and with radius 1
2. A closed circular disc with center at (2,2) and with radius 1
3. A closed circular disc with center at (1,1) and with radius 0
4. A closed circular disc with center at (-1,-1) and with radius 0.5

8. If (2+i) is a root of the equation $x^3-5x^2+9x-5=0$, then the other roots are

[AP EAMCET 21-09-20_Shift-1]

1. 1 and (2-i) 2. -1 and (3+i)
3. 0 and 1 4. -1 and (-2+i)

9. The locus of z satisfying $\left|\frac{z-i}{z-2i}\right| = 2$ is a

[AP EAMCET 21-09-20_Shift-2]

1. Hyperbola 2. Circle
3. Straight line 4. Ellipse

10. For how many natural numbers 'n' such that $1 \le n \le 2021$ is $\left(\frac{1+i}{1-i}\right)^n = 1$?

[AP EAMCET 21-09-20_Shift-2]

1. 504 2. 505
3. 506 4. 503

11. If $x+iy = \frac{3}{2+\cos\theta + i\sin\theta}$, then $x^2+y^2 =$

[AP EAMCET 21-09-20_Shift-2]

1. $4x-3$ 2. $4x+3$
3. 0 4. 1

12. Find the conjugate of $\frac{5i}{7+i}$

[AP EAMCET 22-09-20_Shift-2]

1. $\frac{1}{10}(1-7i)$ 2. $\frac{1}{10}(7i-1)$
3. $\frac{1}{10}(7i+1)$ 4. $\frac{1}{\sqrt{50}}(1-7i)$

13. If $\left|\frac{z-25}{z-1}\right|=5\ then\ |z|$

[AP EAMCET 22-09-20_Shift-2]

1. 5 2. 3
3. 4 4. 10

14. If '2i' is a root $f(z)=z^4+z^3+2z^2+4z-8=0$, then which among the following cannot be a root of f(z)=0? **[AP EAMCET 22-09-20_Shift-1]**

1. -2i 2. 1
3. -2 4. 2

15. Suppose $z \in C$ has argument θ such that $0<\theta<\frac{\pi}{2}$ and satisfy the equation |z-3i|=3. Then what is the value of $\cot\theta-\frac{6}{z}$?

[AP EAMCET 23-09-20_Shift-1]

1. 2i 2. i
3. -i 4. -2i

16. If $A=\left\{z=x+iy\,/\,real\ part\ of\ \frac{z-1}{z-i}=2\right\}$, then the locus of the point P(x,y) in the Cartesian plane is

[TS EAMCET 09-09-20_Shift-1]

1. A pair of lines passing through (-1,-1)
2. A circle of radius $\frac{1}{\sqrt{2}}$ and the centre $\left(\frac{-1}{2},\frac{3}{2}\right)$
3. A pair of lines passing through (-1,-2)
4. A circle of radius $\frac{1}{2}$

17. Let $z \in C$ and $i=\sqrt{-1}$, if $a,b,c \in (0,1)$ be such that $a^2+b^2+c^2=1$ and $b+ic=(1+a)z$, Then $\frac{1+iz}{1-iz}=$ **[TS EAMCET 09-09-20_Shift-1]**

1. $\frac{a+ib}{1+c}$ 2. $\frac{a-ib}{1+c}$
3. $\frac{a-ib}{1-c}$ 4. $\frac{a+ib}{1-c}$

18. Let $a,b \in R$ and the roots α,β of the equation $z^2+az+b=0$ be complex. If the origin, $\alpha\ and\ \beta$ represent the vertices of an equilateral triangle on the Argand plane, then

[TS EAMCET 09-09-20_Shift-2]

1. $a=b$ 2. $a^2=3b$
3. $a^2=4b$ 4. $a=3b$

19. **Assertion (A):** If the arguments of $\overline{z}_1\ and\ z_2$ are $\frac{\pi}{5}$ and $\frac{\pi}{3}$ respectively, then $\arg(z_1z_2)\ is\ \frac{2\pi}{15}$

.**Reason(R):** For any complex number $z, \arg\overline{z}=\frac{\pi}{2}+\arg z$.

[TS EAMCET 09-09-20_Shift-2]

1. (A)is true,(R) is true and (R) is the correct explanation for (A)
2. (A)is true,(R) is true but (R)is not the correct explanation for (A)
3. (A) is true but (R) is false
4. (A) is false but (R) is true

20. Let $z=x+iy$ be a complex number, $A=\{z\,/\,|z|\le 2\}\ and\ B=\{z\,/\,(1-i)z+(1+i)\overline{z}\ge 4\}$ Then which one of the following options belongs to $A\cap B$? **[TS EAMCET 10-09-20_Shift-1]**

1. $\sqrt{3}+\frac{1}{2}i$ 2. $\frac{1}{2}+\frac{i}{2}$
3. $\sqrt{2}+\frac{i}{2}$ 4. $2+2i$

21. The solutions of the equation $z^2(1-z^2)=16, z\in C$ lie on the curve **[TS EAMCET 10-09-20_Shift-1]**

1. $|z|=1$
2. $|z|=\frac{2}{|z|}$
3. $|z|^2=3|z|+2$
4. $|z|=2$

22. If $z,\bar{z},-z,-\bar{z}$ forms a rectangle of area $2\sqrt{3}$ square units, then one such z is

[TS EAMCET 10-09-20_Shift-1]

1. $\frac{1}{2}+\sqrt{3}i$
2. $\frac{\sqrt{5}+\sqrt{3}i}{4}$
3. $\frac{3}{2}+\frac{\sqrt{3}i}{2}$
4. $\frac{\sqrt{3}+\sqrt{11}i}{2}$

23. If $z_1=x_1+iy_1, z_2=x_2+iy_2, z_3=x_1+\frac{ix_2}{2}$, $z_4=2y_1+iy_2$ are complex numbers such that $|z_1|=1, |z_2|=2$ and $\text{Re}(z_1z_2)=0$, then

[TS EAMCET 10-09-20_Shift-2]

1. $|z_3|=1, |z_4|=2, \text{Im}(z_3z_4)=0$
2. $|z_3|=2, |z_4|=1, \text{Re}(z_3z_4)=0$
3. $|z_3|=1, |z_4|=2, \text{Re}(z_3z_4)=0$
4. $|z_3|=2, |z_4|=1, \text{Re}(z_1z_2)=\text{Im}(z_2z_4)=0$

24. **Assertion (A):** If z is a complex number such that $|z|\geq 3$, then the least value of $\left|z+\frac{3}{z}\right|$ is 1.

Reason(R): $|z_1-z_2|\leq|z_1|+|z_2|$, for any two complex numbers z_1,z_2. The correct option among the following is

[TS EAMCET 10-09-20_Shift-2]

1. (A) is true,(R) is true and (R) is the correct explanation for (A)
2. (A) is true,(R) is true but (R) is not the correct explanation for (A)
3. (A) is true but (R) is false
4. (A) is false but (R) is true

25. $A(z_1=2+2i), B(z_2), C(z_3)$ are three points on the argand plane satisfying $|z_k-2i|=2, (k=1,2,3)$. If ΔABC encloses the maximum area, then the sum of imaginary parts of the z_2 and z_3 is

[TS EAMCET 11-09-20_Shift-1]

1. 1
2. 0
3. 4
4. -4

26. The number of points z on the Argand plane which satisfy the conditions $\text{Re}\left(\frac{z-2}{z-4i}\right)=0$ and $\text{Im}\left(\frac{z-2}{z-4i}\right)=1$ simultaneously is

[TS EAMCET 11-09-20_Shift-2]

1. 0
2. 1
3. 2
4. Infinitely many

27. Let $a=1+i$ and $z=x+iy$. If the curve $z\bar{z}+az+\bar{a}\bar{z}-4=0$ is cut by the straight line $(z+\bar{z})-i(z-\bar{z})+2=0$ at two points A and B, then the equation of the circle passing through the origin, A and B is

[TS EAMCET 11-09-20_Shift-2]

1. $x^2+y^2+3x-4y=0$
2. $x^2+y^2+x+y=0$
3. $x^2+y^2+6x+2y=0$
4. $x^2+y^2-7x-12y=0$

28. If z is a complex number such that $z^2+z+1=0$, then $\left(z+\frac{1}{z}\right)^3+\left(z^2+\frac{1}{z^2}\right)^3+\left(z^3+\frac{1}{z^3}\right)^3+...+\left(z^{2020}+\frac{1}{z^{2020}}\right)^3=$

[TS EAMCET 11-09-20_Shift-2]

1. 4037
2. -2020
3. 4038
4. 2020+673i

29. Let z be a complex number such that $|z|-z=2+i$, where $i=\sqrt{-1}$ then $|z|=$

[TS EAMCET 14-09-20_Shift-2]

1. $\frac{5}{2}$
2. $\frac{\sqrt{41}}{4}$
3. $\frac{5}{3}$
4. $\frac{5}{4}$

30. If the amplitude of $Z-2-3i$ is $\frac{\pi}{4}$, then the locus of $Z=x+iy$ is

[TS EAMCET 14-09-20_Shift-2]

1. $x+y-1=0$
2. $x-y-1=0$
3. $x+y+1=0$
4. $x-y+1=0$

31. Real part of $(\cos 4 + i\sin 4 + 1)^{2020}$ is.....

[AP EAMCET 19-08-2021_Shift-1]

1. $2^{2020}\cos^{2020} 2\cos 2020$
2. $2^{2020}\cos^{2020} 2\cos 4040$
3. $2^{1020}\cos^{2020} 2\cos 4040$
4. $2^{2020}\cos^{2020} 1\cos 2020$

32. If $|z-2| = |z-1|$, where z is a complex number, then locus 'z' is a straight line

[AP EAMCET 19-08-2021_Shift-2]

1. Parallel to x-axis
2. Parallel to y-axis
3. Parallel to $y=x$
4. Parallel to $y=-x$

33. The radius of the circle represented by $(1+i)(1+3i)(1+7i) = x+iy$ is $\left(i=\sqrt{-1}\right)$

[AP EAMCET 20-08-2021_Shift-1]

1. 1000
2. $10\sqrt{10}$
3. 10000
4. 100

34. If $a>0$ and $z = x+iy$, then, $\log_{\cos^2\theta}|z-a| > \log_{\cos^2\theta}|z-ai|, (\theta\in R)$ implies

[AP EAMCET 20-08-2021_Shift-1]

1. $x>y$
2. $x<y$
3. $x+y=\cos\theta$
4. $x+y<0$

35. If $|z_1+z_2|^2 = |z_1|^2 + |z_2|^2$, where z_1 & z_2 are two complex numbers, then

[AP EAMCET 20-08-2021_Shift-2]

1. $\frac{z_1}{z_2}$ is purely real
2. $\frac{z_1}{z_2}$ is purely imaginary
3. $\arg\left(\frac{z_1}{z_2}\right) = \frac{\pi}{4}$
4. $\left|\frac{z_1}{z_2}\right| = 1$

36. A real value of x will satisfy the equation $\left(\frac{3-4ix}{3+4ix}\right) = \alpha - i\beta\,(\alpha,\beta \textit{ are real})$, if

[AP EAMCET 20-08-2021_Shift-2]

1. $\alpha^2-\beta^2=-1$
2. $\alpha^2-\beta^2=1$
3. $\alpha^2+\beta^2=1$
4. $\alpha^2-\beta^2=2$

37. If $z\in C$, then the minimum value of $|z|+|2z-3|+|z-1|$ is

[AP EAMCET 23-08-2021_Shift-1]

1. 2
2. 1
3. 3
4. 0

38. If $a,b\in R$ and $i=\sqrt{-1}$, then the number of ordered pairs of real numbers (a,b) satisfying the condition $(a+bi)^3 = a-bi$ is

[AP EAMCET 23-08-2021_Shift-1]

1. 3
2. 2
3. 4
4. 5

39. If z is a complex number, the curves $|z|=1, |z-2|=1$ and $|z-1|=0$ have a common point at

[AP EAMCET 24-08-2021_Shift-1]

1. (0, 1)
2. (2, 0)
3. (1, 0)
4. (0, 2)

40. Let $z = x+iy$ be a complex number $(x,y\in R)$, Let A and B be two sets such that $A=\{z:|z|\le 2\}$ and $B=\{z:(z+2y)+\bar{z}\ge 4\}$ the are of region $A\cap B$ is

[AP EAMCET 24-08-2021_Shift-1]

1. 4
2. $\pi-4$
3. π
4. $\pi-2$

41. $\left|\frac{1}{i^{2020}}+\frac{2}{i^{2021}}+\frac{3}{i^{2022}}+\frac{4}{i^{2023}}\right| =$

[AP EAMCET 25-08-2021_Shift-2]

1. $3\sqrt{2}$
2. $4\sqrt{2}$
3. $2\sqrt{2}$
4. $\sqrt{2}$

42. Define $f:C\to R$ by $f(z)=|z|\,\forall z\in C$. Then which of the following is false?

[AP EAMCET 25-08-2021_Shift-1]

1. $f(-z)=f(z)\,\forall z\in C$
2. $f(\bar{z})=f(z)\,\forall z\in C$
3. $f(z^2)=(f(z))^2\,\forall z\in C$
4. $f(z_1^2+z_2^2)=f(z_1^2)+f(z_2^2)\,\forall z_1,z_2\in C$

43. The value of $\left\{i^{22}-\left(\frac{1}{i}\right)^{35}\right\}^2$ is

[AP EAMCET 25-08-2021_Shift-1]

1. $2i$
2. i
3. $-i$
4. $-2i$

44. If z_1, z_2 are conjugate complex numbers. Match the items under the following columns?

Column –I	Column-II
(i) $z_1 z_2$	(a) imaginary axis
(ii) $z_1 + z_2 = 0$	(b) $I_m(-z_2)$
(iii) $I_m(z_1)$	(c) $\lvert z_1 \rvert^2$
(iv) $\mathrm{Re}(z_1)$	(d) $\mathrm{Re}(z_2)$

[AP EAMCET 23-08-2021_Shift-1]

1. $(i-c)(ii-a)(iii-d)(iv-b)$
2. $(i-c)(ii-a)(iii-b)(iv-d)$
3. $(i-a)(ii-b)(iii-d)(iv-c)$
4. $(i-b)(ii-d)(iii-c)(iv-a)$

45. If $x+iy = \dfrac{1+7i}{(2-i)^2}$; then $\operatorname{cosec}\left(\tan^{-1}\dfrac{y}{x} - \dfrac{\pi}{4}\right) =$

[TS EAMCET 04-08-2021_Shift-2]

1. 1 2. ∞
3. -1 4. 0

46. If $(a+ib)^{\frac{1}{4}} = 2+3i$, then $3b-2a =$

[TS EAMCET 04-08-2021_Shift-1]

1. -22 2. -122
3. -598 4. -698

47. If $z_1 = 1-2i,\ z_2 = 1+i$ and $z_3 = 3+4i$, then $\left|\left(\dfrac{1}{z_1} + \dfrac{2}{z_2}\right)\dfrac{z_3}{z_2}\right| =$

[TS EAMCET 05-08-2021_Shift-1]

1. $\dfrac{\sqrt{7}}{2}$ 2. $\dfrac{\sqrt{5}}{2}$
3. $\sqrt{\dfrac{45}{2}}$ 4. $\dfrac{\sqrt{15}}{2}$

48. If $x = \dfrac{4}{5} + \dfrac{3}{5}i,\ y = \dfrac{\sqrt{3}}{\sqrt{8}} - \dfrac{\sqrt{5}}{\sqrt{8}}i$, then $\left(x^2 + \dfrac{1}{x^2}\right)\left(y^2 - \dfrac{1}{y^2}\right) =$

[TS EAMCET 05-08-2021_Shift-1]

1. $\dfrac{-7\sqrt{3}}{5\sqrt{5}}i$ 2. $\dfrac{7}{125}i$
3. $\dfrac{7\sqrt{3}}{5\sqrt{5}}i$ 4. $\dfrac{\sqrt{15}}{\sqrt{8}}i$

49. If $\left(\sqrt{3}+i\right)^8 - \left(\sqrt{3}-i\right)^8 = \alpha + i\beta$, then $\alpha - \dfrac{\sqrt{3}}{2}\beta =$

[TS EAMCET 05-08-2021_Shift-2]

1. 256 2. $384\sqrt{3}$
3. 384 4. $256\sqrt{3}$

50. If $Z = x+iy$ is a complex number and $\sqrt{x^2-2x+8} + (x+4)i = y(2+i)$, then Z =

[TS EAMCET 06-08-2021_Shift-2]

1. $\dfrac{-28}{9} - \dfrac{16}{9}i$ 2. $-2+2i$
3. $\dfrac{2}{3} - \dfrac{2}{3}i$ 4. $\dfrac{-2}{5} - \dfrac{2i}{5}$

51. The locus of $z = x+iy$ such that $\mathrm{Im}\left(\dfrac{z-3i}{iz+4}\right) = 0$ is

[TS EAMCET 06-08-2021_Shift-1]

1. $x^2 - y^2 + 7y - 12 = 0$
2. $x^2 + y^2 - 7y + 12 = 0$
3. $x^2 + y^2 - 7y + 12 = 0\ \&\ (x,y) \neq (0,4)$
4. $x^2 - y^2 + 7y - 12 = 0\ \&\ (x,y) \neq (0,4)$

52. If z_1 and z_2 are the roots of the equation $x^2 + 2x + 2 = 0$, then $\dfrac{-2^{11}(z_1+1+3i)^{11}}{2^5(z_2+1-3i)^{11}} =$

[TS EAMCET 06-08-2021_Shift-1]

1. 64 2. 32
3. $16\sqrt{2}$ 4. $8\sqrt{2}$

53. Let $f(x) = ax^2 + bx + c$ and GCD of a, b, c is 1. If $\dfrac{-7+\sqrt{11}i}{6}$ is a root of $f(x) = 0$ and $f\left(\dfrac{x}{k}\right) - L = (x+4)(3x-5)$ then k and L are respectively

[TS EAMCET 06-08-2021_Shift-1]

1. 1,-15 2. 1,25
3. 7,-15 4. 7,25

54. $iz^3+z^2-z+i=0\Rightarrow |z|=$

[AP EAMCET 04-07-2022_Shift-1]

1. 1/2
2. 2
3. 3/2
4. 1

55. If $\frac{x-1}{3+i}+\frac{y-1}{3-i}=i$ then the true statement among the following is

[AP EAMCET 04-07-2022_Shift-1]

1. $x<0, y<0$
2. $x<0, y>0$
3. $x>0, y<0$
4. $x>0, y<0$

56. The number of integer solutions of the equation $|1-i|^x=2^x$ is

[AP EAMCET 04-07-2022_Shift-1]

1. 1
2. 0
3. 2
4. 3

57. Multiplicative inverse of the complex number $(\sin\theta,\cos\theta)$

[AP EAMCET 04-07-2022_Shift-2]

1. $(+\sin\theta,+\cos\theta)$
2. $(\sin\theta,-\cos\theta)$
3. $(\cos\theta,-\sin\theta)$
4. $(-\cos\theta,\sin\theta)$

58. $\sum_{k=0}^{440} i^k=x+iy\Rightarrow x^{100}+x^{99}y+x^{242}y^2+x^{97}y^3=$

[AP EAMCET 04-07-2022_Shift-2]

1. 0
2. -4
3. 4
4. 1

59. By simplifying $i^{18}-3i^7+i^2\left(1+i^4\right)(i)^{22}$ we get

[AP EAMCET 05-07-2022_Shift-1]

1. $-1+3i$
2. $1-3i$
3. $1+3i$
4. $-1-3i$

60. The locus of point z satisfying $|z|^2=\mathrm{Re}(z)$ is a circle with centre

[AP EAMCET 05-07-2022_Shift-1]

1. $\left(0,\frac{1}{2}\right)$
2. $\left(-\frac{1}{2},0\right)$
3. $\left(\frac{1}{2},0\right)$
4. $\left(0,-\frac{1}{2}\right)$

61. If $(x-iy)^{1/3}=a-ib$. Then the value of $\frac{x}{2a}+\frac{y}{2b}$ is

[AP EAMCET 05-07-2022_Shift-2]

1. $2\left(a^2-b^2\right)$
2. $4\left(a^2-b^2\right)$
3. a^2-b^2
4. $\frac{1}{2}\left(a^2-b^2\right)$

62. If $(x+iy)=\left(\frac{1+i}{1-i}\right)^3-\left(\frac{1-i}{1+i}\right)^3$ then the true statement among the following is

[AP EAMCET 06-07-2022_Shift-1]

1. $x<y$
2. $x>y$
3. $x\neq 0$
4. $x=y$

63. The number of complex numbers z satisfying $\bar{z}=iz^2$ is

[AP EAMCET 06-07-2022_Shift-1]

1. 3
2. 4
3. 2
4. 5

64. For any complex number z, the minimum value of $|z|+|z-1|$ is

[AP EAMCET 06-07-2022_Shift-2]

1. 1
2. 0
3. 1/2
4. 3/2

65. If the vertices A,B and C of an isosceles triangle ABC are respectively z_1, z_2 and z_3 and if $\angle C=90^0$, then

[AP EAMCET 06-07-2022_Shift-2]

1. $(z_1-z_2)=(z_1-z_3)(z_3-z_2)$
2. $(z_1-z_2)^2=(z_1-z_3)(z_3-z_2)$
3. $(z_1-z_2)^2=2(z_1-z_3)(z_3-z_2)$
4. $Z_1^2+Z_2^2+Z_3^2=z_1z_2z_3+2$

66. Let z and w be two complex numbers such that $\bar{z}+i\bar{w}=0$ and Arg(zw)= π. Then Arg z=

[AP EAMCET 07-07-2022_Shift-1]

1. $\frac{3\pi}{4}$
2. $\frac{\pi}{2}$
3. $\frac{5\pi}{4}$
4. $\frac{\pi}{4}$

67. If the complex number $z_1, z_2, 0$ are vertices of an equilateral triangle, then $z_1^2 + z_2^2 =$

[AP EAMCET 07-07-2022_Shift-1]

1. $2z_1^2 z_2^2$
2. $z_1^2 z_2^2$
3. $2z_1 z_2$
4. $z_1 z_2$

68. Let z=x+iy be a complex number with $x, y \in \mathbb{Z}$ .Then the area (in square units) of the rectangle whose vertices are the roots of the equation $\bar{z} z^3 + z \bar{z}^3 = 350$ is

[AP EAMCET 07-07-2022_Shift-1]

1. 48
2. 32
3. 40
4. 44

69. Area of the triangle formed by the complex numbers z, iz and z + iz in the Argand diagram as vertices is

[AP EAMCET 08-07-2022_Shift-1]

1. $\frac{1}{2}.|z|^2$
2. $\frac{1}{2}.z^2$
3. z^2
4. $|z|^2$

70. If $(x-iy)^{\frac{1}{3}} = 2 - i\sqrt{3}$ and the point z = (x, y) lies on the line $\frac{x}{2} + \frac{y}{\sqrt{3}} = k$, then k=

[AP EAMCET 08-07-2022_Shift-2]

1. 16
2. 2
3. 8
4. 4

71. If $\left|z + \frac{2}{z}\right| = 2$, then the maximum value of $|z|$ is

[AP EAMCET 08-07-2022_Shift-2]

1. $\sqrt{z} + 1$
2. $\sqrt{z} - 1$
3. $\sqrt{z}$
4. infinity

72. A complex number z among the following which does not satisfy which does not satisfy $z^3 + 27i = 0$ is

[AP EAMCET 08-07-2022_Shift-2]

1. $(3\sqrt{3} - 3i)/2$
2. -3i
3. $(3\sqrt{3} + 3i)/2$
4. $(-3\sqrt{3} + 3i)/2$

73. If $|z - 3i| + |z + 5i| = 4$, then the locus of z is

[AP EAMCET 08-07-2022_Shift-2]

1. No such point z exists
2. Ellipse
3. Parabola
4. Circle

74. $\sqrt{(-3+4i)(8+6i)} =$

[TS EAMCET 18-07-2022_Shift-1]

1. $\pm(1+2i)$
2. $\pm(3+i)$
3. $\pm(1+7i)$
4. $\pm(7-i)$

75. If $\left(\frac{\sqrt{3}+i}{\sqrt{3}-i}\right)^m = 1, 2022 < m < 2029$, then m =

[TS EAMCET 18-07-2022_Shift-1]

1. 2022
2. 2024
3. 2028
4. 2026

76. If the point (x,y) satisfies the equation $\frac{x+i(x-2)}{3+i} - i = \frac{2y+i(1-3y)}{i-3}$, then x + y =

[TS EAMCET 18-07-2022_Shift-2]

1. 4
2. 2
3. 0
4. -2

77. If $(2x-y+1)+i(x-2y-1) = 2-3i$, then the multiplicative of $(x-iy)$ is

[TS EAMCET 19-07-2022_Shift-I]

1. $\frac{15}{41} + \frac{12}{41}i$
2. $\frac{6}{29} + \frac{15}{29}i$
3. $\frac{15}{29} + \frac{6}{29}i$
4. $\frac{12}{41} + \frac{15}{41}i$

78. *If* $Z = \alpha + i\beta$ satisfies the equation $|Z| - Z = 1 + 2i$ and $|Z| = \sqrt{\alpha^2 + \beta^2}$ *then* $Z\overline{Z} =$

[TS EAMCET 19-07-2022_Shift-2]

1. $\frac{5}{2}$
2. $\frac{25}{4}$
3. $\frac{16}{9}$
4. $\frac{36}{25}$

79. $\{x \in [0, 2\pi] / \sin x + i\cos 2x$ *and* $\cos x - i\sin 2x$ *are conjugate to each other*$\} =$

[TS EAMCET 20-07-2022_Shift-1]

1. $\left\{\frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}, 2\pi\right\}$
2. $\left\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\right\}$
3. $\left\{\frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi\right\}$
4. ϕ

80. If $\begin{vmatrix} 2+3i & i \\ 1-2i & -i \end{vmatrix} = x + iy$, then $x + y =$

[TS EAMCET 20-07-2022_Shift-2]

1. -2 2. -4
3. -8 4. 4

81. The values of θ, for which $\frac{3+2i\sin\theta}{1-2i\sin\theta}$ is real are **[AP EAMCET 07-07-2022_Shift-2]**

1. $\theta = n\pi + \frac{\pi}{3}$ *for* $n \in \mathbb{Z}$ 2. $\theta = n\pi + \frac{\pi}{4}$ *for* $n \in \mathbb{Z}$
3. $\theta = n\pi + \frac{\pi}{2}$ *for* $n \in \mathbb{Z}$ 4. $\theta = n\pi$ *for* $n \in \mathbb{Z}$

82. If $z_1 = (2, -1)$ and $z_2 = (6, 3)$, then $amp\left(\frac{z_1 - z_2}{z_1 + z_2}\right) =$ **[15th May 2023 Shift 1]**

1. $-\frac{3\pi}{4} - \mathrm{Tan}^{-1}\left(\frac{1}{4}\right)$ 2. $\frac{\pi}{4} - \mathrm{Tan}^{-1}\left(\frac{1}{4}\right)$
3. $\frac{3\pi}{4} + \mathrm{Tan}^{-1}\left(\frac{1}{4}\right)$ 4. $\frac{\pi}{4} + \mathrm{Tan}^{-1}\left(\frac{1}{4}\right)$

83. The number of all possible solutions of the equation $z^3 + \bar{z} = 0$ is **[15th May 2023 Shift 1]**

1. 4 2. 5
3. 3 4. 6

84. $S = \{z \in C / |z - 1 + i| = 1\}$ represents

[15th May 2023 Shift 2]

1. A circle with centre $(-1, 1)$ and radius 1 unit
2. A circle with centre (1,2) and radius 5 units
3. A circle with centre $(1, -1)$ and radius 1 unit
4. An ellipse with centre $(1, -1)$

85. If $\left|z - \frac{2}{z}\right| = 2$, then the greatest value of $|z|$ is

[15th May 2023 Shift 2]

1. $\sqrt{3} - 1$ 2. $\sqrt{3}$
3. $\sqrt{3} + 1$ 4. $\sqrt{3} + 2$

86. *If* $\sqrt{-3-4i} = re^{i\theta}$, *then* $r^2 \tan\theta =$

[16th May 2023 Shift 1]

1. -5 2. 5
3. 10 4. -10

87. *If* $Z_1 = 2 - 3i$ and the roots of the equation $z^3 + bz^2 + cz + d = 0$ *are* i, z_1 *and* $\overline{z_1}$ then b + c+ d = **[16th May 2023 Shift 1]**

1. 13 2. 7
3. 9-10i 4. 10-10i

88. Let the two values of $z = \sqrt{\frac{1-i}{1+i}}$ *be* z_1 *and* z_2. *If* $-\frac{\pi}{2} < Arg(z_1) < Arg(z_2) < \pi$ then $\arg(z_1) + \arg(z_2) =$

[16th May 2023 Shift 1]

1. $\frac{\pi}{4}$ 2. $\frac{3\pi}{2}$
3. $\frac{\pi}{3}$ 4. $\frac{\pi}{2}$

89. If C is a point on the straight line joining the points A(–2+i) and B(3–4i) in the Argand plane and $\frac{AC}{CB} = \frac{1}{2}$, then the argument of C is

[16th May 2023 Shift 2]

1. $Tan^{-1}3$ 2. $Tan^{-1}2 - \pi$
3. $Tan^{-1}2$ 4. $\pi - Tan^{-1}3$

90. If α is the modulus of $z_1 = 4 + 3i$, then a point that does not lie in the region represented by $|z - \overline{z_1}| \le \alpha$ is

[16th May 2023 Shift 2]

1. $z_1 - 2i$ 2. z_1
3. $2z_1 - 7i$ 4. $3z_1 - (10 + 8i)$

91. If z_1, z_2, z_3, are the vertices of an equilateral triangle and z is its circum centre, then

[16th May 2023 Shift 2]

1. $\frac{|z-z_1|}{|z-z_2|}=\frac{|z-z_3|}{|z-z_1|}$ 2. $|z-z_1|+|z-z_2|+|z-z_3|$

3. $\frac{|z-z_1|}{|z-z_2|}=|z-z_3|$ 4. $\frac{|z-z_1|+|z-z_2|}{|z-z_3|}=1$

92. For real numbers a and b, if

$4a+i(3a-b)=b-6i$ and $z=a+\frac{b}{4}i$, then $\frac{|z|}{a}=$

[17th May 2023 Shift 1]

1. $2\sqrt{2}$ 2. $6\sqrt{2}$

3. $\sqrt{2}$ 4. 2

93. If $z=(1-i)^3(x+i)$ is a purely imaginary number for $x=x_1$ *and z is a purely real number for* $x=x_2$, then $x_1x_2=$

[17th May 2023 Shift 1]

1. -1 2. 0

3. 1 4. 2

94. The modulus of the conjugate of

$Z=\frac{-2+i}{(1-2i)^2}$ is **[17th May 2023 Shift 2]**

1. $\frac{1}{5}$ 2. $\frac{1}{\sqrt{5}}$

3. $\frac{1}{25}$ 4. $\sqrt{5}$

95. If $z_1=2+5i, z_2=-1+4i$ and $z_3=i$ then

$\left|\frac{z_1-z_3}{z_3-z_2}\right|=$ **[17th May 2023 Shift 2]**

1. $\sqrt{2}$ 2. $2\sqrt{2}$

3. $5\sqrt{2}$ 4. $4\sqrt{2}$

96. The locus of the variable point z=x+iy whose amplitude is always equal to θ, is

[17th May 2023 Shift 2]

1. $x^2+y^2=\tan^2\theta$ 2. $y=x\tan\theta$

3. $\frac{x^2}{\sin^2\theta}+\frac{y^2}{\cos^2\theta}=1$ 4. $\frac{x^2}{\sin^2\theta}-\frac{y^2}{\cos^2\theta}=1$

97. If z = x + iy represents a point in the Argand plane, then a point which is not in the region represented by $|z-1+i|\le 2$ *is* **[18th May2023 shift -1]**

1. $\frac{1-i}{2}$ 2. 1

3. $\frac{1-i}{4}$ 4. i

98. Let the locus of a point z in the Argand plane satisfying the condition $Re(z^2)=4$ be C_1 and the locus of z satisfying the condition $Im(z^2)=4$ be C_2. Then the number of common points of the two curves C_1 and C_2 are **[18th May2023 shift -1]**

1. 0 2. 3

3. 4 4. 2

99. If z is a point on the circle $|z|=1$ with

Arg(z) = $\frac{\pi}{6}$, *then* $\frac{z^{12}+1-z^6}{z^{12}+iz^6-1}=$ **[18th May2023 shift -1]**

1. 2 + 3i 2. 3i

3. 3 + 2i 4. 4 + 3i

100. If $-3+ix^2y$ and x^2+y+4i are complex conjugates, then X = **[18th May 2023 Shift 2]**

1. 0 2. ± 1

3. ± 3 4. ± 4

101. If Z = 1+ cosθ-isinθ 0<θ<π, then

$\left[|z-1|^2-\frac{|z|^2}{4}\right]^{1/2}=$ **[18th May 2023 Shift 2]**

1. $\sqrt{2}\cos\theta$ 2. $\sqrt{2}\sin\theta$

3. $\cos\left(\frac{\theta}{2}\right)$ 4. $\sin\left(\frac{\theta}{2}\right)$

102. In the Argand plane, the values of Z satisfying the equation $|z-1|=|i(z+1)|$ lie on **[18th May 2023 Shift 2]**

1. The Y – axis 2. A Parabola

3. A Hyperbola 4. The X – axis

103. $Arg\left(\frac{4+2i}{1-2i}+\frac{3+4i}{2+3i}\right)$ lies in the interval **[19th May 2023 Shift 1]**

1. $\left(\frac{\pi}{4},\frac{\pi}{2}\right)$
2. $\left(-\pi,\frac{-\pi}{2}\right)$
3. $\left(\frac{-\pi}{2},0\right)$
4. $\left(0,\frac{\pi}{4}\right)$

104. The multiplicative inverse of z is **[19th May 2023 Shift 1]**

1. $\frac{1}{z+\bar{z}}$
2. $\frac{z}{|\bar{z}|}$
3. $\frac{\bar{z}}{|z|^2}$
4. $\frac{1}{\bar{z}}$

105. If $z_1=2+3i,\ z_2=4-5i$ and z_3 are three points in the Argand plane such that $5z_1+xz_2+yz_3=0\ (x,y\in R)$ *and* z_3 is the midpoint of the line segment joining the points z_1 *and* z_2, then $x+y=$ **[19th May 2023 Shift 1]**

1. -5
2. 0
3. 4
4. -1

106. If z_1 *and* z_2 are complex numbers such that $|z_1+z_2|=|z_1|+|z_2|$ then the difference in the amplitudes of z_1 *and* z_2 is

[12TH MAY 2023 SHIFT-1]

1. $\frac{\pi}{4}$
2. $\frac{\pi}{3}$
3. $\frac{\pi}{2}$
4. 0

107. If $i=\sqrt{-1}$ *then* $1+i^2+i^4+i^6+...+i^{2024}=$

[12TH MAY 2023 SHIFT-1]

1. i
2. -i
3. 1
4. -1

108. If $\frac{1+i\cos\theta}{1-2i\cos\theta}$ is purely real then $\cos^3\theta+\sin^2\theta+\cos\theta+1=$

[12TH MAY 2023 SHIFT-1]

1. 0
2. 1
3. 2
4. $\frac{3}{4}(2+\sqrt{2})$

109. If α,β are non-zero integers and $z=(\alpha+i\beta)(2+7i)$ is a purely imaginary number, then minimum value of $|z|^2$ is

[12TH MAY 2023 SHIFT-1]

1. 0
2. 2809
3. 2808
4. 1

110. $\text{Arg}\left(\sin\frac{6\pi}{5}+i\left(1+\cos\frac{6\pi}{5}\right)\right)=$

[12TH MAY 2023 SHIFT -2]

1. $\frac{5\pi}{6}$
2. $\frac{6\pi}{5}$
3. $\frac{2\pi}{5}$
4. $\frac{9\pi}{10}$

111. If $x+iy=\sqrt{\frac{3+i}{1+3i}}$, then $\left(x^2+y^2\right)^2=$

[12TH MAY 2023 SHIFT -2]

1. 0
2. 1
3. 2
4. 3

112. If the imaginary part of $\frac{2z+1}{iz+1}$ is -2, then the locus of the point representing z in the Argand plane is **[12TH MAY 2023 SHIFT -2]**

1. a circle
2. a straight line
3. a parabola
4. an ellipse

113. If the value of $\sqrt{-5-12i}+\sqrt{7+24i}$ is a negative real number k, then k=

[13TH MAY 2023 SHIFT-1]

1. -5
2. -7
3. -6
4. -4

114. Let $z=x+iy$ be a point in the Argand plane. if the amplitude of $\left(\frac{z-3}{z+2i}\right)$ *is* $\frac{\pi}{2}$, then the locus of z is **[13TH MAY 2023 SHIFT-1]**

1. A circle
2. A straight line
3. A semicircular arc not containing the origin
4. A semicircular arc containing the origin

115. If a point P denotes the complex number $z=x+iy$ in the Argand plane and if $\frac{z-(2+i)}{z+(1-2i)}$ is purely real, then the locus of P is [13TH MAY 2023 SHIFT-1]

1. The line $x+3y-5=0$ excluding the point (-1, 2)

2. The circle $x^2+y^2-x-3y=0$ excluding the point (-1, 2)

3. The line $x+3y-5=0$ and the circle $x^2+y^2-x-3y=0$ excluding the point (-1, 2)

4. The circle $x^2+y^2-2x-6y+5=0$ excluding the point (-1, 2)

116. If $i=\sqrt{-1}$ then $\sum_{n=0}^{\infty}\left(\frac{i}{3}\right)^n=$ **[EAPCET 14-05-23 SHIFT-1]**

1. $\frac{9-3i}{10}$ 2. $9-3i$

3. $9+3i$ 4. $\frac{9+3i}{10}$

117. If $i=\sqrt{-1}$ then $\operatorname{Arg}\left[\frac{(1+i)^{2025}}{(1-i)^{2022}}\right]=$

[EAPCET 14-05-23 SHIFT-1]

1. $\frac{-\pi}{4}$ 2. $\frac{\pi}{4}$

3. $\frac{3\pi}{4}$ 4. $\frac{-3\pi}{4}$

118. The locus of z such that $\left|\frac{z-i}{z+i}\right|=2$, where $z=x+iy$, is **[EAPCET 14-05-23 SHIFT-1]**

1. $3x^2+3y^2+10y+3=0$

2. $3x^2-3y^2-10y-3=0$

3. $3x^2+3y^2+10y-3=0$

4. $x^2+y^2-5y+3=0$

119. If the roots of the equation $z^2-i=0$ are α and β, then $|Arg\,\beta-Arg\,\alpha|=$ **[EAPCET 14-05-23 SHIFT-1]**

1. 2π 2. $\frac{\pi}{2}$

3. π 4. $\frac{\pi}{4}$

120. If $i^2=-1$ then $\left(1+\sqrt{3}i\right)^{2022}-\left(\sqrt{3}-i\right)^{2022}=$

[EAPCET 13-05-23 SHIFT-2]

1. 2^{2023} 2. 0

3. 2^{2022} 4. 3^{1011}

121. If $\left(\frac{\sqrt{3}+i}{\sqrt{3}-i}\right)^4+\left(\frac{\sqrt{3}-i}{\sqrt{3}+i}\right)^4=r\,cis\theta$, then one of the values of $\sqrt{r\,cis\theta}$ is

[EAPCET 13-05-23 SHIFT-2]

1. $cis\left(\frac{3\pi}{4}\right)$ 2. $cis\left(\frac{3\pi}{2}\right)$

3. $cis\left(\frac{\pi}{3}\right)$ 4. $cis\pi$

122. If $z=x+iy$ and the point P in the Argand plane represents z, then the locus of z satisfying the equation $|z-2|+|z-2i|=4$ is

[EAPCET 13-05-23 SHIFT-2]

1. $4x^2+3xy+4y^2-6x-6y+8=0$

2. $3x^2+2xy+3y^2-8x-8y+6=0$

3. $3x^2+2xy+3y^2-8x-8y=0$

4. $4x^2+3xy+4y^2-6x-6y=0$

KEY

1)	1	2)	3	3)	3	4)	2	5)	3
6)	3	7)	2	8)	1	9)	2	10)	2
11)	1	12)	1	13)	1	14)	4	15)	2
16)	2	17)	1	18)	2	19)	3	20)	1
21)	4	22)	1	23)	3	24)	4	25)	3
26)	3	27)	2	28)	1	29)	4	30)	4
31)	2	32)	2	33)	2	34)	1	35)	2
36)	3	37)	1	38)	4	39)	3	40)	4

41)	3	**42)**	4	**43)**	1	**44)**	2	**45)**	1
46)	3	**47)**	3	**48)**	1	**49)**	3	**50)**	2
51)	3	**52)**	1	**53)**	2	**54)**	4	**55)**	2
56)	1	**57)**	2	**58)**	4	**59)**	3	**60)**	3
61)	1	**62)**	2	**63)**	2	**64)**	1	**65)**	3
66)	1	**67)**	4	**68)**	1	**69)**	1	**70)**	4
71)	1	**72)**	1	**73)**	1	**74)**	3	**75)**	3
76)	2	**77)**	4	**78)**	2	**79)**	4	**80)**	1
81)	4	**82)**	1	**83)**	2	**84)**	3	**85)**	3
86)	4	**87)**	3	**88)**	4	**89)**	2	**90)**	2
91)	1	**92)**	3	**93)**	1	**94)**	2	**95)**	1
96)	2	**97)**	4	**98)**	4	**99)**	2	**100)**	2
101)	4	**102)**	1	**103)**	1	**104)**	3	**105)**	1
106)	4	**107)**	3	**108)**	3	**109)**	2	**110)**	4
111)	2	**112)**	2	**113)**	3	**114)**	4	**115)**	1
116)	4	**117)**	1	**118)**	1	**119)**	3	**120)**	1
121)	2	**122)**	3						

SOLUTIONS

COMPLEX NUMBERS

1. $\frac{z-1+i}{z+1-i}=\frac{(x-1)+i(y+1)}{(x+1)+i(y-1)}\times\frac{(x+1)-i(y-1)}{(x+1)-i(y-1)}$

$=\frac{\left((x^2-1)+(y^2-1)\right)+i\left((y+1)(x+1)-(x-1)(y-1)\right)}{(x+1)^2+(y-1)^2}$

given $\frac{(x-1)^2+(y+1)^2}{(x+1)^2+(y-1)^2}=\frac{x^2-1+y^2-1}{(x+1)^2+(y-1)^2}$

$\Rightarrow x^2+y^2-2x+2y+2=x^2+y^2-2$

$\Rightarrow x-y-2=0$

2. Given

$\bar{z}_1=i\bar{z}_2$ & $\arg(z_1)+\arg(z_2)=\frac{3\pi}{4}$.......(1)

$\left(\frac{\bar{z}_1}{\bar{z}_2}\right)=i$

$\frac{z_1}{z_2}=-i$

$\arg(z_1)-\arg(z_2)=\frac{-\pi}{2}$......(2)

add (1)&(2)

$2\arg(z_1)=\frac{3\pi}{4}-\frac{\pi}{2}=\frac{\pi}{4}$

$\arg(z_1)=\frac{\pi}{8}$

3. Given $x+iy=\frac{(3+2i)(4-7i)(12+13i)}{(13-12i)(2-3i)(11+3i)}$

$\Rightarrow |x+iy|=\left|\frac{4-7i}{11+3i}\right|$

$\Rightarrow \sqrt{x^2+y^2}=\frac{\sqrt{65}}{\sqrt{130}}=\frac{1}{\sqrt{2}}$

$\therefore x^2+y^2=\frac{1}{2}$

4. Modulus of complex number $(1+2i)(-2+i)$

$=|(1+2i)(-2+i)|=|1+2i||-2+i|$

$\sqrt{1^2+2^2}.\sqrt{-2^2+1^2}$

$=\sqrt{5}.\sqrt{5}$

$=5$

5. α *and* $\frac{1}{\bar{\alpha}}$ lies on the circles

$|z-z_0|=r$ and $|z-z_0|=2r$

$|\alpha-z_0|=r\Rightarrow|\alpha-z_0|^2=r^2$

hence $\Rightarrow(\alpha-z_0)(\bar{\alpha}-\bar{z}_0)=r^2$

$\Rightarrow z_0\bar{\alpha}+\alpha\bar{z}_0=|\alpha|^2+|z_0|^2-r^2$.......(1)

similarly

$\left|\frac{1}{\bar{\alpha}}-z_0\right|^2=4r^2$

$\Rightarrow|1-z_0\bar{\alpha}|^2=4r^2|\alpha|^2$

$\Rightarrow(1-z_0\bar{\alpha})(1-\bar{z}_0\alpha)=4r^2|\alpha|^2$

$\Rightarrow z_0\bar{\alpha}+\bar{z}_0\alpha=1+|\alpha|^2|z_0|^2-4r^2|\alpha|^2$.....(2)

from (1) and (2)

$|\alpha|^2+|z_0|^2-r^2=1+|\alpha|^2|z_0|^2-4r^2|\alpha|^2$

$\Rightarrow\left(\textit{given } 2|z_0|^2=r^2+2\right)\Rightarrow|\alpha|=\frac{1}{\sqrt{7}}$

6. $\bar{z}z\left(z^2+\left(\bar{z}\right)^2\right)=350$

$\Rightarrow\left(x^2+y^2\right)\left(x^2-y^2\right)=175=25\times 7$

$\Rightarrow\left(x^2+y^2\right)=25\ and\ \left(x^2-y^2\right)=7$

By solving, we get

$A(-4,3), B(4,3), C(4,-3), D(-4,-3)$

$\therefore$ area of rectangle ABCD is $8\times 6=48$

7. $|z-2-2i|\leq 1$

Let $z=x+iy$

$|x+iy-2-2i|\leq 1$

$|(x-2)+i(y-2)|\leq 1$

$\sqrt{(x-2)^2+(y-2)^2}\leq 1$

$(x-2)^2+(y-2)^2\leq 1$

A closed circular disc with centre at (2,2) and with radius '1'.

8. (2+i) is a root then 2-i is also root of the given eq.

Let 'α' be a third root $2+i+2-i+\alpha=5$

$\alpha=1$ Remaining roots are 1,2-i

9. $\left|\dfrac{z-i}{z-2i}\right|=2$

$\left|\dfrac{x+iy-i}{x+iy-2i}\right|=2$

$\sqrt{x^2+(y-1)^2}=2\sqrt{x^2+(y-2)^2}$

s.o.b.s

$3x^2+3y^2+14y-15=0$

is a circle

10. $Given\left(\dfrac{1+i}{1-i}\right)^n=1$

It satisfies values $i^4, i^8, i^{12}, \ldots, i^{2020}=1$

$t_n=a+(n-1)d$

$2020=4+(n-1)4$

$2016=4(n-1)\Rightarrow n=505$

11. Given $x+iy=\dfrac{3}{2+\cos\theta+i\sin\theta}$

$=\dfrac{3(2+\cos\theta+i\sin\theta)}{5+4\cos\theta}$

Now $x=\dfrac{3(2+\cos\theta)}{5+4\cos\theta}$, $y=\dfrac{-3\sin\theta}{5+4\cos\theta}$

$x^2+y^2=\dfrac{9}{5+4\cos\theta}$.........(1)

$4x-3=\dfrac{9}{5+4\cos\theta}$........(2)

From $(1)\&(2), x^2+y^2=4x-3$

12. $Z=\dfrac{5i}{7+i}\times\dfrac{7-i}{7-i}$

$Z=\dfrac{1+7i}{10}\Rightarrow\overline{Z}=\dfrac{1-7i}{10}$

13. Conceptual

14. Conceptual

15. Let $z=x+iy$

$\theta=Arg\,z=\mathrm{Tan}^{-1}\left(\dfrac{y}{x}\right)$

$\mathrm{Tan}\,\theta=\dfrac{y}{x}\Rightarrow\cot\theta=\dfrac{x}{y}$

$|z-3i|=3$

$x^2+(y-3)^2=3^2$

$x^2+y^2=6y$

$\cot\theta-\dfrac{6}{z}=\dfrac{x}{y}-\dfrac{6}{x+iy}\times\dfrac{x-iy}{x-iy}=\dfrac{x}{y}-\dfrac{x}{y}+i=i$

16. $\dfrac{z-1}{z-i}=\dfrac{(x-1)+iy}{x+i(y-1)}\times\dfrac{x-i(y-1)}{x-i(y-1)}$

Re part of $\left(\dfrac{z-1}{z-i}\right)=2$

$\dfrac{x(x-1)+y(y-1)}{x^2+(y-1)^2}=2$

$x^2+y^2+x-3y+2=0$

Centre =(-1/2,3/2),radius = $\dfrac{1}{\sqrt{2}}$

17. $z=\dfrac{b+ic}{a}$

$\dfrac{1+iz}{1-iz}=\dfrac{1+i\left(\dfrac{b+ic}{1+a}\right)}{1-i\left(\dfrac{b+ic}{1+a}\right)}$

$$=\frac{1+a^2+2a-c^2-b^2+i2b(1+a)}{1+a^2+b^2+c^2+2a+2ac+2c}$$

$$=\frac{2(1+a)(a+ib)}{2(1+a)(1+c)}=\frac{a+ib}{1+c}$$

18. $\alpha+\beta=-a,\ \alpha\beta=b$

Let $\alpha=x+iy,\ \beta=x-iy$

$$-a=2x\Rightarrow x=-\frac{a}{2}$$

$$\alpha\beta=b$$

$$x^2+y^2=b\Rightarrow y^2=b-\frac{a^2}{4}$$

$$|\alpha|=|\beta|=|\alpha-\beta|$$

$$|\alpha|=|\alpha-\beta|$$

$$\sqrt{x^2+y^2}=\sqrt{4y^2}$$

$$x^2=3y^2 \qquad a^2=3b$$

19. Conceptual

20. $A=\{z/\,x^2+y^2\le 4\}$

$$B=\{z/\,2x+2y\ge 4\}$$

$$=\{z/\,x+y\ge 2\}\Rightarrow\left(\sqrt{3},\frac{1}{2}\right)\in A\cap B$$

21. Conceptual

22. *Let* $z=x+iy\Rightarrow \bar{z}=x-iy$

$$-z=-x-iy\Rightarrow -\bar{z}=-x+iy$$

$$A=(x,y),B=(x,-y)$$

$$C=(-x,-y),D=(-x,y)$$

$$\text{Area of the rectangle}=l\times b$$

$$2\sqrt{3}=AB\times AD\Rightarrow 2\sqrt{3}=\sqrt{4x^2\times 4y^2}$$

$$xy=\frac{\sqrt{3}}{2}$$

By option verification

$$(i)\ x=\frac{1}{2},y=\sqrt{3}\Rightarrow xy=\frac{\sqrt{3}}{2}$$

23. $|z_1|=1\Rightarrow x_1^2+y_1^2=1\ldots\ldots(1)$

$$|z_2|=2\Rightarrow x_2^2+y_2^2=4\ldots\ldots.(2)$$

$$\text{Re}(z_1z_2)=0\Rightarrow x_1^2x_2^2=y_1^2y_2^2$$

$$x_1^2x_2^2=(1-x_1^2)(4-x_2^2)$$

$$4x_1^2+x_2^2=4\ \ldots\ldots.(3)$$

Similarly $4y_1^2+y_2^2=4\ \ldots\ldots.(4)$

$$|z_3|=\sqrt{x_1^2+\frac{x_2^2}{4}}=1$$

$$|z_4|=\sqrt{4y_1^2+y_2^2}=2$$

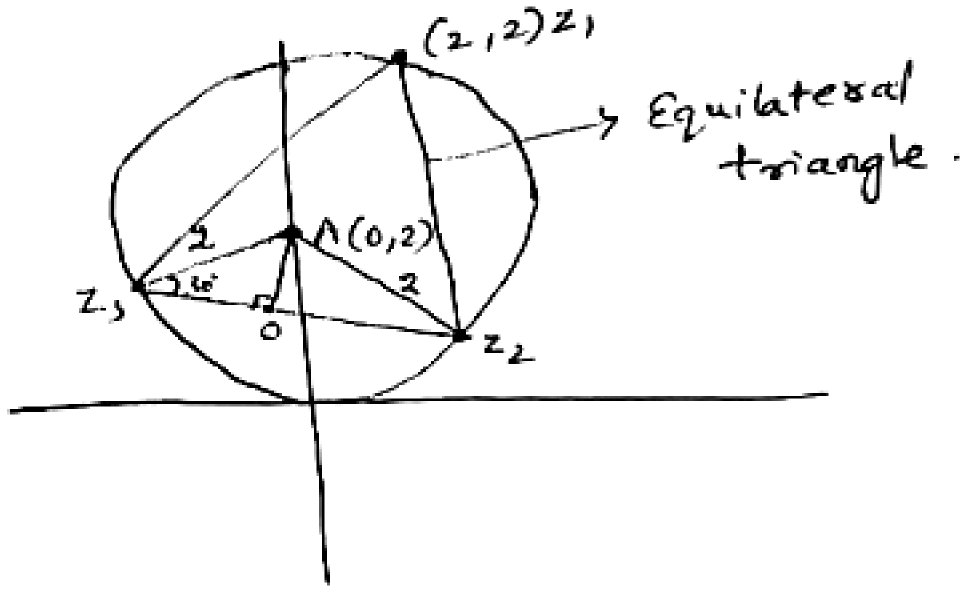
$\text{Re}(z_3z_4)=0$,since solving (1)&(3),(2)&(4)

24. $|z|\ge 3.\Rightarrow\left|z+\frac{3}{z}-\frac{3}{z}\right|\ge 3$

$$\Rightarrow 3\le\left|z+\frac{3}{z}\right|+\frac{3}{|z|}$$

$$\Rightarrow 2\le\left|z+\frac{3}{z}\right|$$

25.



Distance between Z_2 and Z_1 is

$$(x-2)^2+(y-2)^2=12$$

$$\Rightarrow x^2+y^2-4x-4y-4=0\ldots(1)$$

Equation of circle with $C(0,2)$ and $r=2$ is

$$x^2+y^2-4y=0\ldots(2)$$

$$(1)-(2)\Rightarrow x=-1$$

Substitute in (2)

$$\Rightarrow (y-2)^2=3$$

$$\Rightarrow y-2=\pm\sqrt{3}\Rightarrow y=2\pm\sqrt{3}$$

$$\therefore Z_2=\left(-1,2+\sqrt{3}\right)\ \textit{and}\ \ Z_3=\left(-1,2-\sqrt{3}\right)$$

Now, $\text{img}\{Z_2+Z_3\}=2+\sqrt{3}+2-\sqrt{3}=4$

26. $\frac{Z-2}{Z-4i}=\frac{(x-2)+iy}{x+(y-4)i}\times\frac{x-(y-4)i}{x-(y-4)i}$

$$\frac{Z-2}{Z-4i}=\frac{x(x-2)+y(y-4)+i[xy-(x-2)(y-4)]}{x^2+(y-4)^2}$$

$$\text{Re}\left(\frac{Z-2}{Z-4i}\right)=0\Rightarrow x^2+y^2-2x-4y=0$$

$C_1=(1,2)\ r_1=\sqrt{5}$

$$im\left(\frac{Z-2}{Z-4i}\right)=1\Rightarrow\frac{xy-xy+4x+2y-8^2}{x^2+(y-4)^2}=1$$

$\Rightarrow 4x+2y-8=x^2+y^2+16-8y$

$\Rightarrow x^2+y^2-4x-10y+24=0$

$C_1=(1,2)\quad C_2=(2,5)$

$r_1=\sqrt{5}\qquad r_2=\sqrt{5}$

$C_1C_2<r_1+r_2$

No. of point of intersections =2

27. $a=1+i,\quad Z=x+iy$

$\Rightarrow Z\bar{z}+az+\bar{a}\bar{z}-4=0$

$\Rightarrow x^2+y^2+2x-2y-4=0$

Straight line:

$2x-i(2yi)+2=0\Rightarrow 2x+2y+2=0$

The equation of circle passing through A and B is $S+\lambda L=0$

$\Rightarrow x^2+y^2+2x-2y-4+\lambda(2x+2y+2)=0$

It is passing through origin

$\Rightarrow -4+2\lambda=0$

$\lambda=2$

Required circle equation is

$\Rightarrow x^2+y^2+2x-2y-4+4x+4y+4=0$

$\Rightarrow x^2+y^2+6x+2y=0$

28. $Z=w\ \ or\ \ w^2$

$$Z^n+\frac{1}{Z^n}=\begin{cases}-1, & if\ \ n\neq 3n\\ 2, & if\ \ n=3n\end{cases}$$

From 1 to 2020 there are 673 multiples of 3

and 1347 are non -multiples of 3

$=673(2)^3+1347(-1)^3=4037$

29. Let

$z=a+ib$

$\sqrt{a^2+b^2}-(a+ib)=2+i$

Compare real and imaginary parts we get

$a=\frac{-3}{4},b=-1\qquad \therefore |z|=\frac{5}{4}$

30. $amp(z-2-3i)=\frac{\pi}{4}$

$\tan^{-1}\left[\frac{y-3}{x-2}\right]=\frac{\pi}{4}\Rightarrow x-y+1=0$

$$\sum_{i=1}^{n}\frac{\cot^{-1}(2|\text{Im}\,z_i|)-1}{2\,\text{Re}\,z_i}=\sum_{i=1}^{n}\frac{\left(\pi-\frac{i\pi}{n}-1\right)}{-1}$$

$$=\frac{1}{2}[\pi-(\pi-2)n]$$

31. $1+\cos\theta=2\cos^2\frac{\theta}{2},\sin\theta=2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$

$(\cos\theta+i\sin\theta)^n=\cos n\theta+i\sin n\theta$

32. Let $z=x+iy$ and $|z|=\sqrt{x^2+y^2}$, then simplify.

33. $|(1+i)(1+3i)(1+7i)|=|x+iy|$

$\sqrt{2}.\sqrt{10}.\sqrt{50}=\sqrt{x^2+y^2}$

$(\because |z_1z_2z_3|=|z_1||z_2||z_3|)$

s. o. b. s

$x^2+y^2=1000\Rightarrow r^2=1000\Rightarrow r=10\sqrt{10}$

34. $\cos\theta\in[0,1]$ and $z=x+iy$

$|z-a|<|z-ai|$

$|(x-a)+iy|<|x+i(y-a)|$

$\Rightarrow(x-a)^2+y^2<x^2+(y-a)^2$

$\Rightarrow x>y$

35. $|z_1+z_2|^2=|z_1|^2+|z_2|^2$

Let $z_1=x_1+iy_1,z_2=x_2+iy_2$

$\Rightarrow x_1x_2+y_1y_2=0$

$$\frac{z_1}{z_2}=\frac{(x_1x_2+y_1y_2)}{x_2^2+y_2^2}+i\frac{(x_1y_2-x_2y_1)}{x_2^2+y_2^2}$$

$$=0+i\frac{(x_1y_2-x_2y_1)}{x_2^2+y_2^2}$$

It is purely imaginary.

36. $\left|\frac{3-4ix}{3+4ix}\right| = |\alpha - i\beta| \Rightarrow \alpha^2+\beta^2=1$

$\left(\because \left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}\right)$

37. $|z|+|2z-3|+|z-1| \geq |z-(2z-3)+(z-1)| \geq 2$

Formulae: $|z_1|+|z_2|+|z_3| \geq |z_1-z_2+z_3|$

38. $(a+ib)^3 = a-ib.....(1)$

$\overline{(a+ib)^3} = \overline{(a-ib)} \quad \left(\because \overline{z}^3 = \overline{z^3}\right)$

$(a-ib)^3 = a+ib.......(2)$

Equations $(1)\times(2) \Rightarrow (a^2+b^2)^3 = a^2+b^2$

Let $x=a^2+b^2 \Rightarrow x^3 = x \Rightarrow x=0,1,-1$

$a^2+b^2=0, then\ (a,b)=(0,0)$

$a^2+b^2=1, then\ (a,b)=(0,1)=(1,0)$

$a^2+b^2=-1, then\ (a,b)=(-1,0)$

Not possible $(a,b)=(0,-1)$

Total number of ordered pairs are 5.

39. Let $z=x+iy$, then $|z|=1 \Rightarrow x^2+y^2=1......(1)$

$|z-2|=1 \Rightarrow (x-2)^2+y^2=1.....(2)$

$|z-1|=0 \Rightarrow (x-1)^2+y^2=0.....(3)$

The above three equality satisfied (1, 0).

40. Area of $arcOAB$ -Area of $\Delta^{le}OAB$

$\frac{\pi 4}{4} - \frac{1}{2}2\times 2 \Rightarrow \pi - 2 = required\ region$

41. $i^4=1, i^{2020}=1, i^{2021}=i, i^{2022}=-1, i^{2023}=-i$

$\left|\frac{1}{1}+\frac{2}{i}+\frac{3}{-1}+\frac{4}{-i}\right| = \left|-2-\frac{2}{i}\right| = |-2+2i| = 2\sqrt{2}$

42. Verification

43. $\left\{i^{22} - \left(\frac{1}{i}\right)^{35}\right\}^2 = \left\{-1-(-i)^{35}\right\}^2 = (1+i)^2 = 2i$

44. Verification

45. $x+iy = \frac{1+7i}{(2-i)^2} = \frac{1+7i}{4-1-4i}$

$= \frac{1+7i}{3-4i} = \frac{(1+7i)(3+4i)}{25} = \frac{3-28+i(25)}{25}$

$x+iy=-1+i$

$\operatorname{cosec}\left(\tan^{-1} y/x - \frac{\pi}{4}\right)$

$= \operatorname{cosec}\left(\tan^{-1}(-1) - \pi/4\right) = \operatorname{cosec}\left(\frac{3\pi}{4}-\frac{\pi}{4}\right) = 1$

46. $(a+ib)^{\frac{1}{4}} = 2+3i$

$\Rightarrow (a+ib)^{\frac{1}{2}} = (2+3i)^2$

$\Rightarrow (a+ib)^{\frac{1}{2}} = -5+12i$

$\Rightarrow (a+ib) = (-5+12i)^2$

$\Rightarrow (a+ib) = -119-120i$

$a=-119, b=-120$

$3b-2a=-122$

47. Given, $z_1 = 1-2i, z_2 = 1+i$ and $z_3 = 3+4i$

$\left|\left(\frac{1}{z_1}+\frac{2}{z_2}\right)\frac{z_3}{z_2}\right| = \left|\frac{z_2z_3+2z_1z_3}{z_1z_2^2}\right| = \left|\frac{z_3}{z_1z_2}+2.\frac{z_3}{z_2^2}\right|$

Now, $\frac{z_3}{z_1z_2} = \frac{3+4i}{(1-2i)(1+i)} = \frac{3+4i}{3-i} = \frac{1}{2}(1+3i)$

$\frac{z_3}{z_2^2} = \frac{3+4i}{(1+i)^2} = \frac{3+4i}{2i} = \frac{1}{2}(4-3i)$

Now,

$\left|\frac{z_3}{z_1z_2}+2.\frac{z_3}{z_2^2}\right| = \left|\frac{1}{2}+\frac{3}{2}i+4-3i\right| = \left|\frac{9}{2}-\frac{3}{2}i\right| = \sqrt{\frac{45}{2}}$

48. Given, $x=\frac{4}{5}+\frac{3}{5}i, y=\frac{\sqrt{3}}{\sqrt{8}}-\frac{\sqrt{5}}{\sqrt{8}}i$

$\frac{1}{x} = \frac{4}{5}-\frac{3}{5}i$

$x+\frac{1}{x} = \frac{8}{5}$

$\boxed{x^2+\frac{1}{x^2} = \frac{14}{25}} \rightarrow (1)$

$\frac{1}{y} = \frac{\sqrt{3}}{\sqrt{8}}+\frac{\sqrt{5}}{\sqrt{8}}i$

$y+\frac{1}{y} = 2\frac{\sqrt{3}}{\sqrt{8}}$

$y-\frac{1}{y} = -\frac{2\sqrt{5}}{\sqrt{8}}i$

$\left(y+\frac{1}{y}\right)\left(y-\frac{1}{y}\right) = y^2-\frac{1}{y^2} = \frac{-\sqrt{15}}{2}i \rightarrow (2)$

Now,

$$(1)\times(2)\Rightarrow\left(x^2+\frac{1}{x^2}\right)\left(y^2-\frac{1}{y^2}\right)=\left(\frac{14}{25}\right)\times\left(-\frac{\sqrt{15}}{2}i\right)=\frac{-7\sqrt{3}}{5\sqrt{3}}i$$

49. $$2^8\left[\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^8-\left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)^8\right]$$

$$2^8\left[\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)^8-\left(\cos\frac{\pi}{6}-i\sin\frac{\pi}{6}\right)^8\right]$$

$$2^8.2i\sin\frac{8\pi}{6}=2^8.2i\sin\frac{4\pi}{3}=2^9 i\sin\left(\pi+\frac{\pi}{3}\right)$$

$$=-2^9 i\sin\frac{\pi}{3}=-2^9.\frac{\sqrt{3}}{2}=-2^8\sqrt{3}=\alpha+i\beta$$

$$\alpha=0,\beta=-2^8\sqrt{3}$$

$$\alpha-\frac{\sqrt{3}}{2}\beta=0-\frac{\sqrt{3}}{2}\left(-2^8.\sqrt{3}\right)=3\times2^7=384$$

50. $\sqrt{x^2-2x+8}+(x+4)i=y(2+i)$

Comparing real and imaginary parts on B.S.

$2y=\sqrt{x^2-2x+8},y=x+4.......(1)$

$$\Rightarrow 2(x+4)=\sqrt{x^2-2x+4}$$

$$3x^2+34x+56=0$$

$$\Rightarrow x=-2\ (or)\ x=\frac{-28}{3}$$

$$(1)\Rightarrow y=2\ \text{(or)}\ y=-\frac{16}{3}$$

$$\therefore Z=-2+2i\ (or)\ Z=-\frac{28}{3}-\frac{16}{3}i$$

51. $$\mathrm{Im}\left(\frac{z-3i}{iz+4}\right)=0$$

$$\frac{z-3i}{iz+4}=\frac{x+i(y-3)}{ix+4-y}$$

$$=\frac{x+i(y-3)}{(4-y)^2+x^2}\left[(4-y)-ix\right]$$

Imaginary part = 0

$$\frac{-x^2+(4-y)(y-3)}{x^2+(4-y)^2}=0$$

$$-x^2+(4-y)(y-3)=0\ and\ (x,y)\neq(0,4)$$

$$x^2+y^2-7y+12=0\ and\ (x,y)\neq(0,4)$$

52. $x^2+2x+2=0$

$$(x+1)^2=-1$$

$$x+1=\pm\sqrt{-1}=\pm i$$

$$x=-1+i,-1-i$$

$$z_1=-1+i,\ z_2=-1-i$$

$$\frac{-2^{11}(z_1+1+3i)^{11}}{2^5(z_2+1-3i)^{11}}=2^6=64$$

53. Conceptual

54. $iz^3+z^2-z+i=0$

$$\Rightarrow iz^3+z^2+i^2z+i=0$$

$$\Rightarrow(z^2+i)(iz+1)=0$$

$$\Rightarrow iz+1=0\Rightarrow z=\frac{-1}{i}=i$$

$$\therefore|z|=1$$

55. Given $\Rightarrow\dfrac{(x-1)(3-i)}{10}+\dfrac{(y-1)(3+i)}{10}=i$

On simplification we get

$$\left(\frac{3x+3y-6}{10}\right)+i\left(\frac{y-x}{10}\right)=i$$

On comparing, we get

$\Rightarrow x+y=2$ and $y-x=10$

On solving we get y=6, x=–4

$\therefore x<0,y>0$

56. $$|1-i|^x=2^x\Rightarrow\left(\sqrt{2}\right)^x=2^x$$

$$\Rightarrow\frac{x}{2}=x.\Rightarrow x=0$$

$\therefore$number of integral solutions=1

57. Multiplicative inverse of (a, b_) is

$$\left(\frac{a}{a+b^2},\frac{-b}{a^2+b^2}\right)$$

58. $$x+iy=\sum_{k=0}^{440}i^k$$

$$=110\left(1+i+i^2+i^3\right)+1$$

$= 110(0)+1$

$=1$

$\Rightarrow x=1 \,\&\, y=0$

$\therefore x^{100}+x^{99}y+x^{242}\cdot y^2+x^{97}\cdot y^3=1$

59. $wkt\ i^2=-1$ and $i^{4n}=1 \forall n \in N$

G.E $=(-1)^9-3i(-1)^3+(-1)(2)(-1)^{11}$

$=-1+3i+2$

$=1+3i$

60. Put $z=x+iy$

$|x+iy|^2=\text{Re}(x+iy)$

$\Rightarrow x^2+y^2=x$

$\Rightarrow x^2+y^2-x=0$ is a circle with

Centre $(1/2,0)$

61. $x-iy=(a-ib)^3$

$=a(a^2-3b^2)-ib(3a^2-b^2)$

$\Rightarrow \frac{x}{a}+\frac{y}{b}=4(a^2+b^2)$

$\Rightarrow \frac{x}{2a}+\frac{y}{2b}=2(a^2+b^2)$

62. $x+iy=\left(\frac{1+i}{1-i}\right)^3-\left(\frac{1-i}{1+i}\right)^3$

$=\left(\frac{2i}{1+1}\right)^3-\left(\frac{-2i}{1+1}\right)^3$

$=-i-i=-2i$

$\therefore x=0, y=-2 \Rightarrow x>y$

63. $\bar{z}=iz^2 \rightarrow (1)$ $\quad$ *put* $z=x+iy$

$\Rightarrow x-iy=i(x^2-y^2+i2xy)$

$=-2xy-i(y^2-x^2)$

$\therefore x=-2xy$ and $y=y^2-x^2$

$\Rightarrow y=\frac{-1}{2}$ and $x^2=\frac{1}{4}+\frac{1}{2}=\frac{3}{4}$

$\Rightarrow x=\pm\frac{\sqrt{3}}{2}$

$(x,y)=\left(\frac{-\sqrt{3}}{2},\frac{-1}{2}\right),\left(\frac{+\sqrt{3}}{2},\frac{-1}{2}\right)$

$\therefore$Number of complex nos. satisfying (1) is '2'.

64. The minimum value of $|z-a|+|z-b|$ is

$[|b-a|,\infty)$

$\Rightarrow$the minimum value of $|z|+|z-1|$ is

$|z|+|z-1|$ is $[1,\infty)$

65. conceptual

66. $\bar{z}+i\bar{\omega}=0 \Rightarrow \bar{z}=-i\bar{\omega}$

$\Rightarrow z=i\omega$

$\Rightarrow \omega=\frac{z}{i}$

$\therefore Arg(z\omega)=\pi \Rightarrow Arg\left(\frac{z^2}{i}\right)=\pi$

$\Rightarrow 2Argz-Arg(i)=\pi$

$\Rightarrow 2Argz=\pi+\frac{\pi}{2}=\frac{3\pi}{2}$

$\Rightarrow Argz=\frac{3\pi}{4}$

67. If z_1,z_2,z_3 are the vertices of an equilateral triangle. Then $z_1^2+z_2^2+z_3^2=z_1z_2+z_2z_3+z_3z_1$ put $z_3=0$ we get $z_1^2+z_2^2=z_1z_2$

68. $\bar{z}z^3+z\bar{z}^3=350$

$z\bar{z}(z^2+\bar{z}^2)=350$

$(x^2+y^2)(2(x^2-y^2))=350$

$(x^2+y^2)(x^2-y^2)=175$

$x=\pm4$, $y=\pm3$

Area of rectangle $=8\times6=48$

69. Put $z=x+iy$ then the vertices of Δ^{le} are $A(x,y)$, $B(-y,x)$ and $C(x-y,x+y)$

$\therefore$Area $(\Delta)=\frac{1}{2}\begin{vmatrix} x_1-x_2 & x_1-x_3 \\ y_1-y_2 & y_1-y_3 \end{vmatrix}$

70. If $(x-iy)^{\frac{1}{3}}=a-ib$ then $\frac{x}{a}+\frac{y}{b}=4(a^2-b^2)$

$\therefore (x-iy)^{\frac{1}{3}}=2-i\sqrt{3}$ then a=2, $b=\sqrt{3}$

$\therefore \frac{x}{2}+\frac{y}{\sqrt{3}}=4(4-3)=4$

71. $|z|=\left|\left(z+\frac{2}{z}\right)-\frac{2}{z}\right|$

$$\le\left|z+\frac{2}{z}\right|+\frac{2}{|z|}=2+\frac{2}{|z|}$$

$$\Rightarrow |z|^2-2|z|+1\le 3$$

$$\Rightarrow (|z|-1)^2\le 3 \Rightarrow |z|\le\sqrt{3}+1$$

72. conceptual

73. $|z-z_1|+|z-z_2|=k$ represents an empty set

If $k<|z_1-z_2|$

74. $\sqrt{(-3+4i)(8+6i)}=\sqrt{-48+14i}$

$=\sqrt{2}\sqrt{-24+7i}$

$=\pm\sqrt{2}\left(\frac{1}{\sqrt{2}}+i\frac{7}{\sqrt{2}}\right)$

$=\pm(1+7i)$

75. $\left(\frac{\sqrt{3}+i}{\sqrt{3}-i}\right)^m=1\Rightarrow\left(\frac{2+2\sqrt{3}}{3+1}\right)^m=1$

$$\Rightarrow\left(\frac{1+i\sqrt{3}}{2}\right)^m=1$$

$\Rightarrow(-1)^m\left(\frac{-1-i\sqrt{3}}{2}\right)^m=1$

$\Rightarrow(-1)^m.\omega^{2m}=1$

$\Rightarrow m=2028$ (By inspection)

76. $\frac{x+i(x-2)}{3+i}-i=\frac{2y+i(1-3y)}{i-3}$

On simplifying , we get

$(4x-2)+i(2x-16)=(-9y+1)+i(7y-3)$

On solving we get x=3, y=-1

$\therefore x+y=3-1=2$.

77. $2x-y+1=2$ and $x-2y-1=-3$

$\Rightarrow 2x-y=1$ and $2x-4y=-4$

On solving, we get $x=\frac{4}{3}$ and $y=\frac{5}{3}$

$\therefore z=x-iy=\frac{4-5i}{3}$

M.I. of $(x-iy)$ is $\left(\frac{x+iy}{x^2+y^2}\right)=\frac{\frac{1}{3}(4+5i)}{\frac{16}{9}+\frac{25}{9}}$

$=\frac{12+15i}{41}$

$=\frac{12}{41}+\frac{15}{41}i$

78. $|z|-z=1+2i$

$\Rightarrow|\alpha+i\beta|-(\alpha+i\beta)=1+2i$

$\Rightarrow\sqrt{\alpha^2+\beta^2}-\alpha=1$ and $\beta=-2$

$\Rightarrow\alpha^2+\beta^2=(\alpha+1)^2=\alpha^2+2\alpha+1$

$\Rightarrow 2\alpha+1=4.\ \Rightarrow\alpha=\frac{3}{2}$

Now $z.\overline{z}=\alpha^2+\beta^2=\frac{9}{4}+4=\frac{25}{4}$.

79. W k t $\overline{a+ib}=a-ib$

Given$\Rightarrow\sin x-i\cos 2x=\cos x-i\sin 2x$

$\Rightarrow\tan x=1$ and Tan 2x = 1

Which is not possible for any real 'x' solution set = ϕ

80. $x+iy=\begin{vmatrix}2+3i & i\\ 1-2i & -i\end{vmatrix}=\begin{vmatrix}3+i & 0\\ 1-2i & -i\end{vmatrix}$

=1−3i

$\therefore x=1, y=-3\Rightarrow x+y=-2$

81. $\frac{3+2i\sin\theta}{1-2i\sin\theta}$ is real $\Leftrightarrow$ $\sin\theta=0$

$\Leftrightarrow\theta=n\pi, n\in z$

82. $z_1-z_2=(-4,-4)$

$z_1+z_2=(8,2)$

$\text{amp}\left(\frac{z_1-z_2}{z_1+z_2}\right)=\text{amp}(z_1-z_2)+\text{amp}(z_1+z_2)$

$=\tan^{-1}\left(\frac{-4}{-4}\right)-\tan^{-1}\left(\frac{2}{8}\right)$

$=-\left(\pi-\frac{\pi}{4}\right)-\tan^{-1}\left(\frac{1}{4}\right)$

$=-\frac{3\pi}{4}-\tan^{-1}\frac{1}{4}$

83. Put $z = x + iy$ & simplify

84. $|z-1+i|=1$

$|x+iy-1+i|=1$

$|(x-1)+i(y+1)|=1$

$\sqrt{(x-1)^2+(y+1)^2}=1$

$(x-1)^2+(y+1)^2=1$

Is a circle with centre (1,-1) and radius =1 unit

85. $\left|z-\frac{2}{z}\right|=2$ given

$\left|\left(z-\frac{2}{z}\right)+\frac{2}{z}\right|\le\left|z-\frac{2}{z}\right|+\left|\frac{2}{z}\right|$

$|z|\le 2+\frac{2}{|z|}$

$|z|^2\le 2|z|+2$

$|z|^2-2|z|-2\le 0$

$\boxed{|z|=1\pm\sqrt{3}}$

Max value $=\sqrt{3}+1$

86. $\sqrt{-3-4i}=\pm\left[\sqrt{\frac{5-3}{2}}-i\sqrt{\frac{5+3}{2}}\right]$

$=\pm(1-2i)$

$\therefore r=\sqrt{5}$

$\tan\theta=-2$

$r^2\tan\theta=-10$

87. Given z_1 =2-3i and the roots of the eq

$z^3+bz^2+cz+d=0$ are $i, z_1, \overline{z}_1$

now $i+z_1+\overline{z}_1=i+4=-b=s_1$

$iz_1+z_1\overline{z}_1+\overline{z}_1\, i=c=s_2$

$4i+13=c$

$s_3=iz_1\overline{z}_1=-d\Rightarrow d=-13i$

$now\, b+c+d=-i-4+4i+13-13i=9-10i$

88. Given $z=\sqrt{\frac{1-i}{1+i}}$

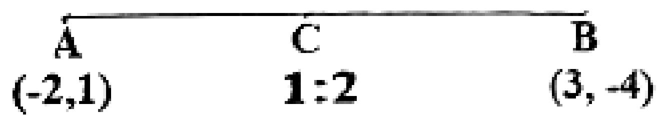
$z=\sqrt{\frac{-\cancel{2}i}{\cancel{2}}}=\sqrt{-i}=$

$=\pm\left(\frac{1}{\sqrt{2}}-\frac{i}{\sqrt{2}}\right)$

$Argz_1=-\frac{\pi}{4}$

$Argz_2=\frac{3\pi}{4}$

89.



$C=\left(\frac{3-4}{3},\frac{-4+2}{3}\right)$

$=\left(\frac{-1}{3},\frac{-2}{3}\right)\rightarrow 3^{rd}$ $Quadrant$

$Arg\,(c)=-\pi+\tan^{-1}\left(\frac{\left|\frac{-2}{3}\right|}{\left|\frac{-1}{3}\right|}\right)$

$=-\pi+\tan^{-1}(2)$

90. $z_1=4+3i$

$|z_1|=\alpha=5$

$z_1=4-3i$

$|z-\overline{z_1}|\le\alpha$

$|x+iy-4+3i|\le\alpha$

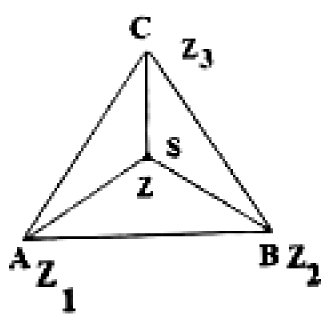
$(x-4)^2+(y+3)^2\le 25$

Clearly, z(4,3) doesn't lies in above region

$\therefore 36^2\le 25$

not possible

91.



We know, SA = SB = SC

$$|z-z_1| = |z-z_2| = |z-z_3|$$

$$\Rightarrow |z-z_1|^2 = |z-z_1|.|z-z_1|$$

$$\Rightarrow |z-z_1|^2 = |z-z_3| \times |z-z_2|$$

$$\Rightarrow \frac{|z-z_1|}{|z-z_2|} = \frac{|z-z_3|}{|z-z_1|}$$

92. $4a + i(3a-b) = b - 6i$

$4a = b$, $3a - b = -6$

$$\Rightarrow 3a - 4a = -6 \Rightarrow a = 6$$

$$\Rightarrow \boxed{b = +24}$$

$$z = a + i\frac{b}{4} = 6 + i6$$

$$|z| = \sqrt{72} = 6\sqrt{2}$$

$$\frac{|z|}{a} = \frac{6\sqrt{2}}{6} = \sqrt{2}$$

93. $z = (1-i)^3 (x+i)$

$$z = (2-2x) - i(2+2x)$$

purely Imaginary $\Rightarrow 2 - 2x = 0 \Rightarrow x = 1 = x_1$

Purely real $\Rightarrow 2 + 2x = 0 \Rightarrow x = -1 = x_2$

$$x_1 x_2 = -1$$

94. $z = \dfrac{-2+i}{-3-4i} \times \dfrac{-3+4i}{-3+4i}$

$$= \frac{6 - 8i - 3i - 4}{9 - 16i^2} = \frac{2 - 11i}{25}$$

$$|z| = \sqrt{\frac{4+121}{625}} = \frac{1}{\sqrt{5}}$$

95. $Z_1 - Z_3 = 2 + 4i$

$$Z_3 - Z_2 = 1 - 3i$$

$$\frac{|Z_1 - Z_3|}{|Z_3 - Z_2|} = \frac{\sqrt{20}}{\sqrt{10}} = \sqrt{2}$$

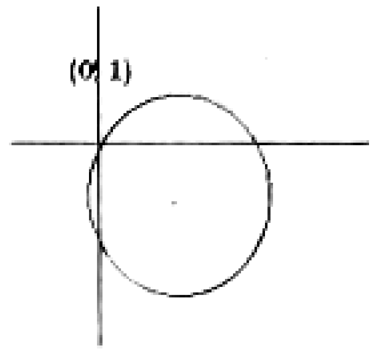
96.

$$\theta = Tan^{-1}\frac{y}{x}$$

$$y = xTan\theta$$

97. Z=x+iy

$$|z - 1 + i| \le 2$$



$$\sqrt{(x-1)^2 + (y+1)^2} \le 2$$

$$(x-1)^2 + (y+1)^2 \le 4$$

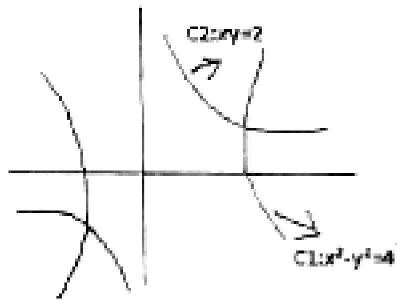
It is a reason represented by

$$(x-1)^2 + (y+1)^2 = 4$$

By verification (0, 1) not in the region

$\therefore$ Required point =i

98. Let z=x+iy



$$z^2 = x^2 - y^2 + 2xyi$$

$$\operatorname{Re}(z^2) = 4$$

$$C_1 : x^2 - y^2 = 4$$

It is a rectangular hyperbola

$$\operatorname{Im}(z^2) = 4$$

$$2xy = 4$$

$$C_2 : \text{xy} = 2$$

It is a rectangular hyperbola having coordinate axes as its asymptotes.

$\therefore$ No of common points of curves C1 and C2=2

99. $r = |z| = 1$

$and\ z = \theta = \pi/6$

$$z = re^{i\theta} = 1.(\cos\pi/6 + i\sin\pi/6)$$

$$\frac{z^{12}+1-z^6}{z^{12}+iz^6-1} = \frac{\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)^{12} + 1 - \left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)^6}{\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)^{12} + i\left(\cos\pi/6 + i\sin\pi/6\right)^6 - 1}$$

$$= \frac{\cos 2\pi + i\sin 2\pi + 1 - (\cos\pi + i\sin\pi)}{\cos 2\pi + i\sin 2\pi + i(\cos\pi + i\sin\pi) - 1}$$

$$= \frac{1+1+1}{1-i-1} = \frac{-3}{i} = 3i$$

100. - 3 + ix2y and x2 + y + 4i are complex conjugates, then

- 3 + ix2y = x2 + y – 4i

Comparing real & Imaginary parts

x2 + y =-3 x2y = -4

$$x2+\left(\frac{-4}{x^2}\right)=-3 \qquad y=-\,{}^{4}\!/_{x^2}$$

x4 + 3x2 – 4 = 0

(x2 + 4) (x2 – 1) = 0

x2 = -4   x2 = 1

x = ± 1

101. Given, Z = 1 + cos□ – i sin □

$$\left[|z-1|^2-\frac{|z|^2}{4}\right]^{1/2}$$

$$=\left[|1+\cos\theta-i\sin\theta-1|^2-\frac{|1+\cos\theta-i\sin\theta|^2}{4}\right]^{1/2}$$

$$=\left[\left(\sqrt{\cos^2\theta+\sin^2\theta}\right)^2-\frac{\left(\sqrt{(1+\cos\theta)^2+(\sin\theta)^2}\right)^2}{4}\right]^{1/2}$$

$$=\left[1-\frac{1+\cos^2\theta+2\cos\theta+\sin^2\theta}{4}\right]^{1/2}$$

$$=\left[1-\frac{1+\cos\theta}{2}\right]^{1/2}=\left[\frac{1-\cos\theta}{2}\right]^{1/2}$$

$$=\left(\sin^2\frac{\theta}{2}\right)^{1/2}=\sin\frac{\theta}{2}$$

102. $|z-1|=|i(z+1)|$

$$|x+iy-1|=|i(x+iy+1)|$$

$$|(x-1)+iy|=|ix+i^2y+i|$$

$$\sqrt{(x-1)^2+y^2}=\sqrt{y^2+(x+1)^2}$$

2x + 2x = 0

4x = 0

x = 0

∴ lie on y – axis

103. $$=\frac{8-6+i(16)+11+i(-2)}{8-i}$$

$$=\frac{13+14i}{8-i}\times\frac{8+i}{8+i}$$

$$=\frac{90+i125}{65}$$

$$Arg\left(\frac{90}{65}+i\frac{125}{65}\right)$$

$$\theta=Tan^{-1}\left(\frac{125}{90}\right)\quad \theta\in Q_1 \text{ and more than}$$

$$\therefore \theta\in\left(\frac{\pi}{4},\frac{\pi}{2}\right)$$

104. Multiplicative inverse of $\bar{Z}$ is $\frac{1}{\bar{z}}$

$$\Rightarrow \frac{1}{z}\times\frac{\bar{z}}{\bar{z}}=\frac{\bar{z}}{|z|^2}$$

105. $Z_3=\frac{z_1+z_2}{2}$

$$\Rightarrow 2z_3=6-2i$$

$$\Rightarrow z_3=3-i$$

$$Now\ 5z_1+xz_2+yz_3=0$$

$$\Rightarrow 10+15i+4x-5xi+3y-yi=0$$

$$\Rightarrow 4x+3y+10+i(-5x-y+15)=0$$

$$\therefore 4x+3y+10=0\ldots\ldots\ldots..(1)$$

$$5x+y-15=0\ldots\ldots\ldots\ldots(2)$$

Solve we get $x=5$ & $y=-10$

$$x+y=-5$$

106. $|z_1+z_2|=|z_1|+|z_2|$

$\Rightarrow z_1, z_2$ are collinear

$$Arg z_1 = Arg z_2$$

107. $1+i^2+i^4+i^6+\ldots+i^{2024}$

$$=\cancel{1}-\cancel{1}+\cancel{1}-\cancel{1}+\ldots.\cancel{1}-\cancel{1}+\cancel{1}$$

$$=1$$

108. $\dfrac{1+i\cos\theta}{1-2i\cos\theta}=\dfrac{1+i\cos\theta}{1-2i\cos\theta}\times\dfrac{1+2i\cos\theta}{1+2i\cos\theta}$

$$=\frac{1+2i\cos\theta+i\cos\theta-2\cos^2\theta}{1+4\cos^2\theta}$$

$$=\frac{1-2\cos^2\theta+3i\cos\theta}{1+4\cos^2\theta}\ \text{which is purely real}$$

$$\therefore \frac{3\cos\theta}{1+4\cos^2\theta}=0$$

$$\cos\theta=0 \Rightarrow \sin\theta=-1(or)1$$

$$Now\cos^3\theta+\sin^2\theta+\cos\theta+1=0+1+0+1$$

$$=2$$

109. $Given\ z=(\alpha+i\beta)(2+7i)\rightarrow(1)$

$$=(2\alpha-7\beta)+i(7\alpha+2\beta)$$

Now, z is purely imaginary

$$2\alpha-7\beta=0\Rightarrow 2\alpha=7\beta\Rightarrow\frac{\alpha}{\beta}=\frac{7}{2}$$

$$\frac{\alpha+i\beta}{\beta}=\frac{7+2i}{2}\Rightarrow\alpha+i\beta=7+2i\rightarrow(2)$$

sub 2 in 1, we get

$$z=(7+2i)(2+7i)=\cancel{14}+49i+4i\cancel{-14}$$

$$=53i$$

$$|z|^2=(53)^2=2809$$

110.

$$\sin\left(\frac{6\pi}{5}\right)+i\left(1+\cos\frac{6\pi}{5}\right) or$$

$$-\sin\left(\frac{\pi}{5}\right)+i\left(1-\cos\frac{\pi}{5}\right)\in 2nd\ Q$$

$$\arg z=\pi-Tan^{-1}(y/x)$$

$$=\pi-Tan^{-1}\left|\frac{1-\cos\pi/5}{\sin\pi/5}\right|$$

$$=\pi-Tan^{-1}\left|\frac{2\sin^2\frac{\pi}{10}}{2\sin\frac{\pi}{10}\cos\frac{\pi}{10}}\right|$$

$$=\pi-Tan^{-1}\left|\frac{\sin\frac{\pi}{10}}{\cos\frac{\pi}{10}}\right|=\pi-\frac{\pi}{10}=\frac{9\pi}{10}$$

111.

if $\quad x+iy=\sqrt{\dfrac{3+i}{1+3i}}$

$$x+iy=\sqrt{\frac{a+ib}{c+id}}\quad Then\ (x^2+y^2)^2=\frac{a^2+b^2}{c^2+d^2}$$

$$=\frac{9+1}{1+9}=1$$

112. $Z=x+iy$

$$\frac{2Z+1}{1+iz}=\frac{(2x+1)+2iy}{(1-y)+ix}\times\frac{(1-y)-ix}{(1-y)-ix}$$

imaginary part = -2

$$2y(1-y)-x(2x+1)=-2\left((1-y)^2+x^2\right)$$

$2y+x=2$ is a straight line

113. $\sqrt{-5-12i}+\sqrt{7+24i}$

$$=\pm\left[\sqrt{\frac{\sqrt{a^2+b^2}+a}{2}}-i\sqrt{\frac{\sqrt{a^2+b^2}-a}{2}}\right]$$

$$\pm[2-i3]\pm[4+3i]$$

By simplifying we get 6,-6

Negative real number is -6

114. $\left(\dfrac{Z-3}{Z+2i}\right)$

$$\frac{(x+iy)-3}{(x+iy)+2i}\Rightarrow\frac{(x-3)+iy}{x+i(y+2)}$$

By rationalizing with $x-i(y+2)$ *we get*

$$\Rightarrow\frac{x^2+y^2-3x+2y}{x^2+(y+2)^2}+\frac{i(-2x+3y+6)}{x^2+(y+2)^2}$$

$$\Rightarrow\frac{-2x+3y+6}{x^2+y^2-3x+2y}=\frac{1}{0}$$

$\therefore x^2+y^2-3x+2y=0$ (it satisfies (0,0))

$\therefore -2x+3y+6>0$

$\therefore$ a semicircular arc containing the origin

115. $\dfrac{Z-(2+i)}{Z+(1-2i)}$

$$\frac{(x+iy)-(2+i)}{(x+iy)+(1-2i)}$$

$$\frac{(x-2)+i(y-1)}{(x+1)+i(y-2)}$$

By rationalizing with (x+1)-i(y-2) we get

$$\Rightarrow \frac{x^2+y^2-x-3y}{(x+1)^2+(y-2)^2}+\frac{i(x+3y-5)}{(x+1)^2+(y-2)^2}$$

It is purely real

$$\Rightarrow \frac{x+3y-5}{(x+1)^2+(y-2)^2}=0$$

$\therefore$ Imaginary =0

$x+3y-5=0 \Rightarrow (x,y)\neq(-1,2)$.

116.

Given i= $\sqrt{-1}$, $\left|\sum_{n=0}^{\infty}\left(\frac{i}{3}\right)^n\right|$

$=1+\frac{i}{3}+\left(\frac{i}{3}\right)^2+....$

$=\left(1-\frac{i}{3}\right)^{-1}$

$=\frac{1}{1-\frac{i}{3}}$

$=\frac{3}{3-i}=\frac{3}{3-i}\times\frac{3+i}{3+i}$

$=\frac{9+3i}{10}$

117.

Arg$\left[\frac{(1+i)^{2025}}{(1-i)^{2022}}\right]$

= Arg (1+i)2025- Arg (1-i)2022

$=\left(e^{i\frac{\pi}{4}}\right)^{2025}-\left(e^{-i\frac{\pi}{4}}\right)^{2022}$

$=\frac{\pi}{4}-\frac{\pi}{2}=-\frac{\pi}{4}$

118. $\left|\frac{z-i}{z+i}\right|=2$

$\left|\frac{x+i(y-1)}{x+i(y+1)}\right|=2$

$x^2+(y-1)^2=4(x^2+(y+1)^2)$

$x^2+y^2+1-2y=4x^2+4y^2+8y+4$

$\Rightarrow \quad 3x^2+3y^2+10y+3=0$

119. α,β are roots of $z^2-i=0$

$z^2=i$

$Z=\pm i$

$\therefore\ \alpha=-i,\beta=i$

$|Arg i-Arg(-i)|=|Arg i-Arg(-i)|$

$=\left|\frac{\pi}{2}-\left(-\frac{\pi}{2}\right)\right|=\pi$

120. $\left(1+\sqrt{3}i\right)^{2022}=2^{2022}[\cos 674\pi+i\sin 674\pi]$

$\left(1+\sqrt{3}i\right)^{2022}=-2^{2022}$

$\therefore\left(1+\sqrt{3}i\right)^{2022}-\left(\sqrt{3}-i\right)^{2022}$

$=2^{2022}+2^{2022}$

$=2.2^{2022}=2^{2023}$

121. $\left(\frac{\sqrt{3}+i}{\sqrt{3}-i}\right)^4+\left(\frac{\sqrt{3}-i}{\sqrt{3}+i}\right)=r\,cis\theta$

$\left(-\omega^2\right)^4+(-\omega)^4=\omega^8+\omega^4=\omega^2+\omega=-1$

$\therefore r\,cis\theta=-1$

Find $\sqrt{r\,cis\theta}=(-1)^{\frac{1}{2}}$

$=[\cos\pi+i\sin\pi]^{\frac{1}{2}}$

$=cis\left(\frac{2k\pi+\pi}{2}\right),\ k=0,1$

$=cis\frac{\pi}{2},cis\frac{3\pi}{2}$

122. $z=x+iy$

$|z-2|+|z-2i|=4$

$|x+iy-2|+|x+iy-2i|=4$

$|(x-2)+iy|+|x+i(y-2)|=4$

$\sqrt{(x-2)^2+y^2}=4-\sqrt{x^2+(y-2)^2}$

Squaring on both sides, we get

3x2+2xy+3y2-8x-8y=0

DEMOVIERS THEOREM

1. If $2\alpha=-1-i\sqrt{3}$ and $2\beta=-1+i\sqrt{3}$ then $5\alpha^4+5\beta^4+7\alpha^{-1}\beta^{-1}$ is equal to

[AP EAMCET 17-09-20_Shift-1]

1. -1 2. -2
3. 0 4. 2

2. If $1,\alpha,\alpha^2,....,\alpha^{n-1}$ are the n-th roots of unity, then $\sum_{i=1}^{n-1}\frac{1}{2-\alpha^i}$ is equal to

[AP EAMCET 17-09-20_Shift-1]

1. $(n-2)2^n$ 2. $\frac{(n-2)2^{n-1}+1}{2^n-1}$
3. $\frac{(n-2)2^{n-1}}{2^n-1}$ 4. $\frac{1}{(n-2)2^n}$

3. If α *and* β are non-real roots of $x^3-x^2-x-2=0$ then $\alpha^{2020}+\beta^{2020}+\alpha^{2020}\beta^{2020}=$

[AP EAMCET 18-09-20_Shift-1]

1. 1 2. 2020
3. $1+\alpha+\beta$ 4. -1

4. Find $\alpha^4+\beta^4$, if α,β are the roots of the equation $x^2+x+1=0$ **[AP EAMCET 18-09-20_Shift-1]**

1. $\frac{1}{\alpha\beta}$ 2. $\frac{2}{\alpha\beta}$
3. $\alpha\beta$ 4. $-\alpha\beta$

5. If α,β are the roots of $x^2-2x+4=0$, for $n\in N$, what is the value of $\alpha^n+\beta^n=$

[AP EAMCET 18-09-20_Shift-2]

1. $2^{n+2}\cos\left(\frac{n\pi}{3}\right)$ 2. $2^{n+1}\cos\left(\frac{n\pi}{3}\right)$
3. $2^{n+1}\cos\left(\frac{n\pi}{6}\right)$ 4. $2^{n+2}\cos\left(\frac{n\pi}{6}\right)$

6. $(-i+\sqrt{3})^{300}+(-i-\sqrt{3})^{300}=$

[AP EAMCET 21-09-20_Shift-1]

1. 2^{300} 2. 2^{301}
3. 2^{100} 4. -2^{300}

7. If α,β are the roots of the equation $x^2-2x+4=0$, then $\alpha^n+\beta^n=$ ____ $\times\cos\left(\frac{n\pi}{3}\right)$ for any $n\in N$

[AP EAMCET 22-09-20_Shift-1]

1. 2^n 2. 2^{n+1}
3. 2^{n-1} 4. 2^{n-2}

8. The modulus of the complex number $\left(\frac{2+i\sqrt{5}}{2-i\sqrt{5}}\right)^{10}+\left(\frac{2-i\sqrt{5}}{2+i\sqrt{5}}\right)^{10}$ is

[AP EAMCET 22-09-20_Shift-1]

1. $2\cos\left(20\,\mathrm{Cos}^{-1}\left(\frac{2}{3}\right)\right)$ 2. $2\sin\left(10\,\mathrm{Cos}^{-1}\left(\frac{2}{3}\right)\right)$
3. $2\cos\left(10\,\mathrm{Cos}^{-1}\left(\frac{2}{3}\right)\right)$ 4. $2\sin\left(20\,\mathrm{Cos}^{-1}\left(\frac{2}{3}\right)\right)$

9. $\left(\frac{1+\cos(3\theta)+i\sin(3\theta)}{1+\cos(3\theta)-i\sin(3\theta)}\right)^{20}=?$

[AP EAMCET 23-09-20_Shift-1]

1. $Cos(60^\theta)+i\sin(60^\theta)$ 2. $Cos(60^\theta)-i\sin(60^\theta)$
3. $Cos(20^\theta)+i\sin(20^\theta)$ 4. $Cos(20^\theta)-i\sin(20^\theta)$

10. The value of $|z|^2+|z-3|^2+|z-i|^2$ is minimum when z equals

1. $2-\frac{2}{3}i$ 2. $45+3i$
3. $1+\frac{1}{3}i$ 4. $1-\frac{1}{3}i$

11. If $\sqrt{x}+\frac{1}{\sqrt{x}}=2\cos\theta$, then $x^6+x^{-6}=$

[TS EAMCET 09-09-20_Shift-1]

1. $2\cos6\theta$ 2. $2\cos12\theta$
3. $2\cos3\theta$ 4. $2\sin3\theta$

12. If ω is a complex cube root of unity, then $\left(\frac{1-\sqrt{3}i}{2}\right)^{2020}+\left(\frac{1+\sqrt{3}i}{2}\right)^{2026}+\sin\left(\sum_{j=1}^{6}(j+\omega)(j+\omega^2)\frac{3\pi}{152}\right)=$

[TS EAMCET 09-09-20_Shift-2]

1. -2
2. 2
3. -1
4. 0

13. If $z=e^{i\theta}$ and $\frac{3\cos\theta+2\cos 2\theta+5\cos 5\theta}{3\sin 3\theta+2\sin 2\theta+5\sin 5\theta}=\frac{i\sum_{r=0}^{10}a_r z^r}{\sum_{r=0}^{10}b_r z^r}$

Then $\frac{\left(\sum_{r=0}^{10}a_r+\sum_{r=0}^{10}b_r\right)}{10}=$

[TS EAMCET 09-09-20_Shift-2]

1. 0
2. 1
3. -2
4. 3

14. $\left(\frac{\cos\theta+i\sin\theta}{\sin\theta+i\cos\theta}\right)^8+\left(\frac{1+\cos\theta-i\sin\theta}{1+\cos\theta+i\sin\theta}\right)^{16}=$

[TS EAMCET 10-09-20_Shift-1]

1. $2\cos 8\theta$
2. $2\cos 16\theta$
3. $2\sin 8\theta$
4. $2\sin 16\theta$

15. If $\left(\frac{\cos\theta+i\sin\theta}{\sin\theta+i\cos\theta}\right)^{2020}+\left(\frac{1+\cos\theta+i\sin\theta}{1-\cos\theta+i\sin\theta}\right)^{2021}=x+iy$, then the value of $x+y$ at $\theta=\frac{\pi}{2}$ is

[TS EAMCET 10-09-20_Shift-2]

1. 2
2. 1
3. -1
4. 2020

16. If ω is a complex cube root of unity, then $\sum_{x=1}^{10}\left((\omega x+2)(\omega^2 x+2)-3\right)$

[TS EAMCET 10-09-20_Shift-2]

1. 285
2. 945
3. 1025
4. 705

17. For some $a,b,c\in R$, if $\sin 5\theta=a\cos^4\theta\sin\theta+b\cos^2\theta\sin^3\theta+c\sin^5\theta$, then $abc=$ **[TS EAMCET 11-09-20_Shift-1]**

1. -10
2. 10
3. 0
4. -50

18. For $n\in N$, if $A_n=\cos\left(\frac{\pi}{2^n}\right)+i\sin\left(\frac{\pi}{2^n}\right)$, then $(A_1A_2A_3A_4)^4=$

[TS EAMCET 11-09-20_Shift-1]

1. $\frac{-1-i}{\sqrt{2}}$
2. 1
3. 0
4. $\frac{1-i}{\sqrt{2}}$

19. Let $A_r=\left(x+\frac{1}{x}\right)^3\cdot\left(x^2+\frac{1}{x^2}\right)^3\cdot\left(x^3+\frac{1}{x^3}\right)^3\cdots\left(x^r+\frac{1}{x^r}\right)^3$, if $x^2+x+1=0$, then $\frac{1}{A_3}+\frac{1}{A_6}+\frac{1}{A_9}+\frac{1}{A_{12}}+\ldots\infty=$

[TS EAMCET 11-09-20_Shift-1]

1. $\frac{1}{6}$
2. $\frac{2}{5}$
3. 1
4. $\frac{1}{7}$

20. If $\left(\sqrt{3}+i\right)^{10}=a+bi,\ a,b\in R,$ then the values of a and b are respectively

[TS EAMCET 11-09-20_Shift-2]

1. 64 and $-64\sqrt{3}$
2. 128 and $128\sqrt{3}$
3. 256 and $256\sqrt{3}$
4. 512 and $-512\sqrt{3}$

21. If $\alpha,\beta,\gamma,\delta$ are the roots of the equation $x^4+x^2+1=0$, then $\frac{\alpha^3+\beta^3+\gamma^3+\delta^3}{\alpha^6+\beta^6+\gamma^6+\delta^6}=$

[TS EAMCET 11-09-20_Shift-2]

1. 0
2. 1
3. -1
4. $\frac{1}{2}$

22. For n>1 and $n\in N$, if $z_1,z_2,\ldots.z_n$ are the roots of the equation $(z+1)^n=z^n$ then $\sum_{i=1}^{n}\frac{\cot^{-1}\left(2|\operatorname{Im}z_i|\right)-1}{2\operatorname{Re}z_i}=$

[TS EAMCET 14-09-20_Shift-2]

1. 0
2. I
3. $\frac{1}{2}\left[\pi-(\pi-2)n\right]$
4. $\frac{1}{2}\left[\pi+(\pi+2)n\right]$

23. $(\sin\theta - i\cos\theta)^3 =$

[AP EAMCET 19-08-2021_Shift-1]

1. $i^3(\cos 3\theta + i\sin 3\theta)$
2. $(\cos 3\theta + i\sin 3\theta)$
3. $\sin 3\theta - i\cos 3\theta$
4. $(-i^3)(\cos 3\theta + i\sin 3\theta)$

24. If $\left(\frac{1+i}{1-i}\right)^m = 1$, then '$m$' cannot be equal to

[AP EAMCET 19-08-2021_Shift-2]

1. 1934
2. 2024
3. 2172
4. 10^{100}

25. If $1, \alpha_1, \alpha_2, \alpha_3, \alpha_4$ are the roots of $x^5 - 1 = 0$ and ω is a cube root of unity then $(\omega-1)(\omega-\alpha_1)(\omega-\alpha_2)(\omega-\alpha_3)(\omega-\alpha_4)+\omega$ is equal to

[AP EAMCET 20-08-2021_Shift-1]

1. 0
2. -1
3. -2
4. 1

26. What is the value of $(1-i\sqrt{3})^9 =$

[AP EAMCET 20-08-2021_Shift-2]

1. 2^9
2. -2^9
3. $2^9 i$
4. $-2^9 i$

27. $\left(\frac{\sqrt{6}-\sqrt{2}}{4} + \frac{\sqrt{6}+\sqrt{2}}{4}i\right)^{2020} =$

[AP EAMCET 20-08-2021_Shift-2]

1. $\frac{1}{2} + \frac{\sqrt{3}}{2}i$
2. $\frac{-1}{2} + \frac{\sqrt{3}}{2}i$
3. $\frac{-1}{2} - \frac{\sqrt{3}}{2}i$
4. $\frac{1}{2} - \frac{\sqrt{3}}{2}i$

28. $\dfrac{\left(\sin\frac{\pi}{8} + i\cos\frac{\pi}{8}\right)^8}{\left(\sin\frac{\pi}{8} - i\cos\frac{\pi}{8}\right)^8} =$

[AP EAMCET 23-08-2021_Shift-1]

1. i
2. $-i$
3. 1
4. 2

29. If $z^2 + z + 1 = 0$ where z is a complex number, then $\left(z+\frac{1}{z}\right)^3 + \left(z^4 + \frac{1}{z^4}\right)^3$ is equal to

[AP EAMCET 23-08-2021_Shift-1]

1. 1
2. 0
3. -1
4. -2

30. Let $z = \cos\theta + i\sin\theta$. Then the value of $\sum_{m=1}^{15} I_m\left(z^{2m-1}\right)$ at $\theta = 2°$ is

[AP EAMCET 24-08-2021_Shift-1]

1. $\frac{1}{\sin 2°}$
2. $\frac{1}{3\sin 2°}$
3. $\frac{1}{2\sin 2°}$
4. $\frac{1}{4\sin 2°}$

31. What is the product of all the values of $(1-i)^{\frac{2}{5}} =$

[AP EAMCET 24-08-2021_Shift-2]

1. $-2i$
2. $2i$
3. -2
4. 2

32. If n is a positive integer, then $(1+i\sqrt{3})^n + (1-i\sqrt{3})^n =$

[AP EAMCET 25-08-2021_Shift-1]

1. $2^{n-1}\cos\frac{n\pi}{3}$
2. $2^n\cos\frac{n\pi}{3}$
3. $2^{n+1}\cos\frac{n\pi}{3}$
4. $2^{2n}\cos\frac{n\pi}{3}$

33. Let $1, \omega, \omega^2$ be the cube roots of unity. What is the value of $(1-\omega+\omega^{-1})^5 - 2(1+\omega-\omega^{-1})^4 = ?$

[AP EAMCET 25-08-2021_Shift-1]

1. -64ω
2. 64ω
3. $-64\omega^{-1}$
4. $64\omega^{-1}$

34. Which of the following is a fourth root of $\frac{1}{2} + \frac{i\sqrt{3}}{2}$?

[AP EAMCET 25-08-2021_Shift-2]

1. $cis\frac{\pi}{12}$
2. $cis\frac{\pi}{2}$
3. $cis\frac{\pi}{6}$
4. $cis\frac{\pi}{3}$

35. If $1,\omega,\omega^2,....,\omega^{10}$ are 11^{th} roots of unity, then product of these roots is

[AP EAMCET 25-08-2021_Shift-2]

1. ω
2. -1
3. 1
4. ω^2

36. Let $\omega = cis\left(\frac{2\pi}{3}\right) = \cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)$ and $f(x) = x^7 - 2x^4 - 4x^3 + 8.$ which of the following option is correct?

[AP EAMCET 23-08-2021_Shift-1]

1. $\left\{2^{\frac{1}{2}}, 2^{\frac{1}{3}}, \omega, 2^{\frac{1}{3}}\omega\right\}$ is a subset of the solution set of $f(x)$.
2. $\left\{2^{\frac{1}{2}}, -2^{\frac{1}{3}}, 2^{\frac{1}{3}}\omega^2, 2^{\frac{1}{2}}i\right\}$ is a subset of the solution set of $f(x)$.
3. $\left\{2^{\frac{1}{3}}, 2^{\frac{1}{2}}, -2^{\frac{1}{2}}i, 2^{\frac{1}{3}}\omega^2\right\}$ is not a subset of the solution set of $f(x)$.
4. $\left\{2^{\frac{1}{3}}, 2^{\frac{1}{3}}\omega, 2^{\frac{1}{2}}i, -2^{\frac{1}{2}}\right\}$ is a subset of the solution set of $f(x)$.

37. If $\cos\alpha + 3\cos 3\beta + 5\cos 5\gamma = 0, \sin\alpha + 3\sin 3\beta + 5\sin 5\gamma = 0$ and $\cos 3\alpha + 27\cos 9\beta + 125\cos 15\gamma = (\lambda^2 - 4)\cos(\alpha + 3\beta + 5\gamma),$ then $\lambda =$

[AP EAMCET 23-08-2021_Shift-1]

1. $\pm 2\sqrt{2}$
2. $\pm 2\sqrt{5}$
3. $\pm\sqrt{7}$
4. $\pm\sqrt{29}$

38. If ω represents a cube root of unity and $\sum_{k=1}^{n}\left(k+\frac{1}{\omega}\right)\left(k+\frac{1}{\omega^2}\right) = 340,$ then $n =$

[TS EAMCET 04-08-2021_Shift-2]

1. 20
2. 25
3. 10
4. 15

39. $\sum_{k=1}^{6}\left(\sin\frac{2\pi k}{7} - i\cos\frac{2\pi k}{7}\right) =$

[TS EAMCET 04-08-2021_Shift-2]

1. i
2. $-i$
3. $2i$
4. $-2i$

40. If α,β are the roots of the equation $1 + x + x^2 = 0$, then $(2-\alpha)(2-\beta)(2-\alpha^{10})(2-\alpha^{20}) =$

[TS EAMCET 05-08-2021_Shift-2]

1. 36
2. 64
3. 49
4. 81

41. If ω represents a cube root of unity and $\sum_{k=1}^{n}\left(k+\frac{1}{\omega}\right)\left(k+\frac{1}{\omega^2}\right) = 340,$ then $n =$

[TS EAMCET 04-08-2021_Shift-2]

1. 20
2. 25
3. 10
4. 15

42. ω is a complex cube root of unity? Match the items of List – I to the items of List – II

List – I		List – II	
A)	$\omega^{1010} + \omega^{2020}$	I)	0
B)	$(1+\omega-\omega^2)(1-\omega+\omega^2)$	II)	1
C)	$(2+\omega^2+\omega^4)^5$	III)	-1
D)	$(3+5\omega+3\omega^2)^3$	IV)	4
		V)	8

[TS EAMCET 04-08-2021_Shift-1]

1. $A-III, B-IV, C-I, D-V$
2. $A-I, B-IV, C=II, D-V$
3. $A-III, B-IV, C-II, D-V$
4. $A-III, B-I, C-II, D-IV$

43. If the roots of the equation $(z-4)^3 = 8i$ are $a-2i, b+i$ and $c+i$ then $\sqrt{abc} =$

[TS EAMCET 04-08-2021_Shift-1]

1. $13\sqrt{3}$
2. $4\sqrt{13}$
3. $2\sqrt{13}$
4. $5\sqrt{3}$

44. If $2+2\sqrt{3}i=k(\cos\theta+i\sin\theta),(k>0)$, then $\frac{1}{\sqrt{3}}[\cos 6\theta+i\sin 6\theta]=$

[TS EAMCET 05-08-2021_Shift-1]

1. 1 2. $\frac{1}{\sqrt{3}}$

3. $\sqrt{3}$ 4. $\frac{\sqrt{3}}{2}$

---

45. For $n\in N, \left(\frac{1+\cos\theta+i\sin\theta}{1+\cos\theta-i\sin\theta}\right)^n=$

[TS EAMCET 05-08-2021_Shift-2]

1. $\cos(n\theta)-i\sin(n\theta)$ 2. $-\cos(n\theta)+i\sin(n\theta)$

3. $\cos(n\theta)+i\sin(n\theta)$ 4. $-\cos(n\theta)-i\sin(n\theta)$

---

86. n is a positive integer and not a multiple of 3. If ω is non real cube root of unity, then $\omega^n+\omega^{2n}=$ **[TS Eamcet 06-08-2021_Shift-2]**

1. -1 2. 3

3. -3 4. 1

---

47. If n is a positive integer, then $(1+i)^n \div (1-i)^n=-i$ then n will be of the form

[TS EAMCET 06-08-2021_Shift-2]

1. $4k-3, k\in N$ 2. $4k-1, k\in N$

3. $4k-2, k\in N$ 4. $4k, k\in N$

---

48. If ω is a complex cube root of unity, then $\cos\left[\left(\omega^{1234}+\omega^{2021}\right)\pi-\frac{\pi}{4}\right]=$

[TS EAMCET 06-08-2021_Shift-1]

1. $\frac{1}{\sqrt{2}}$ 2. $\frac{1}{2}$

3. $\frac{\sqrt{3}}{2}$ 4. $\frac{-1}{\sqrt{2}}$

---

49. If $e^{i\theta}=cis\,\theta$ then $\sum_{n=0}^{\infty}\frac{\cos(n\theta)}{2^n}=$

[AP EAMCET 04-07-2022_Shift-2]

1. $(4+2\cos\theta)/(5-4\cos\theta)$ 2. $(4-2\cos\theta)/(5+4\cos\theta)$

3. $(4-2\cos\theta)/(5-4\cos\theta)$ 4. $(4+2\cos\theta)/(5+4\cos\theta)$

---

50. The values of x for which sinx + i cos 2x and cosx – i sin 2x are conjugate to each other are

[AP EAMCET 05-07-2022_Shift-1]

1. $x=n\pi\pm\pi/6$ 2. None

3. $x=n\pi\pm\pi/3$ 4. $x\left(n+\frac{1}{2}\right)\pi$

---

51. $\sum_{k=1}^{6}\sin\left(\frac{2\pi k}{7}\right)-i\cos\left(\frac{2\pi k}{7}\right)=$

[AP EAMCET 05-07-2022_Shift-2]

1. 1 2. –i

3. i 4. –1

---

52. The number of distinct solutions of the equation $x^{11}-x^7+x^4-1=0$ is

[AP EAMCET 06-07-2022_Shift-1]

1. 9 2. 11

3. 10 4. 8

---

53. $z=\cos\theta+i\sin\theta \Rightarrow z^r+\left(\bar{z}\right)^r=$

[AP EAMCET 06-07-2022_Shift-1]

1. $\cos r\theta$ 2. $2\cos r\theta$

3. $\sin r\theta$ 4. $2\sin r\theta$

---

54. $(-1+i\sqrt{3})^{60}=$

1. 2^{60} 2. 2^{59}

3. 2^{61} 4. 2^{30}

---

55. If $\alpha_1, \alpha_2, \ldots\ldots\ldots \alpha_{23}$ are the 23rd roots of unity, then $\alpha_1^{47}+\alpha_2^{47}+\cdots\cdots+\alpha_{23}^{47}=$

[AP EAMCET 06-07-2022_Shift-2]

1. 23 2. -1

3. 1 4. 0

---

56. If $1,\omega,\omega^2$ are the cube roots of unity, then $(2-\omega)^2(2-\omega^2)^2(2-\omega^{10})^2(2-\omega^{11})^2=$

[AP EAMCET 07-07-2022_Shift-1]

1. -7^4 2. 7^4

3. 7^8 4. -7^8

---

57. $\frac{(1+i)x-2i}{3+i}+\frac{(2-3i)y}{3-i}=i\Rightarrow x+y=$

[AP EAMCET 07-07-2022_Shift-2]

1. -1 2. 1

3. -2 4. 2

58. Let z and w be two distinct non-zero complex numbers if $|z|^2 w - |w|^2 z = z - w$, then

[AP EAMCET 07-07-2022_Shift-2]

1. $w = z^{-2}$
2. $z\bar{w} = 2$
3. $z\bar{w} = 1$
4. $w = \bar{z}$

59. If $1, \omega, \omega^2$ denote the cube roots of unity, then the value of $(1-\omega+\omega^2)^5 + (1+\omega-\omega^2)^5$ is

[AP EAMCET 07-07-2022_Shift-2]

1. $32\omega^2$
2. 32ω
3. -32
4. 32

60. If the Arg z_1 and Arg $\bar{z}_2$ are $\frac{\pi}{3}$ and $\frac{\pi}{5}$ respectively then the value of $Arg\ z_1 + Arg\ z_2$ is **[AP EAMCET 08-07-2022_Shift-1]**

1. $11\pi/15$
2. $6\pi/15$
3. $2\pi/15$
4. $8\pi/15$

61. $\left(\frac{1-i}{1+i}\right)^{2022} + \left(\frac{1+i}{1-i}\right)^{2021} =$

[AP EAMCET 08-07-2022_Shift-1]

1. -i
2. i
3. i+1
4. i-1

62. If $1, \omega, \omega^2$ are the cube roots of unity then the value of $(x+y)^2 + (x\omega + y\omega^2)^2 + (x\omega^2 + y\omega)^2$ is **[AP EAMCET 08-07-2022_Shift-1]**

1. $2x^2.3y^2$
2. 4xy
3. 6xy
4. $2x^2.2y^2$

63. If $1, \omega, \omega^2$ are the cube roots of unity, $n \in N$ and $n > 2$ then the least value of n such that $1+\omega$ is a root of $x^n - x = 0$ is

[TS EAMCET 18-07-2022_Shift-1]

1. 3
2. 5
3. 7
4. 4

64. If $\cos\alpha + \cos\beta + \cos\gamma = 0$ and $\sin\alpha + \sin\beta + \sin\gamma = 0$ then $\cos 2\alpha + \cos 2\beta + \cos 2\gamma =$

[TS EAMCET 18-07-2022_Shift-2]

1. $\frac{3}{2}$
2. $\cos^2\frac{\alpha}{2} + \cos^2\frac{\beta}{2} + \cos^2\frac{\gamma}{2}$
3. $3\sin(\alpha+\beta+\gamma)$
4. $\cos(\alpha+\beta) + \cos(\beta+\gamma) + \cos(\gamma+\alpha)$

65. One of the values of $(-32i)^{\frac{2}{5}}$

[TS EAMCET 18-07-2022_Shift-2]

1. $4cis\frac{2\pi}{5}$
2. $4cis\frac{3\pi}{5}$
3. $4cis\frac{4\pi}{5}$
4. $4cis\frac{6\pi}{5}$

66. If $cis\,\alpha$ is the common value of $(-1)^{1/4}$ and $(-i)^{1/2}$ then $\tan\alpha =$

[TS EAMCET 19-07-2022_Shift-I]

1. -1
2. 1
3. $\sqrt{3}$
4. $\frac{1}{\sqrt{3}}$

67. If $1, \alpha_1, \alpha_2, \alpha_3.....\alpha_{n-1}$ are n[th] roots of unity then $\sum_{1\le i<j\le n-1} \alpha_i \alpha_j =$

[TS EAMCET 19-07-2022_Shift-2]

1. 1
2. 0
3. -1
4. i

68. If $1, \omega, \omega^2$ are the cube roots of unity and $1, \alpha, \alpha^2, \alpha^3$ are the fourth roots of unity in usual notation then $\alpha + \alpha\omega - \alpha^3\omega^2 =$

[TS EAMCET 20-07-2022_Shift-1]

1. 3
2. 1
3. 0
4. -1

69. If α, β are the roots of the equation $x^2 - 2x + 2 = 0$ then $\alpha^{2020} + \beta^{2020} =$

[TS EAMCET 20-07-2022_Shift-2]

1. 2^{1011}
2. -2^{1011}
3. 2^{2021}
4. 2^{-2021}

70. If $z=\frac{-1-i\sqrt{3}}{2}$, then $\sum_{k=1}^{2022}\left(z^k+\frac{1}{z^k}\right)^2=$

[TS EAMCET 20-07-2022_Shift-2]

1. 0
2. 2022
3. 4044
4. 1011

71. If $|x+iy|=\sqrt{x^2+y^2}$, then $\left|\left(1-\sqrt{3}i\right)^9+\left(\sqrt{3}+i\right)^9\right|=$

[TS EAMCET 20-07-2022_Shift-1]

1. 2^9
2. 2^{18}
3. 2^{10}
4. $2^{\frac{19}{2}}$

72. $\left[\sqrt{2}\left(\cos 56^o15'+i\sin 56^o15'\right)\right]^8=$

[15th May 2023 Shift 1]

1. 1
2. i
3. 16
4. 16i

73. $(1+i)^{2024}+(1-i)^{2024}=$

[15th May 2023 Shift 1]

1. -2^{1012}
2. 2^{1013}
3. $2^{2024}i$
4. $-2^{1012}i$

74. One of the 15^{th} roots of -1 is

[15th May 2023 Shift 2]

1. $cis\,0$
2. $cis\frac{14\pi}{15}$
3. $cis\frac{13\pi}{15}$
4. $cis\frac{8\pi}{15}$

75. The product of the four values of $\left(1+i\sqrt{3}\right)^{\frac{3}{4}}$ is

[15th May 2023 Shift 2]

1. –8i
2. i
3. –8
4. 8

76. *If* ω is complex cube root of unity, then

$$\cos\left(\sum_{k=1}^{7}(k-\omega)\left(k-\omega^2\right)\frac{\pi}{175}\right)=$$

[16th May 2023 Shift 1]

1. -1
2. 0
3. 1
4. 5

77. If a and b are the roots of the equation $y^2+y+1=0$, then the value of $a^4+b^4+a^{-1}b^{-1}$ is

[16th May 2023 Shift 2]

1. 1
2. 0
3. 5
4. 2

78. If $1,\omega,\omega^2$ are the cube roots of unity, K is positive integer and

$\left(1-\omega+\omega^2\right)^{3k}+\left(1-\omega^2+\omega\right)^{3k}$

$=\left(1-\omega+\omega^2\right)^{3k+1}+\left(1+\omega-\omega^2\right)^{3k+1}$, then $k=$

[17th May 2023 Shift 1]

1. $r, r\in N$
2. $2r+1, r\in N$
3. $4r+1, r\in N$
4. $3r, r\in N$

79. If α,β are the two real roots of 4^{th} roots of unity and γ,δ are the other two roots of it, then the sum of the eccentricities of the conics $|z-\alpha|+|z-\beta|=4$ and $|z-\gamma|+|z-\delta|=6$ is

[17th May 2023 Shift 1]

1. $\frac{5}{6}$
2. $\frac{5}{12}$
3. $\frac{3}{7}$
4. $\frac{4}{5}$

80. α is the real root and β,γ are the other roots of the equation $x^3-a^3=0(a>0)$, then the number of common points of the curves given by $|z-\beta|=\frac{\sqrt{3}a}{2}$ and $|z-\gamma|=\frac{\sqrt{3}}{2}\alpha$

[17th May 2023 Shift 2]

1. 0
2. 2
3. 3
4. 1

81. If ω is a complex cube root of unity, then $\sin\left[(\omega^{10}+\omega^{23})\pi-\frac{\pi}{4}\right]=$ **[18^{th} May2023 shift -1]**

1. $1/\sqrt{2}$
2. 1/2
3. 1
4. $\sqrt{3}/2$

82. If 1, ω, ω^2 are the cube roots of unity and $(x+y)\ (x\omega+y\omega^2)(x\omega^2+y\omega) = f(x,y)$, then f (2,3) =**[18th May 2023 Shift 2]**

1. 16
2. 24
3. 35
4. 45

83. If $1, \omega, \omega^2$ are the cube roots of unity, then the roots of the equation $8z^3-12z^2+6z-28=0$ are **[19th May 2023 Shift 1]**

1. $2, 3\omega, 3\omega^2+1$
2. $2, \frac{3\omega+1}{2}, \frac{3\omega^2+1}{2}$
3. $2, \frac{1+3\omega}{3}, \frac{1+3\omega^2}{3}$
4. $2, \frac{1-\omega}{2}, \frac{1-\omega^2}{2}$

84. If $\theta = \frac{\pi}{6}$, then the 10^{th} term of the series $1+(\cos\theta+i\sin\theta)^1+(\cos\theta+i\sin\theta)^2+...$ is

[12TH MAY 2023 SHIFT-1]

1. -1
2. $-i$
3. $\frac{1}{2}+\frac{\sqrt{3}i}{2}$
4. 1

85. If $\omega \neq 1$ is a cube root of unity, then

$$\begin{vmatrix} \omega+\omega^2 & \omega^2+\omega^9 & \omega^9+\omega \\ \omega^{27}+\omega^{31} & \omega^{31}+\omega^{17} & \omega^{17}+\omega^{27} \\ \omega^{30}+\omega^{41} & \omega^{41}+\omega^{19} & \omega^{19}+\omega^{30} \end{vmatrix} =$$

[12TH MAY 2023 SHIFT -2]

1. 3
2. 2
3. 1
4. 0

86. If $i=\sqrt{-1}$, then $(1+i)^{10}+(1-i)^{10}=$

[12TH MAY 2023 SHIFT -2]

1. -64
2. 64
3. 0
4. $64i$

87. If 'i' is the root of the equation $x^2+1=0$ then

$$\left(1+\sqrt{3}i\right)^{2023}+\left(1-\sqrt{3}i\right)^{2023}=$$

[13TH MAY 2023 SHIFT-1]

1. 2^{2022}
2. 2^{2023}
3. $2^{2022}\left(\sqrt{3}\right)$
4. $2^{2023}\left(\sqrt{3}\right)$

88. One of the values of $\left(\sqrt{3}-i\right)^{\frac{1}{6}}$

[13TH MAY 2023 SHIFT-1]

1. $2^{\frac{1}{6}} cis \frac{61\pi}{36}$
2. $2^{\frac{1}{6}} cis \frac{37\pi}{36}$
3. $2^{\frac{1}{6}} cis \frac{59\pi}{36}$
4. $2^{\frac{1}{6}} cis \frac{49\pi}{36}$

89. If $X_n = \cos\frac{\pi}{2^n}+i\sin\frac{\pi}{2^n}$, then $\prod_{n=1}^{\infty} x_n =$

[EAPCET 14-05-23 SHIFT-1]

1. 0
2. 1
3. -1
4. i

90. One of the values of $\left(\sqrt{3}-i\right)^{2/5}$ is

[EAPCET 13-05-23 SHIFT-2]

1. $2^{\frac{2}{5}}\left(1-\sqrt{3}i\right)$
2. $2^{-\frac{3}{5}}\left(\sqrt{3}+i\right)$
3. $2^{\frac{2}{5}}\left(\sqrt{3}-i\right)$
4. $2^{-\frac{3}{5}}\left(1+\sqrt{3}i\right)$

KEY

1)	4	2)	2	3)	3	4)	4	5)	2
6)	2	7)	2	8)	1	9)	1	10)	3
11)	2	12)	1	13)	3	14)	2	15)	1
16)	1	17)	4	18)	4	19)	4	20)	4
21)	1	22)	2	23)	4	24)	1	25)	3
26)	2	27)	4	28)	3	29)	4	30)	4
31)	1	32)	3	33)	3	34)	1	35)	3
36)	4	37)	3	38)	3	39)	1	40)	3
41)	3	42)	3	43)	3	44)	2	45)	3
46)	1	47)	2	48)	4	49)	3	50)	2
51)	3	52)	3	53)	2	54)	1	55)	4
56)	2	57)	4	58)	3	59)	4	60)	3
61)	4	62)	3	63)	3	64)	4	65)	2
66)	1	67)	1	68)	3	69)	2	70)	3
71)	4	72)	4	73)	2	74)	3	75)	4
76)	1	77)	2	78)	1	79)	1	80)	4
81)	1	82)	3	83)	2	84)	2	85)	4
86)	3	87)	2	88)	4	89)	3	90)	4

SOLUTIONS

DEMOVIRE'S

1. Let $\alpha = \omega^2, \beta = \omega$

Now sub in $5\alpha^4 + 5\beta^4 + 7\alpha^{-1}\beta^{-1}$

2. Here $x^n - 1 = (x-1)(x-\alpha)(x-\alpha^2).....(x-\alpha^{n-1})$

Apply log and then diff

$$\frac{1}{x^n-1}nx^{n-1} = \frac{1}{x-1} + \frac{1}{x-\alpha} + \frac{1}{x-\alpha^2} + ... + \frac{1}{x-\alpha^{n-1}}$$

Put $x = 2$

$$\frac{1}{2^n-1}n.2^{n-1} = \frac{1}{2-1} + \frac{1}{2-\alpha} + \frac{1}{2-\alpha^2} + ... + \frac{1}{2-\alpha^{n-1}}$$

$$\frac{n2^{n-1} - 2^n + 1}{2^n - 1} = \sum_{i=1}^{n-1}\frac{1}{2-\alpha^i}$$

$$\frac{(n-2)2^{n-1} + 1}{2^n - 1} = \sum_{i=1}^{n-1}\frac{1}{2-\alpha^i}$$

Or verify option with n=3

3. Let $\alpha = \omega, \beta = \omega^2$

$$\alpha^{2020} + \beta^{2020} + \alpha^{2020}\beta^{2020}$$

$$= \omega^{2020} + \omega^{4040} + (\omega^3)^{2020}$$

$$= \omega + \omega^2 + 1 = 1 + \alpha + \beta$$

4. α, β are roots of $x^2 + x + 1 = 0$ then

$$\alpha = \omega, \beta = \omega^2$$

$$\alpha^4 + \beta^4 = \omega^4 + (\omega^2)^4 = \omega^4 + \omega^8 = \omega + \omega^2 = -1$$

$$-\alpha\beta = -\omega\omega^2 = -1$$

$$\alpha^4 + \beta^4 = -\alpha\beta$$

5. $\alpha = 1 + i\sqrt{3}, \beta = 1 - i\sqrt{3}$

$$\alpha^n + \beta^n = 2^n\left[\cos\frac{n\pi}{3} + i\sin\frac{n\pi}{3} + \cos\frac{n\pi}{3} - i\sin\frac{n\pi}{3}\right]$$

$$= 2^{n+1}\cos\frac{n\pi}{3}$$

6. $(\sqrt{3} - i)^{300} + (\sqrt{3} + i)^{300}$

$$= (2)^{300}\left[\left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)^{300} + \left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^{300}\right]$$

$$= (2)^{300}\left[\left(cis\left(\frac{-\pi}{6}\right)\right)^{300} + \left(cis\left(\frac{\pi}{6}\right)\right)^{300}\right]$$

$$= 2^{300}\left[2\cos\left(\frac{300\pi}{6}\right)\right] = 2^{301}.(1)$$

7. $x^2 - 2x + 4 = 0$

$$(x-1)^2 + 3 = 0$$

$$x - 1 = \pm\sqrt{3}i$$

$$x = \pm\sqrt{3}i + 1$$

$$x = 2e^{\pm i\frac{\pi}{3}}$$

$$let\ \alpha = 2e^{i\frac{\pi}{3}}, \beta = 2e^{-i\frac{\pi}{3}}$$

$$\alpha^n + \beta^n = 2^n\left\{e^{i\frac{n\pi}{3}} + e^{-i\frac{n\pi}{3}}\right\} = 2^{n+1}\cos\frac{n\pi}{3}$$

8. $z = 2 + i\sqrt{5}$ $\quad\quad \bar{z} = 2 - i\sqrt{5}$

$= re^{ie}$ $\quad\quad = re^{-ie}$

$$\Rightarrow \left(\frac{z}{\bar{z}}\right)^{10} + \left(\frac{\bar{z}}{z}\right)^{10}$$

$$\left(\frac{re^{ie}}{re^{-ie}}\right)^{10} + \left(\frac{re^{-ie}}{re^{ie}}\right)^{10}$$

$$\Rightarrow e^{i20\theta} + e^{-i20\theta}$$

$$\Rightarrow cis(20e) + i\sin(20e) + cis(-20e) + i\sin(-20e)$$

$$\Rightarrow 2\cos 20e$$

$$|z| = |2\cos 20e|$$

$$2\cos\left(20\text{Cos}^{-1}\left(\frac{2}{3}\right)\right)$$

9. $$\left[\frac{2\cos\frac{3\theta}{2}\left(\cos\frac{3\theta}{2} + i\sin\frac{3\theta}{2}\right)}{2\cos\frac{3\theta}{2}\left(\cos\frac{3\theta}{2} - i\sin\frac{3\theta}{2}\right)}\right]^{20}$$

$$= \frac{cis30\theta}{cis(-30\theta)} = cis60\theta$$

10. $|z|^2+|z-3|^2+|z-1|^2$

$$\left[\begin{array}{l}\because |z-a|^2+|z-b|^2+|z-c|^2 \ \ is \\ \min.. \ \ when \ \ z=\dfrac{a+b+c}{3}\end{array}\right]$$

This expansion is minimum when

$$z=\frac{0+3+i}{3}=1+\frac{1}{3}i$$

11. $$\sqrt{x}+\frac{1}{\sqrt{x}}=2\cos\theta$$

$$x+\frac{1}{x}=4\cos^2\theta-2=2\cos 2\theta$$

$$x^2+\frac{1}{x^2}=4\cos^2 2\theta-2=2\cos 4\theta$$

$$x^6+\frac{1}{x^6}=\left(x^2+\frac{1}{x^2}\right)^3-3\left(x^2+\frac{1}{x^2}\right)$$

$$=8\cos^3 4\theta-6\cos 4\theta=2\cos 12\theta$$

12. $$(\omega)^{2020}+(\omega^2)^{2026}+\sin\left(\sum_{j=1}^{6}(j+\omega)(j+\omega^2)\frac{3\pi}{152}\right)$$

$$\omega+\omega^2+\sin\left[\frac{3\pi}{152}(1+\omega)(1+\omega^2)+......+(6+\omega)(6+\omega^2)\right]$$

$$=-1+\sin\left[\frac{3\pi\times 76}{152}\right]=-1+\sin\left(\frac{3\pi}{2}\right)=-2$$

13. $z=e^{i\theta}=\cos\theta+i\sin\theta$

$$\frac{3\cos\theta+2\cos 2\theta+5\cos 5\theta}{3\sin 3\theta+2\sin 2\theta+5\sin 5\theta}=\frac{i\sum_{r=0}^{10}a_r z^r}{\sum_{r=0}^{10}b_r z^r}$$

$a_1=-3,\ a_2=-2,\ a_5=-5$

$b_3=-3,\ b_2=-2,\ b_5=-5$

Remaining all zero

$$\frac{\sum_{r=0}^{10}a_r+\sum_{r=0}^{10}b_r}{10}=-2$$

14. $$=\left(\frac{\cos\theta+i\sin\theta}{i(\cos\theta-i\sin\theta)}\right)^8+\left(\frac{2\cos^2\frac{\theta}{2}-i2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}+i2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right)^{16}$$

$$=\left(\frac{\cos\theta+i\sin\theta}{\cos\theta-i\sin\theta}\times\frac{\cos\theta+i\sin\theta}{\cos\theta+i\sin\theta}\right)^8$$

$$+\left(\frac{\cos\frac{\theta}{2}-i\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}+i\sin\frac{\theta}{2}}\times\frac{\cos\frac{\theta}{2}-i\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}-i\sin\frac{\theta}{2}}\right)^{16}$$

$$=(\cos\theta+i\sin\theta)^{16}+\left(\cos\frac{\theta}{2}-i\sin\frac{\theta}{2}\right)^{32}$$

$$=\cos 16\theta+i\sin 16\theta+\cos 16\theta-i\sin 16\theta$$

$$=2\cos 16\theta$$

15. $put\ \ \theta=\dfrac{\pi}{2}$

$$\left(\frac{i}{1}\right)^{2020}+\left(\frac{1+i}{1+i}\right)^{2020}=2\Rightarrow x+y=2$$

16. $$\sum_{x=1}^{10}\left(x^2-2x-2\omega^2x+2\omega^2x+10\right)$$

$$=\frac{10\times 11\times 21}{6}-\frac{2\times 10\times 11}{2}+10=285$$

17. Consider $(cis\theta)^5=(\cos\theta+i\sin\theta)^5$

On expanding, $(cis\theta)^5=(\cos\theta+i\sin\theta)^5$

$$cis5\theta={}^5C_0\cos^5\theta+{}^5C_1\cos^4\theta.i\sin\theta-{}^5C_2\cos^3\theta.\sin^2\theta$$

$$+{}^5C_3\cos^2\theta.i\sin^3\theta+{}^5C_4\cos\theta.\sin^4\theta+{}^5C_5 i\sin^5\theta$$

By comparing, imaginary part on both sides

$$\sin 5\theta=5\cos^4\theta\sin\theta-10\cos^2\theta\sin^3\theta+\sin^5\theta$$

$$a=5, b=-10, c=1;\ \therefore abc=5(-10)=-50$$

18. Given

$$A_n=\cos\left[\frac{\pi}{2^n}\right]+i\sin\left[\frac{\pi}{2^n}\right]$$

$$A_1=\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}=cis\frac{\pi}{2}$$

$$A_2=\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}=cis\frac{\pi}{4}$$

$$A_3=\cos\frac{\pi}{8}+i\sin\frac{\pi}{8}=cis\frac{\pi}{8}$$

$$A_4=\cos\frac{\pi}{16}+i\sin\frac{\pi}{16}=cis\frac{\pi}{16}$$

$$(A_1.A_2.A_3.A_4)^4=\left(cis\frac{\pi}{2}.cis\frac{\pi}{4}.cis\frac{\pi}{8}.cis\frac{\pi}{16}\right)^4$$
$$=\left[cis\left(\frac{\pi}{2}+\frac{\pi}{4}+\frac{\pi}{8}+\frac{\pi}{16}\right)\right]^4$$
$$=cis\left(4\times\frac{15\pi}{16}\right)=cis\frac{15\pi}{4}$$
$$=\cos\left(4\pi-\frac{\pi}{4}\right)+i\sin\left(4\pi-\frac{\pi}{4}\right)$$
$$=\cos\frac{\pi}{4}-i\sin\frac{\pi}{4}=\frac{1-i}{\sqrt{2}}$$

19. Given

$$A_r=\left(x+\frac{1}{x}\right)^3\cdot\left(x^2+\frac{1}{x^2}\right)^3\cdot\left(x^3+\frac{1}{x^3}\right)^3.....\left(x^r+\frac{1}{x^r}\right)^3$$

Given equation $x^2+x+1=0$

Roots are ω,ω^2

$A_3=(2^1)^3$, $A_6=(2^2)^3$, $A_9=(2^3)^3$

Now,

$$\frac{1}{A_3}+\frac{1}{A_6}+\frac{1}{A_9}+....+\infty$$
$$=\frac{1}{8}+\left(\frac{1}{8}\right)^2+\left(\frac{1}{8}\right)^3+....+\infty=\frac{\frac{1}{8}}{1-\frac{1}{8}}=\frac{1}{7}$$

20.
$$\left(\sqrt{3}+i\right)^{10}=a+ib\Rightarrow\left[2\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)\right]^{10}=a+ib$$
$$\Rightarrow 2^{10}\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)^{10}=a+ib$$
$$\Rightarrow 2^{10}\left(\cos\frac{5\pi}{3}+i\sin\frac{5\pi}{3}\right)=a+ib$$
$$a=2^{10}.\cos\frac{5\pi}{3}\qquad b=2^{10}.\sin\frac{5\pi}{3}$$
$$a=2^{10}.\cos\left(2\pi+\frac{\pi}{3}\right)\qquad b=2^{10}.\sin\frac{\pi}{3}$$
$$a=2^{10}.\cos\frac{\pi}{3}\qquad b=-2^9.\sqrt{3}$$
$$=2^{10}.\frac{1}{2}\ =2^9$$

21. $\alpha,\beta,\gamma,\delta$ *are the roots of* $x^4+x^2+1=0$

$$\Rightarrow(x^2+x+1)(x^2-x+1)=0$$
$$\alpha=\omega,\beta=\omega^2,\gamma=-\omega,\delta=-\omega^2$$
$$\frac{\alpha^3+\beta^3+\gamma^3+\delta^3}{\alpha^6+\beta^6+\gamma^6+\delta^6}=\frac{\omega^3+\omega^6-\omega^3-\omega^6}{\omega^6+\omega^{12}+\omega^6+\omega^{12}}=0$$

22.
$$(z+1)^n=z^n\Rightarrow\left(1+\frac{1}{z}\right)^n=1$$
$$\Rightarrow 1+\frac{1}{z}=cis\frac{2k\pi}{n},\ \textit{where}\ k=0,1,2,....,n-1$$
$$\Rightarrow z=-\frac{i}{2}\cot\frac{k\pi}{n}-\frac{1}{2}$$

23.
$$(-i)^3(\cos\theta+i\sin\theta)^3\quad\left(\because\frac{-1}{1}=i\right)$$
$$-i^3(\cos3\theta+i\sin3\theta)$$

24. $\left(\frac{1+i}{1-i}\right)^m=i^m=1$, if m is multiple of 4. m cannot be equal to 1934.

25. Given $x^5-1=0$

$$(\omega-1)(\omega-\alpha_1)(\omega-\alpha_2)(\omega-\alpha_3)(\omega-\alpha_4)+\omega$$
$$=\omega^5-S_1\omega^4+S_2\omega^3-S_3\omega^2+S_4\omega-S_5+\omega$$

But $S_1=S_2=S_3=S_4=0$ and $S_5=1$.

$$=\omega^2-1+\omega=-1-1=-2$$

26.
$$2^9\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)^9=2^9\left(\cos\frac{\pi}{3}-i\sin\frac{\pi}{3}\right)^9$$
$$=2^9(\cos3\pi-i\sin3\pi)=2^9(-1)=-2^9$$

27.
$$\left(\frac{\sqrt{3}-1}{2\sqrt{2}}+i\frac{\sqrt{3}+1}{2\sqrt{2}}\right)^{2020}=\left(\cos\frac{5\pi}{12}+i\sin\frac{5\pi}{12}\right)^{2020}$$
$$=\cos\left(842\pi-\frac{\pi}{3}\right)+i\sin\left(842\pi-\frac{\pi}{3}\right)$$
$$=\cos\frac{\pi}{3}-i\sin\frac{\pi}{3}=\frac{1}{2}-\frac{i\sqrt{3}}{2}$$

28.
$$\frac{\left(\sin\frac{\pi}{8}+i\cos\frac{\pi}{8}\right)^8}{\left(\sin\frac{\pi}{8}-i\cos\frac{\pi}{8}\right)^8}=\frac{\cos8\pi}{\cos8\pi}=1$$

29. Let $z=\omega$

$$\left(\omega+\frac{1}{\omega}\right)^3+\left(\omega^4+\frac{1}{\omega^4}\right)^3$$

$$=\left(\omega+\omega^2\right)^3+\left(\omega+\omega^2\right)^3=-1-1=-2$$

30.

$$z=cis\theta,\sum_{m=1}^{15}\sin(2m-1)\theta$$

$$=\sin\theta+\sin 3\theta+\sin 5\theta+....+\sin 29\theta$$

$$=\frac{1}{2\sin\theta}\left[2\sin^2\theta+2\sin\theta\sin 3\theta+...+2\sin\theta\sin 29\theta\right]$$

$$=\frac{1}{2\sin\theta}\begin{bmatrix}1-\cos 2\theta+\cos 2\theta-\cos 4\theta\\+\cos 4\theta-\cos 6\theta.......\\+\cos 28\theta-\cos 30\theta\end{bmatrix}$$

$$=\frac{1}{2\sin 2^0}\left[1-\cos 60^0\right]=\frac{1}{4\sin 2^0}$$

31. IPE question

32. IPE question

33. $\left(1-\omega+\omega^2\right)^5-2\left(1+\omega-\omega^2\right)^4$

$$=(-2\omega)^5-2\left(-2\omega^2\right)^4=-64\omega^2=-64\omega^{-1}$$

34. $\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)^{1/4}=\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)^{1/4}=cis\frac{\pi}{12}$

35. Let $x=\sqrt[11]{1}\Rightarrow x^{11}=1\Rightarrow x^{11}-1=0$

Product of 11 roots = 1.

36. $\omega=\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}$

$$f(x)=x^4\left(x^3-2\right)-4\left(x^3-2\right)$$

$$=\left(x^3-2\right)\left(x^4-4\right)=0$$

$x=2^{1/3}$ or $\left(x^2-2\right)\left(x^2+2\right)=0$.

$$x=\left\{2^{1/3},2^{1/3}\omega,2^{1/3}\omega^2,2^{1/2},-2^{1/2},2^{1/2}i-2^{1/2}i\right\}$$

$\left\{2^{1/3},2^{1/3}\omega,2^{1/2}i,-2^{1/2}\right\}$ is subset of above set.

37. $a=\cos\alpha+i\sin\alpha=cis\alpha$

$$b=3\cos 3\beta+i3\sin 3\beta=3cis3\beta$$

$$c=5\cos 5\gamma+i5\sin 5\gamma=5cis5\gamma$$

$$a+b+c=0\Rightarrow a^3+b^3+c^3=3abc$$

$$\cos 3\alpha+27\cos 9\beta+125\cos 5\gamma=45cis(\alpha+3\beta+5\gamma)$$

$$45=\lambda^2-4\Rightarrow\lambda^2=49\Rightarrow\lambda=\pm 7.$$

38. $\sum_{k=1}^{n}\left(k+\frac{1}{\omega}\right)\left(k+\frac{1}{\omega^2}\right)=340$

$$\sum_{k=1}^{n}\left(k+\omega^2\right)\left(k+\omega\right)=340$$

$$\sum_{k=1}^{n}\left(k^2+k\left(\omega+\omega^2\right)+1\right)=340$$

$$\sum_{k=1}^{n}\left(k^2-k+1\right)=340$$

$$\frac{n(n+1)(2n+1)}{6}-\frac{n(n+1)}{2}+n=340$$

$$n=10$$

By solving we get

39. $\sum_{k=1}^{6}\left(\sin\frac{2\pi k}{7}-i\cos\frac{2\pi k}{7}\right)=\sum_{k=1}^{6}(-i)\left(e^{\frac{2\pi k}{7}}\right)$

$$=-i\left[e^{2\pi/7}+\left(e^{2\pi/7}\right)^2+....+\left(e^{2\pi/7}\right)^6\right]$$

$$=-i[-1]=i$$

40 Given α,β are roots of $1+x+x^2=0$

Let $\alpha=\omega,\beta=\omega^2$

$$(2-\alpha)(2-\beta)\left(2-\alpha^{10}\right)\left(2-\alpha^{20}\right)$$

$$(2-\omega)\left(2-\omega^2\right)(2-\omega)\left(2-\omega^2\right)$$

$$=(2-\omega)^2\left(2-\omega^2\right)^2$$

$$=\left[(2-\omega)\left(2-\omega^2\right)\right]^2\qquad\left(\because\omega+\omega^2=-1\right)$$

$$=\left[4-2\left(\omega+\omega^2\right)+1\right]^2=49$$

41. $\sum_{k=1}^{n}\left(k+\frac{1}{\omega}\right)\left(k+\frac{1}{\omega^2}\right)=340$

$$\sum_{k=1}^{n}\left(k+\omega^2\right)\left(k+\omega\right)=340$$

$$\sum_{k=1}^{n}\left(k^2+k\left(\omega+\omega^2\right)+1\right)=340$$

$$\sum_{k=1}^{n}\left(k^2-k+1\right)=340$$

$$\frac{n(n+1)(2n+1)}{6}-\frac{n(n+1)}{2}+n=340$$

By solving we get $n=10$

42. $(A)\omega^{1010}+\omega^{2020}=\omega^2+\omega=-1$

$(B)(1+\omega-\omega^2)(1-\omega+\omega^2)$

$=(-2\omega^2)(-2\omega)$

$=4\omega^3=4$

$(C)(2+\omega^2+\omega^4)^5$

$=(2+\omega^2+\omega)^5$

$=(2-1)^5=1$

$(D)(3+5\omega+3\omega^2)^3$

$=(3(1+\omega^2)+5\omega)^3$

$=(-3\omega+5\omega)^3$

$=(2\omega)^3=8$

43. $(z-4)^3=8i$

$z-4=(8i)^{1/3}$

$z-4=2\left(cis\frac{\pi}{2}\right)^{1/3}$

$z-4=2\left(cis\frac{2k\pi+\frac{\pi}{2}}{3}\right),k=0,1,2$

$z-4=2cis\frac{\pi}{6},2cis\frac{5\pi}{6},2cis\frac{9\pi}{6}$

$z=4+2cis\frac{\pi}{6},4+2cis\frac{5\pi}{6},4+2cis\frac{3\pi}{2}$

$z=4+\sqrt{3}+i,4-\sqrt{3}+i,4-2i$

$a=4,b=4+\sqrt{3},c=4-\sqrt{3}$

$\sqrt{abc}=2\sqrt{13}$

44. Given, $2+2\sqrt{3}i=k(\cos\theta+i\sin\theta)$

$\Rightarrow 2\left[1+\sqrt{3}i\right]=kcis\theta$

$\Rightarrow 4\left[\frac{1}{2}+\frac{\sqrt{3}}{2}i\right]=kcis\theta$

$\Rightarrow 4\left[\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right]=k\,cis\theta$

$\Rightarrow 4cis\frac{\pi}{3}=kcis\theta$

$i.e.,k=4\ and\ \theta=\frac{\pi}{3}$

Now, $\frac{1}{\sqrt{3}}cis6\theta=\frac{1}{\sqrt{3}}cis6\left(\frac{\pi}{3}\right)=\frac{1}{\sqrt{3}}$

45. $\left(\frac{2\cos^2\frac{\theta}{2}+i2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}-i2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right)^n$

$=\left(\frac{\cos\frac{\theta}{2}+i\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}-i\sin\frac{\theta}{2}}\right)^n$

$=\left(\cos\frac{\theta}{2}+i\sin\frac{\theta}{2}\right)^{2n}=\cos n\theta+i\sin n\theta$

46. $\omega^3=1$ and $1+\omega+\omega^2=0$

Let $n=3m+1$ (or) $n=3m+2$

$\omega^n+\omega^{2n}=\omega^{3m+1}+\omega^{6m+2}$

$=(\omega^3)^m.\omega+(\omega^6)^m.\omega^2=-1$

47. $\left(\frac{1+i}{1-i}\right)^n=-i$

$\left[\frac{(1+i)(1+i)}{2}\right]^n=-i$

$\left(\frac{1-1+2i}{2}\right)^n=-i$

$i^n=-i$

$\therefore n=4k-1,k\in N$

48. $\cos\left[\left(\omega^{1234}+\omega^{2021}\right)\pi-\pi/4\right]$

$=\cos\left[\left(\omega+\omega^2\right)\pi-\pi/4\right]$

$=\cos\left[-\pi-\pi/4\right]=\cos(5\pi/4)=-\frac{1}{\sqrt{2}}$

49. Conceptual

50. $\sin x + i\cos 2x = \cos x - i\sin 2x$

$\Rightarrow \sin x - i\cos 2x = \cos x - \lambda \sin 2x$

Equation real end Img.parts.

Tanx=1 and Tan2x=1

There in no such real x.

$\therefore$Solution set=ϕ

51. G.E=$-i\sum_{k=1}^{6}\left(\cos\frac{2k\pi}{7} + i\sin\frac{2k\pi}{7}\right) = -i(-1) = i$

52. $x^{11} - x^7 + x^4 - 1 = 0$

$\Rightarrow (x^7+1)(x^4-1) = 0$

$\Rightarrow x^4 - 1 = 0 \;\; or \;\; x^7 + 1 = 0$

$x = (1)^{1/4} = cis(2k+1)\frac{\pi}{4}, K = 0,1,2,3 \rightarrow (1)$

Also

$x = (-1)^{1/7} = cis(2K+1)\frac{\pi}{7}, K = 0,1,2,....,6 \rightarrow (2)$

For (1) & (2), cis π is the only common solution.

$\therefore$Number of distinct solutions=10.

53. $z^r + (\overline{Z})^r$

$= (\cos\theta + i\sin\theta)^r + (\cos\theta - i\sin\theta)^r$

$= \cos r\theta + i\sin r\theta + \cos r\theta - i\sin r\theta$

$= 2\cos r\theta$

54. $(-1+i\sqrt{3})^{60} = 2^{60}\left(\frac{-1+i\sqrt{3}}{2}\right)^{60}$

$= 2^{60}\omega^{60}$

$= 2^{60}(1) = 2^{60}$

55. conceptual

56. $1+\omega+\omega^2 = 0 \,\&\, \omega^3 = 1$

$G.E = (2-\omega)^2(2-w^2)^2(2-\omega)^2(2-\omega^2)^2$

$= ((2-\omega)(2-\omega^2))^4$

$= (4 - 2(\omega+\omega^2) + \omega^3)^4$

$= (4+2+1)^4 = 7^4$

57. conceptual

58. $|z|^2\omega - |\omega|^2 z = z - \omega.....(1)$

$(|z|^2+1)\omega = (1+|\omega|^2)z$

$\Rightarrow \frac{z}{\omega} = \frac{(|z|^2+1)}{(1+|\omega|^2)} = real$

$\therefore \frac{z}{\omega} = \frac{\overline{z}}{\overline{\omega}} \Rightarrow z\overline{\omega} = \omega\overline{z}.....(2)$

$(1) \Rightarrow z\overline{z}\omega - z - \omega\overline{\omega}z + \omega = 0$

$\Rightarrow z(\omega\overline{z} - 1) - \omega(\overline{\omega}z - 1) = 0$

$\Rightarrow z(\omega\overline{z} - 1) - \omega(\omega\overline{z} - 1) = 0 \;\; (from(2))$

$\Rightarrow (z-\omega)(\omega\overline{z} - 1) = 0$

$\because z \neq \omega \Rightarrow \omega\overline{z} = 1$

$\Rightarrow \overline{\omega}z = 1$

59. $(1-\omega+\omega^2)^5 + (1+\omega-\omega^2)^5$

$= (-2\omega)^5 + (-2\omega^2)^5$

$= -32(\omega^2+\omega) = -32(-1) = 32$

60. $Argz_1 = \pi/3 \,\&\, Arg\overline{z}_2 = \pi/5$

$\Rightarrow Argz_2 = -\pi/5$

$\therefore Argz_1 + Argz_2 = \frac{\pi}{3} - \frac{\pi}{5} = \frac{2\pi}{15}$

61. $\left(\frac{1-i}{1+i}\right)^{2022} + \left(\frac{1+i}{1-i}\right)^{2021}$

$= \left(\frac{-2i}{1+1}\right)^{2022} + \left(\frac{2i}{1+1}\right)^{2021}$

$= i^{2022} + i^{2021}$

$= (i^2)^{1011} + i(i^2)^{1010}$

$= -1 + i = i - 1$

62. $G \cdot E = x^2(1+\omega^2+\omega) + y^2(1+\omega+\omega^2) + 2xy(1+1+1)$

$= x^2(0) + y^2((c) + 2xy(3)$

=6xy

63. $(1+\omega)$ is a root of $x^n - x = 0$

$\therefore (1+\omega)^n - (1+\omega) = 0$

$\Rightarrow (-\omega^2)^n = -\omega^2 \quad (\because 1+\omega+\omega^2 = 0)$

$\Rightarrow (-1)^n\omega^{2n} = -\omega^2$

From given options, the least value of n is 7

64. Let $a = cis\alpha$, $b = cis\beta$, $c = cis\gamma$

Then $a+b+c=0$ & $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=0$

ab + bc + ca = 0 $\quad a^2+b^2+c^2=0$

$\Rightarrow cis2\alpha + cis2\beta + \cos 2\gamma = 0 \rightarrow (1)$

Equating real parts we get $\sum \text{Cos}\, 2\alpha = 0$

And $ab+bc+ca=0$

$\Rightarrow \sum cis(\alpha+\beta)=0$

$\Rightarrow \sum cos(\alpha+\beta)=0$

Hence $\sum \cos 2\alpha = 0 = \sum \cos(\alpha+\beta)$

65. $(-32i)^{2/5} = 4(-i)^{2/5}$

$$= 4\left[cis\left(\frac{-\pi}{2}\right)\right]^{2/5}$$

$$= 4cis(4K-1)\frac{\pi}{5}, K=0,1,2,3,4$$

$\therefore$one of the value of $(-32i)^{2/5}$ is $4cis\frac{3\pi}{5}$

66. $-1 = cis\pi$

$(-1)^{1/4} = cis(2K+1)\frac{\pi}{4}, K=0,1,2,3$

Also $-i = cis\left(\frac{-\pi}{2}\right)$

$\Rightarrow (-i)^{1/2} = cis(4K-1)\frac{\pi}{4}, K=0,1$

$\therefore$common root is $cis\frac{3\pi}{4}$

Hence $\text{Tan}\,\alpha = \text{Tan}\frac{3\pi}{4} = -1$

67. Conceptual

68. Given that $1,\alpha,\alpha^2,\alpha^3$ are the $\sqrt[4]{1}$ are1,I,=1,–i

So that $\alpha = i$

$\therefore \alpha + \alpha\omega - \alpha^3\omega^2 = i + i\omega + i\omega^2$

$$= i(1+2\omega+\omega^2)$$

$$= i(0) = 0$$

69. Roots of $x^2-2x+2=0$ are $1\pm i$.

Given $\Rightarrow \alpha = 1+i$ & $\beta = 1-i$

$\therefore \alpha = \sqrt{2}\left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i\right) = \sqrt{2}cis\frac{\pi}{4}$

$\therefore \beta = \sqrt{2}\left(\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}i\right) = \sqrt{2}cis\left(\frac{-\pi}{4}\right)$

$\alpha^{2020}+\beta^{2020} = 2^{2010}\left(2\cos 2020\frac{\pi}{4}\right)$

$$= 2^{2011}(-1) = -2^{2011}$$

70. $z=\omega^2 \Rightarrow G.E = \sum_{k=1}^{2022}\left(\omega^{2k}+\frac{1}{\omega^{2k}}\right)^2$

$$= 674\left[\left(\omega^2+\frac{1}{\omega^2}\right)^2+\left(\omega^4+\frac{1}{\omega^4}\right)^2+\left(\omega^6+\frac{1}{\omega^6}\right)^2\right]$$

$$= 674\times 8 = 4044$$

71. $\text{G.E}=2^9\left|\left(\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)^9+\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^9\right|$

$$= 2^9\left|\left(\cos\frac{\pi}{3}-i\sin\frac{\pi}{3}\right)^9+\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)^9\right|$$

$$= 2^9\left|\cos 3\pi - i\sin 3\pi + \cos\frac{3\pi}{2}+i\sin\frac{3\pi}{2}\right|$$

$$= 2^9\left|-1-0+0-i\right| = 2^9\cdot\sqrt{2} = 2^{\frac{19}{2}}$$

72. $\left(\sqrt{2}\right)^8\left[cis\,8\times 56^o15^1\right] = 16\,cis\,450^o$

$= 16(0+i)$

$= 16i$

73. $\left(\sqrt{2}\right)^{2024}\left[\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)^{2024}+\left(\cos\frac{\pi}{4}-i\sin\frac{\pi}{4}\right)^{2024}\right]$

$\left(\sqrt{2}\right)^{2024} 2\cos \overset{506}{\cancel{2024}}\times\frac{\pi}{\cancel{4}}$

$= 2^{1012}\times 2^1\cos 506\pi$

$= 2^{1013}(1)$

74. $\sqrt[15]{-1} = (cis\,\pi)^{\frac{1}{15}}$

$= cis\left(\frac{2k\pi+\pi}{15}\right)$,k=0,1,2,……..14

$$= cis\left(\frac{(2k+1)\pi}{15}\right),$$

$$k=0 \to cis\left(\frac{\pi}{15}\right)$$

$$k=6 \to cis\left(\frac{13\pi}{15}\right)$$

75.

$$\left(1+i\sqrt{3}\right)^{\frac{3}{4}} = 2^{\frac{3}{4}}\left[\frac{1}{2}+i\frac{\sqrt{3}}{2}\right]^{\frac{3}{4}}$$

$$= 2^{\frac{3}{4}}\left[cis\,\frac{\pi}{3}\right]^{\frac{3}{4}}$$

$$= 2^{\frac{3}{4}}(cis\,\pi)^{\frac{1}{4}}$$

$$= 2^{\frac{3}{4}}.\text{cis}\left(\frac{2k\pi+\pi}{4}\right), k=0,1,2,3$$

$$= 2^{\frac{3}{4}}.cis\left(\frac{(2k+1)\pi}{4}\right)$$

$k=0 \to 2^{\frac{3}{4}}.cis\left(\frac{\pi}{4}\right)$, $\quad k=2 \to 2^{\frac{3}{4}}.cis\left(\frac{5\pi}{4}\right)$

$k=1 \to 2^{\frac{3}{4}}.cis\left(\frac{3\pi}{4}\right)$, $\quad k=3 \to 2^{\frac{3}{4}}.cis\left(\frac{7\pi}{4}\right)$

Product of the four values

$$=\left(2^{\frac{3}{4}}\right)^4 cis\left(\frac{\pi}{4}+\frac{3\pi}{4}+\frac{5\pi}{4}+\frac{7\pi}{4}\right)$$

$$= 2^3\, cis\left(\frac{16\pi}{4}\right)$$

$$= 8(1)$$

$=8$

76.

Given $\cos\left(\sum_{k=1}^{7}(k-w)(k-w^2)\frac{\pi}{175}\right)$

$= \cos\left(\frac{\pi}{175}\sum_{k=1}^{7}(k-w)(k-w^2)\frac{\pi}{175}\right)$

$= \cos$

$$\left(\frac{\pi}{175}\sum_{k=1}^{7}\left(k^2+k+1\right)\right) = \cos\left(\frac{\pi}{175}(140+28+7)\right)$$

$$= \cos\frac{\pi}{\cancel{175}}(\cancel{175}) = \cos\pi = -1$$

77. $y^2+y+1=0$

Given : a+b=−1 , ab=1

$$\Rightarrow a=\omega,\ b=\omega^2$$

$$a^4+b^4+\ a^{-1}.b^{-1} = \omega^4+\omega^8+\omega^{-3}$$

$$=\omega+\omega^2+1=0$$

78. $\left(1-w+w^2\right)^{3k}+\left(1-w^2+w\right)^{3k} = \left(1-w+w^2\right)^{3k+1}$

$$+\left(1+w-w^2\right)^{3k+1}$$

$$(-2w)^{3k}+\left(-2w^2\right)^{3k} = (-2w)^{3k+1}+(-2w^2)^{3k+1}$$

$$(-8)^k+(-8)^k = (-2w)(-8)^k+\left(-2w^2\right)(-8)^k$$

$$2(-8)^k = -2(-8)^k(-1)$$

$$2(-8)^k = 2(-8)^k$$

$\therefore$ It is true for all $K \in N$

79. α,β are real

$$\Rightarrow \alpha=1, \beta=-1, \gamma=i, \delta=-i$$

$$|z-\alpha|+|z-\beta|=4$$

$$|z-1|+|z+1|=4$$

$$|(x-1)+iy|+|(x+1)+iy|=4$$

$$sp+s^1p=2a$$

$$s(1,0),\ s^1(-1,0),\quad 2a=4$$

$$ss^1=2ae_1$$

$$2=4e_1 \Rightarrow e_1=\frac{1}{2}$$

$$|z-\gamma|+|Z-\delta|=6$$

$$|z+i|+|z-i|=6$$

$$|x+(y+1)i|+|x+i(y-1)|=6$$

$$s(0,-1), s^1(0,1), 2a=6$$

$$ss^1 = 2ae_2$$

$$2 = 6e_2 \Rightarrow e_2 = \frac{1}{3}$$

$$e_1 + e_2 = \frac{1}{2} + \frac{1}{3} = \frac{5}{6}$$

80. $x^3 - a^3 = 0$

$\alpha = a$ *is a real root*

$$x^3 - a^3 = (x-a)\,(x^2 + ax + a^2)$$

$$\Rightarrow x^2 + ax + a^2 = 0$$

$$x = \frac{-a \pm \sqrt{a^2 - 4a^2}}{2} = \frac{-a + ai\sqrt{3}}{2} = a\omega, a\omega^2$$

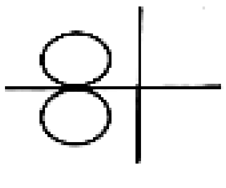
$$|z - a\omega| = \frac{\sqrt{3}a}{2}$$

$$|z - a\omega^2| = \frac{\sqrt{3}}{2}a$$

$$\left(x + \frac{a}{2}\right)^2 + \left(y - \frac{\sqrt{3}a}{2}\right)^2 = \frac{3a^2}{4}$$

$$\left(x - \frac{a}{2}\right)^2 + \left(y + \frac{\sqrt{3}a}{a}\right)^2 = \frac{3a^2}{4}$$

Both circles touch
each other at only one
point number of common points=1



81. $\sin\left[(\omega + \omega^2)\pi - \frac{\pi}{4}\right]$

$$\sin\left(-\pi - \frac{\pi}{4}\right)$$

$$-\sin\frac{5\pi}{4} = \frac{1}{\sqrt{2}}$$

82. $\Rightarrow (x+y)(x\omega + y\omega^2)(x\omega^2 + y\omega)$

$\Rightarrow x^3 + x^2y\omega^2 + x^2y\omega^4 + xy^2 + x^2y + xy^2\omega^2 + xy^2\omega^4 + y^3$

$\Rightarrow x^3 + x^2y\omega^2 + x^2y\omega + xy^2 + x^2y + xy^2\omega^2 + xy^2\omega + y^3$

$= x^3 - x^2y - xy^2 + xy^2 + x^2y + y^3$

$= x^3 + y^3 = 8 + 27 = 35$

83. $8z^3 - 12z^2 + 6z - 28 = 0$

$\Rightarrow (2z)^3 - 3(2z)^2(1) + 3(2z) - 1 = 27$

$\Rightarrow (2z-1)^3 = 27$

$\Rightarrow 2z - 1 = 3(1)^{1/3}$

$3(1, w, w^2)$

$$z = 2, \frac{3w+1}{2}, \frac{3w^2+1}{2}$$

84. $T_{10} = (\cos\theta + i\sin\theta)^9$

$$= \left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)^9$$

$$= \cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}$$

$$= -i$$

85.
$$\begin{vmatrix} w + w^2 & w^2 + w^9 & w^9 + w \\ w^{27} + w^{31} & w^{31} + w^{17} & w^{17} + w^{27} \\ w^{10} + w^{41} & w^{41} + w^{19} & w^{19} + w^{30} \end{vmatrix}$$

use $1 + w + w^2 = 0$

and $w^3 = 1$

86. $(1+i)^{10} + (1-i)^{10}$

$$\left((1+i)^2\right)^5 + \left((1-i)^2\right)^5$$

$$= (1 - 1 + 2i)^5 + (1 - 1 - 2i)^5 = 0$$

87. $(1+\sqrt{3}i)^{2023} + (1-\sqrt{3}i)^{2023}$

$$2^{2023}\left(2\cos\frac{2023\pi}{3}\right)$$

$$2^{2023}\left(2\cos\left(674\pi + \frac{\pi}{3}\right)\right)$$

$$2^{2023}.2.\frac{1}{2}=2^{2023}$$

88. $\left(\sqrt{3}-i\right)^{1/6}$

$$\left[2\left[\frac{\sqrt{3}}{2}-\frac{i}{2}\right]\right], \theta\in Q_4$$

$$2^{1/6}\left(\cos\left(\frac{-\pi}{6}\right)+i\sin\left(\frac{-\pi}{6}\right)\right)^{1/6}$$

$$2^{1/6}cis\left(\frac{2k\pi-\pi/6}{6}\right), k=0,1,2\ldots\ldots\ldots$$

Put $k=5$ then

$$2^{1/6}cis\frac{59\pi}{36}$$

89. $x^n=\cos\frac{\pi}{2^n}+i\sin\frac{\pi}{2^n}$

$$x^n=cis\frac{\pi}{2^n}$$

$$\therefore \pi_{n=1}^{\infty}x^n = Cis\ \frac{\pi}{2}\ Cis\ \frac{\pi}{2^2}\ldots\ldots\ldots$$

$$= Cis\ \pi\left(\frac{\frac{1}{2}}{1-\frac{1}{2}}\right)=(\text{cis}\,\pi(1)$$

$$= Cis\ \pi=\cos\pi+i\sin\pi$$

$$= -1$$

90. $\left[\left(\sqrt{3}-i\right)^2\right]^{\frac{1}{5}}=\left(3-1-2\sqrt{3}i\right)^{\frac{1}{5}}$

$$=2^{\frac{1}{5}}\left(1-i\sqrt{3}\right)^{\frac{1}{5}}$$

$$=2^{2/5}cis\left(\frac{2k\pi-\pi/3}{5}\right), k=0,1,2,3,4$$

If k=1

$$\Rightarrow\ =2^{2/5}\ cis\left(\frac{\pi}{3}\right)$$

$$=2^{-3/5}\left(1+\sqrt{3}i\right)$$