42. DEFINITE INTEGRATION

1. $\int_{1}^{e^2} \frac{dx}{x(1+\log x)^2} =$

[AP EAMCET 17-09-20_Shift-1]

1. $\frac{2}{3}$ 2. $\frac{1}{3}$
3. $\frac{3}{2}$ 4. $\log 2$

2. $\int_{-\pi}^{\pi} x^2(\sin x)dx =$

[AP EAMCET 17-09-20_Shift-1]

1. π 2. $\frac{\pi^2}{2}$
3. 0 4. $2\pi^2$

3. $\int_0^1 \text{Sin}^{-1}\left(\frac{2x}{1+x^2}\right) dx =$

[AP EAMCET 17-09-20_Shift-2]

1. $\pi-\log 2$ 2. $\pi+\log 2$
3. $\frac{\pi}{2}-\log 2$ 4. $\frac{\pi}{2}+\log 2$

4. $\int_0^1 \frac{8\log(1+x)}{1+x^2} dx =$

[AP EAMCET 17-09-20_Shift-2]

1. $\pi\log 2$ 2. $\frac{\pi}{2}\log 2$
3. $\frac{\pi}{4}\log 2$ 4. $\log 2$

5. $\int_{-\pi/4}^{\pi/4} x^3 \sin^4(x)\, dx =$

[AP EAMCET 17-09-20_Shift-2]

1. 0 2. π
3. 1 4. 2π

6. $\int_0^{\frac{\pi}{4}} \frac{dx}{\cos^3(x).\sqrt{2\sin(2x)}} =$

[AP EAMCET 18-09-20_Shift-1]

1. $\frac{6}{5}$ 2. $\frac{3}{5}$
3. $\frac{4}{5}$ 4. $\frac{8}{5}$

7. $\int_{-1}^{2} |x|dx =$ **[AP EAMCET 18-09-20_Shift-1]**

1. 1 2. 2
3. $\frac{5}{2}$ 4. $\frac{3}{2}$

8. Find the value of 'k' , if it is given that

$$\int_0^{b-c} f(x+c)dx = k\int_c^b f(x)$$

[AP EAMCET 18-09-20_Shift-1]

1. 1 2. 2
3. 0 4. -2

9. $\int_0^{\pi/2} e^{\sin x}.\cos x\, dx =$

[AP EAMCET 18-09-20_Shift-1]

1. $1-e$ 2. $1+e$
3. e-1 4. e

10. $\int_0^k (\sqrt{k}-\sqrt{t})^2\, dt =$

[AP EAMCET 18-09-20_Shift-2]

1. $\frac{k^2}{2}$ 2. $\frac{k^2}{4}$
3. $\frac{k^2}{6}$ 4. $\frac{k^2}{8}$

11. If f is integrable on [0,a], then the function h defined on [0,a] as h(x)=___ $\forall x \in [0,a]$ is integrable on [0,a]

[AP EAMCET 18-09-20_Shift-2]

1. f(a-x) 2. f(x-a)
3. f(x) 4. f(a)

12. $\int_0^{\frac{\pi}{4}} \left(\tan^2 x + \tan^4 x\right)dx =$

[AP EAMCET 18-09-20_Shift-2]

1. 3 2. 2
3. $\frac{1}{3}$ 4. 0

13. $I_n = \int_0^{\frac{\pi}{2}} \sin^n(x)dx$ and $I_n = (k)I_{n-2}$ then what will be the value of k?

[AP EAMCET 18-09-20_Shift-2]

1. $\frac{n}{n-1}$
2. $\frac{n-1}{n}$
3. $\frac{n+1}{n}$
4. $\frac{n}{n+1}$

14. $\int_0^{\frac{\pi}{4}} \tan^2(x)dx =$

[AP EAMCET 21-09-20_Shift-1]

1. $\frac{\pi}{4}$
2. $\frac{\pi}{4}$-1
3. 1-$\frac{\pi}{4}$
4. 0

15. $\int_0^{\frac{\pi}{2}} |\sin t - \cos t| dt =$

[AP EAMCET 21-09-20_Shift-2]

1. $2\left(\sqrt{2}+1\right)$
2. $2\left(\sqrt{2}-1\right)$
3. $\left(\sqrt{2}+1\right)$
4. $\left(\sqrt{2}-1\right)$

16. $\int_0^{\frac{\pi}{2}} \frac{1}{1+\tan^{2020}(x)} dx =$

[AP EAMCET 22-09-20_Shift-1]

1. π
2. $\frac{\pi}{2}$
3. $\frac{\pi}{4}$
4. 0

17. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(2\sin|x| + \cos|x|\right) dx =$

[AP EAMCET 22-09-20_Shift-1]

1. 3
2. 6
3. 8
4. 2

18. $\int_0^{2\pi} \frac{x\cos x}{1+\cos x} dx =$

[AP EAMCET 22-09-20_Shift-1]

1. $\frac{\pi}{6}$
2. π^2
3. $\frac{\pi}{4}$
4. $2\pi^2$

19. Choose the correct option regarding following definite integrals

A. $\int_0^{\frac{\pi}{2}} \sin^m(x)\cos(x)\, dx = \frac{1}{m+1}$

B. $\int_0^{\frac{\pi}{2}} \sin(x)\cos^n(x)\, dx = \frac{1}{n+1}$

[AP EAMCET 22-09-20_Shift-2]

1. A is true, B is false
2. A is false, B is true
3. Both A and B are false
4. Both A and B are true

20. $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{1+\sqrt{\cot x}} dx =$

[AP EAMCET 22-09-20_Shift-2]

1. $\frac{\pi}{12}$
2. $\frac{\pi}{6}$
3. $\frac{\pi}{4}$
4. $\frac{\pi}{13}$

21. The value of f(1) , given the equation $\int_0^{x^2} xf(t)dt = x^5 - x^3$ is

[AP EAMCET 22-09-20_Shift-2]

1. 4
2. 3
3. 2
4. 1

22. If $I_1 = \int_0^{\pi/2} \frac{x}{\sin x} dx$ and $I_2 = \int_0^1 \frac{\mathrm{Tan}^{-1} x}{x} dx$, then $I_1 : I_2$ is

[AP EAMCET 22-09-20_Shift-2]

1. 1:1
2. 2:1
3. 3:1
4. 4:1

23. $\int_0^{\pi/2} \frac{x+\sin x}{1+\cos x} dx =$

[AP EAMCET 23-09-20_Shift-1]

1. $\frac{\pi}{4}$
2. $\frac{\pi}{3}$
3. $\frac{\pi}{2}$
4. $\frac{\pi}{6}$

24. $\int_{-a}^{a} \sqrt{\frac{a-x}{a+x}}dx =$

[AP EAMCET 23-09-20_Shift-1]

1. $\frac{a\pi}{2}$ 2. 1
3. $2a\pi$ 4. $a\pi$

25. $\int_0^{\frac{\pi}{2}} \frac{2\sin x + 3\cos x}{\sin x + \cos x}dx =$

1. $\frac{\pi}{4}$ 2. $\frac{3\pi}{4}$
3. $\frac{5\pi}{4}$ 4. 0

26. $\int_0^{2a} f(x)dx - \int_a^{2a} f(x)dx =$

1. $\int_0^a f(x)dx$ 2. $-\int_0^a f(x)dx$
3. $-\int_0^{2a} f(x)dx$ 4. $\int_0^{a/2} f(x)dx$

27. If $A = \int_1^{\sin\theta} \frac{t}{1+t^2}dt$ and $B = \int_1^{\mathrm{cosec}\theta} \frac{1}{t(1+t^2)}dt$ then the value of $\begin{vmatrix} A & A^2 & B \\ e^{A+B} & B^2 & -1 \\ 1 & A^2+B^2 & -1 \end{vmatrix} =$

1. $\sin\theta$ 2. $\mathrm{cosec}\,\theta$
3. 0 4. 1

28. $\int_0^{\frac{\pi}{2}} x^3 \sin x dx =$

[TS EAMCET 09-09-20_Shift-1]

1. $\frac{3\pi^2}{4} - 3\pi + 6$ 2. $\frac{3\pi^2}{4} + 3\pi - 6$
3. $\frac{3\pi^2}{4} + 6$ 4. $\frac{3\pi^2}{4} - 6$

29. $\lim_{n\to\infty} \prod_{r=1}^{n} \left(1 + \frac{r^2}{n^2}\right)^{\frac{2r}{n^2}} =$

[TS EAMCET 09-09-20_Shift-1]

1. $\log\left(\frac{4}{e}\right)$ 2. $\log\left(\frac{2}{e}\right)$
3. $\frac{2}{e}$ 4. $\frac{4}{e}$

30. If $I_n = \int_0^a \frac{x^n}{\sqrt{a^2 - x^2}} dx$, then $\frac{I_8}{I_4} =$

[TS EAMCET 09-09-20_Shift-2]

1. $\frac{48}{35a^2}$ 2. $\frac{35}{48}a^4$
3. $\frac{19}{72}a^6$ 4. $\frac{29}{56}a^4$

31. $\int_{-5}^{5} x^4 \left(25 - x^2\right)^{\frac{5}{2}} dx =$

[TS EAMCET 09-09-20_Shift-2]

1. $\frac{5^9}{2}\frac{\pi}{2}$ 2. $\frac{16\left(5^9\right)}{63}$
3. $\frac{3\left(5^{10}\right)}{256}\pi$ 4. $\frac{16\left(5^{10}\right)}{693}$

32. $\lim_{n\to\infty}\left[\left(1+\frac{1}{n^2}\right)\left(1+\frac{2^2}{n^2}\right)....\left(1+\frac{n^2}{n^2}\right)\right]^{\frac{1}{n}} =$

[TS EAMCET 10-09-20_Shift-1]

1. e 2. $2e$
3. $2e^{\frac{\pi-2}{2}}$ 4. $2e^{\frac{\pi-4}{2}}$

33. $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{3dx}{1 + e^{\sqrt{8}\sin\left(x - \frac{3\pi}{8}\right)}} =$

[TS EAMCET 10-09-20_Shift-1]

1. $\frac{3\sqrt{2}}{4}\pi$ 2. $\frac{3}{4}\pi$
3. $\frac{\pi}{8}$ 4. $\frac{3}{8}\pi$

34. $\lim_{n\to\infty} \frac{\pi}{2n}\left[\sin\frac{\pi}{2n} + \sin\frac{2\pi}{2n} + ... + \sin\frac{\pi}{2}\right] =$

[TS EAMCET 10-09-20_Shift-2]

1. 1 2. 0
3. 4 4. 3

35. $\int_0^{\frac{\pi}{2}} \frac{dx}{4+5\sin x} =$

[TS EAMCET 10-09-20_Shift-2]

1. $\frac{1}{2}\log 3$
2. $\frac{1}{3}\log 2$
3. $2\log 3$
4. $\frac{1}{2}\log\frac{3}{2}$

36. The positive integer $n \le 5$ for which $\int_0^1 e^x (x-1)^n\, dx = 16-6e$ is

[TS EAMCET 11-09-20_Shift-1]

1. 5
2. 4
3. 3
4. 2

37. If $f(x) = \sin^6 x + \cos^6 x + 2\sin^3 x\cos^3 x$, then $\int_0^{\frac{\pi}{4}} \frac{\sin^2 2x}{f(x)} dx =$

[TS EAMCET 11-09-20_Shift-1]

1. 2
2. $\frac{2}{3}$
3. $\frac{-2}{3}$
4. $\frac{1}{6}$

38. $\int_3^5 (x-3)^3 (5-x)^5\, dx =$

[TS EAMCET 11-09-20_Shift-1]

1. $\frac{64}{63}$
2. $\frac{25}{7}$
3. $\frac{3}{32}$
4. $\frac{16}{25}$

39. If $f(x) = \begin{vmatrix} 1+\sin x+\sin 2x+\sin 3x & \frac{3+\sin 2x}{2} & \frac{-2+\sin 3x}{3} \\ 3+4\sin x & \frac{3}{2} & \frac{4}{3}\sin x \\ 1+\sin x & \frac{1}{2}\sin x & \frac{1}{3} \end{vmatrix}$,

then $\int_0^{\pi/2} \left(f(x)+f'(x)\right) dx =$

[TS EAMCET 11-09-20_Shift-2]

1. $\frac{-1}{6}$
2. $\frac{-1}{9}$
3. $\frac{-2}{9}$
4. $\frac{1}{27}$

40. $\lim_{n\to\infty} \frac{1}{n}\left[\frac{1}{n}\mathrm{Sin}^{-1}\frac{1}{n} + \frac{2}{n}\mathrm{Sin}^{-1}\frac{2}{n} + \ldots.. + \frac{\pi}{2}\right] =$

[TS EAMCET 11-09-20_Shift-2]

1. $\frac{\pi}{2}$
2. $\frac{\pi}{3}$
3. $\frac{\pi}{8}$
4. $\frac{\pi}{4}$

41. $\int_2^4 \left\{|x-2|+|x-3|\right\} dx =$

[AP EAMCET 19-08-2021_Shift-1]

1. 1
2. 2
3. 3
4. 4

42. $\int_{-1/2}^{1/2} \left\{[x] + \log\left(\frac{1+x}{1-x}\right)\right\} dx =$

[AP EAMCET 19-08-2021_Shift-1]

1. $2\log\left(\frac{1}{2}\right)$
2. 0
3. $\frac{-1}{2}$
4. 1

43. If $\int_0^{\pi/2} \tan^n(x)\,dx = k\int_0^{\pi/2} \cot^n(x)\,dx$, then ___

[AP EAMCET 19-08-2021_Shift-2]

1. $k=1$
2. $k=2$
3. $k=\frac{1}{2}$
4. $k=3$

44. $\int_0^2 xe^x dx =$ **[AP EAMCET 19-08-2021_Shift-2]**

1. e^2+1
2. e^2-1
3. $e^{-1}-1$
4. $e^{-1}+1$

45. If $\int_0^a \frac{dx}{4+x^2} = \frac{\pi}{8}$, then the value of $a =$ ___

[AP EAMCET 20-08-2021_Shift-1]

1. 1
2. 2
3. 3
4. 4

46. $\int_1^2 \frac{x^3-1}{x^2} dx =$

[AP EAMCET 20-08-2021_Shift-1]

1. $\frac{5}{3}$
2. $\frac{3}{5}$
3. 1
4. -1

47. If $\int_0^1 f(x)dx = 1, \int_0^1 xf(x)dx = a, \int_0^1 x^2 f(x)dx = a^2$

then $\int_0^1 (x-a)^2 f(x)dx =$

[AP EAMCET 23-08-2021_Shift-1]

1. a^2
2. a^2+1
3. a^2-1
4. 0

48. If $\int_0^{\pi} \log(\sin x)dx = 8k$, then $\int_0^{\pi/4} \log(1+\tan x)dx =$ _____

[AP EAMCET 20-08-2021_Shift-2]

1. k
2. $-k$
3. $\frac{k}{2}$
4. $4k$

49. If $\int_0^1 x^m (1-x)^n dx = k\int_0^1 x^n (1-x)^m dx$, then the value of k equals____

[AP EAMCET 20-08-2021_Shift-2]

1. m
2. n
3. $\frac{1}{mn}$
4. 1

50. $\int_{-1}^{1} \frac{|x|}{x} dx =$

[AP EAMCET 23-08-2021_Shift-1]

1. 1
2. -1
3. 0
4. 1/2

51. $\int_{2/e}^{1/e} \frac{1}{x(\log x)^{1/3}} dx =$

[AP EAMCET 23-08-2021_Shift-2]

1. $\frac{3}{2}\left\{1+(\log(2)-1)^{2/3}\right\}$
2. 1
3. $\frac{3}{2}\left\{1+(\log(2)+1)^{3/2}\right\}$
4. $\frac{3}{2}\left\{1-(\log(2)-1)^{2/3}\right\}$

52. $\int_0^{\pi} \frac{x\tan x}{\sec x+\tan x} dx =$

[AP EAMCET 24-08-2021_Shift-1]

1. $\frac{\pi(\pi-2)}{2}$
2. $\frac{\pi+2}{2}$
3. $\frac{\pi(\pi+2)}{2}$
4. $\frac{\pi-2}{2}$

53. If $b>a$, then $\int_a^b \frac{dx}{\sqrt{(x-a)(b-x)}} =$

[AP EAMCET 24-08-2021_Shift-1]

1. $\frac{\pi}{2}$
2. $\frac{\pi}{3}$
3. $\frac{\pi}{6}$
4. π

54. $\int_{-\pi/4}^{\pi/4} x\tan(1+x^2)dx =$

[AP EAMCET 24-08-2021_Shift-2]

1. 0
2. $\frac{\pi}{4}$
3. $-\frac{\pi}{4}$
4. 1

55. If $I = \int_0^{\pi} x\{\sin^2(\sin x)+\cos^2(\cos x)\}dx$, then $[I] =$ ______. Here $[.]$ denotes G.I.F

[AP EAMCET 24-08-2021_Shift-2]

1. 3
2. 4
3. 5
4. 6

56. If $\int_a^b x^3 dx = 0$ and $\int_a^b x^2 dx = \frac{2}{3}$ then

[AP EAMCET 25-08-2021_Shift-1]

1. $a=-1 \& b=1$
2. $a=1 \& b=-1$
3. $a=2 \& b=-2$
4. $a=-2 \& b=2$

57. If $\int_1^4 x\sqrt{x^2-1}dx = \alpha(k)^{\beta}$, then $\alpha\beta =$ _____

[AP EAMCET 25-08-2021_Shift-2]

1. $\frac{9}{2}$
2. $\frac{1}{2}$
3. $\frac{1}{3}$
4. $\frac{3}{2}$

58. $\int_0^1 \frac{xe^x}{(x+1)^2}dx =$

[AP EAMCET 25-08-2021_Shift-2]

1. $\frac{e}{2}$
2. $\frac{e}{2}-1$
3. $\frac{e}{2}+1$
4. $2e$

19. $\lim_{n\to\infty}\frac{1}{n}\left[\sin\frac{\pi}{4}+\sin\frac{\pi}{12}\left(3+\frac{1}{n}\right)+\sin\frac{\pi}{12}\left(3+\frac{2}{n}\right)+...+\sin\frac{\pi}{3}\right]=$

[TS EAMCET 04-08-2021_Shift-2]

1. $\frac{\sqrt{2}-1}{2\sqrt{2}}$
2. $\frac{6(\sqrt{2}-1)}{\pi}$
3. $\frac{\sqrt{2}-1}{6\pi}$
4. 0

60. $\int_0^{\pi/2}\frac{\sin^{3/2}x}{\sin^{3/2}x+\cos^{3/2}x}dx =$

[TS EAMCET 04-08-2021_Shift-2]

1. π
2. $\frac{\pi}{2}$
3. $\frac{\pi}{4}$
4. 0

61. $\int_5^9 \frac{\log 3x^2}{\log 3x^2+\log(588-84x+3x^2)}dx$

[TS EAMCET 04-08-2021_Shift-1]

1. 2
2. 1
3. $\frac{1}{2}$
4. 4

62. $\int_{-1}^{1}\frac{\log(1+x)}{1+x^2}dx=\int_0^1\frac{\log(1+x)}{1+x^2}dx+\int_0^1 f(x)dx$

then $f(x)$ =

[TS EAMCET 04-08-2021_Shift-1]

1. $\frac{\log(1+x)}{1+x^2}$
2. $\frac{-\log(1+x)}{1+x^2}$
3. $\frac{\log(1-x)}{1+x^2}$
4. 0

63. $\lim_{n\to\infty}\frac{1}{n}\log\left(\frac{(2n)!}{n^n\cdot n!}\right)=\int_1^2 f(x)dx,$ then $f(x)=$

[TS EAMCET 05-08-2021_Shift-1]

1. $\log(1+x)$
2. $\log\left(\frac{1}{x}\right)$
3. $\log x$
4. $\log\left(\frac{x+1}{x}\right)$

64. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\sin(x-[x])dx =$

[TS EAMCET 05-08-2021_Shift-1]

1. $3(1-\cos 1)+\sin 2-\sin 1$
2. $\cos 2-\sin 2$
3. $3(1-\cos 1)+\cos 2-\sin 1$
4. 0

65. Let $\{x\}$ denotes the fractional part of a real number x. Then $\int_0^2\{x\}dx =$

[TS EAMCET 05-08-2021_Shift-2]

1. 1
2. 2
3. 3
4. 0

66. $\int_0^2\left|1-x^2\right|dx =$

[TS EAMCET 05-08-2021_Shift-2]

1. 1
2. 2
3. 3
4. $\frac{1}{2}$

67. Let $\tan^0 x=1$. If $\int\left(\sum_{k=0}^{7}\tan^k x\right)dx=\sum_{k=1}^{7}A_k\tan^k x+C$, then $\sum_{k=1}^{7}A_k =$

[TS EAMCET 06-08-2021_Shift-2]

1. $\frac{76}{25}$
2. $\frac{28}{15}$
3. $\frac{38}{35}$
4. $\frac{124}{75}$

68. $\int_0^{\frac{\pi}{2}} \frac{\pi \sin x}{1+\cos^2 x} dx =$

[TS EAMCET 06-08-2021_Shift-2]

1. π^2
2. $\frac{\pi^2}{2}$
3. $\frac{\pi^2}{4}$
4. $\frac{\pi^2}{6}$

69. $\int_0^{\pi} \sqrt{1+4\sin^2 \frac{x}{2}+4\sin\frac{x}{2}}\, dx =$

[TS EAMCET 06-08-2021_Shift-2]

1. π
2. $\pi+2$
3. $\pi+4$
4. 0

70. $\int_{\pi/4}^{3\pi/4} \frac{dx}{1+\cos x} =$

[TS EAMCET 06-08-2021_Shift-1]

1. $\pi-2$
2. $\pi+2$
3. $\frac{\pi}{4}$
4. $2\sin\frac{\pi}{2}$

71. $\int_0^{\pi} x f(\sin x) dx =$

[TS EAMCET 06-08-2021_Shift-1]

1. $2\pi \int_0^{\pi/4} f(\sin x) dx$
2. $\pi \int_0^{\pi/4} f(\sin x) dx$
3. $2\pi \int_0^{\pi/2} f(\sin x) dx$
4. $\pi \int_0^{\pi/2} f(\sin x) dx$

72. $\int_0^1 a^k x^k dx =$

[AP EAMCET 04-07-2022_Shift-1]

1. $\lim_{n\to\infty} \frac{a^k\left(1+2^k+3^k...+n^k\right)}{n^{k+1}}$
2. $\lim_{n\to\infty} \frac{a^k+a^k+...+a^k}{n^{k+1}}$
3. $\lim_{n\to\infty} \frac{1}{n}\sum\left(\frac{r}{n}\right)^k$
4. $\lim_{n\to\infty} \frac{1}{n}\sum\left(\frac{2r}{n}\right)^k$

73. Let α and $\beta(\alpha<\beta)$ are roots of $18x^2-9\pi x+\pi^2=0, f(x)=x^2, g(x)=\cos x$. Then $\int_\alpha^\beta x(gof(x))dx =$

[AP EAMCET 04-07-2022_Shift-1]

1. $\frac{\sqrt{3}-1}{4}$
2. $\frac{\sqrt{3}}{4}$
3. $\frac{2+\sqrt{3}}{2}$
4. $\frac{1}{2}\left(\sin\frac{\pi^2}{9}-\sin\frac{\pi^2}{36}\right)$

74. $\int_0^{\pi} x\left(\sin^2(\sin x)+\cos^2(\cos x)\right)dx =$

[AP EAMCET 04-07-2022_Shift-1]

1. π^2
2. $\pi^2/2$
3. 2π
4. $\pi/4$

75. $\lim_{n\to\infty}\left(\frac{1}{1+n^5}+\frac{2^4}{2^5+n^5}+\frac{3^4}{3^5+n^5}+...+\frac{n^4}{n^5+n^5}\right)=$

[AP EAMCET 04-07-2022_Shift-1]

1. $\frac{1}{5}\log 3$
2. $\frac{1}{3}\log 5$
3. $\frac{1}{2}\log 5$
4. $\log\sqrt[5]{2}$

76. *If* $I_n = \int_0^{\pi/4} \tan^n x\, dx$, *then* $\frac{1}{I_2+I_4}+\frac{1}{I_3+I_5}+\frac{1}{I_4+I_6}=$

[AP EAMCET 04-07-2022_Shift-2]

1. $\frac{1}{I_9+I_{11}}$
2. $\frac{1}{I_{10}+I_{12}}$
3. $\frac{1}{I_{12}+I_{14}}$
4. $\frac{1}{I_{11}+I_{13}}$

77. $\int_0^{\pi/4} e^{\tan^2\theta}\sin^2\theta\tan\theta\, d\theta =$

[AP EAMCET 04-07-2022_Shift-2]

1. $\frac{1}{2}\left(\frac{e}{2}-1\right)$
2. $\frac{e}{2}-1$
3. $\frac{\pi}{2}$
4. $2\left(\frac{\pi}{2}-e\right)$

78. $\int_{\pi/4}^{5\pi/4} \left(|\cos t|\sin t + |\sin t|\cos t\right) dt =$

[AP EAMCET 04-07-2022_Shift-2]

1. 0
2. 1
3. 1/2
4. $\sqrt{3}/2$

79. *If* $f(x) = Max\{\sin x, \cos x\}$ *and* $g(x) = Min\{\sin x, \cos x\}$, *then* $\int_0^{\pi} f(x)dx + \int_0^{\pi} g(x)dx =$

[AP EAMCET 04-07-2022_Shift-2]

1. $2\sqrt{2}+2$
2. $2\sqrt{2}-2$
3. 2
4. $2\sqrt{2}$

80. Let T>0 be a fixed number. $f: R \to R$ is a continuous function such that $f(x+T) = f(x)$ $x \in R$. If I = $\int_o^T f(x)dx$, then $\int_o^{5T} f(2x)dx =$

[AP EAMCET 05-07-2022_Shift-1]

1. 10 I
2. $\frac{5}{2}I$
3. 5 I
4. 2 I

81. $\int_1^3 x\sqrt[n]{x^2-1}\ dx = 6$ then n =

[AP EAMCET 05-07-2022_Shift-1]

1. 2
2. 3
3. 4
4. 5

82. [.] represents greatest integer function, then $\int_{-1}^{1} \left(x[1+\sin \pi x]+1\right) dx =$

[AP EAMCET 05-07-2022_Shift-1]

1. 1
2. 2
3. 5/2
4. 3/2

83. $\lim_{n\to\infty}\left[\frac{n}{(n+1)\sqrt{2n+1}} + \frac{n}{(n+2)\sqrt{2(2n+2)}} + \frac{n}{(n+3)\sqrt{3(2n+3)}} + \ldots n \text{ terms}\right] = \int_o^1 f(x)dx$,

[AP EAMCET 05-07-2022_Shift-1]

1. $\frac{1}{(1+x)\sqrt{x^2+2x}}$
2. $\frac{1}{(1+x)\sqrt{x+2}}$
3. $\frac{1}{(1+x)\sqrt{x^2+x+1}}$
4. $\frac{1}{(1+x)\sqrt{x^2-2x}}$

84. [.] represents a greatest integer function. If $\int_{\sqrt{3}}^{\sqrt{18}} [x]dx = a + b\sqrt{2} + c\sqrt{3}$, then $a+b+c=$

[AP EAMCET 05-07-2022_Shift-2]

1. 0
2. 1
3. -1
4. 2

85. $\int_0^4 \frac{x+2}{\sqrt{4x-x^2}} dx =$

[AP EAMCET 05-07-2022_Shift-2]

1. 2π
2. 0
3. π
4. $\pi/2$

86. $\int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x} dx =$

[AP EAMCET 05-07-2022_Shift-2]

1. 2π
2. π^2
3. $\pi + 2$
4. $\pi/2$

87. $\int_{-1}^{1} \frac{\sin x - x^2}{3-|x|} dx =$

[AP EAMCET 05-07-2022_Shift-2]

1. 7 + 18 log 3/2
2. 18 log 9/4
3. 7 + 9 log 9/4
4. 7–18 log 3/2

88. $\int_{-a}^{a} f(x)dx - \int_0^a f(-x)dx =$

[AP EAMCET 06-07-2022_Shift-1]

1. $\int_{-a}^{a} f(a-x)dx$
2. $\int_{-a}^{a} f(x)+f(a-x)dx$
3. $\int_0^a f(x)+f(a-x)dx$
4. $\int_0^a f(a-x)dx$

89. $\int_0^{\pi} \frac{x}{\sin x}\left(3\cos^2 x + 2\sin x + \sin^3 x - 3\right)dx =$

[AP EAMCET 06-07-2022_Shift-1]

1. $\frac{\pi(5\pi-12)}{4}$
2. $\frac{\pi}{2}$
3. $\frac{\pi}{2}(5\pi-6)$
4. $\frac{\pi(5\pi-12)}{6}$

90. If [.] represents greatest integer function, then $\int_{\frac{3\pi}{4}}^{\pi}\left[\sin x+\left[\frac{4x}{\pi}\right]\right]dx=$

[AP EAMCET 06-07-2022_Shift-1]

1. $\frac{\pi}{4}$
2. $\frac{\pi}{2}$
3. $\frac{3\pi}{4}$
4. π

91. $\int_0^{\pi}\left(\cos^2\left(\frac{3\pi}{8}-\frac{x}{4}\right)\right)-\cos^2\left(\frac{11\pi}{8}+\frac{x}{4}\right)dx=$

[AP EAMCET 06-07-2022_Shift-1]

1. $1/\sqrt{2}$
2. $2\sqrt{2}$
3. $\sqrt{2}$
4. 2

92. **Assertion (A):**

$\int_2^e\left(\frac{1}{\log_e x}-\frac{1}{(\log_e x)^2}\right)dx=e-2\log_2 e$

Reason (R):

$\int_a^b e^x\left(f(x)+f'(x)\right)dx=e^b f(b)-e^a f(a)$

[AP EAMCET 06-07-2022_Shift-2]

1. (A) and (R) are true, (R) is the correct explanation to (A)
2. (A) and (R) are false (R) is not the correct explanation to (A).
3. (A) is true and (R) is false, R is not the correct explanation to (A)
4. (A) is false and (R) is true, R is not the correct explanation to (A)

93. $\int_0^{NT} f(t)dt=N\int_0^T f(t)dt$ (N is a natural number). Then $\int_0^{50\pi}\sqrt{1-\cos 2x}\ dx=$

[AP EAMCET 06-07-2022_Shift-2]

1. $50\sqrt{2}$
2. $100\sqrt{2}$
3. $\frac{50}{\sqrt{2}}$
4. $\frac{100}{\sqrt{2}}$

94. If $\int_0^{\pi}\frac{dx}{1+2\sin^2 x}=K$, then greatest integer less than or equal to k is

[AP EAMCET 06-07-2022_Shift-2]

1. 2
2. 0
3. 1
4. -1

95. $\int_0^{\frac{\pi^2}{4}}(2\sin\sqrt{x}+\sqrt{x}\cos\sqrt{x})dx=$

[AP EAMCET 06-07-2022_Shift-2]

1. $\pi/2$
2. π
3. $\pi^2/2$
4. π^2

96. $\lim_{n\to\infty}\left(\frac{1^2}{n^3+1^3}+\frac{2^2}{n^3+2^3}+\ldots+\frac{1}{2n}\right)=$

[AP EAMCET 07-07-2022_Shift-1]

1. log2
2. 2log2
3. $\frac{1}{2}\log 2$
4. $\log\sqrt[3]{2}$

97. If $\int f(x)dx=F(x)+C$, then $\frac{d}{dt}\int_{g(t)}^{h(t)} f(x)dx=$

[AP EAMCET 07-07-2022_Shift-1]

1. $f(h(t))-f((t))$
2. $F(h(t))-F(g(t))$
3. $F(h(t))h'(t)-F(g(t))g'(t)$
4. $f(h(t))h'(t)-f(g(t))g'(t)$

98. $\int_0^{\frac{\pi}{2}}\sin^m x\cos^4 x dx=\frac{7\pi}{2048}\Rightarrow m=$

[AP EAMCET 07-07-2022_Shift-1]

1. 8
2. 6
3. 10
4. 12

99. $\int_{-\pi}^{\pi}\frac{\cos^{2022}x}{1+(2022)^x}dx=$

[AP EAMCET 07-07-2022_Shift-1]

1. $\frac{(2022)!}{2^{2022}((1011)!)^2}\pi$
2. $\left(2022_{C_{1011}}\right)\pi$
3. $\left(2022_{C_{1011}}\right)\frac{\pi}{2^{1011}}$
4. $\frac{(2022)!}{(1011)!2^{2022}}\pi$

100. $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\cos^{-8}x dx=$

[AP EAMCET 07-07-2022_Shift-2]

1. $\frac{14}{15}$
2. $\frac{174}{35}$
3. $\frac{192}{35}$
4. $\frac{198}{35}$

101. $\int_{\alpha+1}^{\alpha} \frac{e^x(\alpha-x)}{(x-\alpha+1)^2}dx =$

[AP EAMCET 07-07-2022_Shift-2]

1. $2e^{\alpha}+e$ 2. $\frac{2e^{\alpha+2}}{e-2}$
3. $e^{\alpha}\frac{(e+2)}{2}$ 4. $e^{\alpha}\left(\frac{e-2}{2}\right)$

102. For $n \in N, If\ I_n = \int \frac{\sin nx}{\sin x}dx = \frac{2}{n-1}\sin(n-1)x + I_{n-2}$ and $\int_0^{\pi} \frac{\sin nx}{\sin x}dx = \frac{k\pi}{2}$ Then k =

[AP EAMCET 07-07-2022_Shift-2]

1. $(-1)^n - 1$ 2. $1-(-1)^n$
3. $(-1)^n$ 4. $(-1)^{n+1}$

103. If $f(x) = \sin(\tan^{-1}x), then \int_0^1 x.f''(x)dx =$

[AP EAMCET 07-07-2022_Shift-2]

1. $1-\frac{3}{2\sqrt{2}}$ 2. $-\frac{1}{2\sqrt{2}}$
3. $\frac{1}{\sqrt{2}}$ 4. $-\sqrt{2}$

104. $\int_{\pi/11}^{9\pi/22} \frac{dx}{1+\sqrt{\tan x}} =$

[AP EAMCET 08-07-2022_Shift-1]

1. $\pi/4$ 2. $\pi/22$
3. $\pi/11$ 4. $7\pi/44$

105. $\int_0^{2\pi} \cos mx \cos nx dx + \int_{-\pi}^{\pi} \sin mx \cos nx dx =$

[AP EAMCET 08-07-2022_Shift-1

1. 0, if m = n and m, $n \in Z$
2. π, m =n, m, $n \in Z$
3. $\pi\ if\ m \neq n, m, n \in Z$
4. $2\pi\ \forall m, n \in R$

106. $\int_2^5 \sqrt{x+2\sqrt{x-1}} + \sqrt{x-2\sqrt{x-1}}dx$

[AP EAMCET 08-07-2022_Shift-1]

1. 16/3 2. 32/3
3. 28/3 4. 4/3

107. $\lim_{n\to\infty} \frac{1}{\sqrt{n}}\left[1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{4}}+...+\frac{1}{\sqrt{n}}\right] =$

[AP EAMCET 08-07-2022_Shift-1]

1. $\sqrt{2}$ 2. 2
3. $\sqrt{2}-1$ 4. $2\sqrt{2}$

108. $\int_0^{\infty} (x^{12}+x^{-12})\frac{\log x}{x}dx =$

[AP EAMCET 08-07-2022_Shift-2]

1. 0 2. 1
3. log 2 4. e^2

109. $\int_1^2 \left(\tan^{-1}\left(\frac{x}{x^2+1}\right)+\tan^{-1}\left(\frac{x^2+1}{x}\right)\right)dx =$

[AP EAMCET 08-07-2022_Shift-2]

1. $\frac{\pi}{4}$ 2. $\frac{3\pi}{4}$
3. $\frac{5\pi}{4}$ 4. $\frac{\pi}{2}$

110. $\int_{\frac{-\pi}{2}}^{2\pi} \sin^{-1}(\sin x)dx =$

[AP EAMCET 08-07-2022_Shift-2]

1. $\frac{15\pi^2}{8}$ 2. $\frac{-\pi^2}{8}$
3. $\frac{-7\pi^2}{8}$ 4. $\frac{7\pi^2}{8}$

111. $\int_9^x \frac{f(y)}{y^2}dy = 2\sqrt{x}-6 \Rightarrow f(x) =$

[AP EAMCET 08-07-2022_Shift-2]

1. $\sqrt{x}$ 2. $x\sqrt{x}$
3. $x^2\sqrt{x}$ 4. $x+\sqrt{x}$

112. $\int_1^2 x\sqrt{4-x^2}dx$

[TS EAMCET 18-07-2022_Shift-2]

1. $\sqrt{3}$ 2. 2
3. $\frac{1}{\sqrt{3}}$ 4. $\frac{1}{2}$

113. If [x] denotes the greatest integer function of x and $\int_{-3/2}^{3/2}[2x-3]dx = k, \text{ then } \left|k+\frac{1}{2}\right| =$

[TS EAMCET 18-07-2022_Shift-2]

1. 7
2. 8
3. 10
4. 12

114. $\int_1^4\left(x+\sqrt{x}+\frac{1}{x}\right)dx - \int_1^{2\log 2} dx =$

[TS EAMCET 18-07-2022_Shift-1]

1. $\frac{79}{6}$
2. $\frac{643}{6}$
3. $\frac{321}{5}$
4. 64

115. Let $I = \int_{-\pi/4}^{\pi/4}\frac{1}{2-\cos 2x}\left(\frac{3}{\pi}+\log\left(\frac{4+\sin x}{4-\sin x}\right)\right)dx$.

Given that $\int\frac{dx}{1+kx^2} = \frac{1}{\sqrt{k}}Tan^{-1}\left(\sqrt{k}x\right)+c$,

$Tan^{-1}(0)=0$ and $Tan^{-1}\left(\sqrt{3}\right)=\frac{\pi}{3}$. Then $3I^2=$

[TS EAMCET 18-07-2022_Shift-1]

1. 4
2. 9
3. 16
4. 1

116. It is given that $\frac{d}{dt}(t\log t - t) = \log t$ then

$\exp\left(\int_0^1 2x\log\left(1+x^2\right)dx\right) =$

[TS EAMCET 19-07-2022_Shift-I]

1. e
2. 2
3. $\frac{4}{e}$
4. $\frac{e}{4}$

117. $\int_0^{2a} f(x)dx =$

[TS EAMCET 19-07-2022_Shift-I]

1. $2\int_0^a f(x)dx$
2. $\int_0^a (f(x)+f(x+a))dx$
3. 0
4. $\int_0^{2a} f(2a+x)dx$

118. $\int_0^3\left(\sin\left(\frac{\pi}{3}x\right)-\cos\left(\frac{\pi}{3}x\right)\right)dx =$

[TS EAMCET 19-07-2022_Shift-2]

1. $\frac{-6}{\pi}$
2. 0
3. $\frac{-3}{\pi}$
4. $\frac{6}{\pi}$

119. $\int_0^{\frac{\pi}{2}}\sin^4\theta\cos^3\theta d\theta =$

[TS EAMCET 19-07-2022_Shift-2]

1. $\frac{1}{35}$
2. $\frac{2}{35}$
3. $\frac{4}{35}$
4. $\frac{8}{35}$

120. Give that $\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{np} f\left(\frac{r}{n}\right) = \int_0^p f(x)dx$

$f: R \to R$ is defined by $f(x) = x^2+2$,then

$\lim_{n\to\infty}\frac{3}{n}\left[f\left(\frac{7}{n}\right)+f\left(\frac{14}{n}\right)+f\left(\frac{21}{n}\right)+........+f(7)\right] =$

[TS EAMCET 20-07-2022_Shift-1]

1. 55
2. 57
3. 104
4. 7

121. If $f(x) = \begin{vmatrix} 2\cos^2 x & \sin 2x & \sin x \\ \sin 2x & 2\sin^2 x & -\cos x \\ \sin x & -\cos x & 0 \end{vmatrix}$ then

$\int_0^{\frac{\pi}{4}}\left(2|f(x)|+5f'(x)\right)dx =$

[TS EAMCET 20-07-2022_Shift-1]

1. 0
2. $\frac{\pi}{4}$
3. $\frac{\pi}{2}$
4. π

122. $\int_0^4 \left||x-2|-x\right|dx =$

[TS EAMCET 20-07-2022_Shift-2]

1. 2
2. 3
3. 6
4. 12

123. If $\int_{-a}^{a} f(x)\,dx = \int_{0}^{a} f(x)\,dx + \int_{0}^{a} g(x)\,dx$ then $g(x) =$

[TS EAMCET 20-07-2022_Shift-2]

1. $-f(x)$
2. $f(x)$
3. $f(-x)$
4. $f(x)+f(-x)$

124 If $I = \int_{-a}^{a} \left(x^4 - 2x^2\right)dx$, then I is minimum at a=

[15th May 2023 Shift 1]

1. 2
2. $-\sqrt{2}$
3. $\sqrt{2}$
4. -2

125

Assertion (A) : $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{(\sin x)^{\sqrt{2}}\,dx}{(\sin x)^{\sqrt{2}} + (\cos x)^{\sqrt{2}}} = \frac{\pi}{12}$

Reason (R) : $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{f(x)\,dx}{f(x) + f\left(\frac{\pi}{2} - x\right)} = \frac{\pi}{12}$

[15th May 2023 Shift 1]

1. A is true. R is true and R is the correct explanation of A.
2. A is true, R is true but R is not the correct explanation of A
3. A is true. R is false
4. A is false. R is true

126 If [x] is the greatest integer not exceeding x, then $\int_{-0.5}^{1.5} x^2 [x]\,dx =$ **[15th May 2023 Shift 1]**

1. $\frac{4.5}{4}$
2. $\frac{3}{4}$
3. $\frac{3.5}{4}$
4. $\frac{2.375}{2}$

127 $\int_{0}^{50\pi} \sqrt{1-\cos 2x}\;dx =$

[15th May 2023 Shift 1]

1. $-100\sqrt{2}$
2. $100\sqrt{2}$
3. $50\sqrt{2}$
4. $-50\sqrt{2}$

128 If $f(x) = \frac{x^3+5}{\sqrt{12+x}}$ and $\int_{5}^{5} f(x)\,dx = \int_{0}^{5} \left(f(x) + g(x)\right)dx$, then $g(x) =$

[15th May 2023 Shift 1]

1. $\frac{5-x^3}{\sqrt{12-x}}$
2. $-\left(\frac{5+x^3}{\sqrt{12+x}}\right)$
3. $\frac{-x^3+5}{\sqrt{12+x}}$
4. $\frac{5+x^3}{\sqrt{12-x}}$

129 $\int_{0}^{\frac{\pi}{4}} \frac{\cos^2 x}{\cos^2 x + 4\sin^2 x}dx =$

[15th May 2023 Shift 2]

1. $\frac{\pi}{4} + \frac{2}{3}Tan^{-1}2$
2. $-\frac{\pi}{3} - \frac{2}{3}Tan^{-1}3$
3. $-\frac{\pi}{12} + \frac{2}{3}Tan^{-1}2$
4. $\frac{\pi}{6} - \frac{2}{3}Tan^{-1}4$

130 If $f(x) = \int_{0}^{x} \left[(a+1)(t+1)^2 - (a-1)\left(t^2+t+1\right)\right]dt$, then a possible positive value of 'a'. for which $f'(x) = 0$ has equal roots, is

[15th May 2023 Shift 2]

1. 1
2. -1
3. 7
4. 0

131. $\int_{0}^{\frac{\pi}{2}} \frac{\sin\left(\frac{\pi}{4} + x\right) + \sin\left(\frac{3\pi}{4} + x\right)}{\cos x + \sin x}dx =$

[15th May 2023 Shift 2]

1. $\frac{\pi}{2}$
2. $\frac{\pi}{2\sqrt{2}}$
3. $\frac{\pi}{3\sqrt{2}}$
4. $\frac{\pi}{4\sqrt{2}}$

132 $\int_{0}^{1} \left(\sqrt{10}\right)^{2x} dx =$

[15th May 2023 Shift 2]

1. $\frac{10}{\log 10}$
2. $\frac{9}{\log 10}$
3. $\frac{1}{\log 10}$
4. $\frac{4}{\log 5}$

133. $f(x)=\begin{cases} x^2, 0 \le x < 1 \\ \sqrt{x}, 1 \le x \le 2 \end{cases} \Rightarrow \int_0^2 f(x)dx =$

[16th May 2023 Shift 1]

1. $\frac{4\sqrt{2}-1}{3}$
2. $\frac{4\sqrt{2}+1}{3}$
3. $\frac{4\sqrt{2}-1}{6}$
4. $\frac{4\sqrt{2}+1}{6}$

134. $\int_0^{\frac{1}{2}} \frac{x\sin^{-1}x}{\sqrt{1-x^2}}dx =$

[16th May 2023 Shift 1]

1. $\left(\frac{1}{2}+\frac{\sqrt{3}}{12}\pi\right)$
2. $\left(\frac{1}{2}-\frac{\sqrt{3}}{12}\pi\right)$
3. $\left(-\frac{1}{2}+\frac{\sqrt{3}}{12}\pi\right)$
4. $\left(-\frac{1}{2}-\frac{\sqrt{3}}{12}\pi\right)$

135. Given that $\frac{d}{dx}\left[\int_0^{\phi(x)} f(t)dt\right] = f(\phi(x))f'(x)$.

$If \int_0^{x^3} f(t)dt = x^2 \sin 2\pi x,$

Then the value of f (8) is

[16th May 2023 Shift 1]

1. $\frac{2\pi}{3}$
2. $\frac{4\pi}{3}$
3. $\frac{\pi}{3}$
4. $\frac{\pi}{12}$

136. $\lim_{n\to\infty}\frac{1}{n}\left(\frac{1}{e^{1/n}}+\frac{1}{e^{2/n}}+\frac{1}{e^{3/n}}+....+\frac{1}{e^2}\right)=$

[16th May 2023 Shift 1]

1. $1-e^{-2}$
2. $1+e^{-2}$
3. e^2-1
4. e^2+1

137. $If \int_0^{2024\pi} \frac{2023^{\sin^2 x}}{2023^{\sin^2 x}+2023^{\cos^2 x}}dx = k,$

$then\left(\frac{2k}{\pi}+1\right)=$

[16th May 2023 Shift 1]

1. 2023
2. 2025
3. 2022
4. 2024

138.

Assertion (A) : $\int_0^{\frac{\pi}{2}} (\sin^6 x + \cos^6 x)dx$ lies in the interval $\left(\frac{\pi}{8},\frac{\pi}{2}\right)$

Reason (R) : $\sin^6 x + \cos^6 x$ is periodic function with period $\frac{\pi}{2}$

[16th May 2023 Shift 2]

1. Both A and R are true and R is the correct explanation of A
2. Both A and R are true but R is not the correct explanation of A
3. A is true , R is false
4. A is false , R is true

139. $\int_0^{\pi} \frac{\cos x}{\sqrt{1-\sin^2 x}}dx =$ **[16th May 2023 Shift 2]**

1. π
2. $\frac{\pi}{2}$
3. $-\frac{\pi}{2}$
4. 0

140. If $\int_3^b \frac{x-1}{2x-x^2}dx = \frac{1}{2}, then (b-1)^2 =$

[16th May 2023 Shift 2]

1. 2
2. $\sqrt{2}$
3. $1+\frac{3}{e}$
4. $\sqrt{\frac{3}{e}-1}$

141. If u(n)= $\int_0^{\frac{\pi}{2}} (1+\sin t)^n \sin 2t\, dt$, n$\in N$, then u(4) = **[16th May 2023 Shift 2]**

1. $\frac{28\pi}{5}$
2. $\frac{128}{35}$
3. $\frac{129}{15}$
4. $\frac{68\pi}{15}$

142. If $a \in Z^+, [x]$ is greatest integer not more than x and $\int_0^a 2^{[x]}dx = 127$, then a =

[17th May 2023 Shift 1]

1. 6
2. 7
3. 8
4. 9

143 $\int_{-\pi}^{\frac{\pi}{2}} \sin x.\sin^2(\cos x)dx =$

[17th May 2023 Shift 1]

1. $\frac{1-\sin 2}{4}$
2. $-\left(\frac{1+\sin 2}{4}\right)$
3. $\frac{\sin 2-2}{4}$
4. $-\left(\frac{2+\sin 2}{4}\right)$

144. The positive value of x satisfying the equation $\int_x^1 (1-t)dt = \frac{1}{2}$ is [17th May 2023 Shift 1]

1. 1
2. $\sqrt{2}$
3. 3
4. 2

145. $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{1+\sqrt{\cot x}}dx =$

[17th May 2023 Shift 2]

1. $\frac{\pi}{4}$
2. $\frac{\pi}{2}$
3. $\frac{\pi}{6}$
4. $\frac{\pi}{12}$

146. If $m \in Z^+$, n=2m and

$\int_0^{\frac{\pi}{2}} \sin^m x\cos^n x dx = K(m)\int_0^{\frac{\pi}{2}} \sin^m x\, dx$, then

$\frac{2^{m-1}(m-1)!}{(2m-1)!}$ K(m)= [17th May 2023 Shift 2]

1. $\frac{1}{m+2}\frac{1}{m+4}\frac{1}{m+6}\cdots\frac{1}{3m}$
2. $\frac{1}{2m+2}\frac{1}{2m+4}\cdots\frac{1}{3m}$
3. $\frac{\pi}{2}\frac{1}{m+2}\frac{1}{m+4}\frac{1}{m+6}\cdots\frac{1}{3m}$
4. $\frac{\pi}{2}\frac{1}{2m+2}\frac{1}{2m+4}\cdots\frac{1}{3m}$

147. *If* $5f(x)+3f\left(\frac{1}{x}\right) = 2-\frac{1}{x}, x \neq 0$, then

$\int_1^2 f\left(\frac{1}{x}\right)dx =$ [17th May 2023 Shift 2]

1. $\frac{6\log 2-7}{32}$
2. $\frac{6\log 2-17}{32}$
3. $\frac{6\log 2-1}{32}$
4. $\frac{6\log 2-7}{16}$

148 Let f:R→R be a differentiable function such that $|f(x)-f(y)| \leq 2|x-y|^{3/2}, \forall x, y \in R$ if $f(o)=1$, then $\int_0^1 f^2(x)dx =$

[18th May2023 shift -1]

1. -2
2. $\frac{1}{2}$
3. 0
4. 1

149. $\int_0^\pi x\sin^3 x\cos^2 x dx =$

[18th May2023 shift -1]

1. $\frac{2\pi}{15}$
2. $\frac{4\pi}{15}$
3. $\frac{\pi}{30}$
4. $\frac{2\pi}{5}$

150 $\int_0^{2\pi} |x \sin x|dx = k\pi, then\ k =$

[18th May2023 shift -1]

1. 1
2. 2
3. 3
4. 4

151. $\int_{\frac{1}{25}}^{3} \frac{e^{\frac{3}{x}}}{x^2}dx =$

[18th May2023 shift -1]

1. $-\frac{1}{3}(e^{75}-e)$
2. $\frac{1}{3}(e^{50}-e^{25})$
3. $-\frac{1}{3}(e^{50}-e)$
4. $\frac{1}{3}(e^{75}-e)$

152. If [.] denotes the greatest integer function, then $\int_0^{1000} e^{x-[x]}dx =$ **[18th May 2023 Shift 2]**

1. $\frac{e^{1000}-1}{1000}$
2. $1000(e-1)$
3. $\frac{e^{1000}-1}{e-1}$
4. $\frac{e-1}{1000}$

153 If $I=\int_1^3 \sqrt{3+x+x^2}$ dx, then I lies in the interval

[18th May 2023 Shift 2]

1. $(2\sqrt{5}, 2\sqrt{15})$
2. $(\sqrt{3}, 2\sqrt{5})$
3. $(\sqrt{23}, \sqrt{33})$
4. $(2\sqrt{15}, \sqrt{23})$

154. $\int_{-4\pi}^{4\pi}$ $\tan^9 x \sin^6 x \cos^3 x$ dx=

[18th May 2023 Shift 2]

1. $16\times\frac{\pi}{2}$
2. $8\times\frac{2}{3}$
3. $16\times\frac{14}{17}\times\frac{12}{15}\times\ldots\times\frac{2}{3}$
4. 0

155 Given that $\frac{d}{dx}\int_0^{\phi(x)} f(t)dt = f(\phi(x))\phi'(x)$. For all x $\in\left(0,\frac{\pi}{2}\right), if \int_1^{\cos x} t^2 f(t)dt = \cos 2x$, then $f\left(\frac{1}{\sqrt{2}}\right)=$

[18th May 2023 Shift 2]

1. $2\sqrt{2}$
2. $4\sqrt{2}$
3. $\frac{\pi}{4}$
4. $\frac{-\pi}{4}$

156
Assertion (A): $\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} [2\sin x]$ dx=0, where [.] denotes the greatest integer function
Reason (R) : 2 sin x is a decreasing function in $\left[\frac{\pi}{2},\frac{3\pi}{2}\right]$ **[18th May 2023 Shift 2]**

1. Both A and R are true and R is the correct explanation of A
2. Both A and R are true but R is not the correct explanation of A
3. A is true, R is false
4. A is false, R is true

157. $\int_{1/2}^{2} |\log_{10} x| dx =$

[12TH MAY 2023 SHIFT-1]

1. $\log_{10}\left(\frac{8}{e}\right)$
2. $\frac{1}{2}\log_{10}\left(\frac{8}{e}\right)$
3. $\log_{10}\left(\frac{2}{e}\right)$
4. $\log_e\left(\frac{3}{e}\right)$

158 $\int_0^{\pi/2} \frac{\sin^2 x}{\sin x+\cos x}dx =$

[12TH MAY 2023 SHIFT-1]

1. $\sqrt{2}\log(\sqrt{2}+1)$
2. $\frac{1}{\sqrt{2}}\log(\sqrt{2}+1)$
3. $\log(\sqrt{2}+1)$
4. $\frac{1}{\sqrt{2}}\log(\sqrt{2}-1)$

159 $[\bullet]$ is the greatest integer function then $\int_0^{2\pi} [|\sin x|+|\cos x|]dx =$

[12TH MAY 2023 SHIFT-1]

1. $\frac{\pi}{2}$
2. π
3. $\frac{3\pi}{2}$
4. 2π

160 If f is defined on $\mathbb{R}$ such that $f(x)f(-x)=9$, then $\int_{-23}^{23}\frac{dx}{3+f(x)}=$

[12TH MAY 2023 SHIFT-1]

1. $\frac{51}{3}$
2. $\frac{49}{3}$
3. $\frac{46}{3}$
4. $\frac{46}{6}$

161 $\int_0^{\frac{\pi}{4}}\frac{\sec x}{1+2\sin^2 x}dx =$

[12TH MAY 2023 SHIFT -2]

1. $\frac{1}{3}\log(\sqrt{2}+1)+\frac{\pi\sqrt{2}}{12}$
2. $\frac{2}{3}\log(\sqrt{2}+1)+\frac{\pi\sqrt{2}}{6}$
3. $\frac{1}{6}\log(\sqrt{2}-1)+\frac{\pi}{12}$
4. $\frac{1}{4}\log(\sqrt{2}-1)-\frac{\pi\sqrt{3}}{6}$

162. $\lim_{n\to\infty}\left[\frac{1}{n^2}\sec^2\frac{1}{n^2}+\frac{2}{n^2}\sec^2\frac{4}{n^2}+....+\frac{1}{n}\sec^2 1\right]=$

[12TH MAY 2023 SHIFT -2]

1. $\frac{1}{2}\sec(1)$ 2. $\frac{1}{2}\operatorname{cosec}(1)$

3. $\tan(1)$ 4. $\frac{1}{2}\tan(1)$

163. $\int_2^5\sqrt{\frac{5-x}{x-2}}dx=$

[12TH MAY 2023 SHIFT -2]

1. π 2. $\frac{\pi}{2}$

3. $\frac{3\pi}{2}$ 4. $\frac{\pi}{4}$

164 $\int_0^{\frac{\pi}{2}}\sin^6 x\cos^4 x dx=$

[12TH MAY 2023 SHIFT-2]

1. $\frac{\pi}{256}$ 2. $\frac{\pi}{512}$

3. $\frac{3\pi}{512}$ 4. $\frac{5\pi}{512}$

165 $\int_0^3|x^2-3x+2|dx=$

[13TH MAY 2023 SHIFT-1]

1. $\frac{11}{6}$ 2. $\frac{5}{6}$

3. $\frac{3}{2}$ 4. $\frac{2}{3}$

166. $\int_{-\frac{\pi}{8092}}^{\frac{\pi}{8092}}\frac{\sec(2023x)}{1+(2023)^{(2023x)}}dx=$

[13TH MAY 2023 SHIFT-1]

1. $\frac{1}{2023\sqrt{2}}+c$ 2. $\frac{\log(\sqrt{2}+1)}{2023}+c$

3. $\frac{\log 2}{4046}+c$ 4. $\frac{\sqrt{2}}{2023}+c$

167 $\int_0^2 x^{\frac{5}{2}}\sqrt{2-x}\,dx=$

[13TH MAY 2023 SHIFT-1]

1. $\frac{5\pi}{16}$ 2. $\frac{5}{4}$

3. $\frac{5\pi}{8}$ 4. $\frac{5}{8}$

168 $\int_{-1}^{1}x|x|dx=$

[EAPCET 14-05-23 SHIFT-1]

1. 1 2. $\frac{1}{2}$

3. 0 4. $\frac{2}{3}$

169. $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}\sin^2 x\cos^2 x(\sin x+\cos x)dx=$

[EAPCET 14-05-23 SHIFT-1]

1. $\frac{2}{3}$ 2. $\frac{3}{10}$

3. $\frac{4}{15}$ 4. $\frac{5}{18}$

170 If $t_1>t_2>t_3$ then an integer root of

$3x^2-4x+2=\frac{3k}{5}is$

[EAPCET 14-05-23 SHIFT-1]

1. 1 2. 0

3. 15 4. -1

171. $\int_0^{\pi}\frac{x\cos^2 x}{1+\sin x}dx=$

[EAPCET 13-05-23 SHIFT-2]

1. $\frac{\pi(\pi-2)}{2}$ 2. 1

3. $\frac{\pi(\pi+2)}{2}$ 4. $\frac{\pi}{4}$

172 If $[x]$ represents greatest integer function then $\int_{-2}^{2}[2-x]dx =$ **[EAPCET 13-05-23 SHIFT-2]**

1. 10 2. 6
3. 4 4. 3

173 $\int_0^2 \frac{x}{(2-x)^{\frac{3}{4}}}dx =$

[EAPCET 13-05-23 SHIFT-2]

1. $\frac{24}{5}2^{\frac{1}{4}}$ 2. $\frac{5}{24}2^{\frac{3}{4}}$
3. $\frac{32}{5}2^{\frac{1}{4}}$ 4. $\frac{5}{12}2^{\frac{3}{4}}$

174. $\int_0^2 x^3(2-x)^4\,dx =$

[EAPCET 13-05-23 SHIFT-2]

1. 128/105 2. 16/35
3. 256/105 4. 32/35

KEY

1) 1	2) 3	3) 3	4) 1	5) 1
6) 1	7) 3	8) 1	9) 3	10) 3
11) 1	12) 3	13) 2	14) 3	15) 2
16) 3	17) 2	18) 4	19) 4	20) 1
21) 4	22) 2	23) 3	24) 4	25) 3
26) 1	27) 3	28) 4	29) 4	30) 2
31) 3	32) 4	33) 4	34) 1	35) 2
36) 3	37) 2	38) 1	39) 2	40) 3
41) 3	42) 3	43) 1	44) 1	45) 2
46) 3	47) 4	48) 2	49) 4	50) 3
51) 4	52) 1	53) 4	54) 1	55) 2
56) 1	57) 2	58) 2	59) 2	60) 3
61) 1	62) 3	63) 3	64) 1	65) 1
66) 2	67) 2	68) 3	69) 3	70) 4
71) 4	72) 1	73) 4	74) 2	75) 4
76) 4	77) 1	78) 1	79) 3	80) 3
81) 2	82) 3	83) 1	84) 4	85) 3
86) 2	87) 4	88) 4	89) 1	90) 3
91) 3	92) 1	93) 2	94) 3	95) 3
96) 4	97) 4	98) 1	99) 1	100) 3
101) 4	102) 2	103) 2	104) 4	105) 2
106) 3	107) 2	108) 1	109) 4	110) 2
111) 2	112) 1	113) 3	114) 1	115) 1
116) 3	117) 2	118) 4	119) 2	120) 1
121) 4	122) 3	123) 3	124) 3	125) 1
126) 2	127) 2	128) 1	129) 3	130) 1
131) 2	132) 2	133) 1	134) 2	135) 1
136) 1	137) 2	138) 2	139) 4	140) 3
141) 3	142) 2	143) 3	144) 4	145) 4
146) 1	147) 1	148) 4	149) 1	150) 4
151) 4	152) 2	153) 1	154) 4	155) 2
156) 4	157) 2	158) 2	159) 4	160) 4
161) 1	162) 4	163) 3	164) 3	165) 1
166) 2	167) 3	168) 3	169) 3	170) 4
171) 1	172) 2	173) 3	174) 4	

SOLUTIONS

1. Let $1+\log x = t$

$$\int_1^{e^2}\frac{dx}{x(1+\log x)^2} = \int_1^3 \frac{1}{t^2}\,dt$$

2. Here $f(a+b-x) = -f(x)$

$$\Rightarrow \int_{-\pi}^{\pi} x^2(\sin x)\,dx = 0$$

3. $\int_0^1 \mathrm{Sin}^{-1}\left(\frac{2x}{1+x^2}\right)dx$

put $x=\tan\theta$

$$\Rightarrow \int_0^1 2\theta\,dx = 2\int_0^1 \tan^{-1}x\,dx$$

$$= 2\int_0^1 \tan^{-1}x.1\,dx$$

$$= 2\left[\tan^{-1}x.x - \int_0^1 \frac{x}{x^2+1}dx\right]$$

$$= 2\left[\tan^{-1}x.x - \frac{1}{2}\log\left(1+x^2\right)\right]_0^1$$

$$= 2\left[\frac{\pi}{4}-\frac{1}{2}\log 2\right] = \frac{\pi}{2}-\log 2$$

4. IPE

5. $\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} x^3\sin^4(x)dx$

$\therefore f(-x) = -x^3\sin^4 x = -f(x)$

$\therefore$ 'f'-odd function

$$\therefore \int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} x^3\sin^4(x) = 0$$

6. $$\int_0^{\frac{\pi}{4}}\frac{dx}{\cos^3(x).\sqrt{4\sin x\cos x}}=\int_0^{\frac{\pi}{4}}\frac{dx}{2\cos^4(x).\sqrt{\tan x}}$$

$$=\frac{1}{2}\int_0^{\frac{\pi}{4}}\frac{(1+\tan^2 x)\sec^2 xdx}{\sqrt{\tan x}}$$

put $\tan x=t$

$\sec^2 xdx=dt$

$$=\frac{1}{2}\int_0^1\frac{1+t^2}{\sqrt{t}}dt=\frac{6}{5}$$

7. $$\int_{-1}^{2}|x|dx=\int_{-1}^{0}-xdx+\int_0^2 xdx$$

$$=-\int_{-1}^{0}xdx+\int_0^2 xdx$$

$$=-\left(\frac{x^2}{2}\right)_{-1}^{0}+\left(\frac{x^2}{2}\right)_0^2=\frac{5}{2}$$

8. Let $x+c=t\Rightarrow dx=dt$

$$\int_c^b f(t)dt=k\int_c^b f(x)dx=k\int_c^b f(t)dt$$

$k=1$

9. *Let* $\sin x=t$

$\cos x.dx=dt$

$$\int_0^{\frac{\pi}{2}}e^{\sin x}.\cos x.dx=\int_0^1 e^t dt=e-1$$

10. Put $\sqrt{k}-\sqrt{t}=x$

$$\therefore I=\int_{\sqrt{k}}^{0}-2x^2\left(\sqrt{k}-x\right)dx=\frac{k^2}{6}$$

11. $\int_0^a f(x)dx=\int_0^a f(a-x)dx$

$\therefore h(x)=f(a-x)$

12. $$I=\int_0^{\frac{\pi}{4}}\left(\tan^2 x\sec^2 x\right)dx$$

13. $$I_n=\int_0^{\frac{\pi}{2}}\sin^n(x)dx=\frac{n-1}{2}.I_{n-2}$$

14. $$\int_0^{\frac{\pi}{4}}\left(\sec^2 x-1\right)dx$$

$$=\left[\tan x-x\right]_0^{\frac{\pi}{4}}$$

$$=1-\tfrac{\pi}{4}$$

15. Given $\int_0^{\frac{\pi}{2}}|\sin t-\cos t|dt$

$$=\int_0^{\frac{\pi}{4}}(\cos t-\sin t)dt+\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}(\sin t-\cos t)dt$$

$$=(\sin t+\cos t)_0^{\frac{\pi}{4}}+(-\cos t-\sin t)_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$=2\sqrt{2}-2=2\left(\sqrt{2}-1\right)$$

16. $$I=\int_0^{\frac{\pi}{2}}\frac{1}{1+\tan^{2020}(x)}dx$$

$$=\int_0^{\frac{\pi}{2}}\frac{1}{1+\cot^{2020}(x)}dx$$

$$2I=\int_0^{\frac{\pi}{2}}1dx=\frac{\pi}{2}$$

$$I=\frac{\pi}{4}$$

17. $$I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left(2\sin|x|+\cos|x|\right)dx$$

$$I=\int_{-\frac{\pi}{2}}^{0}(-2\sin x+\cos x)dx+\int_0^{\frac{\pi}{2}}(2\sin x+\cos x)dx$$

=6

18. $$I=\int_0^{2\pi}\frac{x\cos x}{1+\cos x}dx$$

$$I=\int_0^{2\pi}\frac{(2\pi-x)\cos(2\pi-x)}{1+\cos(2\pi-x)}dx$$

19. $$\int_0^{\frac{\pi}{2}} \sin^m(x)\cos(x)\ dx = \left.\frac{(\sin x)^{m+1}}{m+1}\right|_0^{\frac{\pi}{2}} = \frac{1}{m+1}$$

$$\int_0^{\frac{\pi}{2}} \sin(x)\cos^n(x)\ dx = \left.\frac{-\cos^{n+1} x}{n+1}\right|_0^{\frac{\pi}{2}} = \frac{1}{n+1}$$

20. $$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx......(1)$$

$$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx......(2)$$

$$\Rightarrow 2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} dx = \frac{\pi}{6}$$

$$\Rightarrow I = \frac{\pi}{12}$$

21. $$\int_0^{x^2} xf(t)dt = x^5 - x^3$$

diff., w.r.to 'x' on B.S

$$xf\left(x^2\right).2x = 5x^4 - 3x^2$$

put $x = 1$

$$f(1) = \frac{5-3}{2} = 1$$

22. IPE

23. $$I = \int_0^{\frac{\pi}{2}} \frac{(x+\sin x)(1-\cos x)}{\sin^2 x}dx$$

$$= \int_0^{\frac{\pi}{2}} x\,\mathrm{cosec}^2 x + \int_0^{\frac{\pi}{2}} x(-\mathrm{cosec}\,x\cot x)dx + \int_0^{\frac{\pi}{2}} \mathrm{cosec}\,x dx - \int_0^{\frac{\pi}{2}} \cot x dx$$

$$= (-x\cot x)_0^{\pi/2} + \int_0^{\pi/2} \cot x dx + (x\,\mathrm{cosec}\,x)_0^{\pi/2}$$

$$-\int_0^{\pi/2} \mathrm{cosec}\,x dx + \int_0^{\pi/2} \mathrm{cosec}\,x dx - \int_0^{\pi/2} \cot x dx$$

$$= 0 + \frac{\pi}{2}\mathrm{cosec}\frac{\pi}{2} - 0 = \frac{\pi}{2}$$

24. $$\int_{-a}^{a} \frac{a-x}{\sqrt{a^2-x^2}}\ dx = \int_{-a}^{a} \frac{a}{\sqrt{a^2-x^2}}\ dx - \int_{-a}^{a} \frac{x}{\sqrt{a^2-x^2}}\ dx$$

$$= 2a\int_0^a \frac{1}{\sqrt{a^2-x^2}}\ dx = 2a\left[\mathrm{Sin}^{-1}\left(\frac{x}{a}\right)\right]_0^a = \pi a$$

25. Let $I = \int_0^{\pi/2} \frac{2\sin x + 3\cos x}{\sin x + \cos x}dx.......(1)$

$$\left(\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$$

Again $I = \int_0^{\pi/2} \frac{2\cos x + 3\sin x}{\cos x + \sin x}dx.....(2)$

$$(1)+(2) \Rightarrow 2I = \int_0^{\pi/2} 5\,dx = \frac{5\pi}{4}$$

26. We know that,

$$\int_0^{2a} f(x)\ dx = \int_0^{a} f(x)\ dx + \int_a^{2a} f(x)\ dx$$

$$\therefore \int_0^{2a} f(x)\ dx - \int_a^{2a} f(x)\ dx = \int_0^{a} f(x)\ dx$$

27. $$A = \int_1^{\sin\theta} \frac{t}{1+t^2}\ dt$$

Let $1+t^2 = u \ \Rightarrow t\ dt = \frac{1}{2}du$

U.L:- $1+\sin^2\theta$; L.L:- 2

$$A = \int_2^{1+\sin^2\theta} \frac{1}{2u}\ du$$

$$A = \frac{1}{2}\left[\log\left(1+\sin^2\theta\right) - \log 2\right]$$

$$A = \frac{1}{2}\log\left(\frac{1+\sin^2\theta}{2}\right)$$

$$B = \int_1^{\mathrm{cosec}\theta} \frac{1}{t(1+t^2)}\ dt$$

$$B = \int_1^{\mathrm{cosec}\theta} \frac{dt}{t^3\left(\frac{1}{t^2}+1\right)}\ dt$$

Let

$$\frac{1}{t^2}+1 = v \ \Rightarrow \frac{dt}{t^3} = \frac{-1}{2}dv$$

$$B=\frac{-1}{2}\int_{2}^{1+\sin^2\theta}\frac{dv}{v}$$

$$B=\frac{-1}{2}\log\left(\frac{1+\sin^2\theta}{2}\right)$$

$$\therefore A+B=0 \ \Rightarrow A=-B$$

$$\begin{vmatrix} A & A^2 & B \\ e^{A+B} & B^2 & -1 \\ 1 & A^2+B^2 & -1 \end{vmatrix} = -\begin{vmatrix} A & A^2 & A \\ 1 & A^2 & 1 \\ 1 & 2A^2 & 1 \end{vmatrix} = 0$$

---

28. $$\int_0^{\frac{\pi}{2}} x^3\sin x\,dx=\left[-x^3\cos x\right]_0^{\frac{\pi}{2}}+3\int_0^{\frac{\pi}{2}}x^2\cos x dx$$

$$=3\left[\left(x^2\sin x\right)_0^{\frac{\pi}{2}}-\int_0^{\frac{\pi}{2}}2x\sin x dx\right]$$

$$=\frac{3\pi^2}{4}-6\left[\left(-x\cos x\right)_0^{\pi/2}+\left(\sin x\right)_0^{\pi/2}\right]$$

$$=\frac{3\pi^2}{4}-6$$

---

29. $$y=\underset{r=1}{\overset{n}{\pi}}\left(1+\frac{r^2}{n^2}\right)^{\frac{2r}{n^2}}$$

$$\log y=\frac{2}{n}\left[\frac{1}{n}\log\left(1+\frac{1}{n^2}\right)+\frac{2}{n}\log\left(1+\frac{2^2}{n^2}\right)+....+\frac{n}{n}\log\left(\frac{1+n^2}{n^2}\right)\right]$$

$$=\frac{2}{n}\sum_{r=1}^{n}\frac{r}{n}\log\left(1+\frac{r^2}{n^2}\right)$$

$$\lim_{n\to\infty}\underset{r=1}{\overset{n}{\pi}}\left(1+\frac{r^2}{n^2}\right)^{\frac{2r}{n^2}}=2\int_0^1 x\log\left(1+x^2\right)dx$$

$$=\log_e^4-1$$

$$\log y=e^{\log_e^4-1}$$

$$y=\frac{4}{e}$$

---

30. $$I_n=\int_0^a\frac{x^n}{\sqrt{a^2-x^2}}\,dx$$

$$x=a\sin\theta$$
$$dx=a\cos\theta d\theta$$

$$I_n=\int_0^{\pi/2}\frac{a^n\sin^n\theta}{a\cos\theta}a\cos\theta\ d\theta$$

$$I_8=a^8\int_0^{\pi/2}\sin^8\theta\ d\theta$$

$$=a^8\frac{7}{8}.\frac{5}{6}I_4$$

$$\frac{I_8}{I_4}=\frac{35a^8}{48}$$

---

31. I.P.E model

Hint:- $x=5\sin\theta \ \ or \ \ 5\cos\theta$

---

32. $$Let\ y=\lim_{n\to\infty}\left[\left(1+\frac{1}{n^2}\right)\left(1+\frac{2^2}{n^2}\right)....\left(1+\frac{n^2}{n^2}\right)\right]^{\frac{1}{n}}=$$

$$\log y=\lim_{n\to\infty}\frac{1}{n}\left[\begin{array}{l}\log\left(1+\frac{1}{n^2}\right)+\log\left(1+\frac{2^2}{n^2}\right)+. \\ ..+\log\left(1+\frac{n^2}{n^2}\right)\end{array}\right]$$

$$=\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\log\left(1+\frac{r^2}{n^2}\right)$$

$$=\int_0^1\log\left(1+x^2\right)dx$$

$$=\left[x\log\left(1+x^2\right)\right]_0^1-\int_0^1\frac{2x^2}{1+x^2}dx$$

$$=\log 2-2\int_0^1\left(1-\frac{1}{1+x^2}\right)dx$$

$$=\log 2-2\left[x-\tan^{-1}x\right]_0^1$$

$$=\log 2-2\left(1-\frac{\pi}{4}\right)$$

$$\text{logy}=\log 2-2+\frac{\pi}{2}$$

$$=\log 2+\log e^{\frac{\pi}{2}-2}$$

$$y=2.e^{\frac{\pi}{2}-2}$$

33. $\int_a^b f(x)\,dx = \frac{b-a}{2}\left[\because f(a+b-x) = f(x)\right]$

$$question = 3\left(\frac{\frac{\pi}{2}-\frac{\pi}{4}}{2}\right)$$

$$= 3\left(\frac{\pi}{8}\right)$$

34. $\frac{\pi}{2}\left[\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\sin\left(\frac{\pi r}{2n}\right)\right]$

$$\frac{\pi}{2}\int_0^1 \sin\left(\frac{\pi x}{2}\right)dx$$

Let

$$\frac{\pi x}{2} = t$$

$$\frac{\pi}{2}dx = dt$$

$$x = 1 \Rightarrow t = \frac{\pi}{2}$$

$$x = 0 \Rightarrow t = 0$$

$$= \int_0^{\frac{\pi}{2}} \sin t\,dt$$

$$= \left[-\cos t\right]_0^{\frac{\pi}{2}} = 1$$

35. Put

$$\tan\frac{x}{2} = t$$

$$dx = \frac{2dt}{1+t^2}$$

$$\sin x = \frac{2t}{1+t^2}$$

$$x = 0 \to t = 0$$

$$x = \frac{\pi}{2} \to t = 1$$

$$\int_0^1 \frac{\frac{2dt}{1+t^2}}{4+5\left(\frac{2t}{1+t^2}\right)} = \int_0^1 \frac{2dt}{4+4t^2+10t}$$

$$= \frac{1}{2}\int_0^1 \frac{dt}{\left(t+\frac{5}{4}\right)^2 - \left(\frac{3}{4}\right)^2}$$

$$\frac{1}{2}\frac{1}{2\left(\frac{3}{4}\right)}\log\left|\frac{t+\frac{5}{4}-\frac{3}{4}}{t+\frac{5}{4}+\frac{3}{4}}\right|_0^1$$

$$= \frac{1}{3}\left[\log\left(\frac{3}{6}\right) - \log\left(\frac{1}{4}\right)\right] = \frac{1}{3}\log 2$$

36. $\int_0^1 e^x (x-1)^n\,dx$

$$= \{e^x[(x-1)^n - n(x-1)^{n-1} + n(n-1)(x-1)^{n-2} - n(n-1)(n-2)(x-1)^{n-3} + \dots\dots]\}_0^1$$

$if\ n = 3$

$$= \left[e^x\left[(x-1)^3 - 3(x-1)^2 + 6(x-1) - 6\right]\right]_0^1$$

$$= -6e + 16$$

37. $\int_0^{\frac{\pi}{4}} \frac{\sin^2 2x}{f(x)}dx$

$$Given\ f(x) = \sin^6 x + \cos^6 x + 2\sin^3 x\cos^3 x$$

$$= \left(\sin^3 x + \cos^3 x\right)^2$$

$$= 4\int_0^{\frac{\pi}{4}} \frac{\tan^2 x\sec^2 x}{\left(\tan^3 x + 1\right)^2}dx$$

$Put\ \tan^3 x + 1 = t$

$$3\tan^2 x\sec^2 x\,dx = dt$$

$$= 4\int_1^2 \frac{dt}{t^2}$$

$$= \frac{2}{3}$$

38. $\int_3^5 (x-3)^3 (5-x)^5\,dx$

$$= \int_3^5 \left[(x-5)+2\right]^3 (5-x)^5\,dx$$

$put\ 5 - x = t$

$$-dx = dt$$

$$= \int_2^0 (2-t)^3 t^5 (-dt)$$

$$= \frac{64}{63}$$

$$B=\frac{-1}{2}\int_{2}^{1+\sin^2\theta}\frac{dv}{v}$$

$$B=\frac{-1}{2}\log\left(\frac{1+\sin^2\theta}{2}\right)$$

$$\therefore A+B=0 \Rightarrow A=-B$$

$$\begin{vmatrix} A & A^2 & B \\ e^{A+B} & B^2 & -1 \\ 1 & A^2+B^2 & -1 \end{vmatrix} = -\begin{vmatrix} A & A^2 & A \\ 1 & A^2 & 1 \\ 1 & 2A^2 & 1 \end{vmatrix}=0$$

28. $$\int_0^{\frac{\pi}{2}} x^3\sin x\,dx=\left[-x^3\cos x\right]_0^{\frac{\pi}{2}}+3\int_0^{\frac{\pi}{2}} x^2\cos x dx$$

$$=3\left[\left(x^2\sin x\right)_0^{\frac{\pi}{2}}-\int_0^{\frac{\pi}{2}}2x\sin x dx\right]$$

$$=\frac{3\pi^2}{4}-6\left[\left(-x\cos x\right)_0^{\pi/2}+\left(\sin x\right)_0^{\pi/2}\right]$$

$$=\frac{3\pi^2}{4}-6$$

29. $$y=\prod_{r=1}^{n}\left(1+\frac{r^2}{n^2}\right)^{\frac{2r}{n^2}}$$

$$\log y=\frac{2}{n}\left[\frac{1}{n}\log\left(1+\frac{1}{n^2}\right)+\frac{2}{n}\log\left(1+\frac{2^2}{n^2}\right)+....+\frac{n}{n}\log\left(\frac{1+n^2}{n^2}\right)\right]$$

$$=\frac{2}{n}\sum_{r=1}^{n}\frac{r}{n}\log\left(1+\frac{r^2}{n^2}\right)$$

$$\lim_{n\to\infty}\prod_{r=1}^{n}\left(1+\frac{r^2}{n^2}\right)^{\frac{2r}{n^2}}=2\int_0^1 x\log\left(1+x^2\right)dx$$

$$=\log_e^4-1$$

$$\log y=e^{\log_e^4-1}$$

$$y=\frac{4}{e}$$

30. $$I_n=\int_0^a\frac{x^n}{\sqrt{a^2-x^2}}\,dx$$

$$x=a\sin\theta$$
$$dx=a\cos\theta d\theta$$

$$I_n=\int_0^{\pi/2}\frac{a^n\sin^n\theta}{a\cos\theta}a\cos\theta\ d\theta$$

$$I_8=a^8\int_0^{\pi/2}\sin^8\theta\ d\theta$$

$$=a^8\frac{7}{8}\cdot\frac{5}{6}I_4$$

$$\frac{I_8}{I_4}=\frac{35a^8}{48}$$

31. I.P.E model

Hint:- $x=5\sin\theta$ *or* $5\cos\theta$

32. *Let* $y=\lim_{n\to\infty}\left[\left(1+\frac{1}{n^2}\right)\left(1+\frac{2^2}{n^2}\right)....\left(1+\frac{n^2}{n^2}\right)\right]^{\frac{1}{n}}=$

$$\log y=\lim_{n\to\infty}\frac{1}{n}\left[\begin{array}{l}\log\left(1+\frac{1}{n^2}\right)+\log\left(1+\frac{2^2}{n^2}\right)+. \\ ..+\log\left(1+\frac{n^2}{n^2}\right)\end{array}\right]$$

$$=\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\log\left(1+\frac{r^2}{n^2}\right)$$

$$=\int_0^1\log\left(1+x^2\right)dx$$

$$=\left[x\log\left(1+x^2\right)\right]_0^1-\int_0^1\frac{2x^2}{1+x^2}dx$$

$$=\log 2-2\int_0^1\left(1-\frac{1}{1+x^2}\right)dx$$

$$=\log 2-2\left[x-\tan^{-1}x\right]_0^1$$

$$=\log 2-2\left(1-\frac{\pi}{4}\right)$$

$$\text{logy}=\log 2-2+\frac{\pi}{2}$$

$$=\log 2+\log e^{\frac{\pi}{2}-2}$$

$$y=2.e^{\frac{\pi}{2}-2}$$

33. $\int_a^b f(x)dx = \frac{b-a}{2}\left[\because f(a+b-x)=f(x)\right]$

$$question = 3\left(\frac{\frac{\pi}{2}-\frac{\pi}{4}}{2}\right)$$

$$=3\left(\frac{\pi}{8}\right)$$

34. $\frac{\pi}{2}\left[\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\sin\left(\frac{\pi r}{2n}\right)\right]$

$$\frac{\pi}{2}\int_0^1 \sin\left(\frac{\pi x}{2}\right)dx$$

Let

$$\frac{\pi x}{2}=t$$

$$\frac{\pi}{2}dx = dt$$

$$x=1 \Rightarrow t=\frac{\pi}{2}$$

$$x=0 \Rightarrow t=0$$

$$=\int_0^{\frac{\pi}{2}} \sin t dt$$

$$=\left[-\cos t\right]_0^{\frac{\pi}{2}} = 1$$

35. Put

$$\tan\frac{x}{2}=t$$

$$dx=\frac{2dt}{1+t^2}$$

$$\sin x = \frac{2t}{1+t^2}$$

$$x=0 \to t=0$$

$$x=\frac{\pi}{2}\to t=1$$

$$\int_0^1 \frac{\frac{2dt}{1+t^2}}{4+5\left(\frac{2t}{1+t^2}\right)} = \int_0^1 \frac{2dt}{4+4t^2+10t}$$

$$=\frac{1}{2}\int_0^1 \frac{dt}{\left(t+\frac{5}{4}\right)^2-\left(\frac{3}{4}\right)^2}$$

$$\frac{1}{2}\frac{1}{2\left(\frac{3}{4}\right)}\log\left|\frac{t+\frac{5}{4}-\frac{3}{4}}{t+\frac{5}{4}+\frac{3}{4}}\right|_0^1$$

$$=\frac{1}{3}\left[\log\left(\frac{3}{6}\right)-\log\left(\frac{1}{4}\right)\right]=\frac{1}{3}\log 2$$

36. $\int_0^1 e^x (x-1)^n dx$

$$=\{e^x[(x-1)^n - n(x-1)^{n-1} + n(n-1)(x-1)^{n-2} - n(n-1)(n-2)(x-1)^{n-3} + \ldots\ldots]\}_0^1$$

$$if\ n=3$$

$$=\left[e^x\left[(x-1)^3-3(x-1)^2+6(x-1)-6\right]\right]_0^1$$

$$=-6e+16$$

37. $\int_0^{\frac{\pi}{4}} \frac{\sin^2 2x}{f(x)}dx$

$$Given\ f(x)=\sin^6 x+\cos^6 x+2\sin^3 x\cos^3 x$$

$$=\left(\sin^3 x+\cos^3 x\right)^2$$

$$=4\int_0^{\frac{\pi}{4}} \frac{\tan^2 x\sec^2 x}{\left(\tan^3 x+1\right)^2}dx$$

$$Put\ \tan^3 x+1=t$$

$$3\tan^2 x\sec^2 x dx = dt$$

$$=4\int_1^2 \frac{dt}{t^2}$$

$$=\frac{2}{3}$$

38. $\int_3^5 (x-3)^3(5-x)^5 dx$

$$=\int_3^5\left[(x-5)+2\right]^3(5-x)^5 dx$$

$$put\ 5-x=t$$

$$-dx=dt$$

$$=\int_2^0 (2-t)^3 t^5(-dt)$$

$$=\frac{64}{63}$$

39. $C_1 \to C_1 - 2C_2 - 3C_3$

$$f(x) = \begin{vmatrix} \sin x & \frac{3+\sin 2x}{2} & \frac{-2+\sin 3x}{3} \\ 0 & \frac{3}{2} & \frac{4}{3}\sin x \\ 0 & \frac{1}{2}\sin x & \frac{1}{3} \end{vmatrix}$$

$$f(x) = \sin x\left(\frac{1}{2} - \frac{2}{3}\sin^2 x\right) = \sin\left(\frac{3-4\sin^2 x}{6}\right) = \frac{1}{6}\sin 3x$$

$$\frac{1}{6}\int_0^{\frac{\pi}{2}} \sin 3x dx + f\left(\frac{\pi}{2}\right) - f(0) = \frac{1}{6}\left[\frac{-3\cos 3x}{3}\right]_0^{\frac{\pi}{2}} + \frac{1}{6}\left(\sin\frac{3\pi}{2}\right)$$

$$= \frac{-1}{18}\left(\cos\frac{3\pi}{2} - \cos 0\right) + \frac{1}{6}(-1) = \frac{-1}{9}$$

40. $\lim\limits_{n\to\infty} \frac{1}{n}\sum_{r=1}^{n} \frac{r}{n}.\sin^{-1}\left(\frac{r}{n}\right)$

$$= \int_0^1 x.\sin^{-1} x dx$$

$$\left(\frac{x^2}{2}.\sin^{-1} x\right)_0^1 - \int_0^1 \frac{x^2}{2}.\frac{1}{\sqrt{1-x^2}} dx$$

$$= \frac{1}{2}.\frac{\pi}{2} + \frac{1}{2}\int_0^1 \frac{1-x^2-1}{\sqrt{1-x^2}} dx$$

$$= \frac{\pi}{4} + \frac{1}{2}\int_0^1 \sqrt{1-x^2} dx - \frac{1}{2}\left(\sin^{-1} x\right)_0^1$$

$$= \frac{1}{2}\left(\frac{1}{2}.\frac{\pi}{2}\right) = \frac{\pi}{8}$$

41. $\int_2^4 (x-2) dx + \int_2^3 (3-x) dx + \int_3^4 (x-3) dx$

42. $\int_{-1/2}^{1/2} [x] dx + 0 = \frac{1}{2}\int_{-1/2}^{1/2} (-1) dx = \frac{-1}{2}$

43. $\int_0^a f(x) dx = \int_0^a f(a-x) dx$

44. $\left[e^x (x-1)\right]_0^2 = e^2 + 1$

45. $\left[\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)\right]_0^a = \frac{\pi}{8} \Rightarrow \frac{a}{2} = 1 \Rightarrow a = 2$

46. $\left[\frac{x^2}{2} + \frac{1}{x}\right]_1^2 = 1$

47. $\int_0^1 x^2 f(x) dx - 2a\int_0^1 x f(x) + a^2\int_0^1 f(x) = 0$

48. $\int_0^{\pi} \log(\sin x) dx = 2\int_0^{\pi/2} \log(\sin x) dx$

$$= -\pi \log 2$$

$$\int_0^{\pi/4} \log(1+\tan x) = \frac{\pi}{8}\log 2 = -k$$

49. $\int_0^a f(x) dx = \int_0^a f(a-x) dx$

50. $\frac{|x|}{x}$ is odd.

51. Put $\log x = t \Rightarrow \frac{1}{x} dx = dt$

$$= \left[\frac{3}{2}(\log x)^{2/3}\right]_{2/e}^{1/e}$$

$$= \frac{3}{2}\left\{1 - (\log 2 - 1)^{2/3}\right\}$$

52. $\int_0^{\pi} \frac{x\sin x}{1+\sin x} dx = \frac{\pi(\pi-2)}{2}$

53. $x = a\cos^2\theta + b\sin^2\theta$

$$\int_a^b \frac{dx}{\sqrt{(x-a)(b-x)}} = \pi$$

54. $x\tan(1+x^2)$ is odd.

56. $b^4 - a^4 = 0, b = -a$

$$\int_a^{-a} x^2 dx = \frac{2}{3} \quad \therefore a = -1, b = 1$$

57. $\frac{2}{3}\left(\left(x^2-1\right)^{3/2}\right)_1^4 = \frac{2}{3}\left(\frac{15\sqrt{15}}{2}\right) = \sqrt{15\times 25}$

$$\therefore \alpha = 1, \beta = \frac{1}{2} \quad \therefore \alpha\beta = \frac{1}{2}$$

58. $\int_0^1\left(\frac{1}{x+1}-\frac{1}{(x+1)^2}\right)e^x dx=\left(\frac{e^x}{x+1}\right)_0^1=\frac{e}{2}-1$

59. $\lim_{n\to\infty}\frac{1}{n}\left[\sin\frac{\pi}{4}+\sin\frac{\pi}{12}\left(3+\frac{1}{n}\right)+\sin\frac{\pi}{12}\left(3+\frac{2}{n}\right)+\ldots+\sin\frac{\pi}{3}\right]=$

$$=\lim_{n\to\infty}\frac{1}{n}\sum_{r=0}^{n}\text{Sin}\frac{\pi}{12}\left(3+\frac{r}{n}\right)$$

Let $I=\int_0^1\text{Sin}\frac{\pi}{12}(3+x)dx=\frac{6}{\pi}\left(\sqrt{2}-1\right)$

60. Let $I=\int_0^{\pi/2}\frac{\sin^{3/2}x}{\sin^{3/2}x+\cos^{3/2}x}dx.......(1)$

$$I=\int_0^{\pi/2}\frac{\cos^{3/2}x}{\cos^{3/2}x+\sin^{3/2}x}dx.......(2)$$

Now, $(1)+(2),2I=\frac{\pi}{2}\Rightarrow\boxed{I=\frac{\pi}{4}}$

61. $I=\int_5^9\frac{\log 3x^2}{\log 3x^2+\log\left(588-84x+3x^2\right)}dx$

$$=\int_5^9\frac{\log 3\left[(14-x)^2\right]}{\log 3\left[(14-x)^2\right]+\log\left[3\left(14-(14-x)\right)^2\right]}dx$$

$$\left[\because 588-84x+3x^2=3(14-x)^2\right]$$

$$=\int_5^9\frac{\log 3\left[(14-x)^2\right]}{\log 3\left[(14-x)^2\right]+\log 3x^2}dx$$

$2I=[x]_5^9=4\Rightarrow I=2$

62. $\int_{-1}^{1}\frac{\log(1+x)}{1+x^2}dx=\int_0^1\frac{\log(1+x)}{1+x^2}dx+\int_0^1 f(x)dx$

$$\int_{-1}^{0}\frac{\log(1+x)}{1+x^2}dx+\int_0^1\frac{\log(1+x)}{1+x^2}dx=\int_0^1\frac{\log(1+x)}{1+x^2}dx+\int_0^1 f(x)dx$$

$$\Rightarrow\int_{-1}^{0}\frac{\log(1+x)}{1+x^2}dx=\int_0^1 f(x)dx$$

$$\Rightarrow-\int_0^{-1}\frac{\log(1+x)}{1+x^2}dx=\int_0^1 f(x)dx$$

put $x=-t$

$$\Rightarrow-\int_0^1\frac{\log(1-t)}{1+t^2}(-dt)=\int_0^1 f(x)dx$$

$$\Rightarrow\int_0^1\frac{\log(1-x)}{1+x^2}dx=\int_0^1 f(x)dx$$

$$\therefore f(x)=\frac{\log(1-x)}{1+x^2}$$

63. $\lim_{n\to\infty}\frac{1}{n}\log\left(\frac{(2n)!}{n^n\cdot n!}\right)=\lim_{n\to\infty}\frac{1}{n}\log\left[\frac{1.2.3......2n}{n^n.n!}\right]$

$$=\lim_{n\to\infty}\frac{1}{n}\log\left[\frac{(n+1).(n+2).....(n+n)}{n^n}\right]$$

$$=\lim_{n\to\infty}\frac{1}{n}\log\left[\left(1+\frac{1}{n}\right).\left(1+\frac{2}{n}\right).....\left(1+\frac{n}{n}\right)\right]$$

$$=\lim_{n\to\infty}\frac{1}{n}\left[\log\left(1+\frac{1}{n}\right)+\log\left(1+\frac{2}{n}\right)+\ldots+\log\left(1+\frac{n}{n}\right)\right]$$

$$=\int_0^1\log(1+x)dx$$

Let $1+x=t\Rightarrow dx=dt$

$$=\int_1^2\log x.dx$$

Hence comparing $f(x)=\log x$.

64. Let $I=\int_{-\pi/2}^{\pi/2}\sin(x-[x])dx$

$$=\int_{-\pi/2}^{-1}\sin\{x\}dx+\int_{-1}^{0}\sin\{x\}dx+\int_0^1\sin\{x\}dx+\int_1^{\pi/2}\sin\{x\}dx$$

$$=\int_{\frac{-\pi}{2}}^{-1}\sin(x+2)dx+\int_{-1}^{0}\sin(x+1)dx+\int_0^1\sin x dx+\int_1^{\pi/2}\sin(x-1)dx$$

$$=3(1-\cos 1)+\sin 2-\sin 1$$

65. $I=\int_0^2\{x\}dx$

$$=\int_0^2(x-[x])dx$$

$$=\int_0^2 xdx-\int_0^2[x]dx=\left[\frac{x^2}{2}\right]_0^2-\left[\int_0^1 0dx+\int_1^2 dx\right]$$

$$=2-\left[0+[x]_1^2\right]=2-[2-1]=1$$

66. $I=\int_0^2 \left|1-x^2\right| dx$

$=\int_0^1 (1-x^2)dx+\int_1^2 (x^2-1)dx$

$=\left[x-\frac{x^3}{3}\right]_0^1+\left[\frac{x^3}{3}-x\right]_1^2$

$=[1-1/3]+\left[\frac{8}{3}-2-\frac{1}{3}+1\right]=\frac{2}{3}+\left[\frac{2}{3}+\frac{2}{3}\right]=2$

67. If $\int\left(\tan^7 x+\tan^6 x+.... \tan x+1\right)dx$

$$\boxed{\begin{array}{l}\int_0^{\pi/4}\tan^n x=\frac{1}{n-1}-I_{n-2}\\ I_n+I_{n-2}=\frac{1}{n-1}\end{array}}$$

$\frac{1}{6}+\frac{1}{5}+\frac{1}{2}+1=\frac{28}{15}$

68. $I=\int_0^{\pi/2}\frac{\pi\sin x}{1+\cos^2 x}dx$

$\cos x=t\Rightarrow -\sin x dx=dt$

$I=-\int_1^0\frac{\pi dt}{1+t^2}$

$I=\pi\int_0^1\frac{dt}{1+t^2}=\pi\left(\mathrm{Tan}^{-1}t\right)_0^1$

$=\pi\left(\frac{\pi}{4}\right)=\frac{\pi^2}{4}$

69. $\int_0^{\pi}\sqrt{1+4\sin^2\frac{x}{2}+4\sin\frac{x}{2}}\,dx$

$\int_0^{\pi}\left|1+2\sin\frac{x}{2}\right|dx$

$\int_0^{\pi}\left(1+2\sin\frac{x}{2}\right)dx=\left(x-4\cos\frac{x}{2}\right)_0^{\pi}=\pi+4$

70. $\int_{\pi/4}^{3\pi/4}\frac{dx}{1+\cos x}=\int_{\pi/4}^{3\pi/4}\left(\frac{1-\cos x}{\sin^2 x}\right)dx$

$\int_{\pi/4}^{3\pi/4}\left(\cos ec^2 x-\cos ecx.\cot x\right)dx$

$=1+\sqrt{2}+1-\sqrt{2}=2$

Option (4) satisfy the given condition. And

$2.\sin\frac{\pi}{2}=2(1)=2$

71. Let $I=\int_0^{\pi}x f(\sin x)dx$

w.k.t $\int_0^{nT}x.f(x)dx=n\int_0^{T}f(x).dx$

(or)

$\int_0^a f(x).dx=\int_0^a f(a-x).dx$

$=\int_0^{\pi}(\pi-x)f(\sin(\pi-x))dx$

$=\int_0^{\pi}(\pi-x).f(\sin x).dx$

$=\pi\int_0^{\pi}f(\sin x)dx-I$

$2I=\pi\int_0^{\pi}f(\sin x).dx\therefore I=\pi\int_0^{\pi/2}f(\sin x).dx$

72. $a^k\left.\frac{x^{k=1}}{k+1}\right]_0^1=\frac{a^k}{k+1}$

$a^k\ \underset{n\to\infty}{Lt}\frac{1}{n}\sum_{\lambda=1}^{n}(r/n)^k=a^k-\int_0^1 x^k dx$

$=a^k\left.\frac{x^{k+1}}{k+1}\right]_0^1=\frac{a^k}{k+1}$

73. $x=\pi/6,\quad \pi/3$

$\alpha=\frac{\pi}{6},\quad\frac{\pi}{3}$

$\int_{\pi/6}^{\pi/3}x\cos x^2 dx$

put $x^2=t$

$\Rightarrow xdx=\frac{dt}{2}$

$\int_{\frac{\pi^2}{36}}^{\frac{\pi^2}{9}}\cos t\frac{dt}{2}=\frac{1}{2}\left(\sin\frac{\pi^2}{9}-\sin\frac{\pi^2}{36}\right)$

74. $= \frac{\pi}{2}\int_0^{\pi}\left(\sin^2(\sin x)+\cos^2(\cos x)\right)dx$

$= \frac{\pi}{\cancel{2}}.\cancel{2}\int_0^{\pi/2}\left(\sin^2(\sin x)+\cos^2(\cos x)\right)dx$

$= \pi\int_0^{\pi/2}\left(\sin^2(\cos x)+\cos^2(\cos x)\right)dx$

$= \pi\left(\frac{\pi}{2}\right) = \frac{\pi^2}{2}$

75.

$$\underset{n\to\infty}{Lt}\sum_{r=1}^{n}\frac{r^4}{r^5+n^5} = \underset{n\to\infty}{Lt}\frac{1}{n}\sum_{r=1}^{n}\frac{\left(\frac{r}{n}\right)^4}{\left(\frac{r}{n}\right)^5+1}$$

$$= \frac{1}{5}\int_0^1\frac{x^4dx}{x^5+1} = \frac{1}{5}|n|x^5+11\Big]_0^1 = \frac{1}{5}ln2 = ln\sqrt[5]{2}$$

76. $I_{n+2}+I_n = \frac{1}{n-1}$

$3+4+5 = 12 = \frac{1}{I_{11}+I_{13}}$

77. $\int_0^{\pi/4} e^{\tan^2\theta}\sin^2\theta\tan\theta\ d\theta$

$\int_0^{\pi/4} e^{\tan^2\theta}\frac{\tan^2\theta}{\sec^4 0}\tan\theta\sec^2\theta d\theta$

$\tan^2\theta = t$

$2\tan\theta\cdot\sec^2\theta d\theta = dt$

$$= \frac{1}{2}\int_0^1 e^t\left(\frac{t^2}{(t^2+1)^2}\right)dt$$

$$= \frac{1}{2}\int_0^1 e^t\left(\frac{t^2+1-1}{(t^2+1)^2}\right)dt = \frac{1}{2}\frac{e^t}{2(2+t^2)}\Big]_0^1 = \frac{1}{2}\left[\frac{e}{2}-1\right]$$

78. $\int_{\pi/4}^{\pi/2}\sin 2t dt + \int_{\pi/2}^{\pi}(o)dx + \int_{\pi}^{5\pi/4}\sin 2t\,dt$

$= \frac{1}{2}+0-\frac{1}{2} = 0$

79. $\int_0^{\pi} f(x)dx + \int_0^{\pi} g(x)dx$

$\int_0^{\pi/4}\cos xdx + \int_{\pi/4}^{\pi}\sin xdx + \int_0^{\pi/4}\sin xdx + \int_{\pi/4}^{\pi}\cos xdx$

$= \left(\frac{1}{\sqrt{2}}-0\right)+1+\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}+1+0-\frac{1}{\sqrt{2}} = 2$

80. $\frac{1}{2}\int_0^{10T} f(x)dx = \frac{10}{2}\int_0^{T} f(x)dx = 5I$

81. $\int_1^3 x\sqrt[n]{x^2-1}dx$

$x^2-1 = t^n$

$2xdx = nt^{n-1}dt$

$$= \frac{n}{t}\int_0^{8^{1/n}} t^n dt = \frac{n}{2}\frac{t^{n+1}}{n+1}\Big]_0^{8^{1/n}} = \frac{n}{2(n+1)}8^{\frac{n+1}{n}}$$

If n = 3

$\frac{3}{2(4)}8^{4/3} = \frac{3}{\cancel{8}}(\cancel{16})^2 = 6.$

82. $\int_{-1}^{1}\left(x[1+\sin\pi x]+1\right)dx$

$\pi x = t$

$\pi dx = dt$

$= \int_{-\pi}^{\pi}\frac{t}{\pi}[1+\sin t]\frac{dt}{\pi}+2$

$= \frac{1}{\pi^2}\int_{-1}^{1} tdt + \frac{1}{\pi^2}\int_{-\pi}^{\pi} t[\sin t]dt + 2$

$= 0+\frac{1}{\pi^2}\int_{-\pi}^{0} t(-1)dt + 2 = \frac{5}{2}$

83. $\underset{n\to\infty}{Lt}\left[\frac{n}{(n+1)\sqrt{2n+1}}+\frac{n}{(n+2)\sqrt{3(2n+2)}}+\frac{n}{(n+3)\sqrt{3(2n+3)}}+\text{.......n terms}\right]$

$$\underset{n\to\infty}{Lt}\frac{1}{n}\sum_{n=1}^{n}\frac{1}{(1+r/n)}\sqrt{\frac{r}{n}\left(2+\frac{2r}{n}\right)}$$

$$= \int_0^1\frac{1}{(1+x)\sqrt{x(2+x)}}dx$$

$$f(x) = \frac{1}{(x+1)\sqrt{2x+x^2}}$$

84. $\int_{\sqrt{3}}^{\sqrt{18}}[x]dx = \int_{\sqrt{3}}^{2}(1)dx + \int_2^3(2)dx + \int_3^4(3)dx + \int_4^{3\sqrt{2}}(4)dx$

$= -9+12\sqrt{2}-\sqrt{3}$

$a+b+c = -9+12-1 = 2$

85. $\int_0^4\frac{x+2}{\sqrt{4x-x^2}}dx$

$$x+2=A(4-2x)+B$$

$$A=\frac{-1}{2},\ B=4$$

$$=\frac{-1}{2}2\sqrt{4x-x^2}\Big]_0^4+4\int_0^4\frac{1}{\sqrt{x(4-x)}}dx$$

$$=0+4\pi=4\pi$$

86. $$=0+\int_{-\pi}^{\pi}\frac{2x\sin x}{1+\cos^2 x}dx$$

$$=2\int_0^{\pi}\frac{2x\sin x}{1+\cos^2 x}dx$$

$$=4\left(\frac{\pi}{2}\right)\int_0^{\pi}\frac{\sin x}{1+\cos^2 x}dx$$

Put $\cos x=t$

$$=2\pi\left(\frac{\pi}{2}\right)=\pi^2$$

87. $$=0-\int_{-1}^{1}\frac{x^2}{3-|x|}dx$$

$$=-2\int_0^1\frac{x^2}{3-x}dx$$

$$=-2\int_0^1\frac{\left[\left(x^2-9\right)+9\right]dx}{3-x}$$

$$=+2[x+3]_0^1-18[\ln(3-x)]_0^1$$

$$=2\left[\frac{1}{2}+3\right]+18(\ln 2-\ln 3)$$

$$=7-18\ln\frac{3}{2}$$

88. $$\int_{-a}^{a}f(x)dx-\int_o^a f(-x)dx$$

$$=\int_{-a}^{0}f(x)dx+\int_0^a f(x)dx-\int_0^a f(-x)dx$$

$$x=-t$$

$$=\int_a^0 f(-t)dt+\int_0^a f(x)dx-\int_0^a f(-x)dx$$

$$=\int_o^a f(a-x)dx$$

89. $$=\frac{\pi}{2}\int_0^{\pi}\left(\frac{3\cos^2 x+2\sin x+\sin^3 x-3}{\sin x}\right)dx$$

$$=\pi\int_0^{\pi_2}\left(\frac{\sin^3 x-3\sin^2 x+2\sin x}{\sin x}\right)dx$$

$$=\pi\left[\frac{2-1}{2}\times\frac{\pi}{2}-3+2\left(\frac{\pi}{2}\right)\right]$$

$$=\pi\left[\frac{5\pi-12}{4}\right)$$

90. $$\frac{3\pi}{4}<x<\pi\Rightarrow 3<\frac{4x}{\pi}<4$$

$$\int_{3\pi/4}^{\pi}(3+[\sin x])dx=3\left(\pi-\frac{3\pi}{4}\right)=\frac{3\pi}{4}$$

91. $$=\int_0^{\pi}\sin\left(\frac{7\pi}{4}\right)\sin\left(\pi+\frac{x}{2}\right)dx=-\sqrt{2}(0-1)=\sqrt{2}$$

92. $$\log_e^x=t\Rightarrow x=e^t\Rightarrow dx=e^t dt$$

$$\int_{\ln 2}^{1}\left(\frac{1}{t}-\frac{1}{t^2}\right)e^t dt=\left[\frac{1}{t}e^t\right]_{\ln 2}^{1}=e-2\log_2^e$$

$$\int_a^b e^x\left(f(x)+f^1(x)\right)dx=\left[e^x f(x)\right]_a^b=e^b f(b)-e^a f(a)$$

93. $$\int_0^{50\pi}\sqrt{2}|\sin x|dx=50\int_0^{\pi}\sqrt{2}\sin x\ \ dx$$

$$=50\sqrt{2}[-\cos x]_0^{\pi}$$

$$=50\sqrt{2}(2)=100\sqrt{2}$$

94. $$\int_0^{\pi}\frac{\csc^2 x dx}{1+\cot^2 x+2}=-2\int_0^{\frac{\pi}{2}}\frac{\operatorname{cosec}^2 x}{\left(\sqrt{3}\right)^2+(\cot x)^2}dx$$

$$=-\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{\cot x}{\sqrt{3}}\right)_0^{\frac{\pi}{2}}=\frac{-2}{\sqrt{3}}\left[0-\frac{\pi}{2}\right]=\frac{\pi}{\sqrt{3}}=k$$

$$[k]=1$$

95. put $x=t^2\Rightarrow dx=2tdt$

$$\int_0^{\pi/2}(2\sin t+t\cos t)2tdt$$

$$=\int_0^{\pi/2}4t\sin tdt+\left[2t^2(\sin t)\right]_0^{\pi/2}-\int_0^{\pi/2}4t\sin t\,dt$$

$$=\frac{\pi^2}{2}$$

96. $$\underset{n\to\infty}{Lt}\sum_{r=1}^{n}\frac{r^2}{n^3+r^3}$$

$$\underset{n\to\infty}{Lt}\frac{1}{n}\sum_{r=1}^{n(1)}\frac{(r/n)^2}{1+(r/n)^3}$$

$$=\frac{1}{3}\int_0^1 \frac{3x^2}{1+x^3}dx=\frac{1}{3}\ln(3)$$

97. $=f(h(f))h^1(t)-f(g(t))g^1(t)$

98. If m = 8

$$\frac{7}{\cancel{12}_4}\times\frac{5}{\cancel{10}_2}\times\frac{3}{8}\times\frac{1}{\cancel{6}_2}\times\frac{3}{4}\times\frac{1}{2}\times\frac{\pi}{2}=\frac{7\pi}{2048}$$

$\therefore m=8$

99. $$I=\int_{-\pi}^{\pi}\frac{\cos^{2022}x}{1+(2022)^{-x}}$$

$$2I=\int_{-\pi}^{\pi}\cos^{2022}x\ dx$$

$$I=2\int_0^{\pi/4}\cos^{2022}x\ dx$$

$$=\frac{(2022)!}{2^{2022}((1011)!)^2}\pi$$

100 $$2\int_0^{\frac{\pi}{4}}(1+\tan^2 x)^3\sec^2 x\ dx$$

Put $\tan x=t$

$$=2\int_0^1(2+t^6+3t^2+3t^4)dt$$

$$=2\left[t+\frac{t^7}{7}+t^3+\frac{3t^5}{5}\right]_0^1$$

$$=2\left[1+\frac{1}{7}+1+\frac{3}{5}\right]$$

$$=2\left[2+\frac{26}{35}\right]$$

$$=2\left[\frac{96}{35}\right]=\frac{192}{35}$$

101. $$\int_{\alpha+1}^{\alpha}e^x\left(\frac{-1}{(x-\alpha+1)}+\frac{1}{(x-\alpha+1)^2}\right)dx$$

$$=\left[\frac{-e^x}{x-\alpha+1}\right]_{\alpha+1}^{\alpha}=\frac{e^\alpha}{2}(e-2)$$

102. $$I_n=\frac{2}{n-1}\sin(n-1)x+I_{n-2}$$

If n is even

$$\int_0^\pi\frac{\sin x}{\sin x}dx=\frac{2}{n-1}\sin(n-1)x\Big]_0^\pi+I_{n-2}$$

$$=0+I_2=0$$

k = 0

If n is odd.

$$\int_0^\pi\frac{\sin nx}{\sin x}dx=0+I_n\text{ --------}$$

$$=0+I_1=\int_0^\pi dx=\pi$$

$$\frac{k\pi}{2}=\pi$$

k = 2

$k=1-(-1)^n$

103. $$\left[xf'(x)-(1)f(x)\right]_0^1$$

$$=f'(1)-f(1)+f(0)$$

$$f'(x)=\frac{\cos(\tan^{-1}x)}{1+x^2}\Rightarrow f'(1)=\frac{1}{2\sqrt{2}}$$

$$f(1)=\frac{1}{\sqrt{2}},f(x)=0$$

$$=\frac{1}{2\sqrt{2}}-\frac{1}{\sqrt{2}}=-\frac{1}{2\sqrt{2}}$$

104. $$\frac{9\pi}{22}-\frac{\pi}{11}=\frac{7\pi}{44}$$

105. $$=\int_0^{2\pi}\cos mx\cos nx\ dx+0$$

If m = n

$$=\int_0^{2\pi}\left[\frac{1+\cos 2mx}{2}\right]dx$$

$$=\frac{x}{2}+\frac{\sin 2mx}{4m}\Big]_0^{2\pi}=\pi$$

106. $$=\int_2^5\left[(\sqrt{x-1}+1)+(\sqrt{x-1}-1)\right]dx$$

$$=2\frac{(x-1)^{3/2}}{3/2}\Big]_2^5=\frac{28}{3}$$

157. $$\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\frac{1}{\sqrt{r/n}}=\int_0^1\frac{1}{\sqrt{x}}\ dx$$

$$=2\sqrt{x}\Big]_0^1=2$$

108. Put $x=\frac{1}{z}\Rightarrow dx=\frac{-1}{z^2}dz$

$$\int_0^\infty x^{11}\log x dx+\int_\infty^0 z^{13}\log z^{-1}\left(\frac{-1}{z^2}\right)dz=0$$

109. $$\int_1^2\frac{\pi}{2}dx=\frac{\pi}{2}(1)=\frac{\pi}{2}$$

110. $\int_{-\pi/2}^{\pi/2} x dx + \int_{\pi/2}^{3\pi/2} (\pi - x) dx + \int_{3\pi/2}^{2\pi} (x - 2\pi) dx$

$= 0 + \pi(\pi) - \left[\frac{x^2}{2}\right]_{\pi/2}^{3\pi/2} + \left[\frac{x^2}{2}\right]_{3\pi/2}^{2\pi} - \cancel{2}\pi\left(\frac{\pi}{\cancel{2}}\right)$

$= \pi^2 + \frac{1}{2}\left(\frac{7\pi^2}{4}\right)$

$= \frac{-\pi^2}{8}$

111. $\frac{f(x)}{x^2} = \frac{\cancel{2}}{\cancel{2}\sqrt{x}}$

$f(x) = x\sqrt{x}$

112. $\int_1^2 x\sqrt{4 - x^2} dx$

Put

$4 - x^2 = t^2 \Rightarrow xdx = -tdt$

$= -\int_{\sqrt{3}}^{0} t^2 dt = \sqrt{3}$

113. $= \int_{-3/2}^{3/2} [2x - 3] dx$

$2x - 3 = t$

$2dx = dt$

$= \int_{-6}^{0} [t]\frac{dt}{2} = \frac{1}{2}(-6-5-4-3-2-1) = \frac{-21}{2}$

$\left|\frac{-21}{2} + \frac{1}{2}\right| = 10$

114. $\left.\frac{x^2}{2} + \frac{2x^{3/2}}{3} + \log x\right]_1^4 - (2\log 2 - 1)$

$= 8 + \frac{16}{3} + \cancel{\log} 4 - \frac{1}{2} - \frac{2}{3} - 2\cancel{\log} 2 + 1$

$= \frac{79}{6}$

115. $I = \int_{-\pi/4}^{\pi/4} \frac{3}{\pi(2 - \cos 2x)} dx + 0$

$= \frac{6}{\pi}\int_0^{\pi/4} \frac{dx}{2 - \cos 2x}$

$= \frac{6}{\pi}\int_0^{\pi/4} \frac{\sec^2 x}{1 + 3\tan^2 x} dx$

$\tan x = t$

$= \frac{c}{\pi}\int_0^1 \frac{dt}{1 + 3t^2}$

$= \frac{6}{\pi}\frac{-1}{\sqrt{3}}\tan^{-1}\left(\sqrt{3}t\right)\Big]_0^1$

$= \frac{\cancel{6}2}{\sqrt{3}\cancel{\pi}}\left(\frac{\cancel{\pi}}{\cancel{3}}\right) = \frac{2}{\sqrt{3}}$

$3I^2 = 3\left(\frac{4}{3}\right) = 4$

116. $e^{\int_0^1 2x\log(1+x^2)dx}$

$1 + x^2 = t \Rightarrow 2xdx = dt$

$= e^{\int_1^2 \log t dt}$

$= e^{[t\log t - t]_1^2}$

$= e^{2\log 2 - 2 + 1}$

$= e^{\log(4/e)} = \frac{4}{e}$

117. $\int_0^a f(x) dx + \int_a^{2a} f(x) dx$

$= \int_0^a \big(f(x) + f(x + a)\big) dx$ (2)

118. $\frac{3}{\pi}\int_0^{\pi} (\sin x - \cos x) dx$

$= \frac{3}{\pi}[-\cos x]_0^{\pi} = \frac{3}{\pi}(1 + 1) = \frac{6}{\pi}$

119. $\frac{3}{7} \times \frac{1}{5} \times \frac{2}{3} = \frac{2}{35}$

120. $\lim_{n\to\infty} \frac{1}{n} 3(7^2)\sum_{r=1}^{n}\left(\frac{r}{n}\right)^2 + 6$

$= 3(49) \int_0^1 x^2 dx + 6$

$= 49 + 6 = 55$ (1)

121. $f(x) = -2\sin^2 x(1) - 2\cos^2 x(1) = -2$

$\int_0^{\pi/4} 4dx = \cancel{4}\frac{\pi}{\cancel{4}} = \pi$

122. $\int_0^2 |2 - x - x| dx + \int_2^4 |x - 2 - x| dx$

$= 2\int_0^1 (1 - x) dx + 2\int_1^2 (x - 1) dx + 2(2)$

$= 2\left[x - \frac{x^2}{2}\right]_0^1 + 2\left[\frac{x^2}{2} - x\right]_1^2 + 4$

$= 2\left(\frac{1}{2}\right) + 2\left(\frac{1}{2}\right) + 4 = 6$

123. $\int_{-a}^{0} f(x)dx + \int_{0}^{a} f(x)dx$

$= -\int_{a}^{0} f(-x)dx + \int_{0}^{a} f(x)dx$

$= \int_{0}^{a} f(-x)dx + \int_{0}^{a} f(x)dx$

$g(x) = f(-x)$

124. $\mathrm{I} = \int_{-a}^{a} \left(x^4 - 2x^2\right)dx$

$= 2\int_{0}^{a} \left(x^4 - 2x^2\right)dx$

$= 2\left(\frac{x^5}{5} - 2\frac{x^3}{3}\right)_0^a$

$= 2\left(\frac{a^5}{5} - \frac{2a^3}{3}\right)$

$f(a) = 3a^5 - 10a^3$

$f'(a) = 15a^4 - 30a^2$

$= 15a^2\left(a^2 - 2\right) = 1$

$a^2 - 2 = 0 \quad a = \pm\sqrt{2}$

$f'(a) = 15\left(a^4 - 2a^2\right)$

$f''(a) = 15\left(4a^3 - 4a\right)$

$f''\left(\sqrt{2}\right) = 15\left(4(2)^{\frac{3}{2}} - 4\left(\sqrt{2}\right)\right)$

$\therefore$ I is mid point $\boxed{x = \sqrt{2}}$

125. $\mathrm{I} = \frac{1}{2}\left(\frac{\pi}{3} - \frac{\pi}{6}\right) = \frac{1}{2}\left(\frac{2\pi - \pi}{6}\right) = \frac{\pi}{12}$

126. $\mathrm{I} = \int_{-0.5}^{1.5} x^2\left[x^2\right]dx$

$= \int_{-0.5}^{0} x^2[x]dx + \int_{0}^{1} x^2[x]dx + \int_{1}^{1.5} x^2[x]dx$

127. $\mathrm{I} = \int_{0}^{50\pi} \sqrt{1 - \cos 2x}dx = 50\int_{0}^{\pi} \sqrt{2}|\sin x|$

$= 50\int_{0}^{\pi} \sqrt{2}(\sin x)dx = 50\sqrt{2}(1+1) = 100\sqrt{2}$

128. G.T $\int_{-5}^{5} f(x)dx = \int_{0}^{5}\left(f(x) + g(x)\right)dx$

$\Rightarrow \int_{-5}^{5} f(x)dx - \int_{0}^{5} f(x)dx = \int_{0}^{5} g(x)dx$

$\Rightarrow \int_{-5}^{5} f(x)dx + \int_{5}^{0} f(x)dx = \int_{0}^{5} g(x)dx$

$\Rightarrow \int_{-5}^{0} f(x)dx = \int_{0}^{5} g(x)dx$

$\therefore f(x) = \frac{x^3 + 5}{\sqrt{12 + x}} \quad$ put $-x = t$

$-dx = dt$

$-\int_{5}^{0} f(-x)dx = \int_{0}^{5} g(x)dx$

$\int_{0}^{5} \frac{5 - x^3}{\sqrt{12 - x}}dx = \int_{0}^{5} g(x)dx$

$\therefore g(x) = \frac{5 - x^3}{\sqrt{12 - x}}$

129. $I = \int_{0}^{\frac{\pi}{4}} \frac{\cos^2 x}{\cos^2 x + 4\sin^2 x}dx$

$= \int_{0}^{\frac{\pi}{4}} \frac{dx}{1 + 4Tan^2 x}$

Put $2\,Tan x = t \Rightarrow 2\sec^2 x dx = dt$

$\Rightarrow 4\left(1 + Tan^2 x\right)dx = 2dt$

$\Rightarrow dx = \frac{2dt}{4 + t^2}$

$\therefore I = \int_{0}^{2} \frac{1}{1 + t^2}\left(\frac{2dt}{4 + t^2}\right)$

$= 2\int_{0}^{2}\left(\frac{A}{1 + t^2} + \frac{B}{4 + t^2}\right)dt$

130. $f(x) = \int_{0}^{x}\left[(a+1)(t+1)^2 - (a-1)\left(t^2 + t + 1\right)\right]dt$

$f^1(x) = 2x^2 + (a+3)x + 2$

it has equal roots then $\Delta = 0$

$$(a+3)^2 = 4(2)2 = 16$$
$$\Rightarrow a+3 = \pm 4$$
$$\Rightarrow a = -3\pm 4 = 1,-7$$
$$\therefore a>0 \quad \therefore a=1$$

131.
$$I = \int_0^{\frac{\pi}{2}} \frac{\sin\left(\frac{\pi}{4}+x\right)+\sin\left(\frac{3\pi}{4}+x\right)}{\cos x+\sin x}dx$$
$$= \int_0^{\frac{\pi}{2}} \frac{\frac{\sin x+\cos x}{\sqrt{2}}+\frac{\cos x-\sin x}{\sqrt{2}}}{\cos x+\sin x}dx$$

132.
$$I = \int_0^1 \left(\sqrt{10}\right)^{2x}.dx$$
$$= \int_0^1 10^x.dx = \left[\frac{10^x}{\ln 10}\right]_0^1$$
$$\Rightarrow \frac{10-1}{\ln 10}$$
$$\Rightarrow \frac{9}{\ln 10}$$

133. Given
$$f(x) = \begin{cases} x^2, 0\le x\le 1 \\ \sqrt{x},\ 1\le x\le 2 \end{cases}$$
$$\int_0^2 f(x)dx$$
$$= \int_0^1 f(x)dx + \int_1^2 f(x)dx$$
$$= \int_0^1 x^2 dx + \int_1^2 \sqrt{x}\,dx$$

134. Given
$$\int_0^{\frac{1}{2}} \frac{x\sin^{-1}x}{\sqrt{1-x^2}}dx \Rightarrow \sin^{-1}x = t \Rightarrow \int_0^{\frac{\pi}{6}} t\sin t\ dt$$

135. Given
$$\int_0^{x^3} f(t)dt = x^2\sin 2\pi x$$
Diff.w.r.to'x'
$$f\left(x^3\right)3x^2 = x^2\cos 2\pi x.2\pi + 2x\sin 2\pi x$$
$$x=2$$
$$f(8)\times 12 = 4\cos 4\pi.2\pi + 8\sin 4\pi$$
$$f(8)\times 12 = 8\pi$$
$$f(8) = \frac{2}{3}\pi$$

136. Given
$$\lim_{n\to\infty}\frac{1}{n}\left(\frac{1}{e^{\frac{1}{n}}}+\frac{1}{e^{\frac{2}{n}}}+\dots\dots+\frac{1}{e^2}\right)$$
$$= \int_0^2 \frac{1}{e^x}\ dx$$
$$= \int_0^2 e^{-x}\ dx$$

137.
$$I = \int_0^{2024\pi} \frac{2023^{\sin^2 x}}{2023^{\sin^2 x}+2023^{\cos^2 x}}dx$$
$$2I = 2024\pi$$
$$k = 1012\pi\ then\left(\frac{2k}{\pi}+1\right) = \left(\frac{2024\cancel{\pi}}{\cancel{\pi}}+1\right) = 2025$$

138.
$$\int_0^{\frac{\pi}{2}}\left(\sin^6 x+\cos^6 x\right)dx = \int_0^{\frac{\pi}{2}}\left[(\sin^2 x)^3+(\cos^2 x)^3\right]dx$$
$$= \int_0^{\frac{\pi}{2}}\left(1-3\sin^2 x.\cos^2 x\right)dx = \int_0^{\frac{\pi}{2}} 1dx - \frac{3}{4}\int_0^{\frac{\pi}{2}}\sin^2 2x\,dx$$
$$= [x]_0^{\frac{\pi}{2}} - \frac{3}{4}\int_0^{\frac{\pi}{2}}\frac{1-\cos 4x}{2}dx$$
$$= \frac{\pi}{2}-\frac{3}{8}\left[x-\frac{\sin 4x}{4}\right]_0^{\frac{\pi}{2}} = \frac{\pi}{2}-\frac{3}{8}\left[\left(\frac{\pi}{2}-0\right)-(0-0)\right]$$
$$= \frac{\pi}{2}-\frac{3\pi}{16} = \frac{5\pi}{16} \quad \in\left(\frac{\pi}{8},\frac{\pi}{2}\right)$$

Period of $\sin^6 x+\cos^6 x = \frac{\pi}{2}$

Both A&R are true but R is not correct explanation of A

139.
$$I = \int_0^{\pi}\frac{\cos x}{\sqrt{1-\sin^2 x}}dx = \int_0^{\pi}\frac{\cos(\pi-x)}{\sqrt{1-\sin^2(\pi-x)}}dx$$

140.
$$\int_3^b \frac{x-1}{2x-x^2}dx = \frac{1}{2}$$

$$\Rightarrow -\cancel{\frac{1}{2}}\log\left|2x-x^2\right|_3^b = \cancel{\frac{1}{2}}$$

$$\Rightarrow \log\left(\frac{2b-b^2}{-3}\right) = -1 \Rightarrow \frac{2b-b^2}{-3} = e^{-1}$$

$$\Rightarrow (b-1)^2 = 1+\frac{3}{e}$$

141.

$$\text{u}(n) = \int_0^{\frac{\pi}{2}} (1+\sin t)^n \sin 2t\, dt$$

$$1+\sin t = x \Rightarrow \cos t\, dt = dx$$

$$= \int_1^2 x^n\ 2(x-1)dx$$

$$= \int_1^2 \left(2x^{n+1}-2x^n\right)dx$$

$$\text{u}(4) = \int_1^2 \left(2x^5-2x^4\right)dx$$

$$= 2\left[\frac{x^6}{6}-\frac{x^5}{5}\right]_1^2 = \frac{129}{15}$$

142.

$$\int_0^a 2^{[x]}dx = \int_0^1 2^0 dx + \int_1^2 2^1 dx + \ldots . + \int_{a-1}^a 2^{a-1}dx$$

$$= 2^0(1-0)+2^1(2-1)+\ldots+2^{a-1}(a-a+1)$$

$$= 1+2+2^2+\ldots+2^{a-1} = \frac{2^a-1}{2-1} = 127$$

$$2^a = 128 = 2^7$$

a=7

143.

$$\int_{-\pi}^{\pi/2} \sin x \sin^2(\cos x)\, dx$$

Let $\cos x = t$

$-\sin x\, dx = dt$

$$\int_{-1}^{0} -\sin^2 t\, dt = \int_{-1}^{0} \frac{\cos 2t - 1}{2}dt$$

$$= \frac{1}{2}\int_{-1}^{0}(\cos 2t - 1)\,dt$$

$$= \frac{1}{2}\left[\frac{\sin 2t}{2} - t\right]_{-1}^{0}$$

$$= \frac{1}{2}\left[0-0-\left(\frac{\sin 2(-1)}{2}+1\right)\right] = \frac{1}{2}\left(\frac{\sin 2}{2}-1\right)$$

$$= \frac{\sin 2 - 2}{4}$$

144.

$$\int_x^1 (1-t)\,dt = \frac{1}{2}$$

$$\left(t-\frac{t^2}{2}\right)_x^1 = \frac{1}{2}$$

$$\left(1-\frac{1}{2}\right)-\left(x-\frac{x^2}{2}\right) = \frac{1}{2}$$

$$-\left(x-\frac{x^2}{2}\right) = 0$$

$$x^2-2x = 0$$

$x = 0$ *or* $2 \;\therefore x = 2$

145.

$$I = \int_{\pi/6}^{\pi/3} \frac{1}{1+\sqrt{\cot x}}dx = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx \to (1)$$

$$\because \int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

$$\therefore I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx \to (2)$$

$$(1)+(2)$$

$$2I = \int_{\pi/6}^{\pi/3} dx = (x)\Big|_{\pi/6}^{\pi/3} = \frac{\pi}{3}-\frac{\pi}{6} = \frac{\pi}{6}$$

$$\therefore I = \frac{\pi}{12}$$

146. Given $m \in Z^+, n = 2m$

$$\int_0^{\frac{\pi}{2}} \sin^m x \cos^n x\, dx = K(m)\int_0^{\frac{\pi}{2}} \sin^m x\, dx$$

Let $m = 2 \Rightarrow n = 4$

$$\frac{2^{m-1}(m-1)!}{(2m-1)!}k(m) = \frac{2(1)!}{3!}\left(\frac{1}{8}\right) = \frac{1}{24}$$

By verification

$$\frac{1}{m+2}.\frac{1}{m+4}.\frac{1}{m+6}\ldots\ldots\frac{1}{3m}$$

$$= \frac{1}{4}.\frac{1}{6} = \frac{1}{24}$$

147.
$$5f(x)+3f\left(\frac{1}{x}\right)=2-\frac{1}{x}........(1)$$

Replace x by $\frac{1}{x}$

$$\Rightarrow 3f(x)+5f\left(\frac{1}{x}\right)=2-x.........(2)$$

From (1) *and* (2)

$$f\left(\frac{1}{x}\right)=\frac{1}{4}-\frac{5}{16}x+\frac{3}{16}\frac{1}{x}$$

$$\int_1^2 f\left(\frac{1}{x}\right)dx=\int_1^2\left(\frac{1}{4}-\frac{5}{16}x+\frac{3}{16}\frac{1}{x}\right)dx=\frac{6\log 2-7}{32}$$

148. $|f(x)-f(y)|\le 2|x-y|^{3/2}$

$$\frac{|f(x)-f(y)|}{|x-y|}\le 2|x-y|^{1/2}$$

$$\lim_{x\to y}\left|\frac{f(x)-f(y)}{x-y}\right|\le \lim_{x\to y}|x-y|^{1/2}$$

$|f'(y)|\le 0$

$f'(y)=0\Rightarrow f(y)=k$

$\Rightarrow f(x)=k$

but $f(0)=1$

$\Rightarrow f(x)=1$

$$\int_0^1 f^2(x)\,dx=\int_0^1 1\,dx=(x)_0^1=1$$

149
$$I=\int_0^{\pi} x\sin^3 x\cos^2 x\,dx$$

$$2I=\pi\int_0^{\pi}\sin^3\cos^2 x\,dx$$

$$I=\frac{\pi}{\cancel{2}}\cancel{2}\int_0^{\pi/2}\sin^3\cos^2 x\,dx=\frac{\pi\times 2}{5\times 3\times 1}=\frac{2\pi}{15}$$

150 $\int_0^{2\pi}|x\sin x|dx=\int_0^{\pi}x\sin x\,dx-\int_{\pi}^{2\pi}x\sin x\,dx=\pi+2\pi+\pi$

151 $\int_{1/25}^{3}\frac{e^{3/x}}{x^2}dx=$

$$\frac{3}{x}=t\Rightarrow\frac{-3}{x^2}dx=dt\Rightarrow\frac{1}{x^2}dx=\frac{-dt}{3}$$

L.L $\Rightarrow t=75$

U.L $\Rightarrow t=1$

$$\int_{75}^{1}e^t\left(\frac{-dt}{3}\right)=\frac{-1}{3}\left(e^1-e^{75}\right)=\frac{e^{75}-e}{3}$$

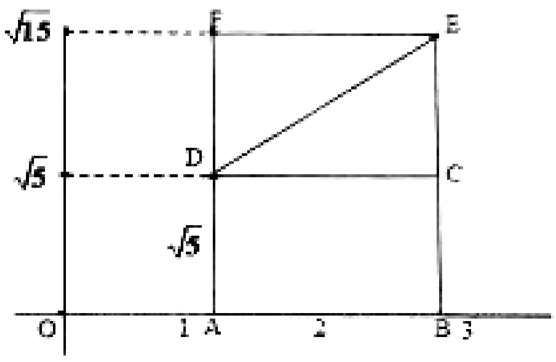
152 $\int_0^{1000} e^{x-[x]}$

$1000\int_0^1 e^{x-[x]}$ (periodic property)

$1000(e^x)_0^1$

$1000(e^1-1)=1000(e-1)$

153. area of rectangle ABCD < area of ABCED < area of rectangle ABEF



$2\sqrt{5}<$ *area c*

$\left(2\sqrt{5},2\sqrt{15}\right)$

154. $\int_{-4\pi}^{4\pi}\tan^9 x\sin^6 x\cos^3 x$

$$f(-x)=\left(\tan(-x)\right)^9\left(\sin(-x)\right)^6\left(\cos(-x)\right)^3$$

$= -\tan^9 x\sin^6 x\cos^3 x$

$=$ - f(x)

$\therefore f(-x)=-f(x)\quad (odd)$

$=0$

155 $-\cos^2 x\, f(\cos x)\sin x=-\sin 2x.2$

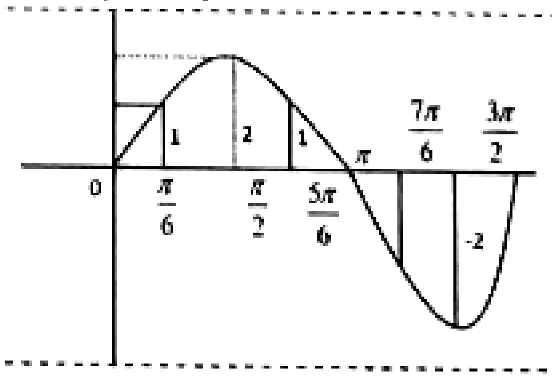
$\cos^2 x\, f(\cos x)\sin x=4\sin x\cos x$

f(cos x) = 4 sec x

put x = $\frac{\pi}{4}$ $f\left(\cos\frac{\pi}{4}\right)=4\sec\frac{\pi}{4}$

$$f\left(\frac{1}{\sqrt{2}}\right)=4\sqrt{2}$$

156



$$\int_{\pi/2}^{5\pi/6} 2\,dx + \int_{5\pi/6}^{\pi} 0\,dx + \int_{\pi}^{7\pi/6} -1\,dx + \int_{7\pi/16}^{3\pi/2} -2\,dx$$

$$2\left[\frac{5\pi}{6}-\frac{\pi}{2}\right]-1\left[\frac{7\pi}{6}-\pi\right]-2\left[\frac{3\pi}{2}-\frac{7\pi}{6}\right]$$

$$=\frac{2\pi}{3}-\frac{\pi}{6}-\frac{2\pi}{3}=\frac{-\pi}{6}$$

A is false

R is true from the graph

157 $$\int_{\frac{1}{2}}^{2}\left|\log_{10}^{x}\right|dx=\int_{\frac{1}{2}}^{2}\frac{\left|\log_{e}^{x}\right|}{\log_{e}10}dx$$

$$=\frac{1}{\log_e 10}\left[\int_{\frac{1}{2}}^{1}\left|\log_e^{x}\right|dx+\int_{1}^{2}\left|\log e^{x}\right|\right]dx$$

$$=\frac{1}{\log_e 10}\left[\int_{\frac{1}{2}}^{1}-\log_e x\,dx+\int_{1}^{2}\log_e x\,dx\right]$$

apply integration by parts

158 $$I=\int_{0}^{\pi/2}\frac{\sin^2 x}{\sin x+\cos x}dx \rightarrow(1)$$

Given

$$I=\int_{0}^{\pi/2}\frac{\cos^2 x}{\text{sinn}+\text{cosn}}dx \rightarrow(2)$$

$$2I=\int_{0}^{\pi/2}\frac{1}{\text{sinx}+\text{cosx}}dx$$

put $Tan\,{}^{x}\!/_{2}=t$, $\sin x=\frac{2t}{1+t^2}$, $\cos x=\frac{1-t^2}{1+t^2}$

$$\cancel{2}I=\int_{0}^{1}\frac{1}{\frac{2t+1-t^2}{1+t^2}}\frac{2dt}{1+t^2}\Rightarrow I=\frac{1}{\sqrt{2}}\log\left(\sqrt{2}+1\right)$$

159 $$\int_{0}^{2\pi}\left[\left|\sin x\right|+\left|\cos x\right|\right]dx$$

$w.k.$t $\left[\left|\sin x\right|+\left|\cos x\right|\right]=1$

$$\int_{0}^{2\pi}1.dx=\left(x\right)_0^{2\pi}=2\pi$$

160 *Given* $f(x).f(-x)=9$

$$I=\int_{-23}^{23}\frac{dx}{3+f(x)}\rightarrow eq(1)$$

$$I=\int_{-23}^{23}\frac{dx}{3+f(-x)}=\int_{-23}^{23}\frac{dx}{3+\frac{9}{f(x)}}$$

$$=\int_{-23}^{23}\frac{f(x).dx}{3\left[f(x)+3\right]}\rightarrow(2)$$

$(1)+(2)$ *we get*

$$4I=\int_{-23}^{23}\frac{1+f(x)}{f(x)+3}\;dx$$

$$3I=\int_{-23}^{23}\frac{f(x)}{f(x)+3}dx\rightarrow(3)$$

$$=\int_{-23}^{23}\frac{f(-x)}{3+f(-x)}\;dx$$

$$=\int_{-23}^{23}\frac{\frac{9}{f(x)}}{3+\frac{9}{f(x)}}\,dx$$

$$3I=\int_{-23}^{23}\frac{\cancel{9}^{3}}{\cancel{3}\left[f(x)+3\right]}dx\rightarrow(4)$$

$$(3)+(4),\;6I=\int_{-23}^{23}\frac{f(x)+3}{f(x)+3}\;dx$$

$$=\left(x\right)_{-23}^{23}=46$$

$$6I=46\quad\Rightarrow I=\frac{46}{6}$$

161 $$\int_{0}^{\frac{\pi}{4}}\frac{\sec x}{1+2\sin^2 x}dx$$

$req\;area=\int_0^3(2x-x^2+x)dx=\frac{9}{2}$

$\sin x=t$

$\cos x\,dx=t$

$x=0\Rightarrow t=0$

$x=\frac{\pi}{4}\Rightarrow t=\frac{1}{\sqrt{2}}$

$$\int_0^{\frac{1}{\sqrt{2}}} \frac{dt}{(1-t^2)(1+2t^2)}$$

$$\frac{1}{(1+t)(1-t)(1+2t^2)} = \frac{A}{1+t} + \frac{B}{1-t} + \frac{cx+d}{1+2t^2}$$

$$A = B = \frac{1}{6}, C = 0, D = \frac{2}{3}$$

$$\therefore \int_0^{\frac{\pi}{4}} \frac{\sec x}{1+2\sin^2 x} dx = \frac{1}{3}\log(\sqrt{2}+1) + \frac{\sqrt{2}\pi}{12}$$

162 $$\lim_{n\to\infty} \frac{1}{n^2}\sec^2\frac{1}{n^2} + \frac{2}{n^2}\sec^2\frac{4}{n^2} + ---+ \frac{n}{n^2}\sec^2\frac{n^2}{n^2}$$

$$\lim_{n\to\infty} \frac{1}{n}\sum_{r=1}^{n} \frac{r}{n}\sec^2\left(\frac{r}{n}\right)^2$$

$$= \int_0^1 x\sec^2 x^2\, dx = \frac{1}{2}\tan(1)$$

163 $$\int_2^5 \sqrt{\frac{5-x}{x-2}}\, dx$$

$$\int_2^5 \frac{5-x}{\sqrt{(x-2)(5-x)}} dx = \frac{3\pi}{2}$$

164 $$\int_0^{\frac{\pi}{2}} \sin^6 x\cos^4 x\, dx$$

$$= \frac{5}{10}.\frac{3}{8}.\frac{1}{6}.\frac{3}{4}.\frac{1}{2}.\frac{\pi}{2} = \frac{3\pi}{512}$$

165 $$\int_0^1 |x^2-3x+2|dx + \int_1^2 |x^2-3x+2|dx + \int_2^3 |x^2-3x+2|dx$$

$$\int_0^1 (x^2-3x+2)dx - \int_1^2 (x^2-3x+2)dx + \int_2^3 (x^2-3x+2)dx$$

$$\left[\frac{x^3}{3} - 3\frac{x^2}{2} + 2x\right]_0^1 - \left[\frac{x^3}{3} - \frac{3x^2}{2} + 2x\right]_1^2 + \left[\frac{x^3}{3} - \frac{3x^2}{2} + 2x\right]_2^3$$

$= 11/6$

166 $\pi/8092$

$$I = \int_{\frac{-\pi}{8092}}^{\frac{\pi}{8092}} \frac{\sec(-2023x)}{1+(2023)^{-2023x}} dx$$

$$I = \int_{\frac{-\pi}{8092}}^{\pi/8092} \frac{2023^{2023x}\sec(2023x)}{2023^{2023}+1} dx$$

$$I + I = \int_{\frac{-\pi}{8092}}^{\pi/8092} \sec(2023x)dx$$

$$= 2\int_0^{\pi/8092} \sec(2023x)dx$$

$$= \frac{\log(\sqrt{2}+1)}{2023} + C$$

167 $Put\ x = 2\sin^2\theta$

$dx = 4\sin\theta\cos\theta d\theta$

$x = 0 \Rightarrow \theta = 0$

$x = 2 \Rightarrow \theta = \pi/2$

$$\int_0^{\pi/2} (2\sin^2\theta)^{5/2} \sqrt{2-2\sin^2\theta}\ 4\sin\theta\cos\theta\ d\theta$$

$$= 2^5\int_0^{\frac{\pi}{2}} \sin^6\theta\cos^2\theta d\theta$$

$$2^5 \times \frac{(5\times3\times1)}{8\times6\times4\times2} \times \frac{\pi}{2} = \frac{5\pi}{8}$$

168 $$\int_{-1}^{1} x|x|dx = \int_{-1}^{0} -x^2\, dx + \int_0^1 x^2\, dx$$

$$= -\left(\frac{x^3}{3}\right)_{-1}^{0} + \left(\frac{x^3}{3}\right)_0^1$$

$$= -\frac{1}{3}(0+1) + \frac{1}{3}(1-0)$$

$= 0$

169 $$I = \int_{-\pi/2}^{\pi/2} \sin^2 x\cos^2 x(\sin x + \cos x)\, dx$$

$$I = \int_{-\pi/2}^{\pi/2} \sin^2 x\cos^2 x\ (-\sin x + \cos x)\ dx$$

$$2I = 2\int_{-\pi/2}^{\pi/2} \sin^2 x\ .\cos^3 x\ dx$$

$$I = 2\int_0^{\pi/2} \sin^2 x\cos^3 x\, dx$$

$$I = 2.\ \frac{1}{5} \times \frac{2}{3} \times \frac{4}{15}$$

170 $$\int_0^3 (3x^2-4x+2)dx = k$$

$$= \left(x^3 - 2x^2 + 2x\right)_0^3$$
$$= 27 - 18 + 6$$
$$= 15 = k$$
$$3x^2 - 4x + 2 = \frac{3k}{5} = 9$$
$$3x^2 - 4x - 7 = 0$$
$$3x^2 - 7x + 3x - 7 = 0$$
$$(x+1)\ (3x-7) = 0$$
$$x = -1 \ \ (\text{integer})$$

171 $\int_0^{\pi} \frac{x\cos^2 x}{1+\sin x}dx$

$$\int_0^{\pi} \frac{x\left(1-\sin^2 x\right)}{1+\sin x}dx = \int_0^{\pi} x\left(1-\sin x\right)dx$$

$$= \frac{\pi^2}{2} - \pi = \frac{\pi}{2}(\pi - 2)$$

172 $\int_{-2}^{2} [2-x]dx$

$$\int_{-2}^{-1}[2-x]dx + \int_{-1}^{0}[2-x]dx + \int_{0}^{1}[2-x]dx + \int_{1}^{2}[2-x]dx$$
$$3\int_{-2}^{-1} dx + 2\int_{-1}^{0} dx + 1\int_{0}^{1} dx + 0\int_{1}^{2} dx$$
$$3(x)^{-1} + 2(x)_{-1}^{0} + (x)_0^1$$
3(1)+2(1)+1= 6

173 $\int_0^2 \frac{x}{(2-x)^{\frac{3}{4}}}dx$

$$\int_0^2 \frac{x}{2-x}(2-x)^{\frac{1}{4}}dx$$
$$(2-x)^{\frac{1}{4}} = t \Rightarrow 2-x = t^4$$
$$\Rightarrow 2 - t^4 = x$$
$$x = 0 \Rightarrow t = 2^{\frac{1}{4}}, x = 2 \Rightarrow t = 0$$
$$\frac{-1}{4}(2-x)^{\frac{-3}{4}}dx = dt$$
$$dx = -4(2-x)^{\frac{3}{4}}dt$$

$$-4\int_{2^{\frac{1}{4}}}^{0}\left(2-t^4\right)dt = -4\left(2t - \frac{t^5}{5}\right)_{2^{\frac{1}{4}}}^{0}$$
$$= 4.2^{\frac{1}{4}}\left(2 - \frac{2}{5}\right)$$
$$= 4.\frac{8}{5}.2^{\frac{1}{4}} = \frac{32}{5}.2^{\frac{1}{4}}$$

174 Conceptual