20. DIFFERENTIATION

1. If $\sqrt{1-x^6}+\sqrt{1-y^6}=a\left(x^3-y^3\right)$, *then* $\frac{dy}{dx}=$

(2020)

1. $\sqrt{\frac{1-y^6}{1-x^6}}$
2. $x^2\sqrt{\frac{1-y^6}{1-x^6}}$
3. $\frac{x^2}{y^2}\sqrt{\frac{1-y^6}{1-x^6}}$
4. $\frac{1}{x^2}\sqrt{\frac{1-y^6}{1-x^6}}$

2. If $\sqrt{\frac{y}{x}}+4\sqrt{\frac{x}{y}}=4$, then $\frac{dy}{dx}=$

(2020)

1. xy
2. $\frac{x}{y}$
3. -4
4. 4

3. If $3f(x)-2f\left(\frac{1}{x}\right)=x$, then $f'(2)$ is

[AP EAMCET 17-09-20_Shift-1]

1. $\frac{7}{2}$
2. $\frac{1}{2}$
3. $\frac{2}{7}$
4. 2

4. If $\left(1-x+x^2\right)^{10}=a_0+a_1x+a_2x^2+.....+a_{20}x^{20}$

Then $2a_2+3a_3+4a_4+....+20a_{20}=$

[AP EAMCET 17-09-20_Shift-1]

1. 0
2. 10
3. 20
4. -20

5. If $y=\sin^{98}(x).\cos^{39}(x)$, find $\frac{dy}{dx}$

[AP EAMCET 17-09-20_Shift-1]

1. $\left(98\cos^{99}x.\sin^{38}x\right)+\left(39\sin^{40}x.\cos^{97}x\right)$
2. $\left(99\cos^{98}x.\sin^{39}x\right)-\left(40\sin^{39}x.\cos^{98}x\right)$
3. $\left(98\sin^{97}x.\cos^{40}x\right)-\left(39\sin^{99}x.\cos^{38}x\right)$
4. $\left(99\cos^{98}x.\sin^{39}x\right)-\left(39\sin^{40}x.\cos^{97}x\right)$

6. The derivative of

$\tan^{-1}\left[\frac{x}{1+\sqrt{1-x^2}}\right]$ w.r. to $\sec^{-1}\left(\frac{1}{2x^2-1}\right)$ is

[AP EAMCET 17-09-20_Shift-1]

1. $\frac{1}{2}$
2. $\frac{1}{4}$
3. $-\frac{1}{4}$
4. $-\frac{1}{2}$

7. If $x^{2019}.y^{2020}=(x+y)^{4039}$, *then* $\frac{dy}{dx}=$

[AP EAMCET 17-09-20_Shift-1]

1. 0
2. $\frac{x}{y}$
3. $\frac{y}{x}$
4. 1

8. Differentiation of $\left(x^2-5x+8\right)\times\left(x^3+7x+9\right)$ can be done by _____

[AP EAMCET 17-09-20_Shift-2]

1. Only by using product rule
2. Only by obtaining a single polynomial, expanding it
3. Only by using logarithmic differentiation
4. All of the options are correct

9. $\frac{d}{dx}\left(\text{Cos}^{-1}\left(\frac{4x^3}{27}-x\right)\right)=$

[AP EAMCET 17-09-20_Shift-2]

1. $\frac{3}{\sqrt{9-x^2}}$
2. $\frac{1}{\sqrt{9-x^2}}$
3. $\frac{-3}{\sqrt{9-x^2}}$
4. $\frac{-1}{\sqrt{9-x^2}}$

10. The function $y=\text{Sin}^{-1}(\cos x)$ is not differential at

[AP EAMCET 17-09-20_Shift-2]

1. π only
2. 2π only
3. -2π only
4. All options are correct.

11. If $\log(x+y)-2xy=0$ then $y'(0)=$

[AP EAMCET 17-09-20_Shift-2]

1. $2y^2+1$
2. $2y^2-1$
3. 0
4. $2y^2$

12. If $y=\sqrt{2x+\cos^2\left(2x+\frac{\pi}{4}\right)}$ then $\frac{dy}{dx}$ at $x=\frac{\pi}{4}$

[AP EAMCET 18-09-20_Shift-1]

1. $\frac{2\sqrt{2}}{\sqrt{\pi+1}}$
2. $2\sqrt{\pi+1}$
3. $\frac{2}{\sqrt{\pi+1}}$
4. $\frac{\sqrt{2}}{\sqrt{\pi+1}}$

13. If $f(x)=\sqrt{x-2\sqrt{2x-4}}+\sqrt{x-2\sqrt{2x-4}}$ then the value of $10\times f'(102)$

[AP EAMCET 18-09-20_Shift-1]

1. 1
2. 2
3. 102
4. -1

14. $\frac{d}{dx}\left(e^{\log_e\sqrt{1+\tan^2 x}}\right)=$

[AP EAMCET 18-09-20_Shift-1]

1. $\sec^2(x).\tan(x)$
2. $\sec(x).\tan^2(x)$
3. $\sec(x).\tan(x)$
4. $\tan^2(x)$

15. The derivative of $f(x)=\cos^{-1}\left[\sin\sqrt{\frac{1+x}{2}}\right]+x^x$ with respect to x at x=1 is equal to

[AP EAMCET 18-09-20_Shift-2]

1. 1
2. $\frac{1}{4}$
3. $\frac{3}{4}$
4. $\frac{2}{3}$

16. If $2f(\sin x)+f(\cos x)=x$ then $f'(x)$

[AP EAMCET 21-09-20_Shift-1]

1. $\frac{1}{\sqrt{1-x^2}}$
2. $\frac{-1}{\sqrt{1-x^2}}$
3. $\frac{x}{\sqrt{1-x^2}}$
4. $\frac{-x}{\sqrt{1-x^2}}$

17. If $y=\text{cosec}^{-1}(x)$ and $\frac{dy}{dx}=\frac{-1}{|x|\sqrt{x^2-1}}$, then

[AP EAMCET 21-09-20_Shift-1]

1. $y\in\left(-\frac{\pi}{2},0\right)$
2. $y\in\left(-\frac{\pi}{2},2\pi\right)$
3. $y\in\left(-\frac{\pi}{2},0\right)\cup\left(0,\frac{\pi}{2}\right)$
4. $y\in R$

18. The derivative of

$$y=\text{Tan}^{-1}\left[\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right]$$

with respect to x is equal to

[AP EAMCET 21-09-20_Shift-1]

1. -1
2. 0
3. ± 2
4. $\pm\frac{1}{2}$

19. If $x=e^{y+e^{y+e^{y+\cdots\cdots}}}$, then $\frac{dy}{dx}=$

[AP EAMCET 21-09-20_Shift-1]

1. $\frac{1-x}{x}$
2. $\frac{1}{x}$
3. $\frac{x}{1+x}$
4. $\frac{1+x}{x}$

20. The derivate of the $y=\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ is equal to **[AP EAMCET 21-09-20_Shift-2]**

1. $\frac{2}{(1+x^2)}$
2. $\frac{1}{2(1+x^2)}$
3. $(1+x^2)$
4. $2(1+x^2)$

21. If $f(x)=x^4-x^3+7x^2+14$, then what is the value of $f''(5)$? **[AP EAMCET 21-09-20_Shift-2]**

1. 842
2. 248
3. 284
4. 482

22. If $y=\sqrt{\frac{1+\tan x}{1-\tan x}}$, then $\frac{dy}{dx}=$

[AP EAMCET 21-09-20_Shift-2]

1. $\frac{1}{2}\left(\sqrt{\frac{1-\tan x}{1+\tan x}}\right)\sec^2\left(\frac{\pi}{4}+x\right)$
2. $\frac{1}{2}\left(\sqrt{\frac{1-\tan x}{1+\tan x}}\right)\sec\left(\frac{\pi}{4}+x\right)$
3. $\left(\sqrt{\frac{1-\tan x}{1+\tan x}}\right)\sec^2\left(\frac{\pi}{4}+x\right)$
4. $\frac{1}{2}\left(\sqrt{\frac{1+\tan x}{1-\tan x}}\right)\sec^2\left(\frac{\pi}{4}+x\right)$

23. Find $\frac{dy}{dx}$ *if* $2x^2-3xy+y^2+x+2y-8=0$

[AP EAMCET 21-09-20_Shift-2]

1. $\frac{3y-4x-1}{2y-3x+2}$
2. $\frac{3y+4x-1}{2y-3x+2}$
3. $\frac{3y-4x-1}{2y+3x+2}$
4. $\frac{3y-4x-1}{2y-3x-2}$

24. If $y=\frac{e^x\log x}{x^2}$, then $\frac{dy}{dx}=$

[AP EAMCET 22-09-20_Shift-1]

1. $\frac{e^x\{1+(x+2)\log x\}}{x^3}$
2. $\frac{e^x\{1-(x-2)\log x\}}{x^4}$
3. $\frac{e^x\{1-(x-2)\log x\}}{x^3}$
4. $\frac{e^x\{1+(x-2)\log x\}}{x^3}$

25. If $\text{Cos}^{-1}\left(\frac{x^2-y^2}{x^2+y^2}\right)=\text{Sin}^{-1}(a)$ then $\frac{dy}{dx}$ is equal to

[AP EAMCET 22-09-20_Shift-1]

1. y/x
2. $-y/x$
3. x/y
4. $-x/y$

26. The derivative of $y=\text{Sin}^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{2}\right)$ is

[AP EAMCET 22-09-20_Shift-2]

1. $\frac{1}{\sqrt{1+x^2}}$
2. $\frac{1}{\sqrt{1-x^2}}$
3. $\frac{1}{2\sqrt{1+x^2}}$
4. $\frac{1}{2\sqrt{1-x^2}}$

27. If $y=\log(\cosh x)$ then $\frac{d^2y}{dx^2}=$

[AP EAMCET 22-09-20_Shift-2]

1. $\text{sech}^2 x$
2. $-\text{sech}^2 x$
3. $\sinh x$
4. $-\sinh x$

28. Express $\frac{dt}{dx}=\frac{t}{\left(x+te^{-2x/t}\right)}$ in the form of $\frac{dx}{dt}=\phi\left(\frac{x}{t}\right)$

[AP EAMCET 22-09-20_Shift-2]

1. $\frac{x}{t}+e^{-2\left(\frac{x}{t}\right)}$
2. $\frac{x}{t}-e^{-2\left(\frac{x}{t}\right)}$
3. $\frac{x}{t}+e^{2\left(\frac{x}{t}\right)}$
4. $\frac{x}{t}-e^{2\left(\frac{x}{t}\right)}$

29. $\frac{d}{dx}\{(1+x^2)\tan^{-1}x\}=$

[AP EAMCET 22-09-20_Shift-2]

1. $x\tan^{-1}x$
2. $2\tan^{-1}x$
3. $2x\tan^{-1}x+1$
4. $x\tan^{-1}x+1$

30. If $f(x)=\log_{x^2}(\log x)$, then $f'(e)$ is equal to

[AP EAMCET 23-09-20_Shift-1]

1. e
2. $\frac{1}{e}$
3. $\frac{1}{2e}$
4. $2e$

31. If $\frac{d}{dx}\left(\frac{x^4+x^2+1}{x^2+x+1}\right)=ax+b$, then a – b=

[AP EAMCET 23-09-20_Shift-1]

1. 3
2. 4
3. 1
4. 2

32. If $y=\text{Tan}^{-1}\left(\frac{4x}{1+5x^2}\right)+\text{Tan}^{-1}\left(\frac{2+3x}{3-2x}\right)$, then $\frac{dy}{dx}=$ **[AP EAMCET 23-09-20_Shift-1]**

1. $\frac{1}{1+25x^2}+\frac{2}{1+x^2}$
2. $\frac{5}{1+25x^2}+\frac{2}{1+x^2}$
3. $\frac{5}{1+25x^2}$
4. $\frac{1}{1+25x^2}$

33. *If* $f(x)=3e^{x^2}$ *then* $f'(x)-2xf(x)+\frac{1}{3}f(0)-f'(0)=$

1. 0
2. 1
3. $\frac{7}{3}e^{x^2}$
4. $12xe^{x^2}+1$

34. *If* $y=\sin^{-1}\left(\frac{4+5\sin x}{5+4\sin x}\right)$ *then find* $\frac{dy}{dx}$

1. $\frac{3}{5+4\sin x}$ *only*
2. $\frac{-3}{5+4\sin x}$ *only*
3. $\frac{\pm 3}{5+4\sin x}$
4. $\frac{3}{4+5\sin x}$ *only*

35. $\frac{d}{dx}\left(\sin^{-1}\frac{3+4x}{5\sqrt{1+x^2}}\right)=$

1. $\frac{1}{1+x^2}$
2. $\frac{-1}{1+x^2}$
3. $\frac{1}{\sqrt{1+x^2}}$
4. $\frac{-1}{\sqrt{1+x^2}}$

36. For $x\neq -1, y\neq -1$ if $x=\frac{1-\sqrt[3]{y}}{1+\sqrt[3]{y}}$ then $\frac{dy}{dx}=$ **[TS EAMCET 09-09-20_Shift-1]**

1. $\frac{-6(1-x)^2}{(1+x)^4}$
2. $\frac{-(1+x)^4}{6(1-x)^2}$
3. $\frac{4(1-x)^4}{(1+x)^6}$
4. $\frac{-6(1+x)^2}{(1-x)^4}$

37. Assertion (A):For $x<0$, $\frac{d^2}{dx^2}(\log|x|)=\frac{1}{|x|^2}$

Reason(R): For $x<0$, $|x|=-x$ **[TS EAMCET 09-09-20_Shift-1]**

1. (A) is true, (R) is true and (R) is the correct explanation to (A)
2. (A)is true,(R)is true but (R) is not a correct explanation to (A)
3. (A)is true,(R)is false
4. (A)is false,(R)is true

38. If $ax^2+2hxy+by^2=0, then \frac{d^2y}{dx^2}=$ **[TS EAMCET 09-09-20_Shift-1]**

1. $\frac{h^2-ab}{(hx+by)^3}$
2. $\frac{2(h^2-ab)}{(hx+by)^3}$
3. $\frac{(hx+by)^3}{h^2-ab}$
4. 0

39. If $f(x)=\frac{(x+1)\sinh x}{e^{2x}\tan x}$ *and* $\frac{f'(x)}{f(x)}=\frac{1}{x+1}+\coth x+g(x)$, then $g(x)=$ **[TS EAMCET 09-09-20_Shift-2]**

1. $-2+\frac{1}{\sin x\cos x}$
2. $2-2\,\text{cosec}\,2x$
3. $-2(1+\text{cosec}\,2x)$
4. $2-\frac{1}{\sin x\cos x}$

40. If $x^y=y^{\sin x}(\tan x)^{\cos x}$, *then* $\left(\log x-\frac{\sin x}{y}\right)\frac{dy}{dx}=$ **[TS EAMCET 09-09-20_Shift-2]**

1. $\cos x\log y-\sin x\log(\tan x)+\text{cosec}\,x-\frac{y}{x}$
2. $\cos x\log y-\sin x\log(\tan x)+\cos^2 x\,\text{cosec}\,x-\frac{y}{x}$
3. $\frac{\cos x}{x}-\sin^2 x\sec x$
4. $\cos x-x\sin^2 x\sec x$

41. If $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$,

$then\left[\left(1-x^2\right)^2\frac{d^2y}{dx^2}+y\left(1-x^2\right)\right]\frac{dy}{dx}=$

[TS EAMCET 09-09-20_Shift-2]

1. 0
2. $x\left(1-y^2\right)$
3. $y\left(1-x^2\right)$
4. $\sqrt{1-x^2}\sqrt{1-y^2}$

42. Let $f:[2,5]\to R$ be a differentiable function and $\frac{f(5)}{f(2)}=1$ .If there is a $c\in(2,5)$ such that $cf'(c)=2f(c)-2c^3$ then $f(x)=$

[TS EAMCET 10-09-20_Shift-2]

1. $-2x^3+\frac{78}{7}x^2$
2. $x^3-8x^2+17x-10=0$
3. $x^3-6x^2+3x+10=0$
4. x^3-7x^2+10x

43. Match the functions of List -I with their derivatives in List-II

List - I	List - II
A) $\mathrm{Sec}^{-1}x$	I) $\frac{1}{1-x^2},x\in(-1,1)$
B) $\mathrm{Tanh}^{-1}x$	II) $\frac{-1}{\lvert x\rvert\sqrt{x^2+1}},x\neq0$
C) $\mathrm{Coth}^{-1}x$	III) $\frac{1}{\lvert x\rvert\sqrt{x^2-1}},\lvert x\rvert>1$
D) $\mathrm{Cosech}^{-1}x$	IV) $\frac{1}{1-x^2},x\in R-[-1,1]$
	V) $\frac{-1}{\lvert x\rvert\sqrt{1-x^2}},\lvert x\rvert<1,x\neq0$

The correct match is

[TS EAMCET 11-09-20_Shift-2]

1. $A\to V,\ B\to II,\ C\to I,\ D\to III$
2. $A\to I,\ B\to III,\ C\to V,\ D\to II$
3. $A\to III,\ B\to I,\ C\to II,\ D\to V$
4. $A\to III,\ B\to I,\ C\to IV,\ D\to II$

44. If $f(x)=\frac{x-1}{e^x}$, then $f'(0)+f''(0)=$

[TS EAMCET 11-09-20_Shift-2]

1. 0
2. 1
3. -1
4. 2

45. If $\left(\frac{dy}{dx}\right)=\frac{1}{\left(\frac{dx}{dy}\right)}$ and $\frac{d^2x}{dy^2}\left(\frac{dy}{dx}\right)^3+\frac{d^2y}{dx^2}=k$, then $e^{kf(x)}-kf(x)=$

[TS EAMCET 11-09-20_Shift-2]

1. 1
2. 0
3. $\frac{1}{2}$
4. 2

46. If $\log\sqrt{x^2+y^2}=\tan^{-1}\left(\frac{x}{y}\right)$, then $\frac{dy}{dx}$ is equal to **[AP EAMCET 22-09-20_Shift-2]**

1. $\frac{y-x}{y+x}$
2. $\frac{x+y}{x-y}$
3. $\frac{1}{y+x}$
4. $\frac{1}{x-y}$

47. If p(x) be a polynomial satisfying

$p(2x)=p'(x).p''(x),\text{then}\sum_{x=1}^{5}p(x)=$

[TS EAMCET 11-09-20_Shift-1]

1. 200
2. 100
3. 50
4. 450

48. If $2^x+2^y=2^{x+y}$,then $\frac{dy}{dx}$

[TS EAMCET 11-09-20_Shift-1]

1. $1-2^y$
2. $1-2^{-y}$
3. $1+2^y$
4. $1+2^{-y}$

49. If $x\sqrt{y+1}+y\sqrt{1+x}=0,\text{then}\frac{dy}{dx}=$

[TS EAMCET 11-09-20_Shift-1]

1. $\frac{-1}{(1+x)^2}$
2. $\frac{1}{(1+x)^2}$
3. $\frac{2}{(1+x)^{\frac{3}{2}}}$
4. $\frac{-2}{(1+x)^{\frac{1}{2}}}$

50. If $y=x+\frac{1}{x}$, then which among the following holds? **[AP EAMCET 19-08-2021_Shift-1]**

1. $x^2y'+xy=0$
2. $x^2y'+xy+2=0$
3. $x^2y'-xy+2=0$
4. $x^2y'+xy-2=0$

51. If $y=\tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right)$, where $x^2\le 1$. then find $\frac{dy}{dx}$

[AP EAMCET 19-08-2021_Shift-1]

1. $\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\left(x^2\right)$
2. $\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\left(x^2\right)$
3. $\frac{-x}{\sqrt{1-x^4}}$
4. $\frac{-2x}{\sqrt{1-x^4}}$

52. If $3\sin xy+4\cos xy=5$, then $\frac{dy}{dx}$ is equal to

[AP EAMCET 19-08-2021_Shift-1]

1. $\frac{3\sin xy+4\cos xy}{3\cos xy-5\sin xy}$
2. $\frac{3\cos xy+4\sin xy}{4\cos xy-3\sin xy}$
3. $\frac{-y}{x}$
4. $\frac{x}{y}$

53. $f(x)=\sqrt{x^2+1}: g(x)=\frac{x+1}{x^2+1}: h(x)=2x-3\cdot$

Then the value of $f'\left[h'\left(g'(x)\right)\right]=$

[AP EAMCET 19-08-2021_Shift-1]

1. $\sqrt{5}$
2. $\frac{2}{\sqrt{5}}$
3. $\frac{\sqrt{5}}{2}$
4. $\frac{1}{\sqrt{5}}$

54. If $\log\left(\sqrt{1+x^2}-x\right)=y\left(\sqrt{1+x^2}\right)$, then

$\left(1+x^2\right)\frac{dy}{dx}+xy=$

[AP EAMCET 19-08-2021_Shift-2]

1. 0
2. 1
3. 2
4. -1

55. If $y=e^{x^2+e^{x^2}+e^{x^2}\cdots}$ then $\frac{dy}{dx}=$

[AP EAMCET 19-08-2021_Shift-2]

1. $\frac{2x}{1-y}$
2. $\frac{2xy}{y-1}$
3. $\frac{2xy}{1-y}$
4. $\frac{2y}{y-1}$

56. $\frac{d}{dx}\left(\mathrm{Tan}^{-1}\left(\frac{\cos x}{1+\sin x}\right)\right)=$

[AP EAMCET 19-08-2021_Shift-2]

1. $\frac{1}{2}$
2. $\frac{-1}{2}$
3. 1
4. -1

57. If $f(x)=2x^2+3x-5$, then the value of $f'(0)+3f'(-1)$ is equal to _____

[AP EAMCET 20-08-2021_Shift-1]

1. 1
2. 0
3. 3
4. 2

58. If $y=\left(1+\frac{1}{x}\right)\left(1+\frac{2}{x}\right)+\ldots.\left(1+\frac{n}{x}\right)$ and $x\ne 0$.

When $x=-1$, $\frac{dy}{dx}=$ _____

[AP EAMCET 20-08-2021_Shift-1]

1. $n!$
2. $(n-1)!$
3. $(-1)^n(n-1)!$
4. $(-1)^n n!$

59. $\frac{d}{dx}\left\{\sin^2\left(\cot^{-1}\sqrt{\frac{1+x}{1-x}}\right)\right\}=$

[AP EAMCET 20-08-2021_Shift-1]

1. 0
2. $\frac{1}{2}$
3. $-\frac{1}{2}$
4. -1

60. If $y = \text{Tan}^{-1}\left\{\frac{ax-b}{bx+a}\right\}$, then $y' =$

[AP EAMCET 20-08-2021_Shift-1]

1. $\frac{1}{1+x^2}+\frac{a^2}{a^2+b^2}$
2. $\frac{1}{1+x^2}$
3. $\frac{1}{1+\left(\frac{ax-b}{bx+a}\right)^2}$
4. $\frac{bx+a}{1+(ax-b)^2}$

61. If $x = \sec\theta - \cos\theta$ & $y = \sec^n\theta - \cos^n\theta$, then $\left(x^2+4\right)\left(\frac{dy}{dx}\right)^2 =$____

[AP EAMCET 20-08-2021_Shift-2]

1. $n(y+4)$
2. $n^2\left(y^2+4\right)$
3. $n(y+2)$
4. $n^2\left(y^2+2\right)$

62. If $f(x)=\begin{vmatrix} x & x^2 & x^3 \\ 1 & 2x & 3x^2 \\ 0 & 2 & 6x \end{vmatrix}$, then the ratio

$f''(x) : f'(x) =$____

[AP EAMCET 20-08-2021_Shift-2]

1. 2:x
2. $x^2 : x$
3. 3x:2
4. 6:x

63. If $y = \log_{\cot x}\tan x - \log_{\tan x}\cot x + \tan^{-1}\left(\frac{4x}{4-x^2}\right)$, then $\frac{dy}{dx} =$

[AP EAMCET 20-08-2021_Shift-2]

1. $\frac{1}{4+x^2}$
2. $\frac{4}{4+x^2}$
3. $\frac{1}{4-x^2}$
4. $\frac{4}{4-x^2}$

64. If $y = \sin(\sin x)$ & $y''+f(x).y'+g(x).y=0$, then $f(x).g(x) =$

[AP EAMCET 20-08-2021_Shift-2]

1. $\frac{1}{2}\sin 2x$
2. $\frac{1}{2}\cos 2x$
3. $\sin 2x$
4. $\cos 2x$

65. If $y = \tan^{-1}\left(\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right)$, then $\frac{dy}{dx} =$____

[AP EAMCET 23-08-2021_Shift-1]

1. 0
2. a/b
3. -1
4. 2

66. If $8f(x)+6f\left(\frac{1}{x}\right) = x+5$ and $y = x^2 f(x)$, then $\frac{dy}{dx}$ at x = -1 equals

[AP EAMCET 23-08-2021_Shift-1]

1. 0
2. $\frac{1}{14}$
3. $\frac{-1}{14}$
4. 1

67. If $y = \log\left(\frac{1+x}{1-x}\right)^{1/4} - \frac{1}{2}\tan^{-1}(x)$, then $\frac{dy}{dx}$ at $x=\frac{1}{\sqrt{2}}$ equals

[AP EAMCET 23-08-2021_Shift-1]

1. $\frac{-4}{3}$
2. $\frac{4}{3}$
3. $\frac{-2}{3}$
4. $\frac{2}{3}$

68. If $f(x) = \sin x.\sin 2x.\sin 3x$ and $f''(x) = a(\sin bx)+c(\sin dx)+e(\sin kx)$, then the value of $(a+c+e)-(b+d+k)$ equals__

[AP EAMCET 23-08-2021_Shift-1]

1. 8
2. -8
3. 16
4. 12

69. If $y = \log_y x$, then $\frac{dy}{dx} =$____

[AP EAMCET 23-08-2021_Shift-2]

1. $\frac{1}{x\log y}$
2. $\frac{\log y}{x(1+\log y)}$
3. $\frac{1}{x(1+\log y)}$
4. $\frac{1}{1+\log y}$

70. If $x = a\left[\cos\theta + \log\left(\tan\left(\theta/2\right)\right)\right]$ and $y = a\sin\theta$, then $\frac{dy}{dx} =$ _____

[AP EAMCET 23-08-2021_Shift-2]

1. $\cot\theta$
2. $\tan\theta$
3. $\sin\theta$
4. $\cos\theta$

71. If $y = 4\cos^3(t)$ and $x = 4\sin^3(t)$, then $\frac{dy}{dx} =$

[AP EAMCET 23-08-2021_Shift-2]

1. $-\tan(t)$
2. $\tan(t)$
3. $-\cot(t)$
4. $\cot(t)$

72. If $f(x) = \log_{x^2}(\log x)$, then $f'(x)$ at $x = e$ is _____

[AP EAMCET 23-08-2021_Shift-2]

1. 0
2. 1
3. e^{-1}
4. $(2e)^{-1}$

73. $\frac{d}{dx}\left\{\frac{2}{\sqrt{2+\sqrt{2+\sqrt{2+\cos(4x)}}}}\right\} = k\sec\left(\frac{x}{2}\right)\tan\left(\frac{x}{2}\right)$

[AP EAMCET 24-08-2021_Shift-1]

1. $\frac{1}{2}$
2. 2
3. 1
4. $\frac{1}{8}$

74. If $2f(x) - 3f\left(\frac{1}{x}\right) = x+1$, then $f'\left(\sqrt{3}\right) =$

[AP EAMCET 24-08-2021_Shift-1]

1. $\frac{-1}{5}$
2. $\frac{3}{5}$
3. $\frac{-\sqrt{3}}{5}$
4. $\frac{\sqrt{3}}{5}$

75. A function $F: R \to R$ satisfies the relation $f(x+y) = f(x).f(y),\ \forall x, y \in R$ and $f(x) \neq 0, \forall x \in R$. If '$f$' is differentiable at $x = 0$, $f'(0) = 4$ and $f(6) = 3$, then $f'(6) =$

[AP EAMCET 24-08-2021_Shift-1]

1. 0
2. 12
3. 3
4. 6

76. If $f'(x) = \sqrt{2x^2 - 1}$ and $y = f\left(x^3\right)$, then find the value of $\frac{dy}{dx}$ at $x = 1$

[AP EAMCET 24-08-2021_Shift-1]

1. -1
2. 3
3. 0
4. -3

77. If $y = \tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}$, then the values of $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ respectively are _____

[AP EAMCET 24-08-2021_Shift-2]

1. 1,0
2. $\frac{x}{2}, \frac{1}{2}$
3. $\frac{1}{2}, 0$
4. $-\frac{1}{2}, 0$

78. If $y = (\tan x)^{\sin x}$, then $\frac{dy}{dx} =$

[AP EAMCET 24-08-2021_Shift-2]

1. $(\tan x)^{\sin x}\left\{\sec x + (\cos x)(\log(\tan x))\right\}$
2. $(\sin x)^{\tan x}\left\{\sec x + (\cos x)(\log(\tan x))\right\}$
3. $(\tan x)^{\sin x}\left\{\sec x - (\cos x)(\log(\tan x))\right\}$
4. $(\sin x)^{\tan x}\left\{\sec x - (\cos x)(\log(\tan x))\right\}$

79. If $y = \cos^{-1}\left\{\frac{a\cos x - b\sin x}{\sqrt{a^2+b^2}}\right\}$, then $\frac{d^2y}{dx^2} =$

[AP EAMCET 24-08-2021_Shift-2]

1. $a-b$
2. $a+b$
3. 1
4. 0

80. The derivative of $\sec^{-1}\left(\frac{1}{2x^2-1}\right)$ with respect to $\sqrt{1-x^2}$ at $x = \frac{1}{2}$ equals _____

[AP EAMCET 24-08-2021_Shift-2]

1. 2
2. $\frac{1}{2}$
3. $\frac{1}{4}$
4. 4

81. If $y=\cos(x^\circ), z=\cos x$ then $\frac{dy}{dx}=$

[AP EAMCET 25-08-2021_Shift-1]

1. $\frac{-\pi}{180}\sin(x^\circ)\cos ec\, x$
2. $\sin(x^\circ)\cos ec\, x$
3. $\frac{\pi}{180}\sin(x^\circ)\cos ec\, x$
4. $\frac{\pi}{180}\cos(x^\circ)\cos x$

82. If $y=\cos^{-1}(\cos x)$ then find $\frac{dy}{dx}$ at $x=\frac{5\pi}{4}$

[AP EAMCET 25-08-2021_Shift-1]

1. 1
2. –1
3. 0
4. $\frac{1}{\sqrt{2}}$

83. If $y=\sin^{-1}\left[x\sqrt{1-x}-\sqrt{x}.\sqrt{1-x^2}\right]$ and

$0<x<1$, then $\frac{dy}{dx}$ is equal to

[AP EAMCET 25-08-2021_Shift-1]

1. $\frac{1}{2\sqrt{1-x^2}}-\frac{1}{x\sqrt{1-x^2}}$
2. $\frac{1}{\sqrt{1-x^2}}-\frac{1}{2\sqrt{x-x^2}}$
3. $\frac{1}{2\sqrt{x-x^2}}+\frac{1}{\sqrt{1-x^2}}$
4. $\frac{-1}{\sqrt{1-x^2}}-\frac{1}{x\sqrt{1-x^2}}$

84. If $y=\sqrt{x+\sqrt{x+\sqrt{x+.....\infty}}}$, then $\frac{dy}{dx}=$

[AP EAMCET 25-08-2021_Shift-1]

1. $\frac{1}{y}$
2. $\frac{1}{x}$
3. $\frac{1}{2x-1}$
4. $\frac{1}{2y-1}$

85. If $p(x)$ is a polynomial of degree 3 whichsatisfies

$p''(1)=0$ and $p'''(1)=6$, then $p''(0)=$_____

[AP EAMCET 25-08-2021_Shift-2]

1. 6
2. –6
3. 0
4. 3

86. If $3f(x)-2f\left(\frac{1}{x}\right)=x$, then $f'(2)=$_____

[AP EAMCET 25-08-2021_Shift-2]

1. $\frac{1}{2}$
2. $\frac{-1}{2}$
3. 1
4. –1

87. If $\frac{d}{dx}\left(\frac{x2^x-x}{1-\cos x}\right)=\left(\frac{x2^x-x}{1-\cos x}\right)(f(x)+\log 2)$,

Then $f(x)$

[TS EAMCET 04-08-2021_Shift-2]

1. $\frac{1}{x}+\frac{\log 2}{2^x}+\tan\frac{x}{2}$
2. $\frac{1}{x}+\frac{2^x\log 2}{2^x-1}+\frac{\sin x}{1-\cos x}$
3. $x+2^x-1+\sin x(1-\cos x)$
4. $\frac{1}{x}+\frac{\log 2}{2^x-1}+\cot\frac{x}{2}$

88. If $(a+bx)e^{\frac{y}{x}}=x$, then $\frac{d^2y}{dx^2}=$

[TS EAMCET 04-08-2021_Shift-2]

1. $\frac{1}{x^3}(xy^1+y)^2$
2. $\frac{1}{x^3}(xy'+y^2)$
3. $\frac{1}{x^3}(xy'-y)$
4. $\frac{1}{x^3}(xy'-y)^2$

89. If $f(x)=\frac{\cos^2 x}{1+\sin^2 x}$, then $f\left(\frac{\pi}{4}\right)-3f'\left(\frac{\pi}{4}\right)=$

[TS EAMCET 04-08-2021_Shift-2]

1. $\frac{5}{3}$
2. $\frac{11}{3}$
3. $\frac{13}{9}$
4. 3

90. If $x=5(1-\sin t), y=5(t+\cos t)$, then $\frac{dx}{dy}=$

[TS EAMCET 04-08-2021_Shift-1]

1. $\frac{\sin t-1}{\cos t}$
2. $\frac{\cos t}{\sin t-1}$
3. $\tan\frac{t}{2}$
4. $\frac{\cos\frac{t}{2}-\sin\frac{t}{2}}{\cos\frac{t}{2}+\sin\frac{t}{2}}$

91. Let $g:[-2,2]\to R$ and $f:[-2,2]\to R$ are two functions defined as

$$g(x)=\begin{cases}-1, & if\ -2\le x<0\\ x^2-1, & if\ 0\le x\le 2\end{cases}$$

and $f(x)=|g(x)|+g(|x|)+2.$

In the interval $(-2,2)$, f is not differentiable at $x=$

[TS EAMCET 04-08-2021_Shift-1]

1. 0
2. 1
3. $\frac{1}{2}$
4. -1

92. Consider the following statements.

a) If a function is differentiable at a point 'p' then it is not continuous at 'p'

b) If a function is not continuous at $x=a$, then it is not differentiable at $x=a$

c) If $f(x)=|x|$ then $f(x)$ is not differentiable but continuous on R

d) If $f(x)=x-[x]$, then $f'(1)=1$

Which of the above statement are (is) correct?

[TS EAMCET 04-08-2021_Shift-1]

1. Only(b)
2. (b) and (c)
3. Only(c)
4. (c) and (d)

93. If $x^2+xy+y^2=k$, then $\frac{d^2y}{dx^2}=$

[TS EAMCET 04-08-2021_Shift-1]

1. $\frac{-6k}{(x+2y)^3}$
2. $\frac{-6k}{(x+2y)^2}$
3. $\frac{x^2+xy+y^2}{(2x+y)^2}$
4. 0

94. If $xe^{xy}=y+\sin^2 x$, then $\frac{dy}{dx}$ at $x=0$ is

[TS EAMCET 05-08-2021_Shift-1]

1. 0
2. -1
3. -2
4. 1

95. If $f(x)=x^2\sin\frac{1}{x}$, when $x\neq 0$ and $f(0)=0$, then $\lim_{x\to 0} f'(x)$

[TS EAMCET 05-08-2021_Shift-1]

1. does not exist
2. 0
3. ∞
4. 1

96. The derivative of $(\log x)^{\sin x}$ with respect to $\cos x$ at $x=\frac{\pi}{2}$ is

[TS EAMCET 05-08-2021_Shift-1]

1. $\frac{-4}{\pi}$
2. $\frac{-\pi}{2}$
3. $\frac{-2}{\pi}$
4. $\frac{-\pi}{4}$

97. Let $f(x)$ be differentiable function for all $x\in R$ and $f(x+y)=f(x)+f(y)-3xy.$

If $\lim_{h\to 0}\frac{f(h)}{h}=7$, then $f'(x)=$

[TS EAMCET 05-08-2021_Shift-1]

1. $-3x+7$
2. $3x-7$
3. $3x+7$
4. $-7-3x$

98. If $x=a(t-\sin t)$ and $y=a(1-\cos t)$ then

$\frac{d^2y}{dx^2}=$ **[TS EAMCET 05-08-2021_Shift-2]**

1. $\frac{1}{4a\sin^4\left(\frac{t}{2}\right)}$
2. $\frac{-1}{4a\sin^4\left(\frac{t}{2}\right)}$
3. $\frac{1}{4a\cos^4\left(\frac{t}{2}\right)}$
4. $\frac{-1}{4a\cos^4\left(\frac{t}{2}\right)}$

99. If $y=\cos^{-1}(\tanh x)+\sinh(\sin 6x)$, then $\frac{dy}{dx}=$

[TS EAMCET 05-08-2021_Shift-2]

1. $\frac{-1}{\cosh x}+6\cos 6x\cosh(\sin 6x)$
2. $\frac{1}{\cosh x}-6\cos 6x\cosh(\sin 6x)$
3. $\frac{-1}{\cosh x}-6\cos 6x\cosh(\sin 6x)$
4. $\frac{1}{\cosh x}+6\cos 6x\cosh(\sin 6x)$

100. If $y=x^{\sqrt{x}}$, then $\frac{dy}{dx}=$

[TS EAMCET 05-08-2021_Shift-2]

1. $\frac{\ln x}{2\sqrt{2}}$
2. $\frac{x^{\sqrt{x}}}{\sqrt{x}}$
3. $\frac{y\ln x}{2\sqrt{x}}$
4. $\frac{y(\ln x+2)}{2\sqrt{x}}$

101. Suppose $f:R\to R$ defined as

$f(x)=\begin{cases}[\cos\pi x], x\le 1\\ 2\{x\}-1, x>1\end{cases}$, where [.] and {.}

denote the greatest integer function and the fractional part of x respectively, then at $x=1$

[TS EAMCET 06-08-2021_Shift-2]

1. Right derivative is 2
2. Left derivative is 2
3. Right derivative is 0
4. Left derivative is -1

102. If $y=\tan\left(\text{Cos}^{-1}x\right)$, then $\frac{dy}{dx}=$

[TS EAMCET 06-08-2021_Shift-2]

1. $\frac{-x^2}{\sqrt{1-x^2}}$
2. $\frac{-1}{\sqrt{1-x^2}}$
3. $\frac{-1}{x\sqrt{1-x^2}}$
4. $\frac{-1}{x^2\sqrt{1-x^2}}$

103. If $y=\frac{1}{x}+\cos 2x$, then $\frac{d^2y}{dx^2}=$

[TS EAMCET 06-08-2021_Shift-2]

1. $\frac{2}{x^3}+4y-\frac{4}{x}$
2. $4y-\frac{4}{x}-\frac{2}{x^3}$
3. $\frac{2}{x^3}+\frac{4}{x}-4y$
4. $4y-\frac{4}{x}-\frac{1}{x^3}$

104. $\frac{d}{dt}\left(\tan t+t^2\cos echt\right)$

[TS EAMCET 06-08-2021_Shift-2]

1. $\sec^2 t+2t\coth t-t^2\cos echt\coth t$
2. $\sec^2 t+2t\cos echt-t^2\cos echt\coth t$
3. $\sec t+2t\cot ht-t^2\cos echt\coth t$
4. $\sec^2 t+2t\cos echt+t^2\cos echt\coth t$

105. If $f(x)=\begin{vmatrix}x^3+x & x+1 & x-2\\ 2x^3+3x-1 & 3x & 3x-3\\ x^3+2x+3 & 2x-1 & 2x-1\end{vmatrix}$, then

$\frac{d}{dx}(f(x))=$ **[TS EAMCET 06-08-2021_Shift-1]**

1. 24
2. 0
3. -6
4. 12

106. Let f and g be two differentiable functions satisfying $g'(5)=\frac{3}{4}, g(5)=6$ and $g=f^{-1}$.

Then $f'(6)=$

[TS EAMCET 06-08-2021_Shift-1]

1. $\frac{1}{2}$
2. $\frac{1}{6}$
3. $\frac{2}{3}$
4. $\frac{4}{3}$

107. If $y = x + \tan x$, then $\cos^2 x \frac{d^2y}{dx^2} + 2x =$

[TS EAMCET 06-08-2021_Shift-1]

1. $-2y$
2. $\frac{2}{3}y$
3. $3y$
4. $2y$

108. If $f(x) = \cot^{-1}\left(\frac{x^x + x^{-x}}{2}\right)$, *then* $f'(1) =$

[AP EAMCET 04-07-2022_Shift-1]

1. 1
2. –1
3. 2
4. –2

109. If $x^3 - 2x^2y^2 + 5x + y - 5 = 0$, *then at* $(1,1)$ $y''(1) =$

[AP EAMCET 04-07-2022_Shift-1]

1. $\frac{-197}{27}$
2. $\frac{125}{31}$
3. 12
4. $\frac{-238}{27}$

110. If $x \neq 0$ and f(x) satisfies 8

f(x)+6f(1/x)=x+5, then $\frac{d}{dx}\left(x^2 f(x)\right)$ at x=1 is

[AP EAMCET 04-07-2022_Shift-2]

1. -1/14
2. 25/14
3. 9/14
4. 19/14

111. $\frac{d}{dx}\left(\lim_{y\to 2} \frac{1}{y-2}\left(\frac{1}{x} - \frac{1}{x+y-2}\right)\right) =$

[AP EAMCET 04-07-2022_Shift-2]

1. $1/x^2$
2. $2/x^3$
3. $-2/x^3$
4. $1/x^3$

112. If $3f(\cos x) + 2f(\sin x) = 5x$, then

$f'(\cos x) + f'(\sin x) =$

[AP EAMCET 05-07-2022_Shift-1]

1. $-5(\sin x + \cos x)$
2. $-5 \sin x \cos x$
3. $\frac{-5}{\sin x} - \frac{5}{\cos x}$
4. $\frac{5}{\sin x} + \frac{5}{\cos x}$

113. **Assertion (A) :**

$$\frac{d}{dx}\left(\frac{x^2 \sin x}{\log x}\right) = \frac{x^2 \sin x}{\log x}\left(\cot x + \frac{2}{x} - \frac{1}{x \log x}\right)$$

Reason (R) : $\frac{d}{dx}\left(\frac{uv}{w}\right) = \frac{uv}{w}\left[\frac{u'}{u} + \frac{v'}{v} - \frac{w'}{w}\right]$

[AP EAMCET 05-07-2022_Shift-1]

1. A is true, R is true and R is correct explanation of A
2. A is true, R is true and R is not correct explanation of A
3. A is true, R is not correct
4. A is not correct, R is correct

114. If $x = f(\theta)$ and $y = g(\theta)$, then $\frac{d^3y}{dx^2} =$

[AP EAMCET 05-07-2022_Shift-1]

1. $g''(\theta) / f'(\theta)$
2. $g''(\theta) / x(\theta)$
3. $\frac{f'(\theta)g''(\theta) - g'(\theta)f''(\theta)}{(f'(\theta))^3}$
4. $\frac{g'(\theta)f''(\theta) - g''(\theta)f''(\theta)}{(g^{-}(\theta))^{\cdot}}$

115. If $\frac{d^n y}{dx^n} = y_n$ *and* $y = e^{\sqrt{x}} + e^{-\sqrt{x}}$, *then* $4xy_2 + 2y_1 =$

[AP EAMCET 05-07-2022_Shift-2]

1. $-y$
2. y
3. $2y$
4. $-2y$

116. If $\frac{y}{x}\cos^4\alpha + \frac{x}{y}\sin^4\alpha = 2\sin^2\alpha, \cos^2\alpha$, *then* $\frac{dy}{dx} =$

[AP EAMCET 06-07-2022_Shift-1]

1. $\sin^3\alpha \cos\alpha$
2. $\sin^2\alpha \cos^2\alpha$
3. $\frac{\sin^2\alpha}{\cos^2\alpha}$
4. $\sin\alpha \cos^3\alpha$

117. If $\frac{d}{dx}\left(A \log\left(\frac{\sqrt{1-x^3} + B}{\sqrt{1-x^3} + 1}\right)\right) = \frac{1}{x\sqrt{1-x^3}}$,

then AB =

[AP EAMCET 06-07-2022_Shift-1]

1. $1/3$
2. $-1/3$
3. $-2/3$
4. $2/3$

118. *If* $x\sin(\alpha+y)=\sin y$ *and* $y=\dfrac{m}{x^2+2nx+1}$,

then $m^2=$ **[AP EAMCET 06-07-2022_Shift-2]**

1. $1-n^2$
2. $1+n$
3. $1-n$
4. n^2-1

119. $[\cdot]$ represents the greatest integer function

$At\ x=-1, \dfrac{d}{dx}\sin\pi[x]=$

[AP EAMCET 06-07-2022_Shift-2]

1. 0
2. 2
3. -2
4. 1/2

120. $\dfrac{d}{dx}\dfrac{(x+1)^2\sqrt{x-1}}{(x+4)^3e^x}$

$=f(x)\left[\dfrac{2}{x+1}+\dfrac{1}{2(x-1)}-\dfrac{3}{x+4}-1\right]$, *then* $f(5)=$

[AP EAMCET 07-07-2022_Shift-1]

1. $\dfrac{72}{81}e^5$
2. $\dfrac{7}{81e^5}$
3. $\dfrac{8}{81e^5}$
4. e^5

121. If $y=\tan\left(3\tan^{-1}x\right)$, *then* $(1-3x)^2\dfrac{d^2y}{dx^2}-12x\dfrac{dy}{dx}=$

[AP EAMCET 07-07-2022_Shift-1]

1. $6(x+y)$
2. $6(y-x)$
3. $6y$
4. $-6x$

122. *If* $\dfrac{d}{dx}\left(\dfrac{x^2+1}{(x^2+5)(x^2+9)}\right)=\dfrac{2x(x^2+1)}{(x^2+5)(x^2+9)}\left[\dfrac{1}{f(x)}-\dfrac{1}{g(x)}-\dfrac{1}{h(x)}\right]$

Then $2h(x)-f(x)-g(x)=$

[AP EAMCET 07-07-2022_Shift-2]

1. 12
2. 16
3. 18
4. 20

123. If $y=\dfrac{ax+b}{cx+b}$ *and* $\dfrac{dx}{dy}=\dfrac{ad-bc}{Py^2+Qy+R}$ then

P + Q +R =

[AP EAMCET 08-07-2022_Shift-1]

1. $(a+c)^2$
2. $(a-c)^2$
3. $\dfrac{ad-bc}{a^2+c^2-2ac}$
4. $\dfrac{1}{(a-c)^2}$

124. If $f(x)=\begin{cases}\tan^{-1}x, when|x|\le 1\\ \dfrac{1}{2}(|x|-1)\ when|x|>1\end{cases}$,

then the domain of $\dfrac{d}{dx}f(x)$ is

[AP EAMCET 08-07-2022_Shift-1]

1. R – {-1, 1}
2. R – (-1,1)
3. R – [-1, 1]
4. R – {-1}

125. If $x=a\cos^3\theta, y=a\sin^3\theta$, *then* $\dfrac{d^2y}{dx^2}$ *at* $\theta=\pi/4$ is

[AP EAMCET 08-07-2022_Shift-1]

1. $\dfrac{4\sqrt{2}}{3a}$
2. $\dfrac{2}{3a}$
3. $\dfrac{2\sqrt{2}}{3a}$
4. $\dfrac{7\sqrt{2}}{3a}$

126. If $y=\sin\left(2\tan^{-1}\left(\sqrt{\dfrac{1-x}{1+x}}\right)\right)$. $x=\cos 2\theta$, *then* $\dfrac{dy}{dx}=$

[AP EAMCET 08-07-2022_Shift-2]

1. $\dfrac{x}{\sqrt{1-x^2}}$
2. $-\cot 2\theta$
3. $\tan 2\theta$
4. $\dfrac{-x}{2\sqrt{1-x^2}}$

127. If $f(x)=|x^2-3x+2|$, *then* $\dfrac{df}{dx}=$

[AP EAMCET 08-07-2022_Shift-2]

1. $2x-3$, *when* $1<x<2$
2. $3-2x$, *when* $x>2$
3. $2x-3$ *when* $x>2$
4. $3+2x$, *when* $1<x<2$

128. If $y=\dfrac{\log x}{x}$, *then* $\dfrac{d^2y}{dx^2}$ *at* $x=1$

[AP EAMCET 08-07-2022_Shift-2]

1. $-e^{-3}$
2. -3
3. 3
4. e^3

129. If $f(x)=\log_e\left(e^{2x}\left(\frac{3x+5}{5-3x}\right)^{2/3}\right), x\neq\frac{-5}{3},\frac{5}{3}$,

then the value of $\frac{df}{dx}$ at $x=1$, is

[TS EAMCET 18-07-2022_Shift-1]

1. $\frac{5}{4}$
2. $\frac{7}{4}$
3. $\frac{11}{4}$
4. $\frac{13}{4}$

130. If $x=\operatorname{cosec}\theta-\sin\theta, y=\operatorname{cosec}^{2022}\theta-\sin^{2022}\theta$

and $\left(\frac{dy}{dx}\right)^2=\frac{k\left(y^2+4\right)}{g(x)}$ where $k\in R$, then

$10+k-g(2022)=$

[TS EAMCET 18-07-2022_Shift-1]

1. 0
2. 6
3. 10
4. 14

131. If $a f(x)+b f\left(\frac{1}{x}\right)=x+1$, *and*

$\frac{d}{dx}\left(x^2 f(x)\right)=2x^2+2x+\frac{1}{3}$, then a – b =

[TS EAMCET 18-07-2022_Shift-2]

1. 2
2. 3
3. 0
4. 1

132. If $f(x)=\sin\left(\cosh\left(\frac{x^2+1}{x^2+2}\right)\right)$ *then* $f'(1)=$

[TS EAMCET 18-07-2022_Shift-2]

1. $\frac{2}{9}\sin h\left(\frac{2}{3}\right)\cos\left(\cosh\left(\frac{2}{3}\right)\right)$
2. $\sin h\left(\frac{2}{3}\right)\cos\left(\cosh\left(\frac{2}{3}\right)\right)$
3. $\frac{2}{9}\cos\left(\cosh\left(\frac{2}{3}\right)\right)$
4. $\frac{2}{9}\cos h\left(\frac{2}{3}\right)\cos\left(\sin h\left(\frac{2}{3}\right)\right)$

133. If $f(x)=\begin{vmatrix}-\sin x & 2\sin 2x & 4\cos^2 x\\ \cos x & 4\sin^2 x & 2\sin 2x\\ 0 & -\cos x & \sin x\end{vmatrix}$ then

$f\left(\frac{5\pi}{4}\right)+f'\left(\frac{5\pi}{4}\right)=$

[TS EAMCET 19-07-2022_Shift-I]

1. 0
2. -1
3. -2
4. -4

134. If $x\cos(k+y)=\cos y$ then $\frac{dy}{dx}$ at $y=\frac{\pi}{2}$ is

[TS EAMCET 19-07-2022_Shift-I]

1. $\sin k$
2. $\cos k$
3. 1
4. 0

135. If $x=a(\cos\theta+\theta\sin\theta), y=f(\theta), f(2\pi)=0$,

$\frac{dy}{dx}=\frac{\tan\theta}{\theta}, \theta\neq 0$ and $\theta\neq(2n+1)\frac{\pi}{2}$, then

$f\left(\frac{\pi}{3}\right)=$ **[TS EAMCET 19-07-2022_Shift-I]**

1. $2a\pi$
2. $\frac{\pi}{2}a$
3. $\frac{a}{2}$
4. $-2a$

136. $\frac{d}{dx}\left[\left(x^{\frac{5}{2}}-x^{\frac{3}{2}}+1\right)\left(x^2-3x+5\right)\right]=$

[TS EAMCET 19-07-2022_Shift-2]

1. $\frac{9}{2}x^{\frac{7}{2}}-14x^{\frac{5}{2}}+20x^{\frac{3}{2}}-\frac{15}{2}x^{\frac{1}{2}}+2x-3$
2. $\frac{9}{2}x^{\frac{7}{2}}-7x^{\frac{5}{2}}+5x^{\frac{3}{2}}-\frac{3}{2}x^{\frac{1}{2}}+2x-3$
3. $9x^{\frac{7}{2}}-14x^{\frac{5}{2}}+20x^{\frac{3}{2}}-15x^{\frac{1}{2}}+2x-3$
4. $\frac{9}{2}x^{\frac{7}{2}}-\frac{7}{2}x^{\frac{5}{2}}+\frac{5}{2}x^{\frac{3}{2}}-\frac{15}{2}x^{\frac{1}{2}}+2x-3$

137. The value of

$$\frac{d}{dx}\left[\log\left(\sin\sqrt{\frac{x^2+1}{x^2+2}}\right)\right] when\ x=\sqrt{2}, is$$

[TS EAMCET 19-07-2022_Shift-2]

1. $\frac{\sqrt{2}\cot\left(\frac{\sqrt{3}}{2}\right)}{6\sqrt{3}}$
2. $\frac{\sqrt{2}\tan\left(\frac{\sqrt{3}}{2}\right)}{6\sqrt{3}}$
3. $\frac{\sqrt{2}\cot\left(\frac{\sqrt{3}}{2}\right)}{8\sqrt{3}}$
4. $\frac{\sqrt{2}\tan\left(\frac{\sqrt{3}}{2}\right)}{8\sqrt{3}}$

138. *If* $f(x)=\frac{1+\sec x}{2(\sec x-1)}$ *for* $0<x<\frac{\pi}{2}$ *and* $f'(x)=f(x).g(x)$, then g(x) =

[TS EAMCET 19-07-2022_Shift-2]

1. cosec x
2. -cosec x
3. 2cosec x
4. -2cosec x

139. If $f(x)=\frac{e^{-x}\sin x}{\log_e x}$ *and* $f'(x)=f(x).g(x)$, *then* $g'(e)=$

[TS EAMCET 20-07-2022_Shift-1]

1. $e^{-2}-\cos ec^2(e)$
2. $2e^2-\cos ec^2(e)$
3. $2e^{-2}-\cos ec^2(e)$
4. $2e^2+\cos ec^2(e)$

140. If $y=\frac{e^{\sin x}+\sinh^3 x}{\cosh x-\tan x}$, then $y'(0)=$

[TS EAMCET 20-07-2022_Shift-1]

1. 0
2. 1
3. –1
4. 2

141. If $f(x)=\sum_{p=1}^{7}p^2 Sin^{-1}\left(\frac{4}{5}\sin(px)-\frac{3}{5}\cos(px)\right)$

then the value of $\frac{df}{dx}$ at x = 1 is (Given that $Sin^{-1}(\sin x)=x$)

[TS EAMCET 20-07-2022_Shift-2]

1. 0
2. 628
3. 1140
4. 784

142. If $y=\frac{ax+b}{cx+d}$, then $\frac{dx}{dy}=$

[TS EAMCET 20-07-2022_Shift-2]

1. $\frac{ad-bc}{(ax+b)^2}$
2. $\frac{ad-bc}{(a-cy)^2}$
3. $\frac{ad+bc}{(cx+d)^2}$
4. $\frac{ad+bc}{(a+cy)^2}$

143. If $x^2+y^2=t-\frac{1}{t}$, $x^4+y^4=t^2+\frac{1}{t^2}$ then $\frac{dy}{dx}=$

[TS EAMCET 20-07-2022_Shift-2]

1. $\frac{x}{y}$
2. $\frac{-x}{y}$
3. $\frac{y}{x}$
4. $\frac{-y}{x}$

144. If $x^x y^y=e^e$, then $\left(\frac{d^2y}{dx^2}\right)_{(e,e)}=$

[15th May 2023 Shift 2]

1. $\frac{1}{e}\left(\frac{dy}{dx}\right)_{(e,e)}$
2. $\left(\frac{dy}{dx}\right)_{(e,e)}+\frac{1}{e}$
3. $\left(\frac{dy}{dx}\right)_{(e,e)}-\frac{1}{e}$
4. $e\left(\frac{dy}{dx}\right)_{(e,e)}$

145. If $f(x)=|x-5|+|x+5|+|x-4|+|x+4|$, then $\frac{f'(1)-f'(-6)}{f'(-1)+f'(6)}=$ **[15th May 2023 Shift 2]**

1. 1
2. 0
3. 4/5
4. 3/2

146. If f(x+ay)+g(x–ay)=0, then $a\frac{dy}{dx}=$

[15th May 2023 Shift 2]

1. $\frac{f'(x-ay)+g'(x+ay)}{g'(x+ay)-f'(x-ay)}$
2. $\frac{f'(x+ay)+g'(x-ay)}{g'(x-ay)-f'(x+ay)}$
3. $\frac{f'(x+ay)g'(x-ay)}{f'(x+ay)+g'(x-ay)}$
4. $\frac{f'(x+ay)+g'(x-ay)}{f'(x+ay)g'(x-ay)}$

147. If $y = x\log\left(\frac{1}{ax} + \frac{1}{a}\right)$, then

$x(x+1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y =$

[16th May 2023 Shift 1]

1. 0
2. 1 +x
3. -1
4. x
3. 3
4. 2

148. If $x^2 Tan^{-1}\frac{y}{x} - y^2 Tan^{-1}\frac{x}{y} = k$, then $\left(\frac{dy}{dx}\right)_{(1,1)}$

[16th May 2023 Shift 2]

1. 0
2. $\frac{\pi}{4}$
3. 1
4. $\frac{\pi}{2}$

149. if $u = \sin\left(\frac{x}{y}\right), x = e^t$ and $y = t^2$, then $t^6\left(\frac{du}{dt}\right)^2 \div e^{2t}(t-2)^2 =$

[16th May 2023 Shift 2]

1. 2u
2. u^2
3. $1-u^2$
4. cosu

150. If $\frac{d}{dx}\left(\frac{x^2}{(x+2)(2x+3)}\right) = \frac{A}{(x+2)^2} + \frac{B}{(2x+3)^2}$

then A+B= **[17th May 2023 Shift 2]**

1. 1/2
2. –5
3. –3/2
4. 9/4

151. If $f(t) = \frac{t}{2} + \frac{1}{4}\log(2t-1)$, then

$f'\left(\frac{t+1}{2t+1}\right) =$ **[17th May 2023 Shift 2]**

1. t
2. 1+t
3. 2t+1
4. t–1

152. If $x^3+y^3=3axy$, then at $\left(\frac{3a}{2}, \frac{3a}{2}\right)$ the value of 3ay"+40 is **[18th May2023 shift -1]**

1. -5
2. 0
3. 8
4. 1

153. If f(x) is a function such that $f'(x) = \sqrt{f^2(x)-1}$ and f(0)=1, then f(1)=

[18th May 2023 Shift 2]

1. $\frac{e^{-2}+1}{2e}$
2. $\frac{e^2+1}{2e}$
3. $\frac{e^2-1}{2e}$
4. $\frac{e^{-2}-1}{2e}$

154. At $x = \frac{\pi^2}{4}$, $\frac{d}{dx}\left(\text{Tan}^{-1}\left(\cos\sqrt{x}\right) + Sec^{-1}\left(e^x\right)\right) =$

[19th May 2023 Shift 1]

1. $\frac{1}{\sqrt{e^{\frac{\pi^2}{2}}-1}} - \frac{1}{\pi}$
2. $\frac{\pi}{4} + \frac{1}{\sqrt{e^{\pi^2}+e^{\frac{\pi^2}{2}}}}$
3. $\frac{1}{\sqrt{e^{\pi^2}+e^{\frac{\pi^2}{2}}}} + \frac{2}{\pi}\cot\left(\frac{\sqrt{\pi}}{2}\right)$
4. $\frac{1}{\sqrt{e^{\pi}}} + \frac{1}{\pi}$

155. The maximum value of 'a' such that the second derivative of $x^4 + ax^3 + \frac{3x^2}{2} + 1$ is positive for all real x is

[19th May 2023 Shift 1]

1. 3
2. -3
3. 2
4. -2

156. If $\tan y = \cot\left(\frac{\pi}{4} - x\right)$ then $\frac{dy}{dx} =$

[12TH MAY 2023 SHIFT-1]

1. $\frac{\cos ec^2(\frac{\pi}{4}-x)}{1+\cot^2(\frac{\pi}{4}+x)}$
2. $\frac{-\cos ec^2(\frac{\pi}{4}-x)}{\sec^2 y}$
3. $\frac{\cos ec^2(\frac{\pi}{4}-x)}{1+\tan^2(\frac{\pi}{4}-x)}$
4. $\frac{\sec^2(\frac{\pi}{4}+x)}{1+\tan^2(\frac{\pi}{4}+x)}$

157. If $x = 3\sqrt{2}\cos^3\theta$ and $y = 4\tan^2\theta$ then $\left(\frac{dy}{dx}\right)_{\theta=\pi/4} =$

[12TH MAY 2023 SHIFT-1]

1. $\frac{32\sqrt{2}}{9}$
2. $\frac{16}{9}$
3. $-\frac{16}{9}$
4. $-\frac{32}{9}$

158. The derivative of $\frac{1-x^2}{1+x^2}$ with respect to $\frac{2x}{1+x^2}$ at $x=2$ is[**12TH MAY 2023 SHIFT-1**]

1. 0
2. $\frac{4}{3}$
3. 1
4. $-\frac{4}{3}$

159. If $\sec\left(\log_2 y^2\right)=\cos ec\left(\log_2 x^2\right)$, then $\frac{dy}{dx}=$

1. $\frac{x}{y}$
2. $\frac{y}{x}$
3. $-\frac{y}{x}$
4. $-\frac{x}{y}$

160. If $e^x=y+\sqrt{y^2-1}$, then $\frac{dy}{dx}=$

[12TH MAY 2023 SHIFT -2]

1. $\sinh x$
2. $\cosh x$
3. $\tanh x$
4. $-\sinh x$

161. If $x=\log p$ and $y=\frac{1}{p}$ then $\frac{dy}{dx}=$

[12TH MAY 2023 SHIFT -2]

1. $-e^{-x}$
2. e^x
3. x
4. y

162. $f:\mathbb{R}\to\mathbb{R}$ is a function defined by

$$f(x)=\frac{1}{e^x+2e^{-x}}$$

Assertion (A): $f(c)=\frac{1}{3}$ for some values of $c\in R$

Reason (R): $0<f(x)\le\frac{1}{2\sqrt{2}}$ for all $x\in R$

Then which of the following options is correct?[**12TH MAY 2023 SHIFT -2**]

1.(A) and (R) are true. (R)is the correct explanation of (A)

2.(A) and (R) are true, but (R) is not the correct explanation for (A)

3.(A) is true but (R) is false

4.(A) is false but (R) is true

163. If $f(x)=|x-1|+|x-2|$,then $f^1(-2023)+f'\left(\frac{2024}{2023}\right)+f'(2023)=$

[13TH MAY 2023 SHIFT-1]

1. 1
2. -1
3. 0
4. 3

164. If $f(x)=\frac{e^{2x}-e^{-2x}}{e^{3x}+e^{-3x}}$,then $f'(0)=$

[13TH MAY 2023 SHIFT-1]

1. -1
2. 0
3. 1
4. 2

165. If $f(x)=x^{\tan x}+(\tan x)^x$,then $f'\left(\frac{\pi}{4}\right)=$**[13TH MAY 2023 SHIFT-1]**

1. $1+\frac{\pi}{2}\log\left(\frac{e\pi}{4}\right)$
2. $\frac{\pi}{2}\left(\log\frac{\pi}{4}+1\right)$
3. 1
4. 0

166.On differentiation if we get $f(x,y)dy-g(x,y)dx=0$ from $2x^2-3xy+y^2+x+2y-8=0$ then $\frac{g(2,2)}{f(1,1)}=$ **[EAPCET 14-05-23 SHIFT-1]**

1. $\frac{11}{7}$
2. -3
3. $\frac{-1}{3}$
4. 7

167. If $f(x)=e^x, h(x)=(fof)(x)$, then $\frac{h'(x)}{h(x)}=$

[EAPCET 14-05-23 SHIFT-1]

1. $h(x)$
2. $\frac{1}{h(x)}$
3. $\log h(x)$
4. $-\log h(x)$

168. If $\sin y = \sin 3t$ and $x = \sin t$, then $\frac{dy}{dx} =$

[EAPCET 14-05-23 SHIFT-1]

1. $\frac{3}{\sqrt{4-x^2}}$
2. $\frac{3}{\sqrt{1-x^2}}$
3. $\frac{1}{\sqrt{4-x^2}}$
4. $\frac{-1}{\sqrt{4-x^2}}$

169. If $f(x) = \sqrt{\log\left(x^2+x+1\right) + \sqrt{\cosh\left(2x-3\right)}}$, then $f^1(0) =$ **[EAPCET 13-05-23 SHIFT-2**

1. $\frac{1}{2\sqrt{\sqrt{\cosh(3)}}}\left(1+\frac{\sinh(3)}{\sqrt{\cosh(3)}}\right)$
2. $\frac{1}{2\sqrt{\sqrt{\cosh(3)}}}\left(\log 3-\frac{\sin h(3)}{\sqrt{\cosh(3)}}\right)$
3. $\frac{\log 3\sqrt{\cosh(3)}-\sinh(3)}{2\left(\cosh(3)\right)^{3/4}}$
4. $\frac{\sqrt{\cosh(3)}-\sinh(3)}{2\left(\cosh(3)\right)^{3/4}}$

170. If $x = \cos^3\theta - \sin^3\theta$ *and* $y = \sqrt[3]{\cos\theta} - \sqrt[3]{\sin\theta}$, then the value of $\frac{dy}{dx}$ *at* $\theta = \frac{\pi}{4}$ is

[EAPCET 13-05-23 SHIFT-2]

1. $\frac{2}{9}\sqrt[3]{2}$
2. $\frac{\sqrt[3]{2}}{3}$
3. $\frac{4}{9}\sqrt[3]{2}$
4. $\frac{\sqrt[3]{2}}{9}$

171. If $2x^2+3xy-y^2+4x-5y+6=0$, then the value of $\frac{dy}{dx}$ at $(x,y) = (1\text{-},2)$ is

[EAPCET 13-05-23 SHIFT-2]

1. 1
2. -1
3. 7/2
4. 0

KEY

1)	2	2)	4	3)	2	4)	3	5)	3
6)	3	7)	3	8)	4	9)	3	10)	4
11)	2	12)	1	13)	1	14)	3	15)	3
16)	1	17)	3	18)	4	19)	1	20)	2
21)	3	22)	1	23)	1	24)	4	25)	1
26)	4	27)	1	28)	1	29)	3	30)	3
31)	1	32)	3	33)	2	34)	1	35)	1
36)	1	37)	4	38)	4	39)	3	40)	1
41)	2	42)	1	43)	4	44)	3	45)	1
46)	1	47)	2	48)	1	49)	1	50)	2
51)	3	52)	3	53)	2	54)	4	55)	3
56)	2	57)	2	58)	3	59)	3	60)	2
61)	2	62)	1	63)	2	64)	1	65)	3
66)	3	67)	4	68)	2	69)	3	70)	2
71)	3	72)	4	73)	1	74)	1	75)	2
76)	2	77)	3	78)	1	79)	4	80)	4
81)	3	82)	2	83)	2	84)	4	85)	2
86)	1	87)	4	88)	4	89)	4	90)	2
91)	2	92)	2	93)	1	94)	4	95)	1
96)	3	97)	1	98)	2	99)	1	100)	4
101)	1	102)	4	103)	3	104)	2	105)	1
106)	4	107)	4	108)	2	109)	4	110)	4
111)	3	112)	3	113)	1	114)	3	115)	2
116)	3	117)	2	118)	1	119)	1	120)	3
121)	2	122)	1	123)	2	124)	4	125)	1
126)	2	127)	3	128)	2	129)	4	130)	2
131)	2	132)	1	133)	4	134)	1	135)	3
136)	1	137)	3	138)	4	139)	3	140)	4
141)	4	142)	2	143)	4	144)	1	145)	1
146)	2	147)	3	148)	3	149)	3	150)	2
151)	2	152)	3	153)	2	154)	1	155)	3
156)	4	157)	4	158)	2	159)	3	160)	1
161)	1	162)	1	163)	3	164)	4	165)	1
166)	2	167)	3	168)	2	169)	4	170)	1
171)	2								

SOLUTIONS

1. $\sqrt{1-x^6}+\sqrt{1-y^6}=a\left(x^3-y^3\right)$

$x^3=\sin\alpha$

$y^3=\sin\beta$

$\cos\alpha+\cos\beta=a(\sin\alpha-\sin\beta)$

$a=\cot\left(\frac{\alpha-\beta}{2}\right)$

$\alpha-\beta=2\cot^{-1}a$

$\sin^{-1}x^3-\sin^{-1}y^3=2\cot^{-1}a$

$\frac{3x^2}{\sqrt{1-x^6}}-\frac{3y^2}{\sqrt{1-y^6}}y'=0$

$y'=\frac{x^2}{y^2}\sqrt{\frac{1-y^6}{1-x^6}}$

---

2. $\frac{y+4x}{\sqrt{xy}}=4$

$y+4x=4\sqrt{xy}$

$y^2+16x^2+8xy=16xy$

$y^2+16x^2=8xy$

$2yy'+32x=8xy'+8y$

$y'(2y-8x)=8y-32x$

$y'=\frac{8y-32x}{2y-8x}=\frac{8(y-4x)}{2(y-4x)}$

$=4$

---

3. $3f(x)-2f\left(\frac{1}{x}\right)=x.....(1)$

Replace x by $\frac{1}{x}$

$3f\left(\frac{1}{x}\right)-2f(x)=\frac{1}{x}....(2)$

$3\times(1)+2\times(2)\Rightarrow 5f(x)=3x+\frac{2}{x}$

$\Rightarrow f'(2)=\frac{1}{2}$

---

4. $\left(1-x+x^2\right)^{10}=a_0+a_1x+a_2x^2+..+a_{20}x^{20}\rightarrow(I)$

Diff w.r.to ' x ' and then put $x=1$

$10=a_1+2a_2+3a_3+..+20a_{20}$

$2a_2+3a_3+..+20a_{20}=10-a_1\rightarrow(1)$

Coeff x in (I) is $-10=a_1$

From (1) $2a_2+3a_3+..+20a_{20}=20$

---

5. Apply

$\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$

---

6. Let $\tan^{-1}\left(\frac{x}{1+\sqrt{1-x^2}}\right)=f(x)$

Put $x=\sin\theta$ then $\tan^{-1}\left(\tan\frac{\theta}{2}\right)=f(x)$

$\Rightarrow f(x)=\frac{\theta}{2}=\frac{1}{2}\sin^{-1}x$

Let $g(x)=\sec^{-1}\left(\frac{1}{2x^2-1}\right)$

Put $x=\cos\theta$

$g(x)=\sec^{-1}(\sec2\theta)$

$=2\theta=2\cos^{-1}x$

$\therefore\frac{d(f(x))}{d(g(x))}=\frac{\frac{1}{2\sqrt{1-x^2}}}{-2\frac{1}{\sqrt{1-x^2}}}=\frac{-1}{4}$

---

7. Use $x^my^n=(x+y)^{m+n}\Rightarrow\frac{dy}{dx}=\frac{y}{x}$

---

8. Conceptual

---

9. Put

$\frac{x}{3}=\cos\theta\Rightarrow\theta=\cos^{-1}\left(\frac{x}{3}\right)$

$\frac{d}{dx}\left[\cos^{-1}(\cos3\theta)\right]=\frac{d}{dx}(3\theta)$

$$=3\frac{d}{dx}\left[\cos^{-1}\frac{x}{3}\right]=\frac{-3}{\sqrt{1-\frac{x^2}{9}}}\cdot\frac{x}{3}=\frac{-3}{\sqrt{9-x^2}}$$

10. $y=\text{Sin}^{-1}(\cos x)$

$$\frac{dy}{dx}=\frac{1}{\sqrt{1-\cos^2 x}}(-\sin x)=\frac{-\sin x}{|\sin x|}$$

It is not differentiable at integral multiple of π

11. $\log(x+y)-2xy=0....(1)$

$\log(x+y)=2xy$

diff., w.r.to 'x', weget

$$\frac{1}{x+y}\left[1+y^1\right]=2\left[xy^1+y\right]$$

$$y^1\left[\frac{1}{x+y}-2x\right]=2y-\frac{1}{x+y}$$

$$\frac{dy}{dx}=-\left(\frac{1-2y^2-2xy}{1-2x^2-2xy}\right)$$

from(1),

if $x=0, y=1$

$$y^1(0)=-\left[\frac{1-2}{1}\right]=1$$

12. $y=\sqrt{2x+\cos^2\left(2x+\frac{\pi}{4}\right)}$

$$\frac{dy}{dx}=\frac{1}{2\sqrt{2x+\cos^2\left(2x+\frac{\pi}{4}\right)}}\left[2-4\cos\left(2x+\frac{\pi}{4}\right)\sin\left(2x+\frac{\pi}{4}\right)\right]$$

$$\left(\frac{dy}{dx}\right)_{x=\frac{\pi}{4}}=\frac{1-\sin\frac{3\pi}{2}}{\sqrt{\frac{\pi}{2}+\cos^2\frac{3\pi}{2}}}=\frac{2\sqrt{2}}{\sqrt{\pi+1}}$$

13. $f(x)=\sqrt{\left(\sqrt{x-2}+\sqrt{2}\right)^2}+\sqrt{\left(\sqrt{x-2}-\sqrt{2}\right)^2}$

$=\sqrt{x-2}+\sqrt{2}+\sqrt{x-2}-\sqrt{2}$

$=2\sqrt{x-2}$

$$f'(x)=\frac{1}{\sqrt{x-2}}$$

$10\times f'(102)=1$

14. $\frac{d}{dx}\left(e^{\log_e\sqrt{1+\tan^2 x}}\right)=\frac{d}{dx}\left(e^{\log_e\sec x}\right)$

$=\frac{d}{dx}(\sec x)=\sec x\tan x$

15. $f(x)=\cos^{-1}\left[\cos\left(\frac{\pi}{2}-\sqrt{\frac{1+x}{2}}\right)\right]+x^x$

$=\frac{\pi}{2}-\sqrt{\frac{1+x}{2}}+x^x$

$\therefore f'(x)=-\frac{1}{2\sqrt{\frac{1+x}{2}}}\times\frac{1}{2}+x^x(1+\log x)$

$f'(1)=\frac{3}{4}$

16. $2f(\sin x)+f(\cos x)=x....(1)$

Replace 'x' by "$\frac{\pi}{2}-x$"

$2f(\cos x)+f(\sin x)=\frac{\pi}{2}-x....(2)$

By solving (1) and (2)

$3f(\sin x)=3x-\frac{\pi}{2}$

$\sin x=y\Rightarrow x=\sin^{-1}y$

$3f(y)=3\sin^{-1}y-\frac{\pi}{2}$

$3f'(y)=3.\frac{1}{\sqrt{1-y^2}}$

$f'(x)=\frac{1}{\sqrt{1-x^2}}$

17. Conceptual

18. Let $\sin x=\cos 2A$

$A=\frac{1}{2}\cos^{-1}(\sin x)$

$$y = Tan^{-1}\left(\tan\left(\frac{\pi}{4} + A\right)\right)$$

$$y = \frac{\pi}{4} + \frac{1}{2}\cos^{-1}(\sin x)$$

$$\frac{dy}{dx} = 0 + \frac{1}{2}\frac{-1}{\sqrt{1-\sin^2 x}}(\cos x)$$

$$= \frac{1}{2}.\frac{-1}{|\cos x|}.\cos x$$

$$= \pm\frac{1}{2}$$

19. $x = e^{y+x}$

$$\log x = y + x$$

$$\frac{1}{x} = \frac{dy}{dx} + 1$$

$$\frac{dy}{dx} = \frac{1}{x} - 1 = \frac{1-x}{x}$$

20. Put $x = \tan\theta \Rightarrow \theta = \tan^{-1} x$

Now $y = \frac{\theta}{2} = \frac{1}{2}\tan^{-1} x$

$$\frac{dy}{dx} = \frac{1}{2}.\frac{1}{1+x^2}$$

21. Given: $f(x) = x^4 - x^3 + 7x^2 + 14$

$$f'(x) = 4x^3 - 3x^2 + 14x$$

$$f''(x) = 12x^2 - 6x + 14$$

$$\therefore f''(5) = 12(5)^2 - 6(5) + 14 = 284$$

22. GIVEN: $y = \sqrt{\frac{1+\tan x}{1-\tan x}} = \sqrt{Tan\left(\frac{\pi}{4} + x\right)}$

Diff w.r.t x

$$\frac{dy}{dx} = \frac{1}{2}\left(\sqrt{\frac{1-\tan x}{1+\tan x}}\right)\sec^2\left(\frac{\pi}{4} + x\right)$$

23. Given f= $2x^2 - 3xy + y^2 + x + 2y - 8 = 0$

Now $\frac{dy}{dx} = -\frac{\left(\frac{\partial f}{\partial x}\right)}{\left(\frac{\partial f}{\partial y}\right)} = \frac{3y - 4x - 1}{2y - 3x + 2}$

24. $y = \frac{e^x \log x}{x^2}$

$$\frac{dy}{dx} = \frac{x^2\left(e^x \log x + \frac{e^x}{x}\right) - e^x \log x(2x)}{x^4}$$

$$= \frac{e^x\left(1 + (x-2)\log x\right)}{x^3}$$

25. Conceptual

26. $y = \mathrm{Sin}^{-1}\left(\frac{\sqrt{1+x} - \sqrt{1-x}}{2}\right)$

put $x = \cos 2\theta$

$$y = \sin^{-1}\left(\frac{\sqrt{2}\cos\theta - \sqrt{2}\sin\theta}{2}\right)$$

$$= \sin^{-1}\sin\left(\frac{\pi}{4} - \theta\right)$$

$$y = \frac{\pi}{4} - \frac{1}{2}\cos^{-1} x$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{1-x^2}}$$

27. $y = \log(\cosh x)$

$$y^1 = \frac{\sinh x}{\cosh x} = \tanh x$$

$$\frac{d^2 y}{dx^2} = \sec^2 hx$$

28. $\frac{dt}{dx} = \frac{t}{\left(x + te^{-2x/t}\right)} \Rightarrow \frac{dt}{dx} = \frac{1}{\left(\frac{x}{t}\right) + e^{-2\left(\frac{x}{t}\right)}}$

$$\frac{dx}{dt} = \left(\frac{x}{t}\right) + e^{-2\left(\frac{x}{t}\right)}$$

$$\phi\left(\frac{x}{t}\right) = \left(\frac{x}{t}\right) + e^{-2\left(\frac{x}{t}\right)}$$

29. $\frac{d}{dx}\{(1+x^2)\tan^{-1}x\}$

$=(1+x^2)\frac{1}{1+x^2}+2x.\tan^{-1}x=1+2x\tan^{-1}x$

30. $f(x)=\frac{\log(\log x)}{2\log x}$

$f'(x)=\frac{1}{2}\left[\frac{\log x\times\frac{1}{\log x}\times\frac{1}{x}-\frac{\log(\log x)}{x}}{(\log x)^2}\right]=\frac{1}{2}\left[\frac{1-\log(\log x)}{x(\log x)^2}\right]$

$f'(e)=\frac{1}{2e}$

31. $\frac{d}{dx}\left[\frac{(x^2+x+1)(x^2-x+1)}{x^2+x+1}\right]=2x-1$

$a-b=3$

32. $y=\text{Tan}^{-1}\left(\frac{5x-x}{1+5x(x)}\right)+\text{Tan}^{-1}\left(\frac{x+\frac{2}{3}}{1-\frac{2x}{3}}\right)$

$y=\text{Tan}^{-1}5x+\text{Tan}^{-1}(2/3)$

$\frac{dy}{dx}=\frac{5}{1+25x^2}$

33. $f(x)=3e^{x^2}\Rightarrow f'(x)=6e^{x^2}(x)$

$f(0)=3\Rightarrow f'(0)=0$

$\therefore f'(x)-2xf(x)+\frac{1}{3}f(0)-f'(0)$

$=6xe^{x^2}-2x\left(3e^{x^2}\right)+\frac{1}{3}(3)-0=1$

34. $y=\sin^{-1}\left(\frac{4+5\sin x}{5+4\sin x}\right)$

$\sin y=\frac{4+5\sin x}{5+4\sin x}$

$\frac{d}{dx}\left(\frac{a+b\sin x}{b+a\sin x}\right)=\frac{(b^2-a^2)\cos x}{(b+a\sin x)^2}$

$\therefore\frac{d}{dx}(\sin y)=\frac{(25-16)\cos x}{(5+4\sin x)^2}$

Diff w.r.to x

$\Rightarrow\cos y\frac{dy}{dx}=\frac{9\cos x}{(5+4\sin x)^2}$

$\Rightarrow\frac{dy}{dx}=\frac{3}{5+4\sin x}$

35. $\frac{d}{dx}\left(\sin^{-1}\frac{3+4x}{5\sqrt{1+x^2}}\right)$

Let $x=\tan\theta$

$=\frac{d}{dx}\left[\sin^{-1}\left(\frac{3+4\tan\theta}{5\sqrt{1+\tan^2\theta}}\right)\right]$

$=\frac{d}{dx}\left[\sin^{-1}\left(\frac{3}{5}\cos\theta+\frac{4}{5}\sin\theta\right)\right]$

$=\frac{d}{dx}\left[\sin^{-1}(\sin(\theta+A))\right]$

$=\frac{d}{dx}(\theta+A)=\frac{d}{dx}(\tan^{-1}x+A)=\frac{1}{1+x^2}$

36. $x\left(1+y^{\frac{1}{3}}\right)=1-y^{\frac{1}{3}}$

$y^{\frac{1}{3}}=\frac{1-x}{1+x}\Rightarrow y=\left(\frac{1-x}{1+x}\right)^3$

$\frac{dy}{dx}=3\left(\frac{1-x}{1+x}\right)^2\left[\frac{(1+x)(-1)-(1-x)(1)}{(1+x)^2}\right]$

$=\frac{-6(1-x)^2}{(1+x)^4}$

37. A: $\frac{d^2}{dx^2}(\log|x|)=\frac{d}{dx}\left[\frac{1}{|x|}\times\frac{|x|}{x}\right]=\frac{-1}{x^2}$

R: $|x|=-x\quad if\ x<0$

A is false, R is true

38. $ax^2+2hxy+by^2=0$

$\frac{dy}{dx}=\frac{-(ax+hy)}{(hx+by)}$

$\frac{d^2y}{dx^2}=\frac{(h^2-ab)(ax^2+2hxy+by^2)}{(hx+by)^3}=0$

39. $f(x)=\dfrac{(x+1)\sinh x}{e^{2x}\tan x}$

$\log f(x)=\log(x+1)+\log\sinh x$

$-2x-\log\tan x$

$$\frac{f'(x)}{f(x)}=\frac{1}{x+1}+\coth x-2-2\cos ec2x$$

$g(x)=-2(1+\cos ec2x)$

40. $x^y=y^{\sin x}(\tan x)^{\cos x}$

$$y\log x=\sin x\log y+\cos x\log\tan x$$

$$\frac{y}{x}+\frac{dy}{dx}\log x=\sin x\frac{1}{y}\frac{dy}{dx}+\cos x\log y$$

$$+\cos x\frac{1}{\tan x}\sec^2 x-\sin x\log\tan x$$

$$\left(\log x-\frac{\sin x}{y}\right)\frac{dy}{dx}$$

$$=\frac{-y}{x}+\cos x\log y+\cos ecx-\sin x\log\tan x$$

41. $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$

$$\frac{dy}{dx}=\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$$

$$(y')^2=\frac{1-y^2}{1-x^2}$$

$$2y'y''=\frac{(1-x^2)(-2yy')-(1-y^2)(-2x)}{(1-x^2)^2}$$

$$(1-x^2)^2y'y''=-(1-x^2)yy'+(1-y^2)x$$

$$(1-x^2)^2y'y''+(1-x^2)yy'=(1-y^2)x$$

42. Conceptual

43. Formulae of differentiation

44. $f(x)=x.e^{-x}-e^{-x}$

$f'(x)=-x.e^{-x}+e^{-x}+e^{-x}\Rightarrow f'(0)=2$

$f''(x)=-\left\{-x.e^{-x}+e^{-x}\right\}-2e^{-x}\Rightarrow f''(0)=-3$

$f'(0)+f''(0)=2-3=-1$

45. $\dfrac{d^2y}{dx^2}=\dfrac{d}{dx}\left(\dfrac{dy}{dx}\right)=\dfrac{d}{dx}\left(\dfrac{1}{\frac{dy}{dx}}\right)$

$$\frac{d^2y}{dx^2}=\frac{d}{dy}\left(\frac{1}{\frac{dx}{dy}}\right)\frac{dy}{dx}=\frac{-1}{\left(\frac{dx}{dy}\right)^2}.\frac{d^2x}{dy^2}\left(\frac{dy}{dx}\right)$$

46. $\log\sqrt{x^2+y^2}=\tan^{-1}\left(\dfrac{x}{y}\right)$

$$\log\left(x^2+y^2\right)=2\tan^{-1}\left(\frac{x}{y}\right)$$

D.w.r.to.'x'

$$\frac{2\left(x+yy^1\right)}{x^2+y^2}=\frac{2y^2}{x^2+y^2}\left(\frac{y-xy^1}{y^2}\right)$$

$\Rightarrow x+yy^1=y-xy^1$

$$\Rightarrow y^1=\frac{y-x}{y+x}$$

47. *Let* $p(x)=ax^3$

$p^1(x)=3ax^2$

$p^{11}(x)=6ax$

$p(2x)=a(2x)^3=8ax^3$

now, $p^1(x).p^{11}(x)=18a^2x^3$

$\Rightarrow 18a^2x^3=8ax^3\Rightarrow a=\dfrac{4}{9}$

$\therefore p(x)=\dfrac{4}{9}\left(x^3\right)$

$$\sum_{x=1}^{5}p(x)=\frac{4}{9}\sum_{x=1}^{5}\left(x^3\right)=\frac{4}{9}\times\left(\frac{5\times6}{2}\right)^2=100$$

48. Given $2^x+2^y=2^{x+y}$

diff..on B.S

$2^x+2^y.y^1=2^{x+y}\left(1+y^1\right)$

$\Rightarrow 2^x+2^y.y^1=\left(2^x+2^y\right)\left(1+y^1\right)$

$\Rightarrow -2^y=2^x.y^1$

$$\Rightarrow -2^y=\left[\frac{2^y}{2^y-1}\right].y^1$$

$\Rightarrow y^1=1-2^y$

49. Given $x\sqrt{y+1}+y\sqrt{1+x}=0$,

$$x\sqrt{y+1}=-y\sqrt{1+x}$$

$S.O.B.S$

$$\Rightarrow x^2+x^2y=y^2+y^2x$$

$$\Rightarrow xy=-(x+y)$$

$$\Rightarrow y=\frac{-x}{1+x}$$

$diff.w.r.to\ 'x'on\ B.S$

$$\Rightarrow y^1=\frac{-1}{(1+x)^2}$$

$$\frac{d^2y}{dx^2}=-\left(\frac{d^2y}{dx^2}\right)^3\cdot\frac{d^2y}{dx^2}\Rightarrow\left(\frac{d^2y}{dx^2}\right)^3\cdot\frac{d^2y}{dx^2}+\frac{d^2y}{dx^2}=0$$

$K=0$

$$e^{K.f(x)}-K.f(x)=e^0-0=1$$

50. Given $y=x+\frac{1}{x}$ and $y'=1-\frac{1}{x^2}$

Consider $x^2y'-xy+2=0$

51. I.P.E question. Put $x^2=\cos 2\theta$

$$\frac{dy}{dx}=\frac{-x}{\sqrt{1-x^4}}$$

52. Given $3\sin(xy)+4\cos(xy)=5$

$$\frac{dy}{dx}=\frac{-\left(\frac{\partial f}{\partial x}\right)}{\left(\frac{\partial f}{\partial y}\right)}=\frac{-y}{x}$$

53. $f'(x)=\frac{x}{\sqrt{x^2+1}},g'(x)=\frac{-x^2-2x+1}{(x^2+1)^2},h'(x)=2$

Consider, $f'(h'(g'(x)))=\frac{2}{\sqrt{5}}$

54. Diff w.r.t 'x' $\therefore(1+x^2)\frac{dy}{dx}+xy=-1$

55. Given $y=e^{x^2+y}$

Taking 'log' on both sides $\Rightarrow\frac{dy}{dx}=\frac{2xy}{1-y}$

56. Conceptual

57. I.P.E question

58. Taking 'log' on both sides,

$$\log y=\log\left(1+\frac{1}{x}\right)+\log\left(1+\frac{2}{x}\right)...+\log\left(1+\frac{n}{x}\right)$$

$$(-1)\left[\left(1+\frac{2}{x}\right)\left(1+\frac{3}{x}\right)+...+\left(1+\frac{n}{x}\right)\right]+$$

$$\frac{dy}{dx}=\frac{2\left[\left(1+\frac{1}{x}\right)\left(1+\frac{3}{x}\right)...\left(1+\frac{n}{x}\right)+..\right]+3..}{x^2}$$

$$\left(\frac{dy}{dx}\right)_{at\ x=-1}=(-1)\left[(-1)(-2)....(1-n)\right]$$

$$=(-1)^n\left[1.2.3....(n-1)\right]=(-1)^n.(n-1)!$$

59. Put $x=\cos 2\theta,y=\sin^2\theta\Rightarrow\frac{dy}{dx}=\frac{-1}{2}$

60. Conceptual

61. $\frac{dy}{dx}=\frac{1+\cos^{2n}\theta}{(1+\cos^2\theta)(\cos^{n-1}\theta)}$,

$$x=\frac{1-\cos^2\theta}{\cos\theta},y=\frac{1-\cos^{2n}\theta}{\cos^n\theta}$$

$$B.V.M\left(x^2+y\right)\left(\frac{dy}{dx}\right)^2=n^2\left(y^2+4\right)$$

62. $f'(x)=6x^2\Rightarrow f''(x)=12x$

$\Rightarrow f''(x):f'(x)=2:x$

63. Conceptual

64. Given $y=\sin(\sin x)$

$$y''+\frac{\sin x}{\cos x}y'+\cos^2 x.y=0$$

Here, $f(x)=\frac{\sin x}{\cos x},g(x)=\cos^2 x$

Now, $f(x).g(x)=\frac{1}{2}(\sin 2x)$

65. Conceptual

66. $8f(x)+6f\left(\frac{1}{x}\right)=x+5....(1)\times 8$

Replace 'x' by '$\frac{1}{x}$'

$$8f\left(\frac{1}{x}\right)+6f(x)=\frac{1}{x}+5....(2)\times 6$$

Solving (1) and (2), we get

$$f(x)=\frac{8x}{28}-\frac{6}{28x}+\frac{10}{28}\Rightarrow f'(x)=\frac{8}{28}+\frac{6}{28x^2}$$

And $y=x^2.f(x)\Rightarrow y'=x^2f'(x)+2x.f(x)$

$$(y')_{at\ x=-1}=\frac{-1}{14}$$

67. $\frac{dy}{dx}=\frac{4}{1+x}+\frac{1}{4(1-x)}-\frac{1}{2(1+x^2)}$

$$\left(\frac{dy}{dx}\right)_{at\ x=\frac{1}{\sqrt{2}}}=\frac{2}{3}$$

68. $f''(x)=-4\sin x+9\sin 6x-\sin 2x$

Here $a=-4,b=4,c=9,d=6,e=-1,k=2$

Consider, $(a+c+e)-(b+d+k)=-8$

69. Given $y=\log_y^x=\frac{\log x}{\log y}\Rightarrow y\log y=\log x$

$$\therefore\frac{dy}{dx}=\frac{1}{x(1+\log y)}$$

70. $\frac{dx}{d\theta}=a\left(\frac{\cos^2\theta}{\sin\theta}\right),\frac{dy}{d\theta}=a.\cos\theta$

$$\frac{dy}{dx}=\operatorname{Tan}\theta$$

71. Conceptual

72. Given $\log_{x^2}^{(\log x)}\quad f'(x)\ at\ x=e$

$$f(x)=\frac{\log(\log x)}{\log x^2}$$

$$f'(x)=\frac{2/e}{(2+\log e)^2}=\frac{2/e}{4}=(2e)^{-1}$$

73. $\frac{d}{dx}\left\{\frac{2}{\sqrt{2+\sqrt{2+\sqrt{2+\cos(4x)}}}}\right\}$

$$=\frac{d}{dx}\left[\frac{2}{\sqrt{2+2\cos x}}\right]=\frac{d}{dx}\left(\sec\frac{x}{2}\right)$$

$$=\frac{1}{2}\sec\left(\frac{x}{2}\right).\tan\left(\frac{x}{2}\right)\ \therefore k=\frac{1}{2}$$

74. $2f(x)-3f\left(\frac{1}{x}\right)=x+1....(1)\times 2$

$$2f\left(\frac{1}{x}\right)-3f(x)=\frac{1}{x}+1....(2)\times 3$$

Solving (1) and (2), we get

$$f(x)=-\frac{2}{5}x+\frac{3}{-5x}-1,\quad f'\left(\sqrt{3}\right)=\frac{-1}{5}$$

75. $y'=f'(x^3).(3x^2)\Rightarrow y'=\sqrt{2x^6-1}.(3x^2)$

$$(y')_{at\ x=1}=3$$

76. $y=\operatorname{Tan}^{-1}\left(\tan\frac{x}{2}\right)=\frac{x}{2}\Rightarrow\frac{dy}{dx}=\frac{1}{2},\frac{d^2y}{dx^2}=0$

77. Conceptual

78. $\frac{dy}{dx}=1\Rightarrow\frac{d^2y}{dx^2}=0$

79. Put $x=\cos\theta$

$$f(x)=2\cos^{-1}x$$

$$f'(x)=\frac{-2}{\sqrt{1-x^2}}\therefore\left(\frac{f'(x)}{g'(x)}\right)_{at\ x=\frac{1}{2}}=4$$

80. $\frac{dy}{dx}=-\sin x^0.\frac{\pi}{180^0},\frac{dz}{dx}=-\sin x$

$$\Rightarrow\frac{dy}{dz}=\frac{\pi}{180^0}\sin(x^0)\operatorname{cosec}x$$

81. $\frac{dy}{dx}=\frac{\sin x}{|\sin x|}\Rightarrow\left(\frac{dy}{dx}\right)_{at\ x=\frac{5\pi}{4}}=\frac{\sin x}{-\sin x}=-1$

82. Given $y=\sin^{-1}\left[x\sqrt{1-x}-\sqrt{x}.\sqrt{1-x^2}\right]$

$y=\sin^{-1}x-\sin^{-1}\sqrt{1-x}$

$\frac{dy}{dx}=\frac{1}{\sqrt{1-x^2}}-\frac{1}{2\sqrt{x-x^2}}$

83. Given $y=\sqrt{x+\sqrt{x+\sqrt{x+\ldots.\infty}}}$

Let $y=\sqrt{x+y}\Rightarrow(2y-1)\frac{dy}{dx}=1$

$\therefore\frac{dy}{dx}=\frac{1}{2y-1}$

85. Let $P(x)=ax^3+bx^2+cx+d$

$P'(x)=3ax^2+2bx+c, P''(x)=6ax+2b$

$P''(1)=0\Rightarrow 3a+b=0$

$P'''(x)=6a=6\Rightarrow a=1, b=-3, P''(0)=-6$

86. Given $3.f(x)-2f\left(\frac{1}{x}\right)=x.....(1)\times 3$

Replace 'x' by '$\frac{1}{x}$', we get

$3f\left(\frac{1}{x}\right)-2f(x)=\frac{1}{x}.....(2)\times 2$

$(1)+(2)\Rightarrow f(x)=\frac{3}{5}x+\frac{2}{5x}$

$\therefore f'(2)=\frac{1}{2}$

87. Let $y=\frac{x2^x-x}{1-\cos x}\Rightarrow\log y=\log(x2^x-x)-\log(1-\cos x)$

$\Rightarrow\frac{1}{y}\frac{dy}{dx}=\frac{1}{x2^x-x}\left[x2^x.\log 2+2^x-1\right]-\frac{1}{1-\cos x}(\sin x)$

$\Rightarrow\frac{dy}{dx}=y\left[\frac{(2^x-1)}{x(2^x-1)}+\frac{x2^x\log 2}{x(2^x-1)}-\frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2\sin^2\frac{x}{2}}\right]$

$\Rightarrow\frac{dy}{dx}=\frac{x2^x-x}{1-\cos x}\left[\frac{1}{x}+\frac{2^x\log 2}{2^x-1}-\cot\frac{x}{2}\right]$

$\Rightarrow\frac{dy}{dx}=\frac{x2^x-x}{1-\cos x}\left[\frac{1}{x}+\frac{2^x\log 2-\log 2+\log 2}{2^x-1}-\cot\frac{x}{2}\right]$

$\Rightarrow\frac{dy}{dx}=\frac{x2^x-x}{1-\cos x}\left[\frac{1}{x}+\log 2+\frac{\log 2}{2^x-1}-\cot\frac{x}{2}\right]$

88. Given $(a+bx)e^{y/x}\Rightarrow e^{y/x}=\frac{x}{a+bx}$

$\Rightarrow\log e^{y/x}=\log x-\log(a+bx)$

$\Rightarrow\frac{xy'-y}{x^2}=\frac{1}{x}-\frac{1}{a+bx}\times b$

$\Rightarrow\frac{xy'-y}{x^2}=\frac{1}{x}-\frac{b}{a+bx}\Rightarrow xy'-y=x-\frac{bx}{a+bx}$

$\Rightarrow xy''+y'-y'=1-\left[\frac{(a+bx)(b)-bx(b)}{(a+bx)^2}\right]$

$\Rightarrow xy''=1-\left[\frac{ab+b^2x-b^2x}{(a+bx)^2}\right]$

89. $f(x)=\frac{\cos^2 x}{1+\sin^2 x}$

$f'(x)=\frac{-\sin 2x.2}{(1+\sin^2 x)^2}$

Now, $f\left(\frac{\pi}{4}\right)-3f'\left(\frac{\pi}{4}\right)=3$

90. $x=5(1-\sin t), y=5(t+\cos t)$

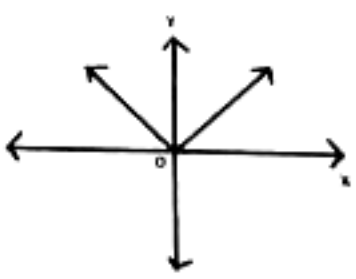
$\frac{dx}{dt}=-5\cos t\quad,\frac{dy}{dt}=5(1-\sin t)$

$\Rightarrow\frac{dx}{dy}=\frac{\frac{dx}{dt}}{\frac{dy}{dt}}=\frac{5\cos t}{5(\sin t-1)}=\frac{\cos t}{\sin t-1}$

91. Conceptual

92. Since every discontinuous function is not differentiable.

Given $f(x)=|x|$



Clearly curve has no gap

So, $f(x)$ is continuous

But at $x=0$ has sharp edge

So, $f(x)$ is not differentiable at $x=0$

93. $x^2+xy+y^2=k$

$2x+2yy'+xy'+y\cdot 1=0$

$y'(x+2y)=-(2x+y)$

$y'=-\left(\frac{2x+y}{x+2y}\right)=-2+\frac{3y}{x+2y}$

$y''=3\left[\frac{(x+2y)y'-y(x+2y)'}{(x+2y)^2}\right]$

$\Rightarrow y''=3\frac{(xy'-y)}{(x+2y)^2}\ldots(1)$

sub y' *in* (1)

$y''=\frac{3}{(x+2y)^2}\left[\frac{-2x-3xy-xy-2y^2}{x+2y}\right]$

$=\frac{-6(x^2+xy+y^2)}{(x+2y)^3}=\frac{-6k}{(x+2y)^3}$

94. $x.e^{xy}=y+\sin^2 x.....(2)$

Diff .w.r.t x

$x.e^{xy}[xy'+y]+e^{xy}=y'+2\sin x\cos x$

$\Rightarrow y'[x^2e^{xy}-1]=\sin 2x-e^{xy}-xye^{xy}$

$\Rightarrow y'=\frac{\sin 2x-e^{xy}-xye^{xy}}{x^2.e^{xy}-1}.....(2)$

Put $x=0$ in (1), $0=y+\sin^2 0\Rightarrow \boxed{y=0}$

From (2), $(y')_{(0,0)}=1$

95. Conceptual

96. $y=(\log x)^{\sin x}$

$\Rightarrow \log y=\sin x.\log(\log x)$

Diff w.r.t x

$\Rightarrow \frac{y'}{y}=\sin x.\frac{1}{\log x}\left(\frac{1}{x}\right)+\log(\log x).\cos x$

$\Rightarrow y'=(\log x)^{\sin x}\left[\frac{\sin x}{x}.\frac{1}{\log x}+\cos x.\log(\log x)\right]$

$\Rightarrow (y')_{x=\pi/2}=\frac{\log \pi/2}{\left(\frac{\pi}{2}\right)\log\frac{\pi}{2}}=\frac{2}{\pi}$

97. $f(x+y)=f(x)+f(y)-3xy$

$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$=\lim_{h\to 0}\frac{f(x)+f(h)-3xh-f(x)}{h}$

$=\lim_{h\to 0}\frac{f(h)}{h}-3x\lim_{h\to 0}\frac{h}{h}=7-3x$

98. Given

$x=a(t-\sin t)$ $\quad$ $y=a(1-\cos t)$

$\frac{dx}{dt}=a(1-\cos t)$ $\quad$ $\frac{dy}{dt}=a(\sin t)$

$\frac{dx}{dt}=a2\sin^2 t/2$ $\quad$ $\frac{dy}{dt}=2a\sin t/2\cos t/2$

$\frac{dy}{dx}=\cot t/2$

$\frac{d^2y}{dx^2}=-\operatorname{cosec}^2 t/2.\frac{1}{2}\frac{dt}{dx}$

$=\frac{-1}{2\sin^2 t/2}.\frac{1}{2a\sin^2 t/2}=\frac{-1}{4a\sin^4 t/2}$

99. Let $y=\cos^{-1}(\tanh x)+\sinh(\sin 6x)$

$\frac{dy}{dx}=\frac{-1}{\sqrt{1+\tanh^2 x}}\operatorname{sec} h^2 x$

$+\cosh[\sin 6x]\cos 6x(6)$

$\frac{dy}{dx}=-\operatorname{sec} hx+6\cos 6x.\cosh(\sin 6x)$

$=\frac{-1}{\cosh x}+6\cos 6x.\cosh(\sin 6x)$

100. Let $y=x^{\sqrt{x}}$

$\log y=\sqrt{x}\log x$

$$\frac{1}{y}\frac{dy}{dx}=\frac{1}{2\sqrt{x}}\log x+\frac{\sqrt{x}}{x}$$

$$\frac{1}{y}\frac{dy}{dx}=\frac{\log x}{2\sqrt{x}}+\frac{1}{\sqrt{x}}$$

$$=\frac{\log x+2}{2\sqrt{x}}=\frac{y(\log x+2)}{2\sqrt{x}}$$

101. $f(1^-)=-1$

$$f(1^+)=2(x-[x])-1=2(x-1)-1$$

$$=2x-3=2$$

Right hand derivative is 2.

102. $y=\tan(\mathrm{Cos}^{-1}x)$

$$\frac{dy}{dx}=\sec^2(\cos^{-1}x)\frac{d}{dx}(\cos^{-1}x)$$

$$=\sec^2(\cos^{-1}x)\frac{-1}{\sqrt{1-x^2}}=\frac{-1}{x^2\sqrt{1-x^2}}$$

103. $y=\frac{1}{x}+\cos 2x$

$$\frac{dy}{dx}=\frac{-1}{x^2}-2\sin x$$

$$\frac{d^2y}{dx^2}=\frac{2}{x^3}-4\cos 2x$$

$$=\frac{2}{x^3}+\frac{4}{x}-4y$$

104. $\frac{d}{dt}(\tan t+t^2\cos echt)$

$$=\sec^2 t-t^2\cos echt\coth t+2t\cos eht$$

105. Conceptual

106. $(fog)(x)=x$

$$f'(g(x))g'(x)=1$$

$$x=5$$

$$f'(g(5))g'(5)=1$$

$$f'(6)\times\frac{3}{4}=1\Rightarrow f'(6)=\frac{4}{3}$$

107. Given $y=x+\mathrm{Tan}\,x.....(1)$

Diff w.r.t 'x'

$$y'=1+\sec^2 x$$

Again diff w.r.t 'x'

$$y''=2\sec x.(\sec x.\tan x)$$

$$\cos^2 x.y''=2\mathrm{Tan}\,x.....(2)$$

Consider $\cos^2 x.\frac{d^2y}{dx^2}+2x$

$=2\tan x+2(y-\tan x)$ ($\because$ from (1) &(2))

$=2y$

108. $f(x)=\mathrm{Tan}^{-1}\left(\frac{2x^x}{(x^x)^2-1}\right)$

$=-2\theta$

$$f(x)=-2\mathrm{Tan}^{-1}x^x$$

$$f'(x)=\frac{-2}{1+(x^x)^2}x^x(1+\log x)$$

$$f'(1)=-1$$

109. $\frac{dy}{dx}=-\frac{\left(\frac{df}{dx}\right)}{\left(\frac{df}{dy}\right)}=-\frac{(3x^2-4xy^2+5)}{4x^2y+1}$

Find $\frac{d^2y}{dx^2}$ using $\left(\frac{U}{V}\right)'=\frac{VU'-UV'}{V^2}$

Put $(x,y)=(1,1)$

110. $9f(x)+6f\left(\frac{1}{x}\right)=x+5\rightarrow(1)$

$$x=\frac{1}{x}$$

$$8f\left(\frac{1}{x}\right)+6f(x)=\frac{1}{x}+5-(2)$$

$$4\times(1)-3\times(2)$$

$$32f(x)-18f(x)=4x+20-\frac{3}{x}-15$$

$$f(x)=\frac{1}{14}[4x-\frac{3}{x}+5]$$

$$f'(x)=\frac{1}{14}\left[4+\frac{3}{x^2}\right]$$

$$\therefore\left[x^2f'(x)+2xf(x)\right]$$

At x = 1

$$=\frac{19}{14}$$

111. $\lim_{y\to 2}\frac{1}{y-2}\left(\frac{\cancel{x}+y-2-\cancel{x}}{x(x+y-2)}\right)=\frac{1}{x^2}$

112. $3f(\cos x)+2f(\sin x)\to(1)$

$$x=\frac{\pi}{2}-x$$

$$3f(\sin x)+2f(\cos x)=\frac{5\pi}{2}-5x-(2)$$

$$3\times(1)-(2)\times 2$$

$$5f(\cos x)=5x-5\pi\Rightarrow f(\cos x)=x-\pi$$

$$f^1(\cos x)(-2\sin x)=1$$

$$f'(\cos x)=-\cos ecx$$

$$3(x-\pi)+2f(\sin x)=5x$$

$$f(\sin x)=\frac{5x-3x+3\pi}{2}$$

$$=x+3\pi/2$$

$$f'(\sin x)=\frac{1}{\cos x}=\sec x$$

$$\therefore\frac{-1}{\sin x}+\frac{1}{\cos x}$$

113. $\frac{d}{dx}\left(\frac{uv}{w}\right)=\frac{w(uv'+vu')-uvw'}{w^2}$

$$=\frac{uv}{w}\left[\frac{v'}{v}+\frac{u'}{v}-\frac{w'}{w}\right]$$

114. $\frac{dy}{dx}=\frac{g'(\theta)}{f'(\theta)}$

$$\frac{d^2y}{dx^2}=\frac{f(\theta)g''(\theta)-g'(\theta)f''(\theta)}{\left[f'(\theta)\right]^2}\times\frac{d}{dx}(\theta)$$

115. $y_1=\left(e^{\sqrt{x}}-e^{-\sqrt{x}}\right)\frac{1}{2\sqrt{x}}-(1)$

$$y_2=\left(e^{\sqrt{x}}+e^{-\sqrt{x}}\right)\frac{1}{4x}+\left(e^{-\sqrt{x}}-e^{-\sqrt{x}}\right)\frac{1}{2}\times\frac{-1}{2}x^{-3/2}$$

$$y_2=\frac{y}{yx}-\frac{1}{2}\frac{y_1}{n}\text{ (from (1) and G.E.)}$$

116. $y^2\cos^4\alpha+x^2\sin^4\alpha=2xy\sin^2\alpha\cos^2\alpha$

$2yy'4\cos^3\alpha+2x\sin^4\alpha=2\sin^2\alpha\cos^2\alpha(xy'+y)$

Take y' common and simplify.

117. Using $\log\frac{A}{B}=\log A-\log B$

Diff w.r.t. x

$$\frac{-3Ax^2}{2\sqrt{1-x^3}}\left[\frac{1-B}{\left[\sqrt{1-x^3}+B\right]\left[\sqrt{1-x^3}+1\right]}\right]=\frac{1}{x\sqrt{1-x^3}}$$

$$\frac{-3A}{2}\left[\frac{1-B}{(1+B)\left[\sqrt{1-x^3}+1\right]}-x^3\right]=\frac{1}{x^3}$$

$$\left.\begin{array}{l}\frac{3A}{2}(1-B)=1\\ \Rightarrow A=\frac{1}{3}\end{array}\right|\begin{array}{l}-(1+B)=0\\ B=-1\end{array}$$

118. $\text{siny} = x\left[\sin\alpha\cos y + \cos\alpha\sin y\right]$

$$\Rightarrow y = \tan^{-1}\left(\frac{x\sin\alpha}{1-x\cos\alpha}\right)$$

$$\frac{dy}{dx} = \frac{\sin\alpha}{1-2x\cos\alpha+x^2}$$

$m = \sin\alpha, n = \cos\alpha$

$m^2 + n^2 = 1$

119. $\frac{d}{dx}\sin\pi$ (integer)=0

120. $\log y = \log(x+1)^2$

$$+\frac{1}{2}\log(x-1) - 3\log(x+4) - x$$

$$\frac{1}{y}y' = \left[\frac{2}{x+1} + \frac{1}{2(x-1)} - \frac{3}{x+4} - 1\right]$$

$$y'_{(x=5)} \Rightarrow \frac{(5+1)^2\sqrt{4}}{9^3 e^5} = \frac{8}{81e^5}$$

121. $\tan^{-1} y = 3\tan^{-1} x$

$$\frac{1}{1+y^2}y^1 = \frac{3}{1+x^2}$$

$$y^1 = \frac{3(1+y^2)}{1+x^2}$$

$$y^{11} = \frac{(1+x^2)(6yy^1) - 3(1+y^2)2x}{(1+x^2)^2}$$

$$= 6\frac{\left((1+x^2)y.3\frac{(1+y^2)}{(1+x^2)}\right) - x(1+y^2)}{(1+x^2)^2}$$

$$= \frac{6\left[(1+y^2)(3y-x)\right]}{(1+x^2)^2}$$

122. $y = \frac{x^2+1}{(x^2+5)(x^2+9)}$

$$\log y = \log(x^2+1) - \log(x^2+5) - \log(x^2+9)$$

$$\frac{1}{y}y' = \frac{2x}{x^2+1} - \frac{2x}{x^2+5} - \frac{2x}{x^2+9}$$

$$y' = \frac{2x(x^2+1)}{(x^2+5)(x^2+9)}\left[\frac{1}{x^2+1} - \frac{1}{x^2+5} - \frac{1}{x^2+9}\right]$$

$$2(x^2+9) - (x^2+1) - (x^2+5)$$

123. $y = \frac{ax+b}{cx+d}$

Find y' using $\frac{u}{v}$ Rule then take Reciprocal

124. Conceptual

125. $\frac{dx}{d\theta} = -3a\cos^2\theta\sin\theta$

$$\frac{dy}{d\theta} = 3a\sin^2\theta\cos\theta$$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = -\text{Tan}\,\theta$$

$$\frac{d^2y}{dx^2} = -\sec^2\theta\cdot\frac{d\theta}{dx}$$

$$= -\sec^2\theta\, x\, \frac{1}{-3a\cos^2\theta\sin\theta}$$

126. $x = 2\cos 2\theta$

$$\tan^{-1}(\tan\theta)$$

$$\left.\begin{matrix} y = \sin 2\theta \\ x = \cos 2\theta \end{matrix}\right\} \frac{dy}{dx} = \frac{2\cos 2\theta}{-2\sin 2\theta}$$

$$= -\cot 2\theta$$

127. $\pm(x^2 - 3x + 2)$

$1 < x < 2 \Rightarrow -x^2 + 3x - 2 \Rightarrow -2x + 3$

$x > 2- \Rightarrow x^2 - 3x + 2 \Rightarrow 2x - 3$

128.
$$y_1 = \frac{x.\frac{1}{x} - \log x \cdot 1}{x^2} = \frac{1-\log x}{x^2}$$

$$y_2 = \left[\frac{.x^2\left(\frac{-1}{x}\right) - (1-\log x).2x}{x^4}\right]$$

Put x=1

129. $f(x) = 2x + \frac{2}{3}\left[\log(3x+5) - \log(5-3x)\right]$

$$f'(x) = 2 + \frac{2}{3}\left[\frac{3}{3x+5} + \frac{3}{5-3x}\right]$$

$$f'(1) = 2 + \frac{2}{3}\left[\frac{3}{8} + \frac{3}{2}\right] = \frac{13}{4}$$

130. $x^2 + 4 = (\cos ec\theta + \sin\theta)^2$

$$y^2 + 4 = \left(c8c^n\theta + \sin^n\theta\right)^2$$

$$\frac{dx}{d\theta} = -\cos ec\theta\cos\theta - \cos\theta$$

$$\frac{dy}{d\theta} = -n\sin^{n-1}\theta\cos\theta$$

$$\frac{dy}{dx} = \frac{n\sqrt{y^2+4}}{\sqrt{x^2+4}}$$

$$g(x) = x^2 + 4$$

$10 + \cancel{(2022)^2} - \cancel{(2022)^2} + 4 = 14$

131. Conceptual

132. $f'(x) = \cos\left(\cosh\left(\frac{x^2+1}{x^2+2}\right)\right)\sinh\left(\frac{x^2+1}{x^2+2}\right)\frac{\cancel{2}x}{(x^2+2)^{\cancel{2}}}$

$x = 1$

133. Expand the det

$$f(x) = (-\sin x)\left[4\sin^3 x + 4\sin x\cos^2 x\right]$$
$$-2\sin x\left[\cos x\sin x\right] + 4\cos^2 x\left[-\cos^2 x\right]$$
$$= -4\sin^4 x - 4\sin^2 x\cos^2 x - 4\sin^2 x\cos^2 x - 4\cos^4 x$$
$$= -4\left(\sin^4 x + \cos^4 x + 2\sin^2 x\cos^2 x\right)$$
$$= -4\left(\sin^2 x + \cos^2 x\right)^2 = -4$$

134. $-x\sin(k+y)y' + \cos(k+y) = -\sin y.y'$

$$y' = \left.\frac{\cos(k+y)}{x\sin(k+y) - \sin y}\right|_{\text{put } y=\frac{\pi}{2}}$$
$$x\sin\left(\frac{\pi}{2}+k\right) = 0$$

$$y'_{\left(\frac{\pi}{2}\right)} = \frac{\cos(k+\pi/2)}{x\sin\left(\frac{\pi}{2}+k\right)-1} = \frac{-\sin k}{-1} = \sin k$$

135. $\frac{dx}{d\theta} = a\left(-\cancel{\sin\theta} + \theta\cos\theta + \cancel{\sin\theta}\right)$

$$= a\theta\cos\theta$$

$$\frac{dy}{d\theta} = f'(\theta) \rightarrow f'(\theta)\cot\theta = dy$$

$$f(\theta) = y + c$$

$$c = -2\pi$$

$$\frac{dy}{dn} = \frac{f'(\theta)}{a\theta\cos\theta} = \frac{\tan\theta}{\theta}$$

$$\Rightarrow f'(\theta) = a\sin\theta$$

$$f(\pi/3) = a/2$$

136. Apply $(uv)' = uv' + vu'$

137. $\text{Cos}\sqrt{\frac{x^2+1}{x^2+2}}.\cos\sqrt{\frac{x^2+1}{x^2+2}}\cdot\frac{1}{2}\times\frac{\sqrt{x^2+2}}{\sqrt{x^2+1}}\times\frac{2x}{(x^2+2)^2}$

$$= \cos\left(\sqrt{\frac{x^2+1}{x^2+2}}\right)\left(\sqrt{\frac{x^2+2}{x^2+1}}\right)\left(\frac{x}{(x^2+2)^2}\right)$$

Put $x = \sqrt{2}$

138.

$$f(x)=\frac{1+\cos x}{2(\cos x-1)}=\frac{1}{2}\left(\frac{2\cos^2\frac{x}{2}}{-2\sin^2\frac{x}{2}}\right)$$

$$=\frac{-1}{2}\cot^2\frac{x}{2}$$

$$f'(x)=\frac{-1}{2}\left(2\cot\frac{x}{2}\right)\left(-\cos ec^2\frac{x}{2}\right).\frac{1}{2}$$

$$=\frac{1}{2}\cot\frac{x}{2}\cos ec^2\frac{x}{2}$$

$$3(x)=\frac{f'(x}{f(x)}=\frac{-\cos ec^2\frac{x}{2}}{\cot\frac{x}{2}}$$

$$=\frac{-1}{\cot x/2\cdot\sin x/2}=-2\text{cosec } x$$

139. $$f'(x)=\frac{\log x\left[-e^{-x}.\sin+e^{-x}\cos x\right]-\frac{e^{-x}\sin x}{x}}{(\log x)^2}$$

$$f'(x)=\frac{f'(x)}{f(x)}$$

$$=\frac{\cancel{e^x}\left\{-\log x\sin x+\cos x\right\}x-\sin x}{x(\log x)^2}\times\frac{\log x}{\cancel{e^x}\sin x}$$

$$=\frac{x\left[-\log x\sin x+\cos x\right]-\sin x}{x\log x\sin x}$$

140. Apply

$$\left(\frac{u}{v}\right)'=\frac{vu'-uv}{v^2}$$

Put x = 0

141. Conceptual

142. Apply $\left(\frac{u}{v}\right)'$ Rule

143. $$\left(x^2+y^2\right)^2=t^2+\frac{1}{+2}-2$$

$$\cancel{x^4}+\cancel{y^4}+2x^2y^2=\cancel{x^4}+\cancel{y^4}-2$$

$$x^2y^2=-1$$

$$x^2y^2+1=0$$

$$x^2\left(2yy'\right)+y^2\left(2x\right)=0$$

$$2xy\left(xy'+y\right)=0$$

$$y'=\frac{-y}{x}$$

144. $$x^x\,y^y=e^e$$

$$x\log x+y\log y=\log e^e$$

Differentiating w.r.t 'x'.

$$x\frac{1}{x}+\ln x(1)+\left(y\frac{1}{y}+\ln y\right)\frac{dy}{dx}=0$$

$$\Rightarrow\frac{dy}{dx}(1+\ln y)=-(1+\ln x)\rightarrow(1)$$

$$\therefore\left(\frac{dy}{dx}\right)_{(e,e)}=\frac{-(1+1)}{1+1}=-1$$

Again diff (1) w.r.t 'x'.

$$\frac{d^2y}{dx^2}(1+\ln y)+\frac{dy}{dx}\left(\frac{1}{y}.\frac{dy}{dx}\right)=\frac{-1}{x}$$

At the point (e,e)

$$\left(\frac{d^2y}{dx^2}\right)_{(e,e)}(1+1)+\frac{1}{e}(-1)^2=\frac{-1}{e}$$

$$\left(\frac{d^2y}{dx^2}\right)_{(e,e)}(2)=\frac{-2}{e}$$

$$\left(\frac{d^2y}{dx^2}\right)_{(e,e)}=\frac{-1}{e}=\frac{1}{e}\left(\frac{dy}{dx}\right)_{(e,e)}$$

145. $f(x)=|x-5|+|x+5|+|x-4|+|x+4|$

$$=\begin{cases}-4x, \text{when } x \le -5\\ 10-2x, \text{when } -5<x<-4\\ 18, when -4 \le x<4\\ 2x+10, when\ 4 \le x<5\\ 4x, when\ x \ge 5\end{cases}$$

$$f'(x)=\begin{cases}-4, \text{when } x \le -5\\ -2, \text{when } -5<x<-4\\ 0, when -4 \le x<4\\ 2, when\ 4 \le x<5\\ 4, when\ x \ge 5\end{cases}$$

$$\frac{f'(1)-f'(-6)}{f'(-1)+f'(6)}=\frac{0-(-4)}{0+4}=1$$

146. f (x+ay)+g(x-ay)=0

Diff. w. rt 'x'

$$f'(x+ay)[1+a\frac{dy}{dx}]+g'(x-ay)\left[1-a\frac{dy}{dx}\right]=0$$

$$\Rightarrow f'(x+ay)+g'(x-ay)=\left[g'(x-ay)-f'(x+ay)\right]a\frac{dy}{dx}$$

$$\Rightarrow a\frac{dy}{dx}=\frac{f'(x+ay)+g'(x-ay)}{g'(x-ay)-f'(x+ay)}$$

147. Given $y = x \log\left[\frac{1}{ax}+\frac{1}{a}\right]$

$$y=x\log\left[\frac{1+x}{ax}\right]$$

$$y=x\log(1+x)-x\log ax$$

$$\frac{dy}{dx}=\frac{x}{1+x}+\log(1+x)-\frac{x}{x}-\log\ ax$$

$$=\frac{x}{1+x}-1+\frac{y}{x}$$

$$=\frac{x^2-x(1+x)+y(1+x)}{x(1+x)}$$

$$x(1+x)\frac{dy}{dx}=y(1+x)-x$$

$$x(1+x)\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=-1$$

148. conceptual

149. $u=\sin\left(\frac{x}{y}\right); x=e^t, y=t^2$

$$\Rightarrow u=\sin\left(\frac{e^t}{t^2}\right)$$

$$\frac{du}{dt}=\cos\left(\frac{e^t}{t^2}\right)\frac{t^2e^t-e^t.2t}{t^4}$$

$$\left(\frac{du}{dt}\right)^2=\frac{\cos^2\left(\frac{e^t}{t^2}\right)e^{2t}(t-2)^2}{t^6}$$

$$t^6\left(\frac{du}{dt}\right)^2=\cos^2\left(\frac{e^t}{t^2}\right)e^{2t}(t-2)^2$$

$$\Rightarrow\frac{t^6\left(\frac{du}{dt}\right)^2}{e^{2t}(t-2)^2}=\cos^2\left(\frac{e^t}{t^2}\right)=1-u^2$$

150. $\frac{x^2}{(x+2)(2x+3)}=\frac{1}{2}+\frac{p}{x+2}+\frac{q}{2x+3}$

$p=-4,\ q=\frac{9}{2}$

$$\therefore\frac{d}{dx}\left(\frac{x^2}{(x+2)(2x+3)}\right)=\frac{d}{dx}\left(\frac{1}{2}\right)+\frac{d}{dx}\left(\frac{-4}{x+2}\right)+\frac{d}{dx}\left(\frac{\frac{9}{2}}{2x+3}\right.$$

$$=0+\frac{4}{(x+2)^2}+\frac{-9}{2}\ \frac{1}{(2x+3)^2}(2)$$

$$=\frac{4}{(x+2)^2}-\frac{9}{(2x+3)^2}$$

$\therefore A=4 \quad B=-9 \quad \Rightarrow A+B=-5$

151. $f^1(t)=\frac{1}{2}+\frac{1}{4}\left(\frac{2}{2t-1}\right)=\frac{1}{2}+\frac{1}{2}\left(\frac{1}{2t-1}\right)$

$$f^1\left(\frac{t+1}{2t+1}\right)=\frac{1}{2}+\frac{1}{2}\left(\frac{1}{2\left(\frac{t+1}{2t+1}\right)-1}\right)=\frac{1}{2}+\frac{1}{2}\left(\frac{2t+1}{1}\right)$$

$$=\frac{1}{2}(2t+2)=t+1$$

$$\therefore f^1\left(\frac{t+1}{2t+1}\right)=t+1$$

152. $x^3+y^3=3axy$

Diff on both sides

$$3x^2+3y^2\frac{dy}{dx}=3a\left(x\frac{dy}{dx}+y(1)\right)$$

$$\frac{dy}{dx}=\frac{x^2-ay}{ax-y^2}$$

$$m=\left(\frac{dy}{dx}\right)_{\left(\frac{3a}{2},\frac{3a}{2}\right)}=-1$$

$$y^{\cdot}=\frac{\left(2x-ay^{\cdot}\right)\left(ax-y^2\right)-\left(x^2-ay\right)\left(a-2yy^{\cdot}\right)}{\left(ax-y^2\right)^2}$$

$$\left(y^{\cdot}\right)_{\left(\frac{3a}{2},\frac{3a}{2}\right)}=\frac{-32}{3a}$$

$$3ay^{\cdot}=-32$$

$$3ay^{\cdot}+40=8$$

153. $$f^{1}(x)=\sqrt{\left(f(x)\right)^2-1}$$

$$\int\frac{f^{1}(x)}{\sqrt{\left(f(x)\right)^2-1}}dx=\int 1dx$$

$$\cos h^{-1}\left[f(x)\right]=x+c$$

Put x = 0

$$\cos h^{-1}\left[f(0)\right]=0+C$$

$$\cos h^{-1}(1)=C$$

$$C=0$$

$$\cos h^{-1}\left[f(x)\right]=x$$

$$f(x) = \cosh x$$

$$f(x)=\frac{e^x+e^{-x}}{2}$$

$$f(1)=\frac{e^{1}+e^{-1}}{2}$$

$$=\frac{e^2+1}{2e}$$

154. $$\frac{d}{dx}\left(Tan^{-1}\left(\cos\sqrt{x}\right)+\sec^{-1}\left(e^x\right)\right)$$

$$=\frac{1}{\sqrt{1-\left(\cos\sqrt{x}\right)^2}}\left(-\sin\sqrt{x}\right)\left(\frac{1}{2\sqrt{x}}\right)+\frac{1}{e^x\sqrt{\left(e^x\right)^2-1}}e^x$$

$$Put\ x=\frac{\pi^2}{4}$$

$$\Rightarrow\frac{-1}{\pi}+\frac{1}{\sqrt{e^{\frac{\pi^2}{2}}-1}}$$

155.

$$Let\ f(x)=x^4+ax^3+\frac{3x^2}{2}+1$$

$$f^{1}(x)=4x^3+3ax^2+3x$$

$$f^{11}(x)=12x^2+6ax+3$$

If $f^{11}(x)$ is positive then

$$f^{11}(x)>0$$

$$12x^2+6ax+3>0$$

$$4x^2+2ax+1>0$$

Discriminate $\Delta=0$

$$\Rightarrow 4a^2-16=0\Rightarrow a^2=4$$

$$a=2$$

156.

Given any=$\cot\left(\frac{\pi}{\mu}-x\right)(-1)$

Diffw.r.t 'x'

$$\mathrm{Sec^2 y}\frac{dy}{dx}=-\cos ec^2\left(\frac{\pi}{\mu}-x\right)(-1)$$

$$\therefore\frac{dy}{dx}=\frac{\sec^2\left(\frac{\pi}{\mu}+x\right)}{1+\mathrm{Tan}^2\left(\frac{\pi}{\mu}+x\right)}$$

157. Given $x=3\sqrt{2}\cos^3\theta$ & $y=4\,\mathrm{Tan}^2\theta$

$$\frac{dx}{d\theta}=3\sqrt{2}\left(3\cos^2\theta(-\sin\theta)\right)$$

$$\&\frac{dy}{d\theta}=4(2\,\mathrm{Tan}\,\theta.\mathrm{Sec}^2\theta)$$

$$\frac{dy}{dx}=\frac{\left(dy|d\theta\right)}{\left(dy|d\theta\right)}=\frac{8\,\mathrm{Tan}\,\theta.\mathrm{Sec}^2\theta}{-9\sqrt{2}\cos^2\theta\sin\theta}$$

$$Now\left(\frac{dy}{dx}\right)_{at\ \theta=\frac{\pi}{4}}=\frac{-8.\tan 45^e.\sec^2 45^e}{9\sqrt{2}\,\mathrm{Cos}^2 45^o\sin 45^e}$$

$$=\frac{-8(1)(2)}{9\sqrt{2}\left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right)}=\frac{-32}{9}$$

158.

$$Let\ f(x)=\frac{1-x^2}{1+x^2}$$

$$put\ x=Tan\theta\Rightarrow\theta=Tan^{-1}x$$

$$= \frac{1-Tan^2\theta}{1+Tan^2\theta} = \cos 2\theta$$

$$f^1(x) = -2\sin 2\theta$$

$$\& g(x) = \frac{2x}{1+x^2} = \frac{2Tan\theta}{1+Tan^2\theta} = \sin 2\theta$$

$$g^1(x) = 2.\cos 2\theta$$

$$\frac{f^1(x)}{g^1(x)} = \frac{-2\sin 2\theta}{2\cos 2\theta} = -Tan2\theta$$

$$= -Tan\left(2\left(Tan^{-1}x\right)\right)$$

$$= -Tan\left(Tan^{-1}\left(\frac{2x}{1-x^2}\right)\right)$$

$$= \frac{-2x}{1-x^2} = \frac{-4}{1-4} = \frac{4}{3}$$

159. $\sec\left(\log_2 y^2\right) = \cos ec\left(\log_2 x^2\right)$

$$\sec\left(\log_2 y^2\right) = \sec\left(\frac{\pi}{2} - \log_2 x^2\right)$$

$$\log_2 y^2 = \frac{\pi}{2} - \log_2 x^2$$

$$\log_2 x^2 + \log_2 y^2 = \frac{\pi}{2}$$

$$= x^2y^2 = 2^{\frac{\pi}{2}} \qquad \Rightarrow \frac{dy}{dx} = \frac{-y}{x}$$

160. $e^x = y + \sqrt{y^2-1}$ $\left(\cos h^{-1}(y) = \log\left(y+\sqrt{y^2-1}\right)\right)$

$$\log e^x = \log\left(y+\sqrt{y^2-1}\right)$$

$$x = \cos h^{-1}(y) \Rightarrow \cos h\, x = y$$

$$\therefore \frac{dy}{dx} = \sin hx$$

161. $x = \log P, \qquad y = \frac{1}{P}$

$$e^x = P \qquad y = \frac{1}{e^x}$$

$$\boxed{\frac{dy}{dx} = -\bar{e}^x}$$

162. Conceptual

163.
$$f(x) = \begin{cases} 3-2x, & if\ x<1 \\ 1, & \text{if } 1 \le x < 2 \\ 2x-3, & if\ x \ge 2 \end{cases}$$

$$f'(x) = \begin{cases} -2, & if \quad x<1 \\ 0, & if \quad 1 \le x \le 2 \\ 2, & if \quad x \ge 2 \end{cases}$$

$$= -2+0+2 = 0$$

164.
$$f(x) = \frac{e^{4x}-1}{e^{2x}} \times \frac{e^{3x}}{e^{6x}+1}$$

$$= \frac{e^{4x}-1}{e^{6x}+1} \times e^x$$

$$f(x) = \frac{e^{4x}-1}{e^{5x}+e^{-x}}$$

$$f'(x) = \frac{\left(e^{5x}+e^{-x}\right)\left[4e^{4x}\right]}{\left(e^{5x}+e^{-x}\right)^2} + \left[e^{4x}-1\right]\left[5.e^{5x}-e^{-x}\right]$$

$$f'(0) = \frac{(1+1)(4)}{(1+1)^2} = \frac{8}{4} = 2$$

165. Let $f(x) = x^{\tan x} + (Tan\, x)^x$

Let $y = u + v$

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \ldots\ldots\ldots(1)$$

Put $x = \frac{\pi}{4}$

166. $2x^2 - 3xy + y^2 + x + 2y - 8 = 0$

$$4x - 3\left(x\frac{dy}{dx} + y\right) + 2y\frac{dy}{dx} + 1 + 2\frac{dy}{dx} = 0$$

$$(4x-3y+1) + \frac{dy}{dx}(-3x+2y+2) = 0$$

$$(-3x+2y+2)dy - (-4x+3y-1)\,dx = 0$$

$$f(x,y) = -3x+2y+2$$

$$f(1,1) = -3+2+2 = 1$$

$$g(x,y) = -4x+3y-1$$

$$g(2,2) = -8+6-1 = -3$$

$$\frac{g(2,2)}{f(1,1)} = -\frac{3}{1} = -3$$

167. $f(x) = e^x$

$h(x) = (fof)(x)$

$= f(f(x))$

$= f(e^x)$

$h(x) = e^{e^x}$

$\log\ h(x) = e^x$

$\frac{1}{h(x)} h^1(x) = e^x = \log\ h(x)$

168. $\sin y = \sin 3t \quad , \quad x = \sin t$

$y = 3\ t \qquad t = \sin^{-1} x$

$y = 3\sin^{-1} x$

$\frac{dy}{dx} = \frac{3}{\sqrt{1-x^2}}$

169. $f(x) = \sqrt{\log\left(x^2+x+1\right) + \sqrt{losh(2x-3)}}, then\ f^1(0)$

$f^1(x) = \frac{1}{2\sqrt{log\left(x^2+x+1\right)+\sqrt{\cosh(2x-3)}}} \cdot \left(\frac{1}{x^2+x+1}(2x+1) + \frac{2\sinh(2x-3)}{2\sqrt{\cosh(2x-3)}}\right)$

$f^1(0) = \frac{1}{2\sqrt{\sqrt{\cosh(3)}}}\left(1 - \frac{\sinh(3)}{\sqrt{cosh(3)}}\right)$

$= \frac{\sqrt{\cosh(3)-\sinh(3)}}{2\left(\cosh(3)\right)^{3/4}}$

170. $x = \cos^3\theta - \sin^3\theta, y = (\cos\theta)^{\frac{1}{3}} - (\sin\theta)^{\frac{1}{3}}$

$\frac{dy}{dx} at\ \theta = \frac{\pi}{4}$

$\frac{dx}{d\theta} = 3\cos^2\theta(-\sin\theta) - 3\sin^2\theta(\cos\theta)$

$= 3\sin\theta\cos\theta[-\cos\theta - \sin\theta]$

$\frac{dy}{d\theta} = \frac{1}{3}(\cos\theta)^{-2/3}(-\sin\theta) - \frac{1}{3}(\sin\theta)^{\frac{-2}{3}}(\cos\theta)$

$= \frac{-1}{3}\left[\frac{(\sin\theta)^{5/3} + (\cos\theta)^{5/3}}{(\sin\theta)^{2/3}(\cos\theta)^{2/3}}\right]$

$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{\frac{-1}{3}\left[\frac{(\sin\theta)^{5/3} + (\cos\theta)^{5/3}}{(\sin\theta)^{2/3}(\cos\theta)^{2/3}}\right]}{3\sin\theta\cos\theta[-\cos\theta - \sigma\iota\nu\theta]}$

$= \frac{\frac{-1}{3}\left[\frac{1}{2^{5/6}} + \frac{1}{5^{5/6}}\right]}{\frac{1}{2^{1/3}}\frac{1}{2^{1/3}}\frac{-3}{\sqrt{2}}}$

$= \frac{+1}{3}\left[\frac{2.2^{5/6}}{2^{5/3}}\right].2^{2/3}.\frac{\sqrt{2}}{3}$

$= \frac{2^{4/3}}{9} = \frac{2}{9}.\sqrt[3]{2}$

171. $2x^2 + 3xy - y^2 + 4x - 5y + 6 = 0$

$4x + 3\left[xy^1 + y\right] - 2yy^1 + 4 - 5y^1 = 0$

$4x + 3xy^1 + 3y - 2yy^1 + 4 - 5y^1 = 0$

$y^1\left[3x - 2y - 5\right] = -4x - 3y - 4$

$y^1 = \frac{-4x-3y-4}{3x-2y-5}$

At P (1, -2)

$\frac{dy}{dx} = \frac{-4+6-4}{3+4-5} = \frac{-2}{2} = -1$