39. ELLIPSE

1. Find the eccentricity of an ellipse, if the length of its latusrectum is 4 units and distance between its vertex and the nearest focus is 3/2 units. **[AP EAMCET 21-09-20_Shift-1]**

1. $\frac{1}{3}$
2. $\frac{2}{3}$
3. $\frac{1}{9}$
4. $\frac{3}{4}$

2. For the ellipse $\frac{x^2}{18}+\frac{y^2}{32}=1$, if a tangent with slope $\frac{-4}{3}$ intersects the major and minor axes at P and Q respectively, find P and Q.

[AP EAMCET 21-09-20_Shift-1]

1. P(6,0), Q(0,8)
2. P(0,6), Q(8,0)
3. P($3\sqrt{2}$,0), Q(0,$4\sqrt{2}$)
4. P(0,$3\sqrt{2}$), Q($4\sqrt{2}$,0)

3. The line $x=m^2$ meets an ellipse $9x^2+y^2=9$ in the real and distinct points if the only if

[AP EAMCET 21-09-20_Shift-2]

1. $|m|>1$
2. $|m|<1$
3. $|m|>2$
4. $|m|<2$

4. Find the condition for the line ax+by+c=0 to be a normal to an ellipse $\frac{x^2}{4}+\frac{y^2}{36}=1$

[AP EAMCET 21-09-20_Shift-2]

1. $\frac{1}{a^2}+\frac{1}{b^2}=\frac{144}{c^2}$
2. $\frac{1}{a^2}+\frac{1}{b^2}=\frac{128}{c^2}$
3. $\frac{1}{a^2}+\frac{9}{b^2}=\frac{256}{c^2}$
4. $\frac{1}{a^2}+\frac{9}{b^2}=\frac{32}{c^2}$

5. The number of values of $'c'$ for which the line $y=4x+c$ touches the ellipse $\frac{x^2}{4}+\frac{y^2}{1}=1$ is

[AP EAMCET 22-09-20_Shift-1]

1. 0
2. 1
3. 2
4. Infinite

6. The equation of the tangent to the ellipse $x^2+16y^2=16$ which makes an angle 600 with the x-axis is ___

[AP EAMCET 22-09-20_Shift-1]

1. $\sqrt{3}x-y+7=0$
2. $\sqrt{3}x-y-7=0$
3. $\sqrt{3}x+y-7=0$
4. $\sqrt{3}x-y=0$

7. The eccentricity of the ellipse $4x^2+25y^2=100$ is **[AP EAMCET 22-09-20_Shift-2]**

1. $\frac{\sqrt{21}}{5}$
2. $\frac{\sqrt{21}}{2}$
3. $\frac{\sqrt{21}}{4}$
4. $\frac{\sqrt{21}}{25}$

8. If the line $y=2x+c$ touches the curve $x^2+4y^2=4$, then $c^2=$

[AP EAMCET 23-09-20_Shift-1]

1. $\sqrt{65}$
2. 17
3. 63
4. 8

9. Let P, Q be the foci of an ellipse and let R be one end of its minor axis. If ΔPQR is an equilateral triangle, then the eccentricity of the ellipse is equal to

1. $\frac{1}{2}$
2. $\frac{1}{4}$
3. $\frac{1}{8}$
4. $\frac{1}{6}$

10. The eccentricity of the ellipse of minor-axis 2b, if the line segment joining the foci subtend an angle 2α at the upper vertex is equal to ___

1. $\cos\alpha$
2. $\sin\alpha$
3. $\tan\alpha$
4. $\sec\alpha$

11 The equation of the ellipse in the standard form whose length of the latus rectum is 4 and whose distance between the foci is $4\sqrt{2}$, is

[TS EAMCET 09-09-20_Shift-1]

1. $\frac{x^2}{2}+\frac{y^2}{3}=1$
2. $2x^2+y^2=8$
3. $x^2+2y^2=16$
4. $x^2+5y^2=25$

12. For k>0,the shortest distance from a point P(1,k) on the ellipse $9x^2+4y^2-18x+16y-11=0$ to one of its directrix is

[TS EAMCET 09-09-20_Shift-2]

1. $3-\sqrt{5}$
2. $3+\sqrt{5}$
3. $\frac{9}{\sqrt{5}}-3$
4. $\frac{9}{\sqrt{5}}-2$

13. A tangent drawn at a point on the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ cuts the X-axis at a point A .If A' be the image of A with respect to the line y=x, then the circle with AA' as its diameter passes through the fixed point

[TS EAMCET 09-09-20_Shift-2]

1. (0,–4)
2. (0,4)
3. (0,0)
4. (1,1)

14. The eccentricity of an ellipse passing through $\left(3\sqrt{2},\sqrt{10}\right)$ with foci at (–4,0) and (4,0) is

[TS EAMCET 10-09-20_Shift-1]

1. $\frac{1}{2}$
2. $\frac{2}{3}$
3. $\frac{\sqrt{2}}{3}$
4. $\frac{1}{\sqrt{3}}$

15. If the product of the lengths of the perpendiculars drawn from the foci to the tangent $y=\frac{-3}{4}x+3\sqrt{2}$ of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is 9 ,then the eccentricity of that ellipse is

[TS EAMCET 10-09-20_Shift-1]

1. $\frac{\sqrt{2}}{3}$
2. $\frac{\sqrt{5}}{6}$
3. $\frac{1}{9}$
4. $\frac{\sqrt{7}}{4}$

16. The ellipse having foci$(0,\pm1)$ and major axis of length $\sqrt{5}$ is

[TS EAMCET 10-09-20_Shift-2]

1. $20x^2+4y^2=5$
2. $36x^2+20y^2=45$
3. $4x^2+20y^2=5$
4. $20x^2+36y^2=45$

17. An ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with eccentricity $\frac{2\sqrt{2}}{3}$ is inscribed in a circle $x^2+y^2=18$ such that the length of its major axis is equal to the diameter of this circle. The locus of the poles of all the tangents of the circle with respect to the ellipse is **[TS EAMCET 10-09-20_Shift-2]**

1. $x^2+y^2=\frac{8}{9}$
2. $18x+\frac{2y}{9}=1$
3. $\frac{x^2}{18}+\frac{y^2}{9}=1$
4. $\frac{x^2}{18}+\frac{9y^2}{2}=1$

18. If the sum of the distances from the foci to the centre O(0,0) of an ellipse is $8\sqrt{6}$ units and the area of the smallest rectangle in which that ellipse is inscribed is 80sq.units,the the equation of such an ellipse is

[TS EAMCET 11-09-20_Shift-1]

1. $\frac{x^2}{100}+\frac{y^2}{64}=1$
2. $\frac{x^2}{100}+\frac{y^2}{16}=1$
3. $\frac{x^2}{10}+\frac{y^2}{4}=1$
4. $\frac{x^2}{100}+\frac{y^2}{4}=1$

19. The equation of the ellipse with directrix 3x+4y-5=0,focus (1,2) and eccentricity $\frac{1}{2}$,is

[TS EAMCET 11-09-20_Shift-1]

1. $x^2+84y^2-24xy-360y+170x+475=0$
2. $91x^2+84y^2-24xy-170x-360y+475=0$
3. $91x^2+84y^2-24xy-170x+360y+475=0$
4. $91x^2+84y^2-24xy-170x-360y-475=0$

20. If $\frac{\pi}{3}, \theta$ are the eccentric angles of the ends of a focal chord of the ellipse $\frac{x^2}{16}+\frac{y^2}{12}=1$ then $\tan\theta =$ **[TS EAMCET 11-09-20_Shift-2]**

1. $-\sqrt{3}$ 2. $\sqrt{3}$
3. -1 4. $\frac{1}{\sqrt{2}}$

21. If x+2y+k=0, k>0 is a tangent to the ellipse $2x^2+y^2=2$, then the equation of the normal to the given ellipse at $\left(\frac{1}{\sqrt{2}}, \frac{k}{3}\right)$, is

[TS EAMCET 11-09-20_Shift-2]

1. $\sqrt{2}x-2y+1=0$ 2. $3\sqrt{2}x-y-2=0$
3. $2\sqrt{2}x-5y+3=0$ 4. $\sqrt{2}x+3y-4=0$

22. If tangents are drawn to the ellipse $x^2+2y^2=2$, then the locus of the mid points of the intercepts made by those tangents between the coordinate axis is

[TS EAMCET 14-09-20_Shift-2]

1. $\frac{x^2}{2}+\frac{y^2}{4}=1$ 2. $\frac{x^2}{4}+\frac{y^2}{2}=1$
3. $\frac{1}{2x^2}+\frac{1}{4y^2}=1$ 4. $\frac{1}{4x^2}+\frac{1}{2y^2}=1$

23. The area (in sq.units) of the quadlateral formed by the tangents drawn at the end points of the latus recta to the ellipse $S \equiv \frac{x^2}{16}+\frac{y^2}{12}=1$

[TS EAMCET 14-09-20_Shift-2]

1. 96 2. 16
3. 128 4. 64

24. If a point $P(x,y)$ moves along the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ and if c is the center of the ellipse, then the sum of maximum and minimum values of CP is__ **[AP EAMCET 19-08-2021_Shift-1]**

1. 25 2. 9
3. 4 4. 5

25. In an ellipse, if the distance between the foci is 6 units and length of its minor axis is 8 units, then its eccentricity is_____

[AP EAMCET 19-08-2021_Shift-2]

1. $\frac{1}{2}$ 2. $\frac{7}{5}$
3. $\frac{1}{\sqrt{5}}$ 4. $\frac{3}{5}$

26. If $\tan\theta_1, \tan\theta_2 = \frac{-a^2}{b^2}$, then the chord joining 2 points θ_1 and θ_2 on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ will subtended a right angle at ______

[AP EAMCET 20-08-2021_Shift-1]

1. Focus 2. Center
3. End of major axis 4. End of minor axis

27. The point $(1,3)$ with respect to the ellipse $4x^2+9y^2-16x-54y+61=0$ lies____

[AP EAMCET 23-08-2021_Shift-1]

1. outside the ellipse 2. on the ellipse
3. on the minor axis 4. on the major axis

28. Find the equation of the ellipse which passes through the points (–3, 1) and (2, –2), whose center lies at (0,0) and major axis lies along the x–axis.

[AP EAMCET 24-08-2021_Shift-1]

1. $3x^2+5y^2=32$ 2. $5x^2+3y^2=32$
3. $5x^2-3y^2=32$ 4. $3x^2+5y^2=132$

29. If the lines $2x-y+3=0$ and $4x+ky+3=0$ are conjugate with respect to the ellipse $5x^2+6y^2-15=0$, then 'k' equals _____

[AP EAMCET 24-08-2021_Shift-2]

1. 1
2. 2
3. 3
4. 6

30 The equation of an ellipse in its standard form, given the distance between its foci is 2 units and the length of its latus rectum is $\frac{15}{2}$ units, is ____ **[AP EAMCET 25-08-2021_Shift-2]**

1. $15x^2+4y^2=15$
2. $4x^2+15y^2=60$
3. $15x^2+16y^2=240$
4. $16x^2+15y^2=40$

31. The centre of ellipse $x^2+2y^2-4x+12y+14=0$ is ___ **[AP EAMCET 25-08-2021_Shift-1]**

1. (–2, –3)
2. (–2, 3)
3. (2, –3)
4. (2, 6)

32. P is a point on the conic $a^2x^2+b^2y^2=a^2\left(a^2+b^2-y^2\right)$ and S is focus of that Conic. M is the foot of the perpendicular from P on to a directrix of that conic nearer to S. If $PM=KSP$, then $K=$

[TS EAMCET 04-08-2021_Shift-2]

1. $\frac{b}{\sqrt{a^2+b^2}}$
2. $\frac{\sqrt{a^2+b^2}}{b}$
3. $\frac{a}{\sqrt{a^2+b^2}}$
4. $\frac{\sqrt{a^2+b^2}}{a}$

33. For the ellipse $4(x-2y+1)^2+9(2x+y+2)^2=25$, which of the following is true?

[TS EAMCET 04-08-2021_Shift-2]

1. equation of major axis is $x-2y+1=0$
2. eccentricity $\frac{\sqrt{5}}{3}$
3. length of major axis is $\frac{1}{2}\sqrt{5}$
4. centre $(-1,0)$

34. A focus of an ellipse having eccentricity $\frac{1}{2}$ is at $(0,0)$ and a directrix is the line $x=4$. Then the equation of one such ellipse is

[TS EAMCET 04-08-2021_Shift-1]

1. $\frac{9x^2}{64}+\frac{3y^2}{16}=1$
2. $\frac{(2x+1)^2}{32}+\frac{y^2}{16}=1$
3. $\frac{(3x+4)^2}{64}+\frac{y^2}{32}=1$
4. $(3x+4)^2+12y^2=64$

35. The area (in square units) of the quadrilateral formed by joining the focii of the two ellipse $\frac{x^2}{9}+\frac{y^2}{5}=1$ and $\frac{x^2}{5}+\frac{y^2}{9}=1$ is

[TS EAMCET 04-08-2021_Shift-1]

1. 4
2. 2
3. 6
4. 8

36. An ellipse has its major axis along y-axis and minor axis along x-axis. If its length of latus rectum is $\frac{2}{3}$ times of its minor axis then the eccentricity of the ellipse is

[TS EAMCET 05-08-2021_Shift-1]

1. $\frac{2}{3}$
2. $\frac{3}{5}$
3. $\frac{\sqrt{5}}{3}$
4. $\frac{\sqrt{2}}{5}$

37. If the length and breadth of a rectangle of maximum area that can be inscribed in an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ are $8\sqrt{2}$ and $4\sqrt{2}$ respectively, then the eccentricity of that ellipse is

[TS EAMCET 05-08-2021_Shift-1]

1. $\frac{1}{2}$
2. $\frac{\sqrt{3}}{2}$
3. $\frac{1}{4}$
4. $\frac{1}{\sqrt{3}}$

38. Given the ellipse $(E)\ 4x^2+9y^2-36=0$, the circle $(C)\ x^2+y^2-9=0$ and two points $A(1,2), B(2,1)$, which of the following is correct?

[TS EAMCET 05-08-2021_Shift-2]

1. B lies inside C but outside E
2. B lies outside both C and E
3. A lies inside both C and E
4. A lies inside C ,but outside E

39. If a circle $(x-1)^2+y^2=r^2$ touches the ellipse $x^2+4y^2=16$ internally, then $r=$

[TS EAMCET 05-08-2021_Shift-2]

1. $\sqrt{\frac{11}{3}}$ 2. $\frac{11}{3}$

3. $\sqrt{\frac{15}{2}}$ 4. 2

40. The sum of the focal distances of the point $\left(\frac{4}{\sqrt{5}},\frac{3}{\sqrt{5}}\right)$ on the ellipse $9x^2+4y^2=36$ is

[TS EAMCET 06-08-2021_Shift-2]

1. 12 2. 4

3. 9 4. 6

41. If F_1 and F_2 are the foci of the ellipse $16x^2+25y^2=400$ and P is any point on it, then the value of the product $PF_1.PF_2$ lie in the interval

[TS EAMCET 06-08-2021_Shift-2]

1. $[16,25]$ 2. $[0,16]$

3. $[25,400]$ 4. $[0,400]$

42. The coordinates of any point, in the parametric form, on the ellipse whose foci are $(-2,0),(0,8)$ and eccentricity $\frac{1}{\sqrt{2}}$, is

[TS EAMCET 06-08-2021_Shift-1]

1. $\left(5\sqrt{2}\cos\theta, 5\sin\theta\right)$
2. $\left(3+5\sqrt{2}\cos\theta, 5\sin\theta\right)$
3. $\left(3+5\cos\theta, 5\sqrt{2}\sin\theta\right)$
4. $\left(5\cos\theta, 3+5\sqrt{2}\sin\theta\right)$

43. The eccentric angle of a point on the ellipse $x^2+3y^2=6$ lying at a distance of 2 units from its centre is

[AP EAMCET 04-07-2022_Shift-1]

1. $\frac{\pi}{6}$ 2. $\frac{\pi}{4}$

3. $\frac{\pi}{3}$ 4. $\frac{\pi}{2}$

44. The focal distances of the point $\left(\frac{4}{\sqrt{5}},\frac{3}{\sqrt{5}}\right)$ on the ellipse $\frac{x^2}{4}+\frac{y^2}{9}=1$ are

[AP EAMCET 04-07-2022_Shift-2]

1. $\frac{10}{3},\frac{2}{3}$ 2. 3,1

3. $\frac{13}{3},\frac{5}{3}$ 4. 4,2

45. If the angle between the straight lines joining the foci and the ends of the minor axis of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is 90^0, then its eccentricity is **[AP EAMCET 05-07-2022_Shift-1]**

1. $\frac{1}{2}$ 2. $\frac{1}{4}$

3. $\frac{1}{3}$ 4. $\frac{1}{\sqrt{2}}$

46. The distance between the directrices of the ellipse $\frac{x^2}{36}+\frac{y^2}{20}=1$ is

[AP EAMCET 05-07-2022_Shift-2]

1. 9
2. $6\sqrt{5}$
3. 18
4. $3\sqrt{5}$

47. If the latus rectum of an ellipse is equal to half of minor axis, then its eccentricity is

[AP EAMCET 06-07-2022_Shift-1]

1. $\frac{\sqrt{3}}{4}$
2. $\frac{3}{4}$
3. $\frac{1}{4}$
4. $\frac{\sqrt{3}}{2}$

48. The values of λ, for which $(\lambda, \lambda\text{-}2)$ lies inside the ellipse $4x^2+9y^2=36$ and outside the parabola $y^2=x$, satisfy

[AP EAMCET 06-07-2022_Shift-2]

1. $0<\lambda<1$
2. $0\le\lambda\le1$
3. $0<\lambda<\frac{36}{13}$
4. $\lambda\notin[1.4]$

49. An ellipse has 6 and 2 as lengths of major and minor axes respectively. If the centre is at (5,6) and the major axis is along x-y+1=0, then the ellipse is

[AP EAMCET 07-07-2022_Shift-1]

1. $(x+y-11)^2+9(x-y+1)^2=18$
2. $(x+y+11)^2+9(x+y-1)^2=18$
3. $(x+y)^2+9(x-y)^2=18$
4. $(x-y-11)^2+9(x+y+1)^2=18$

50. The foci of the ellipse $9x^2+25y^2=225$ are

[AP EAMCET 07-07-2022_Shift-2]

1. $(\pm4,0)$
2. $\left(\pm\frac{4}{5},0\right)$
3. $\left(\pm\frac{2}{5},0\right)$
4. $\left(\pm\frac{12}{5},0\right)$

51. Let $E_1=\frac{x^2}{9}+\frac{y^2}{4}=1$ and $E_2=\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ be two ellipses and R be a rectangle with sides parallel to the coordinate axes. Let E_1 be inscribed ellipse in R and E_2 be circumscribed ellipse on R. if E_2 passes through (0,4) then

[AP EAMCET 08-07-2022_Shift-1]

1. $a=4, b=2\sqrt{3}$
2. $a=12, b=16$
3. $a=16, b=16$
4. $a=2\sqrt{3}, b=4$

52. For α belonging to an interval of length β, suppose $(\alpha,-\alpha)$ is an interior point of the ellipse $4x^2+5y^2=1$. Then $(6\beta-4)^{201}+201=$

[AP EAMCET 08-07-2022_Shift-2]

1. 202
2. 0
3. 402
4. 201

53. If the eccentricity and the length of the latus rectum of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ are $\frac{\sqrt{3}}{2}$ and 1 respectively, then the sum of the lengths of major axis and minor axis of the ellipse is

[TS EAMCET 18-07-2022_Shift-1]

1. 6
2. 3
3. 10
4. 8

54. The parametric equations of the ellipse whose foci are (–3,0), (9,0) and eccentricity is $\frac{1}{3}$, are

[TS EAMCET 18-07-2022_Shift-1]

1. $x=3+12\sqrt{2}\cos\theta, y=18\sin\theta$
2. $x=3+18\cos\theta, y=12\sqrt{2}\sin\theta$
3. $x=18\cos\theta, y=3+12\sqrt{2}\sin\theta$
4. $x=3+4\sqrt{2}\cos\theta, y=18\sin\theta$

55. Let $S \equiv \frac{x^2}{a^2}+\frac{y^2}{b^2}-1=0, S' \equiv \frac{x^2}{\alpha^2}+\frac{y^2}{\beta^2}-1=0$ be two intersecting ellipses. If $p(a\cos\theta, b\sin\theta)$ and $Q\left(a\cos\left(\frac{\pi}{2}+\theta\right), b\sin\left(\frac{\pi}{2}+\theta\right)\right)$ are their points of intersection then $\frac{1}{2}\left(a^2\beta^2+b^2\alpha^2\right)=$

[TS EAMCET 18-07-2022_Shift-2]

1. a^2b^2
2. $\alpha^2+\beta^2$
3. a^2+b^2
4. $\alpha^2\beta^2$

56. $P(\theta_1)$ and $Q(\theta_2)$ are two points on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with eccentricity e. If PSQ is a focal chord and $\tan\left(\frac{\theta_1}{2}\right)\tan\left(\frac{\theta_2}{2}\right)=-\left(2\sqrt{2}+3\right)$, then e and S are

[TS EAMCET 18-07-2022_Shift-2]

1. $\frac{1}{\sqrt{3}},\left(\frac{a}{\sqrt{3}},0\right)$
2. $\frac{1}{\sqrt{3}},\left(\frac{-a}{\sqrt{3}},0\right)$
3. $\frac{1}{\sqrt{2}},\left(\frac{a}{\sqrt{2}},0\right)$
4. $\frac{1}{\sqrt{2}},\left(\frac{-a}{\sqrt{2}},0\right)$

57. If $ax^2+by^2=15$ is the equation of the ellipse for which distance between its foci is 2 and distance between its directrices is 5, then $a+b=$ **[TS EAMCET 19-07-2022_Shift-I]**

1. s10
2. 8
3. 16
4. 12

58. Statement I: The equation of the directrix of the ellipse $4x^2+y^2-8x-4y+4=0$ is $3y=6-4\sqrt{3}$

Statement II: The equation of the latus rectum of the ellipse $x^2+4y^2-4x-8y+4=0$ is $y=2+\sqrt{3}$ Which of the above statement(s) is (are) True?

[TS EAMCET 19-07-2022_Shift-2]

1. Statement I is true, but statement II is false
2. Statement II is true, but statement I is false
3. Both statement I and Statement II are true
4. Both statement I and Statement II are false

59. If S is the focus of the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ lying on the positive X–axis and P(θ) is a point on the ellipse such that SP=1, then $\cos\theta$ = **[TS EAMCET 19-07-2022_Shift-2]**

1. $\frac{1}{\sqrt{5}}$
2. $\frac{2}{\sqrt{5}}$
3. $\frac{1}{2}$
4. $\frac{1}{3}$

60. The length of the latus rectum of an ellipse is 6 units and the distance between a focus and its nearest vertex on the major axis is $\frac{5}{3}$ units. If e is the eccentricity of this ellipse, then e satisfies the equation

[TS EAMCET 20-07-2022_Shift-1]

1. $25x^2-40x+16=0$
2. $25x^2+40x-16=0$
3. $25x^2-40x-16=0$
4. $25x^2+40x-32=0$

61. If the line 2x−3y+4=0 cuts the ellipse $x=3\cos\theta, y=5\sin\theta$ in A and B and (α,β) is the midpoint of $\overline{AB}$, then $3\beta-2\alpha=$

[TS EAMCET 20-07-2022_Shift-1]

1. −4
2. 4
3. −5
4. 5

62. If $P(\theta)$ and $Q\left(\frac{\pi}{2}+\theta\right)$ are two points on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the locus of midpoint of PQ is $\frac{x^2}{\alpha^2}+\frac{y^2}{\beta^2}=1$ then $\frac{a+b}{\alpha+\beta}=$

[TS EAMCET 20-07-2022_Shift-2]

1. $\frac{1}{\sqrt{2}}$
2. $\sqrt{3}$
3. $\frac{1}{\sqrt{3}}$
4. $\sqrt{2}$

63. If m is the length of the latus rectum and n is the length of the major axis of the ellipse $25x^2+16y^2-150x-64y-111=0$, then the ordered pair (m, n) =

[TS EAMCET 20-07-2022_Shift-2]

1. $\left(\frac{16}{5},10\right)$
2. $\left(\frac{32}{5},10\right)$
3. $\left(\frac{25}{2},8\right)$
4. $\left(\frac{25}{4},8\right)$

64 Let $x^2+y^2=20$ be the director circle of an ellipse E whose major axis is X-axis and minor axis is Y-axis. If the length of the latus rectum of E is 2, then the distance between its foci is

[15th May 2023 Shift 1]

1. $4\sqrt{5}$
2. $4\sqrt{3}$
3. $4\sqrt{2}$
4. 3

65 Let the length of the latus rectum of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ be equal to the length of its semi-major axis. If the radius of its director circle is $\sqrt{3}$ and e is its eccentricity then the length of its latus rectum is **[15th May 2023 Shift 2]**

1. $\frac{1}{a}$
2. $\frac{1}{b}$
3. $\frac{1}{e}$
4. $\frac{1}{ab}$

66 If $S=\frac{x^2}{k-7}+\frac{y^2}{11-k}-1=0, k\in R-\{7,11\}$, then which one of the following statements is incorrect? **[15th May 2023 Shift 2]**

1. $S=0$ represents a circle with radius $\sqrt{2}$, when k = 9
2. $S=0$ represents an ellipse with eccentricity $\sqrt{\frac{2}{3}}$, when $k=10$
3. $S=0$ represents a hyperbola with eccentricity $\sqrt{\frac{6}{5}}$, when $k=12$
4. $S=0$ represents a hyperbola with eccentricity $\sqrt{\frac{3}{2}}$, when $k=13$

67 If a focal chord of the ellipse $\frac{X^2}{25}+\frac{Y^2}{16}=1$ meets its minor axis at the point (0,3), then the perpendicular distance from the centre of the ellipse to this focal chord is

[16th May 2023 Shift 2]

1. 5
2. $\frac{2}{\sqrt{5}}$
3. 1
4. $\frac{3}{\sqrt{2}}$

68 Let E be an ellipse whose major axis is X –axis and minor axis is Y –axis. If the distance of a point $\left(\frac{5}{2},2\sqrt{3}\right)$ on E from its foci are $\frac{7}{2}$ and $\frac{13}{2}$, then the eccentricity of the ellipse E is

[17th May 2023 Shift 1]

1. 3/5
2. 1/5
3. $1/\sqrt{5}$
4. $1/\sqrt{2}$

69 Let the point L lying in the first quadrant be one end of a latus rectum of the ellipse $\frac{x^2}{4}+\frac{y^2}{3}=1$. Let P and Q be the points where the normal drawn at L to this given ellipse meets the major axis and the minor axis. Then the distance between P and Q is[**17th May 2023 Shift 2**]

1. $\frac{\sqrt{5}}{4}$ 2. $\frac{1}{\sqrt{2}}$

3. $\frac{1}{2\sqrt{2}}$ 4. $\frac{\sqrt{5}}{2}$

70 Let A, A^1 be the end points of major axis, S, S^1 be the foci and B, B^1 be the end points of minor axis of an ellipse E, If $\angle BAB^1 = 60^o, then \angle SBS^1 =$ [**18^{th} May2023 shift -1**]

1. $Tan^{-1}\left(\sqrt{2}\right)$ 2. $Tan^{-1}\left(-2\sqrt{2}\right)$

3. $Tan^{-1}\left(\sqrt{\frac{2}{3}}\right)$ 4. $Tan^{-1}\left(\sqrt{\frac{3}{2}}\right)$

71 Let the eccentricity of the ellipse $2x^2+ay^2-8x-2ay+(8-a)=0$ be $\frac{1}{\sqrt{3}}$. If the major axis of this ellipse is parallel to Y-axis, then the equation of the tangent to this ellipse with slope 1 is[**18th May 2023 Shift 2**]

1. $x-y-1\pm\sqrt{5}=0$ 2. $x-y-3\pm\sqrt{5}=0$

3. $x-y-3\pm\sqrt{\frac{10}{3}}=0$ 4. $x-y-1\pm\sqrt{\frac{10}{3}}=0$

72 Tangents are drawn from point (1,1) to the ellipse $S \equiv x^2+4y^2-2x+8y+1=0$. If $m_1, m_2 (m_1 > m_2)$ are the slopes of these tangents, then with respect to the given ellipse the point $P(m_1, m_2)$ [**19th May 2023 Shift 1**]

1. Lies inside the ellipse S=0
2. Lies outside the ellipse S=0
3. Lies on the ellipse S=0
4. Is the centre of the ellipse S=0

73 A particle is travelling in clockwise direction on the ellipse $\frac{x^2}{100}+\frac{y^2}{25}=1$. If the particle leaves the ellipse at the point $(-8,3)$ on it and travels along the tangent to the ellipse at that point then the point where the particle crosses the Y-axis is

[**12^{TH} MAY 2023 SHIFT-1**]

1. $\left(0,\frac{7}{3}\right)$ 2. $\left(0,\frac{25}{3}\right)$

3. $(0,9)$ 4. $\left(0,\frac{-25}{3}\right)$

74 If an ellipse with foci at $(3,3)$ *and* $(-4,4)$ is passing through the origin, then the eccentricity of that ellipse is

[**12^{TH} MAY 2023 SHIFT-1**]

1. $\frac{5}{7}$ 2. $\frac{3}{7}$

3. $\frac{1}{7}$ 4. $\frac{4}{7}$

75 The product of the lengths of the perpendiculars drawn from the two foci of the ellipse $\frac{x^2}{9}+\frac{y^2}{25}=1$ to the tangent at any point on the ellipse is[**12^{TH} MAY 2023 SHIFT -2**]

1. 6 2. 7
3. 8 4. 9

76 Tangents are drawn to the ellipse $\frac{x^2}{9}+\frac{y^2}{5}=1$ at all the ends of its latus recta. The area of the quadrilateral so formed (in sq. units) is

[**12^{TH} MAY 2023 SHIFT -2**]

1. 27 2. 36
3. 42 4. 45

77. Let S and S^1 be the foci of an ellipse E and B be one end of its minor axis .Let $\angle S^1SB = \frac{\pi}{6}$ and $(2\sqrt{3},1)$ be a point on E. If X-axis is the major axis and Y-axis is the minor axis of the ellipse E, then the sum of the squares of the lengths of major axis and minor axis is **[13TH MAY 2023 SHIFT-1]**

1. 20
2. 60
3. 80
4. 100

78 If $4x+2y+n=0$ is a normal to the ellipse $\frac{x^2}{36}+\frac{y^2}{16}=1$,then n= **[13TH MAY 2023 SHIFT-1]**

1. $\pm\frac{9}{4}$
2. $\pm\frac{9}{\sqrt{10}}$
3. $\pm\frac{5}{4}$
4. ± 8

79 If the line $x\cos\alpha + y\sin\alpha = 2\sqrt{3}$ is a tangent to the ellipse $\frac{x^2}{16}+\frac{y^2}{8}=1$ and α is an acute angle then $\alpha =$ **[EAPCET 14-05-23 SHIFT-1]**

1. $\frac{\pi}{6}$
2. $\frac{\pi}{4}$
3. $\frac{\pi}{3}$
4. $\frac{\pi}{2}$

80 If $x+\sqrt{3}y=3$ is the tangent to the ellipse $2x^2+3y^2=k$ at a point P then the equation of the normal to this ellipse at P is **[EAPCET 14-05-23 SHIFT-1]**

1. $5x-2\sqrt{3}y=1$
2. $x-\sqrt{3}y=2$
3. $x-\sqrt{3}y+1=0$
4. $3x-\sqrt{3}y=1$

81 In an ellipse, the distance from one of the foci to its corresponding end of the major axis is $4-\sqrt{7}$ and the distance from same focus to one end of the minor axis is 4. Then the cosine of the angle subtended by the line segment joining its foci at one end of its minor axis is **[EAPCET 13-05-23 SHIFT-2]**

1. 1/8
2. ¾
3. $\sqrt{7}/3$
4. $1/3\sqrt{7}$

82. If the equations $x=1+2\cos\theta, y=2+\sin\theta$, $0\le\theta\le 2\pi$ represent an ellipse, then the point of intersection of the normal drawn at $P\left(\frac{\pi}{4}\right)$ to this ellipse and its major axis is **[EAPCET 13-05-23 SHIFT-2]**

1. $\left(\frac{4-\sqrt{3}}{4},0\right)$
2. $\left(\frac{\sqrt{3}+1}{4},0\right)$
3. $\left(\frac{8+\sqrt{3}}{2},0\right)$
4. $\left(\frac{5}{2},0\right)$

KEY

1)	1	2)	1	3)	2	4)	3	5)	3
6)	1	7)	1	8)	2	9)	1	10)	2
11)	3	12)	3	13)	3	14)	2	15)	4
16)	1	17)	4	18)	4	19)	2	20)	1
21)	4	22)	3	23)	4	24)	2	25)	4
26)	2	27)	4	28)	1	29)	4	30)	3
31)	3	32)	2	33)	1	34)	4	35)	4
36)	3	37)	2	38)	4	39)	1	40)	4
41)	1	42)	2	43)	2	44)	4	45)	4
46)	3	47)	4	48)	1	49)	1	50)	1
51)	4	52)	4	53)	1	54)	2	55)	4
56)	3	57)	3	58)	1	59)	2	60)	1
61)	2	62)	4	63)	2	64)	2	65)	3
66)	4	67)	4	68)	1	69)	1	70)	2
71)	4	72)	1	73)	2	74)	1	75)	4
76)	1	77)	3	78)	4	79)	2	80)	4
81)	1	82)	1	83)					

<u>**SOLUTIONS**</u>

1. Length of latusrectum$=\frac{2b^2}{a}=4$

$b^2=2a$

$a^2\left(1-e^2\right)=2a$

$a(1+e)(1-e)=2$

Also given

$a-ae=\frac{3}{2}$

$a(1-e)=\frac{3}{2}$

$\frac{3}{2}(1+e)=2$

$e=\frac{1}{3}$

2. $\frac{x^2}{18}+\frac{y^2}{32}=1$

$m=-\frac{4}{3}$

$y=\left(-\frac{4}{3}\right)x\pm\sqrt{18\left(\frac{16}{9}\right)+32}$

$y=\left(-\frac{4}{3}\right)x\pm 8$

$4x+3y=\pm 24$

$\frac{x}{\pm 6}+\frac{y}{\pm 8}=1$

$(6,0),(0,8)$ or $(-6,0),(0,-8)$

3. Given $x=m^2\rightarrow(1)$ & $\frac{x^2}{1}+\frac{y^2}{9}=1\rightarrow(2)$

Sub 'x' value in eq. (2) , we get

$y^2=9\left(1-m^4\right)\Rightarrow y=\pm 3\sqrt{1-m^4}$

i.e.,$1-m^4>0$

$(1)^2-\left(m^2\right)^2>0$

$\left(1^2-m^2\right)\left(1^2+m^2\right)>0$

$(1-m)(1+m)\left(1^2+m^2\right)>0$

$(m-1)(1+m)\left(1^2+m^2\right)<0$

$-1<m<1$

$\therefore |m|<1$

4. Given ellipse is $\frac{x^2}{4}+\frac{y^2}{36}=1$

And line ax+by+c=0

w.k.t normal condition is

$\frac{4}{a^2}+\frac{36}{b^2}=\frac{(4-36)^2}{c^2}$

$\frac{1}{a^2}+\frac{9}{b^2}=\frac{256}{c^2}$

5. $x^2+4(4x+c)^2=4$

$65x^2+32cx+4c^2-4=0$

Touch $\Rightarrow \Delta=0$

$(32c)^2-4(65)\left(4c^2-4\right)=0$

$\Rightarrow c=\pm\sqrt{65}$

6. $\frac{x^2}{16}+\frac{y^2}{1}=1$

$a=4, b=1,\ a>b$

Equation of tangent

$y=mx\pm\sqrt{a^2m^2+b^2}$

$\sqrt{3}x-y\pm 7=0$

$4x^2+25y^2=100$

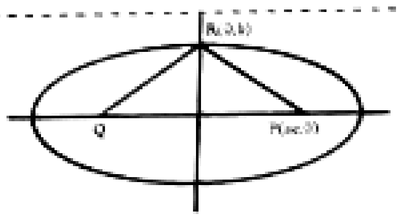
7. $\frac{x^2}{25}+\frac{y^2}{4}=1$

$e=\sqrt{\frac{25-4}{25}}=\frac{\sqrt{21}}{5}$

8. $y=2x+C,\ \frac{x^2}{4}+\frac{y^2}{1}=1$

$c^2=a^2m^2+b^2 \quad \Rightarrow c^2=17$

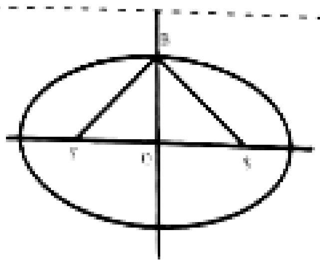
9.



$\tan 30^0=\frac{ae}{b}$

$\frac{1}{\sqrt{3}}=\frac{ae}{b}$

$e=\frac{1}{2}$



$OS^2 + OB^2 = BS^2$

$a^2e^2 + b^2 = BS^2$

$a^2e^2 + a^2 - a^2e^2 = BS^2$

$BS = a$

$\sin\alpha = \frac{ae}{a} \Rightarrow e = \sin\alpha$

1. $\frac{2b^2}{a} = 4 \qquad 2ae = 4\sqrt{2}$

$b^2 = 2a \qquad ae = 2\sqrt{2}$

$a^2 - a^2e^2 = 2a$

$a^2 - 8 = 2a \Rightarrow (a-4)(a+2) = 0$

$a = 4 \quad b^2 = 8$

$\frac{x^2}{16} + \frac{y^2}{8} = 1$

$x^2 + 2y^2 = 16$

2. P(1, k), k>0

$9x^2 + 4y^2 - 18x + 16y - 11 = 0....(1)$

P lies on (1)

$9 + 4k^2 - 18 + 16k - 11 = 0$

$k = -5,\ k = 1$

$k = 1$

$9(x^2 - 2x) + 4(y^2 + 4y) - 11 = 0$

$9(x-1)^2 + 4(y+2)^2 = 36$

$\frac{(x-1)^2}{4} + \frac{(y+2)^2}{9} = 1,\quad e = \sqrt{\frac{9-4}{9}} = \frac{\sqrt{5}}{3}$

Equation of directrix $y = k \pm \frac{b}{e}$

$y = -2 \pm \frac{3}{\sqrt{5}/3}$

P(1,1), $y = -2 + \frac{9}{\sqrt{5}}$, $y = -2 - \frac{9}{\sqrt{5}}$

Shortest distance $|y_1 - y_2| = \frac{9}{\sqrt{5}} - 3$

13. $\frac{x^2}{25} + \frac{y^2}{16} = 1$

Equation of tangent $\frac{x}{5}\cos\theta + \frac{y}{4}\sin\theta = 1 \backslash$

$A\left(\frac{5}{\cos\theta}, 0\right),\ A'\left(0, \frac{5}{\cos\theta}\right)$

Equation of circle AA' as its diameter passes through (0, 0)

14. $P = \left(3\sqrt{2}, \sqrt{10}\right)$

$F_1 = (-4, 0)$

$F_2 = (4, 0)$

$F_1F_2 = 2ae$

$\sqrt{(4+4)^2 + 0} = 2ae$

$2ae = 8.....(1)$

$2a = \sqrt{\left(3\sqrt{2}+4\right)^2 + \left(\sqrt{10}\right)^2} + \sqrt{\left(3\sqrt{2}-4\right)^2 + \left(\sqrt{10}\right)^2}$

$= \left(6 + 2\sqrt{2}\right) + \left(6 - 2\sqrt{2}\right) = 12$

$2a = 12 \Rightarrow a = 6$

$(1) \Rightarrow e = \frac{2}{3}$

15. $\frac{-3}{4}x - y + 3\sqrt{2} = 0.......(1)$

$F_1 = (ae, 0); F_2 = (-ae, 0)$

p_1 =perpendicular distance from F_1 to (1)

$= \frac{-\frac{3ae}{4} + 3\sqrt{2}}{\sqrt{\frac{9}{16} + 1}}$

p_2 = perpendicular distance from F_2 to (1)

$$p_2 = \frac{\frac{3ae}{4} + 3\sqrt{2}}{\sqrt{\frac{9}{16} + 1}}$$

$$p_1 p_2 = 9$$

$$\left(12\sqrt{2}\right)^2 - (3ae)^2 = 225$$

$$(ae)^2 = 7$$

$$ae = \sqrt{7}$$

$$e = \frac{\sqrt{7}}{4}$$

$$c^2 = a^2 m^2 + b^2$$

$$18 = a^2\left(\frac{9}{16}\right) + b^2$$

$$18 = \frac{9a^2}{16} + b^2$$

$$18 = \frac{9a^2}{16} + a^2\left(1 - e^2\right)$$

$$18 = \frac{9a^2}{16} + a^2 - 7$$

$$a = 4$$

---

16. $2b = \sqrt{5}$ and $be = 1$

$$a^2 = b^2 - b^2 e^2 = \frac{1}{4}$$

Required equation is $\dfrac{x^2}{1/4} + \dfrac{y^2}{5/4} = 1$

---

17. $e = \dfrac{2\sqrt{2}}{3}$. and $2a = 2\sqrt{18} \Rightarrow a = \sqrt{18}$

$$\Rightarrow b^2 = a^2(1 - e^2) = 2$$

$\therefore$ ellipse equation is, $\dfrac{x^2}{18} + \dfrac{y^2}{2} = 1$

Equation of polar is $S_1 = 0$

$$\Rightarrow xx_1 + 9yy_1 - 18 = 0$$

It is a tangent to $x^2 + y^2 = 18$

$$\Rightarrow r = d \Rightarrow \frac{x_1^2}{18} + \frac{9y_1^2}{2} = 1$$

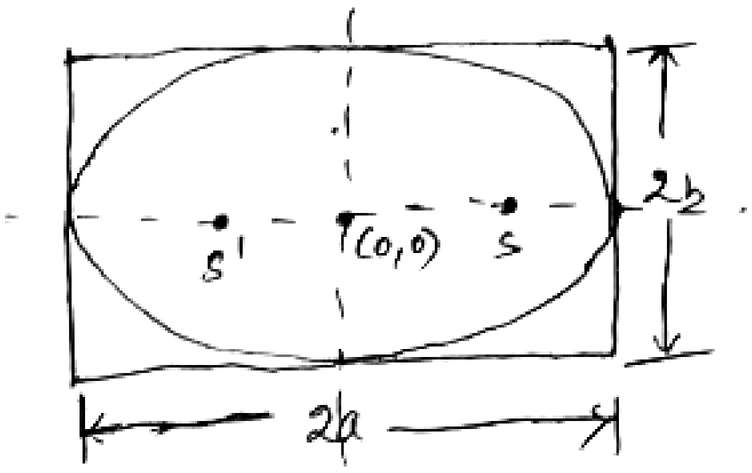
---

18. Given

$$OS + OS' = 8\sqrt{6}$$

$$\Rightarrow 2ae = 8\sqrt{6}$$

$$a^2 e^2 = 96 \ldots.(1)$$



And area of rectangle with length $(2a)$ and breadth $(2b)$ is

$$(2a)(2b) = 80$$

$$\Rightarrow ab = 20 \ldots.(2)$$

From $(1) \Rightarrow a^2\left[1 - \dfrac{b^2}{a^2}\right] = 96 \ldots..(3)$

Solving (2) *and* (3) we get $a = 10$ *and* $b = 2$

Equation of ellipse is $\dfrac{x^2}{100} + \dfrac{y^2}{4} = 1$

---

19. Given directrix $L \cong 3x + 4y - 5 = 0$,

Let $P(x, y)$ any point

Focus, $S(1,2)$ and eccentricity $e = \dfrac{1}{2}$

---

w.k.t

$$SP = ePM \Rightarrow SP^2 = e^2 PM^2$$

$$\Rightarrow (x-1)^2 + (y-2)^2 = \frac{1}{4}\left[\frac{(3x + 4y - 5)^2}{25}\right]$$

By simplifying the above equation. We get

$$91x^2 + 84y^2 - 24xy - 170x - 360y + 475 = 0$$

20. $e=\frac{1}{2}, S=(2,0)$

$P(\theta), q\left(\frac{\pi}{3}\right)$ are ends of focal chord

$Q\left(a\cos\frac{\pi}{3}, b\sin\frac{\pi}{3}\right)=(2,3)$

PQ is LLR of ellipse

$4\cos\theta=2$

$\cos\theta=\frac{1}{2} \Rightarrow \tan\theta=-\sqrt{3}\ \theta\in Q_4$

21. $a^2l^2+b^2m^2=n^2 \Rightarrow k=\pm 3\ (k>0)$

$k=3$

Equation of normal at

$\left(\frac{1}{\sqrt{2}}, \frac{k}{3}\right)=\left(\frac{1}{2}, 1\right) is$

$\frac{a^2x}{x_1}-\frac{b^2y}{y_1}=a^2-b^2$

$\sqrt{2}x-2y=1-2$

$\sqrt{2}x-2y+1=0$

22. The tangent $y=mx\pm\sqrt{2m^2+1}$ to the given ellipse meets the coordinate axes at

$A\left(\frac{\sqrt{2m^2+1}}{m}, 0\right), B=\left(0, \sqrt{2m^2+1}\right)$

$\therefore$Let P(x, y) be locus point

$(x,y)=\left(\frac{\sqrt{2m^2+1}}{2m}, \frac{\sqrt{2m^2+1}}{2}\right)$

$\sqrt{2}x=\frac{\sqrt{2m^2+1}}{\sqrt{2}m}$, $2y=\sqrt{2m^2+1}$

$\frac{1}{2x^2}+\frac{1}{4y^2}=1$

23. $e=\frac{1}{2}$

$the\ ends\ of\ a\ latusrecta\left(\pm ae, \pm\frac{b^2}{a}\right)=(\pm 2, \pm 3)$

$\therefore the$ tan $gents\ at\ latusrecta\ are$

$x+2y-8=0, x-2y+8=0$

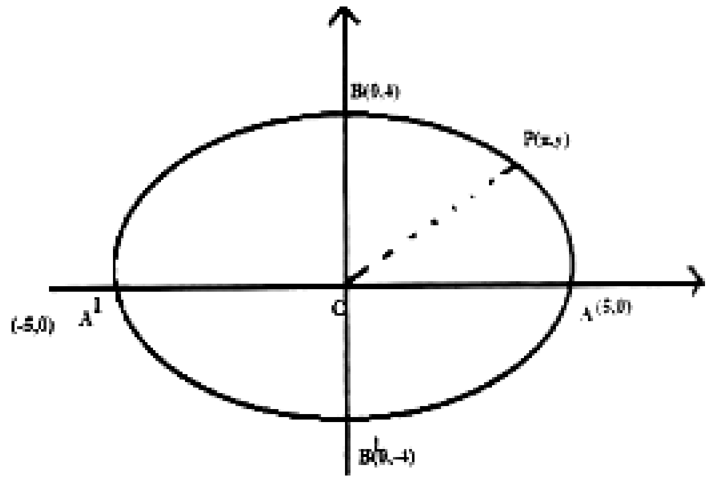
$x+2y+8=0,\ x-2y-8=0$

$\therefore the\ P.O.I\ are(0,4),(-8,0),(0,-4),(8,0)$

$Ar(Quadrileteral)=\frac{1}{2}\begin{vmatrix} x_1-x_3 & x_2-x_4 \\ y_1-y_3 & y_2-y_4 \end{vmatrix}$

$=\frac{1}{2}\begin{vmatrix} 0 & -16 \\ 8 & 0 \end{vmatrix}=64\,sq.units$

24.



Maximum value of CP is 5
Minimum value of CP is 4
$\therefore\ Sum=5+4=9$

25. Conceptual

26. $P(a\cos\theta_1, b\sin\theta_1)$ and $Q(a\cos\theta_2, b\sin\theta_2)$

m_1 =Slope of $CP=\frac{b}{a}\tan\theta_1$

m_2 =Slope of $CQ=\frac{b}{a}\tan\theta_2$

Consider $m_1\times m_2=\frac{b^2}{a^2}.\tan\theta_1\tan\theta_2$

$=\frac{b^2}{a^2}\left(\frac{-a^2}{b^2}\right)=-1$

27. $\frac{(x-2)^2}{9}+\frac{(y-3)^2}{4}=1$

Major axis is $y=3$.

28. Verification

29. $a^2l_1l_2+b^2m_1m_2=n_1n_2$

30. Conceptual

31. Conceptual

32. $a^2x^2+b^2y^2=a^2\left(a^2+b^2\right)-a^2y^2$

$$\frac{x^2}{a^2+b^2}+\frac{y^2}{b^2}=1$$

$$\frac{SP}{PM}=\frac{1}{K}=e$$

$$\frac{1}{K}=\sqrt{\frac{a^2+b^2-a^2}{a^2+b^2}}$$

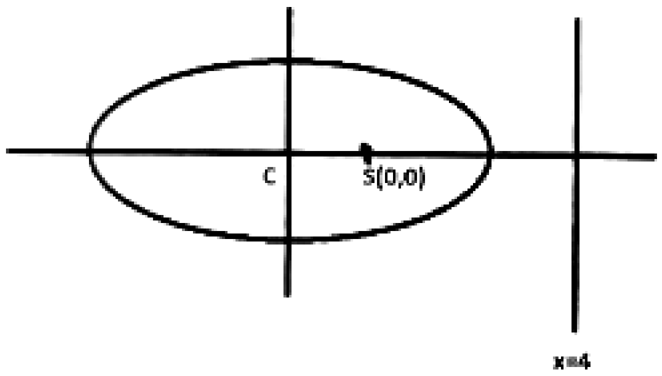
$$K=\frac{\sqrt{a^2+b^2}}{b}$$

33. $\dfrac{(x-2y+1)^2}{\frac{25}{4}}+\dfrac{(2x+y+2)^2}{\frac{25}{9}}=1$

$$\frac{\left(\frac{|x-2y+1|}{\sqrt{5}}\right)^2}{\frac{5}{4}}+\frac{\left(\frac{|2x+y+2|}{\sqrt{5}}\right)^2}{\frac{5}{9}}=1$$

Equation of major axis is $x-2y+1=0$

34.



$e=\dfrac{1}{2} \Rightarrow \dfrac{a}{e}-ae=4$

$a\left(\dfrac{1}{e}-e\right)=4 \Rightarrow a\left(2-\dfrac{1}{2}\right)=4$

$$a=\frac{8}{3}$$

$$b^2=a^2\left(1-e^2\right)=\frac{64}{9}\left(1-\frac{1}{4}\right)=\frac{16}{3}$$

$$(ae,0)=\left(\frac{-4}{3},0\right)$$

$$\frac{\left(x+\frac{4}{3}\right)^2}{\frac{64}{9}}+\frac{y^2}{\frac{16}{3}}=1$$

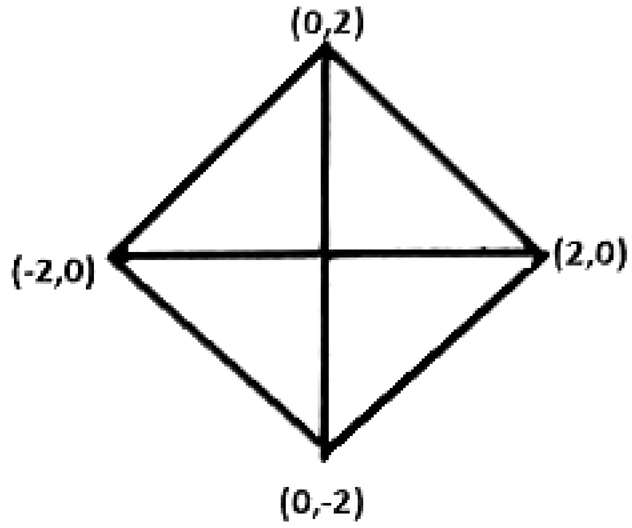
$$(3x+4)^2+12y^2=64$$

35. $\dfrac{x^2}{9}+\dfrac{y^2}{5}=1(a>b)\Rightarrow e=\dfrac{2}{3}\Rightarrow S(\pm2,0)$

$$\frac{x^2}{5}+\frac{y^2}{9}=1(a<b)$$

$$e=\frac{2}{3}$$

$$S(0,\pm2)$$



Area= $\left(2\sqrt{2}\right)^2=8$

36. Diagram

Given $\dfrac{2a^2}{b}=\dfrac{2}{3}(2a)$

$$\frac{a}{b}=\frac{1}{3}$$

$$3a=2b$$

S.O.B.S

$$9a^2=4b^2$$

$$9b^2\left(1-e^2\right)=4b^2$$

$$e=\frac{\sqrt{5}}{3}$$

37. Length $a\sqrt{2}=8\sqrt{2}\Rightarrow a=8$

Breadth $b\sqrt{2}=4\sqrt{2}\Rightarrow b=4$

$$e=\sqrt{\frac{a^2-b^2}{a^2}}=\sqrt{\frac{64-16}{64}}=\frac{\sqrt{3}}{2}$$

38. Given ellipse $(E):4x^2+9y^2-36=0$

Circle $(C):x^2+y^2-9=0$

Points $A(1,2),B(2,1)$ at point

$A\rightarrow S_{11}=C_{11}<0\ \&\ S_{11}=E_{22}>0$

39. Circle

$$(x-1)^2+y^2=r^2 \;\&\; x^2+4y^2=16 \Rightarrow \frac{x^2}{16}+\frac{y^2}{4}=1$$

Normal $P(x_1,y_1)$ is $\frac{a^2x}{x_1}-\frac{b^2y}{y_1}=a^2-b^2$

$\frac{16x}{x_1}-\frac{4y}{y_1}=16-4=12$ passes through $C(1,0)$

$x_1 = 4/3$

P lies on ellipse

$$x_1^2+4y_1^2=16 \Rightarrow \frac{16}{9}+4y_1^2=16$$

$$y_1^2=\frac{32}{9}$$

P lies on circle $(x_1-1)^2+y_1^2=r^2$

$$\Rightarrow \frac{1}{9}+\frac{32}{9}=r^2 \Rightarrow \frac{33}{9}=r^2 \Rightarrow r=\sqrt{\frac{11}{3}}$$

40. $\frac{x^2}{4}+\frac{y^2}{9}=1, \left(\frac{y}{\sqrt{5}}, \frac{3}{\sqrt{5}}\right)=(x_1,y_1)$

$$a=4, b=3, e=\sqrt{\frac{b^2-a^2}{b^2}}=\frac{\sqrt{5}}{3}$$

Focal distance $= b \pm ey_1 = 3 \pm \frac{\sqrt{5}}{3} \times \frac{3}{\sqrt{5}}$

$$= 2 \text{ and } 6$$

41. $16x^2+25y^2=400$

$\frac{x^2}{25}+\frac{y^2}{16}=1$, $\quad e=\sqrt{\frac{25-16}{25}}=\frac{3}{5}$

Focal distances

$$=a(1 \pm e\cos\theta)=5\left(1 \pm \frac{3}{5}\cos\theta\right)$$

$$PF_1 \times PF_2 = 5\left(1+\frac{3}{5}\cos\theta\right)5\left(1-\frac{3}{5}\cos\theta\right)$$

$$=25\left(1-\frac{9}{25}\cos^2\theta\right)=25-9\cos^2\theta$$

Range $=[16, 25]$.

42. Given foci of ellipse are $(-2,0),(8,0)$

Distance between foci $=2ae=10 \Rightarrow a=5\sqrt{2}$

(Given $e=\frac{1}{\sqrt{2}}$)

$\therefore b^2=a^2(1-e^2) \Rightarrow b=5$

Centre $=C(x,y)=(3,0)$

$\therefore$ Required parametric form is

$(x+a\cos\theta, y+b\sin\theta)=(3+5\sqrt{2}\cos\theta, 5\sin\theta)$.

43. $P(\sqrt{6}\cos\theta, \sqrt{2}\sin\theta)$ & $CP^2=2^2$

44. $e=\frac{\sqrt{5}}{3}$; $S(0,\sqrt{5})$, $S'(0,-\sqrt{5})$. Find SP, $S'P$.

45. By data, $\angle CSB=\angle CBS=45^\circ$

From ΔSBC, $\tan 45^\circ=\frac{b}{ac}$

46. Required distance $=\frac{2a}{e}$

47. Conceptual

48. For parabola, $S_{11}>0$ & for ellipse $S_{11}<0$.

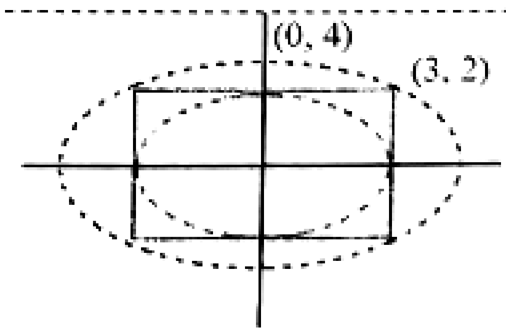
49. Major Axis :– x – y + 1 = 0
Minor Axis :– x + y – 11 = 0
Centre of ellipse = (5, 6).

Ellipse Eq :– $\frac{\left(\frac{Minor}{\sqrt{2}}\right)^2}{3^2}+\frac{\left(\frac{Major}{\sqrt{2}}\right)^2}{1^2}=1$.

$$\Rightarrow (x+y-11)^2+9(x-y+1)^2=18$$

50. Foci $s=(\pm ae, 0)$

51.



$(0,4) \in E_2 \Rightarrow b=4$

$(3,2) \in E_2 \Rightarrow b=\sqrt{12}$

52. $(\alpha,-\alpha)$; $S_{11}<0$ Then $\beta=\frac{2}{3}$

53. Solving $e=\frac{\sqrt{3}}{2}$ & $\frac{2b^2}{a}=1$

54. Centre (6,0) Then ae=6

Here a = 18, using $e=\frac{1}{3}$, we get $b=12\sqrt{2}$

55. Substitute P, Q is $S'=0$. & add the equations.

56. Use $e\cos\left(\frac{\theta_1+\theta_2}{\theta_2}\right)=\cos\left(\frac{\theta_1-\theta_2}{2}\right)$

57. Given $\frac{x^2}{15/a}+\frac{y^2}{15/b}=1$

Use $2AE=2$ & $2\frac{A}{E}=5$

Whose A = semi major axis

E = Eccentricity.

Then $A=\sqrt{\frac{15}{a}}$ & $A=\sqrt{\frac{15}{b}}$

58. For statement (1) : $(x-1)^2+\frac{(y-2)^2}{4}=1$

For Statement (2) : $\frac{(x-2)^2}{4}+(y-1)^2=1$

59. $c=\frac{\sqrt{5}}{3}$; take $S(\sqrt{5},0),P(3\cos\theta,2\sin\theta)$

Given SP=1

60. Use $\frac{2b^2}{a}=6$ & $a-ae=5/3$

61. Ellipse : $\frac{x^2}{9}+\frac{y^2}{25}=1$

Eq. of mid point of chord with (α,β) as mid point $(\alpha,\beta)\Rightarrow S_1=S_{11}$

ie. $\frac{\alpha}{9}x+\frac{\beta}{25}y-\left(\frac{\alpha^2}{9}+\frac{\beta^2}{25}\right)=0\rightarrow(1)$

Given chord $\overline{AB}:2x-3y+4=0\rightarrow(2)$

Solving (1) & (2) we get, $(\alpha,\beta)=\left(-\frac{24}{87},\frac{100}{87}\right)$

62. Here $p(\theta)=(a\cos\theta,b\sin\theta)$

$Q(\theta)=(-a\sin\theta,b\cos\theta)$

Mid point M=

$\left(\frac{a}{2}(\cos\theta-\sin\theta),\frac{b}{2}(\cos\theta+\sin\theta)\right)$

63. conceptual

64 $a^2+b^2=20,\quad \frac{2b^2}{a}=2$

$\Rightarrow a^2+a-20=0\qquad\Rightarrow b^2=a$

$\Rightarrow(a+5)(a-4)=0$

$a=-5x,\quad \boxed{a=4}\quad\Rightarrow b^2=4$

$e=\sqrt{\frac{a^2-b^2}{a^2}}=\sqrt{\frac{12}{16}}=\frac{\sqrt{3}}{2}$

$SS'=2ae=2(4)\left(\frac{\sqrt{3}}{2}\right)=4\sqrt{3}$

65 *Equation of director circle of ellipse is* $x^2+y^2=a^2+b^2$

It's radius $=\sqrt{a^2+b^2}=\sqrt{3}\Rightarrow a^2+b^2=3\rightarrow(1)$

given $\frac{2b^2}{a}=a\Rightarrow b^2=\frac{1}{2}a^2$

$(1)\Rightarrow a^2+\frac{1}{2}a^2=3\Rightarrow a^2=2,b^2=1$

$e=\sqrt{\frac{a^2-b^2}{a^2}}\Rightarrow\sqrt{\frac{2-1}{2}}\Rightarrow\frac{1}{\sqrt{2}}$

Length of the latus rectum $=\frac{2b^2}{a}=\frac{2}{\sqrt{2}}\Rightarrow\sqrt{2}=\frac{1}{e}$

66 Option verification

67 $\frac{x^2}{25}+\frac{y^2}{16}=1$

$a=5,b=4$

$e=\sqrt{\frac{9}{25}}=\frac{3}{5}$

$s(3,0)$

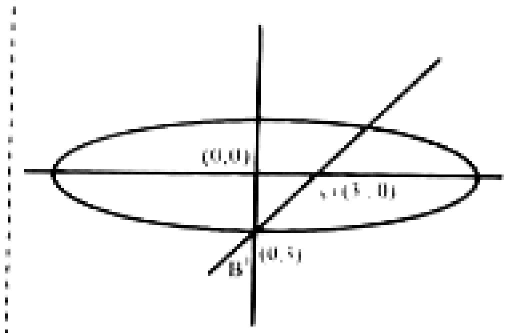
shape, m $=SB^1=-1$

Equation, $y=-(x-3)$

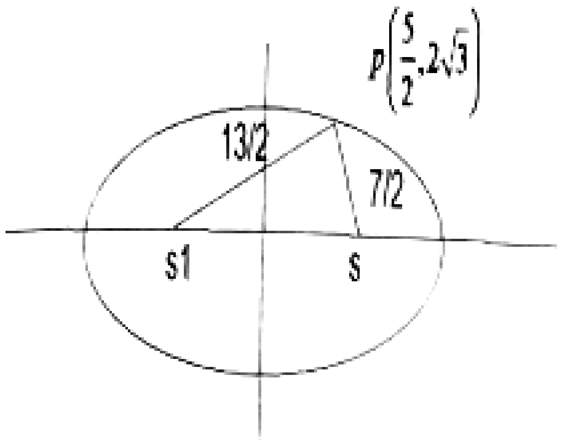
$x+y-3=0$

$\perp^r$ distance from $(0,0)$ to $x+y-3=0$ is

$=\frac{3}{\sqrt{2}}$



68.



$$sp + sp' = 2a$$

$$2a = \frac{13}{2} + \frac{7}{2}$$

$$2a = 10$$

$$a = 5$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{25}{4a^2} + \frac{12}{b^2} = 1$$

$$\frac{25}{4(25)} + \frac{12}{b^2} = 1$$

$$b^2 = 16$$

$$e = \sqrt{\frac{25-16}{25}}$$

$$e = \frac{3}{5}$$

69

$$L = \left(ae, \frac{b^2}{a}\right) = \left(1, \frac{3}{2}\right)$$

Equation of Normal at L is

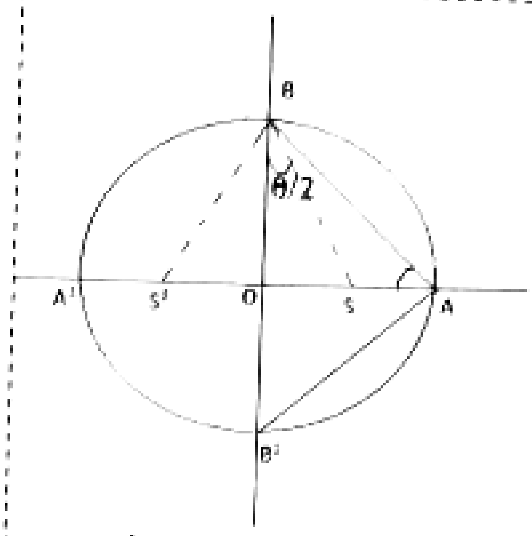
$$\frac{4x}{1} - \frac{3y}{\frac{3}{2}} = 4 - 3$$

$$\Rightarrow 4x - 2y = 1$$

$$P\left(\frac{1}{4}, 0\right) Q\left(0, \frac{-1}{2}\right)$$

$$PQ = \frac{\sqrt{5}}{4}$$

70



ΔBAB^1

from ΔOAB

$$\tan 30^o = \frac{b}{a}$$

$$\Rightarrow a = \sqrt{3}b \text{ and } e = \sqrt{2/3}$$

ΔSBS^1

$$\tan \theta/2 = \frac{ae}{b}$$

$$= \frac{\sqrt{3}b\sqrt{2/3}}{b}$$

$$= \sqrt{2}$$

$$\tan\theta = \left(\frac{2\sqrt{2}}{1-2}\right)$$

$$\theta = \tan^{-1}\left(-2\sqrt{2}\right)$$

71

$$2\left(x^2 - 4x\right) + a\left(y^2 - 2y\right) = a - 8$$

$$2\left(x^2 - 4x + 4\right) + a\left(y^2 - 2y + 1\right) = a - \cancel{8} + \cancel{8} + a$$

$$\frac{2(x-2)^2}{2a} + \frac{a(y-1)^2}{2a} = 1$$

$$\frac{(x-2)^2}{a} + \frac{(y-1)^2}{2} = 1$$

$$e = \sqrt{\frac{2-a}{2}} = \frac{1}{\sqrt{3}}$$

$$\frac{2-a}{2} = \frac{1}{3}$$

$$6 - 3a = 2$$

$$3a = 4$$

$$a = \frac{4}{3} \quad m = 1$$

$$y - 1 = m(x-2) \pm \sqrt{a^2m^2 + b^2}$$

$$x - y - 1 \pm \sqrt{\frac{10}{3}} = 0$$

72 $given\ x^2-2x+1+4\left(y^2+2y+1\right)=4$

$$\frac{(x-1)^2}{4}+\frac{(y+1)^2}{1}=1$$

Equation of tangent is

$y+1=m(x-1)\pm\sqrt{4m^2+1}$, $m=\pm\frac{\sqrt{3}}{2}$, $S_{11}<0$

is point lines inside the ellipse .

73 The equation of tangent at (-8,3) to the ellipse is

$$\frac{x(-\cancel{8})^2}{\cancel{100}_{25}}+\frac{y(3)}{25}-1=0$$

$$\frac{-2x}{25}+\frac{3y}{25}-1=0$$

$$2x-3y+25=0$$

$put\ \ x=0\ \ then\ \ y=\frac{25}{3}$

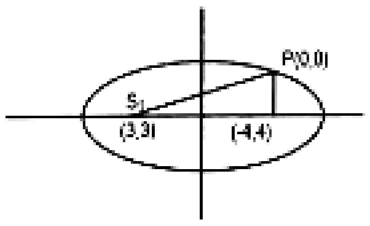
$\therefore$ Required point $=\left(0,\frac{25}{3}\right)$

74. W.K.T $S_1P+S_2P=2a$

$s_1p=3\sqrt{2},$

$s_2p=4\sqrt{2},$

$2a=7\sqrt{2},$



and also $2ae=s_1s_2=$

$5\sqrt{2}$

Now

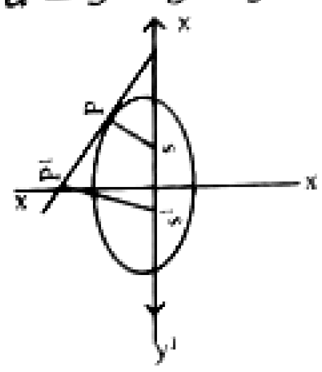
$$\frac{2ae}{2a}=\frac{5\sqrt{\cancel{2}}}{7\sqrt{\cancel{2}}}=\frac{5}{7}$$

75. Given that

$$\frac{x^2}{9}+\frac{y^2}{25}=1$$

$a^2=9\quad b=25$

$a=3\quad b=5$

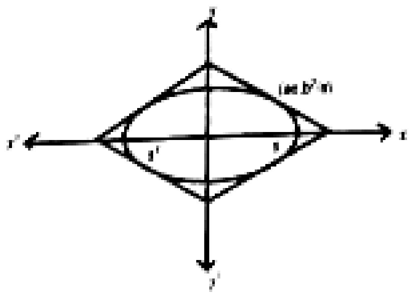


$sp\times s^1p^1=(\text{lenth of the semi minor axes})^2$

$=(a)^2$

$=(3)^2=9$

76 Given that



$$\frac{x^2}{9}+\frac{y^2}{5}=1$$

$a^2=9\quad b^2=5$

$a=3\quad b=\sqrt{5}$

$$e=\sqrt{\frac{a^2-b^2}{a^2}}=\sqrt{\frac{9-5}{9}}=\frac{2}{3}$$

$L\left(2,\frac{5}{3}\right)$

Equation of the tangent to the ellipse at $L\left(2,\frac{5}{3}\right)$

is

$$\frac{x}{\frac{9}{2}}+\frac{y}{3}=1$$

$\therefore$ Area of the $\Delta^{le}=\frac{1}{2}|ab|$

$$=\frac{1}{2}\left|\frac{9}{2}.3\right|=\frac{27}{4}$$

Area of the quadrilateral = 4 Area of the Δ^{le}

$=\cancel{4}\frac{27}{\cancel{4}}=27$ sq units

77 $Tan30=\frac{b}{ae}$

$$\frac{1}{\sqrt{3}}=\frac{b}{ae}$$

$$\cancel{a}^2e^2=3b^2=3\cancel{a}^2\left(1-e^2\right)$$

$$e=\frac{\sqrt{3}}{2}$$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\Rightarrow\frac{12}{a^2}+\frac{1}{b^2}=1$$

$$\frac{12}{a^2}+\frac{1}{a^2\left(1-e^2\right)}=1$$

$$\frac{12}{a^2}+\frac{1}{a^2\left(1-\frac{3}{4}\right)}=1$$

$$b^2=16\left(1-\frac{3}{4}\right)$$

$$\boxed{b=2}$$

$$4a^2+4b^2=64+16$$

$$=80$$

$$(1)\Rightarrow\frac{12}{a^2}+\frac{4}{a^2}=1$$

$$\boxed{a=4}$$

78 $\ell=4, m=2, n=n, a^2=36, b^2=16$

$$\frac{a^2}{\ell^2}+\frac{b^2}{m^2}=\frac{\left(a^2-b^2\right)^2}{n^2}$$

$$\frac{36}{16}+\frac{16}{4}=\frac{(36-16)^2}{n^2}\Rightarrow n=\pm 8$$

79. $x\cos\alpha+y\sin\alpha=2\sqrt{3}$ *is a* tangent

To $\dfrac{x^2}{16}+\dfrac{y^2}{8}=1$

$$n^2=a^2l^2+b^2m^2$$

$$12=16\cos^2\alpha+8\sin^2\alpha$$

$$4=8\cos^2\alpha$$

$$\cos^2\alpha=\frac{1}{2}\Rightarrow\cos\alpha=\pm\frac{1}{\sqrt{2}}$$

$$\alpha=\pi/4$$

80 $x+\sqrt{3}\,y=3$ is a tangent to

$$2x^2+3y^2=k$$

$$\frac{x^2}{(k/2)}+\frac{y}{(k/3)}=1$$

$$n^2=a^2l^2+b^2m^2$$

$$9=\frac{k}{2}\times(1)+\frac{k}{3}\left(\sqrt{3}\right)^2$$

$$9=\frac{k}{2}+k$$

$$9=\frac{3}{2}k$$

$$k=6$$

$\sqrt{3}\,x-y+d=0$ is the normal to the ellipse

$$\frac{x^2}{3}+\frac{y^2}{2}=1$$

$$n^2=\frac{\left(a^2-b^2\right)^2}{\left(\frac{a^2}{l^2}+\frac{b^2}{m^2}\right)}\Rightarrow\frac{1}{\left(\frac{3}{3}+\frac{2}{1}\right)}=d^2$$

$$d=\pm\frac{1}{\sqrt{3}}$$

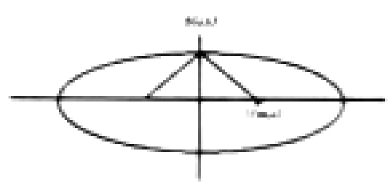
$$\sqrt{3}\,x-y\pm\frac{1}{\sqrt{3}}=0$$

$$3x-\sqrt{3}\,y\pm 1=0$$

81 SB=4, a-ae=4-$\sqrt{7}$

$$\sqrt{a^2e^2+a^2-a^2e^2}=4$$

a=4



$\therefore$ 4-4e=4-$\sqrt{7}$

e=$\dfrac{\sqrt{7}}{4}$

b=$\sqrt{a^2-a^2e^2}=\sqrt{16-16.\dfrac{7}{16}}$

$=3$

$$Cos\theta=\frac{b}{a}=\frac{3}{4}$$

$$Cos2\theta=2\cos^2\theta-1=2\times\frac{9}{16}-1=\frac{1}{8}$$

82 CONCEPTUAL