

# 1. FUNCTIONS

1. Find the domain of the real valued function  $f(x) = ([x]^2 - [x] - 2)^{-1/2}$ , where  $[.]$  is the greatest integer function

[AP EAMCET 17-09-20\_Shift-1]

1.  $R - (-1, 3)$                       2.  $R - [-1, 3)$   
3.  $R - (-1, 3)$                       4.  $R - [-1, 3]$

2. How many bijections  $f: Z \rightarrow Z$  are there such that  $f(x+y) = f(x) + f(y)$  for all  $x, y \in Z$ ?

[AP EAMCET 17-09-20\_Shift-1]

1. One                                      2. Two  
3. Three                                    4. Infinitely many

3. The domain of

$$f(x) = \cos^{-1}\left(\frac{x-3}{2}\right) - \log_{10}(4-x) \text{ is } \underline{\hspace{2cm}}$$

[AP EAMCET 17-09-20\_Shift-1]

1. (1, 4)                                    2. [1, 4)  
3. (1, 4]                                    4. [1, 4]

4. The function  $f: R \rightarrow R$  defined by

$$f(x) = \frac{x}{\sqrt{1+x^2}} \text{ is } \underline{\hspace{2cm}}$$

[AP EAMCET 17-09-20\_Shift-2]

1. Surjective but not injective  
2. Bijective  
3. Injective but not surjective  
4. Neither injective nor surjective

5. For  $f(x) = \frac{\sin \pi[x]}{1+[x]} + \frac{x}{2+3x}$ , where  $[x]$  denotes

the greatest integer function, the domain and range in  $R$  are respectively

[AP EAMCET 17-09-20\_Shift-2]

1.  $R - \left\{-1, \frac{-2}{3}\right\}$  &  $R - \left\{\frac{1}{3}\right\}$   
2.  $R - \left\{-1, \frac{-2}{3}\right\}$  &  $[-1, 1]$   
3.  $R - [-1, 0)$  &  $R - \left\{\frac{1}{3}\right\}$   
4.  $R - [-1, 0)$  &  $[-1, 1]$

6. How many functions  $f: Z \rightarrow Z$  are there such that  $f(x+y) = f(x) + f(y)$  for all  $x, y \in Z$ ?

[AP EAMCET 17-09-20\_Shift-2]

1. 1    2. 2  
3. 3    4. Infinitely many

7. Find  $\sum_{t=1}^{39} f(t)$  if  $f: R \rightarrow R$  is defined as

$$f(x+y) = f(x) + f(y), x, y \in R \text{ and}$$

$$f(1) = 7 \text{ [AP EAMCET 18-09-20_Shift-1]}$$

1. 5187                                        2. 5460  
3. 5740                                        4. 5407

8. Find the function  $g(t)$  if  $f(t) = 3t - 2$  and

$$(g \circ f)^{-1}(t) = t - 2$$

[AP EAMCET 18-09-20\_Shift-1]

1.  $g(t) = \frac{(t-8)}{3}$                                     2.  $g(t) = \frac{(t+8)}{3}$   
3.  $g(t) = \frac{(8-t)}{3}$                                     4.  $g(t) = 3t - 8$

9. If  $f: R \rightarrow R$  is defined by

$$f(x) = \frac{x^6}{x^6 + 2020}, \forall x \in R, \text{ then the range of } f \text{ is}$$

[AP EAMCET 18-09-20\_Shift-2]

1.  $[0, 1]$                                         2.  $[0, \infty)$   
3.  $[0, 1)$                                         4.  $\left[0, \frac{1}{2020}\right)$

10. Exactly how many functions  $f: Q \rightarrow Q$  exist such that  $f(x+y) = f(x) + f(y)$  and

$$f(xy) = f(x)f(y) \text{ for all } x, y \in Q?$$

[AP EAMCET 18-09-20\_Shift-2]

1. one    2. two  
3. three    4. Infinitely many

11. If  $f: R \rightarrow R$  is defined as

$$f(x) = \frac{2020^x}{2020^x + \sqrt{2020}}, \forall x \in R \text{ then } \sum_{r=1}^{4039} 2f\left(\frac{r}{4040}\right) =$$

[AP EAMCET 21-09-20\_Shift-1]

1. 4040                                        2. 4039  
3. 2020                                        4. 1010

12. If  $f(x) = e^x$ ;  $g(x) = \ln(x)$  for all  $x \in [1, \infty)$ , then fog is \_\_\_\_\_

[AP EAMCET 21-09-20\_Shift-1]

1. A one-one function
2. An onto function
3. Not a function
4. Bijective

13. The domain of  $\sqrt{|x| - x}$  is \_\_\_\_\_

[AP EAMCET 21-09-20\_Shift-2]

1.  $(-\infty, 0)$
2.  $(0, \infty)$
3.  $(-\infty, \infty)$
4.  $R - \{0\}$

14. Let S be a finite set, then a non-identity function  $f : S \rightarrow S$  can be .....

[AP EAMCET 21-09-20\_Shift-2]

1. Injective but not surjective
2. Surjective but not injective
3. Bijective but it does not have an inverse function
4. Data insufficient

15. If  $(f(x))^2 = f(x^2) + f(1)$  holds good, then

find  $f(x)$  [AP EAMCET 21-09-20\_Shift-2]

1.  $x + \frac{1}{x}$
2.  $x - \frac{1}{x}$
3.  $x^2 + \frac{1}{x}$
4.  $x - \frac{1}{x^2}$

16. For equality of functions f and g

- i) domain of f = domain of g
- ii)  $f(x) = g(x)$
- iii) x belongs to domain of f

[AP EAMCET 22-09-20\_Shift-1]

1. Only (i) and (ii) are necessary
2. Only (ii) and (iii) are necessary
3. Only (i) and (iii) are necessary
4. All (i),(ii),(iii) are necessary

17. Let  $f : N \times N \rightarrow N$  be a function such that  $f((1,1)) = 2$  and  $f((m+1,n)) = f((m,n)) + 2(m+n)$  and  $f((m,n+1)) = f((m,n)) + 2(m+n-1)$ , for all  $m, n \in N$ , then find  $f(2,2)$ .

[AP EAMCET 22-09-20\_Shift-1]

1. 8
2. 7
3. 9
4. 10

18. For  $f(x) = \sin\left(\frac{1}{|x|\sqrt{x^2-1}}\right)$  the domain and range of  $f(x)$  in R are

[AP EAMCET 22-09-20\_Shift-1]

1.  $R - \{0, \pm 1\}$  and  $[-1, 1]$ , respectively
2.  $R - [-1, 1]$  and  $[-1, 1]$ , respectively
3.  $R - \{0, \pm 1\}$  and  $[0, 1]$ , respectively
4.  $R - [-1, 1]$  and  $[0, 1]$ , respectively

19. The range of the function  $f(x) = x^2 + \frac{1}{x^2 + 1}$  is

[AP EAMCET 22-09-20\_Shift-1]

1.  $[1, \infty)$
2.  $[2, \infty)$
3.  $\left[\frac{3}{2}, \infty\right)$
4.  $(0, 1]$

20. The function  $f(x) = \sin x - \cos x$  is \_\_\_\_\_

[AP EAMCET 22-09-20\_Shift-2]

1. Odd function
2. Even function
3. Neither even nor odd function
4.  $f(x)$  is not a function

21. If  $f : N \times N \rightarrow N$  is defined by

$$f((m,n)) = 2^{m-1}(2n-1), \forall (m,n) \in N \times N$$

then f is [AP EAMCET 22-09-20\_Shift-2]

1. One-one but not onto
2. Onto but not one-one
3. Neither one-one nor onto
4. Both one-one and onto

22. In the following statements \_\_\_\_\_

- (i) Relation is a special case of function
- (ii) Function is special case of relation
- (iii) Both relation & function are same

[AP EAMCET 23-09-20\_Shift-1]

1. (iii) is True, (i) & (ii) are false
2. (i) is True, (ii) & (iii) are false
3. (ii) is True, (i) & (iii) are false
4. All (i), (ii) & (iii) are true

23. If  $f : R \rightarrow R$  is defined as

$$f(x) = x - [x] + 3, \forall x \in R, \text{ then } f \text{ is}$$

[AP EAMCET 23-09-20\_Shift-1]

1. Not a function
2. A periodic function with period  $\pi$
3. A periodic function with period 1
4. An invertible function

24. If  $f: \mathbb{R} \rightarrow \mathbb{R}$  is defined as  $f(x) = (2020 - x^{2019})^{1/2019}$ ,  $\forall x \in \mathbb{R}$ , find  $(f \circ f \circ f \circ f) \left( \frac{2019}{2020} \right)$

[AP EAMCET 23-09-20\_Shift-1]

1. 1  
2. 0  
3.  $\frac{2019}{2020}$   
4.  $\frac{2020}{2019}$

25. If  $f$  is a continuous real valued function, then the range of the function is

[AP EAMCET 23-09-20\_Shift-1]

1.  $[0, 1]$   
2.  $[\text{Minimum}(f), \text{Maximum}(f)]$   
3.  $[0, \infty)$   
4.  $(-\infty, 0]$

26. The function  $f(x) = \frac{e^{|x|} - e^{-x}}{e^x + e^{-x}} + \cos^3 \left( \frac{x}{2} \right)$

from  $\mathbb{R}$  to itself is

1. An injective function  
2. A surjective function  
3. An even function  
4. No bijective

27. Let  $A = \{1, 2, 3, \dots, n\}$  and  $B = \{a, b\}$  if the number of onto functions from  $A$  to  $B$  is 62, then the number of subsets of  $A$  containing exactly three elements is

1. 15  
2. 6  
3. 20  
4. 10

28. Match the items of List-I with those of the items of List-II

LIST-I	LIST-II
A) Range of $\sec^{-1} [1 + \cos^2 x]$ , $[\cdot]$ denotes the greatest integer function	(I) Odd function
B) Domain of $f(x)$ , where $f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2}$	(II) $\left\{0, \frac{1}{2}\right\}$
C) $f(x+y) = f(x) + f(y); f(1) = 5$	(III) $\{\sec^{-1} 5, \sec^{-1} 4\}$
D) $\sin^{-1} x - \cos^{-1} x + \sin^{-1}(1-x) = 0 \Rightarrow x \in$	(IV) $\mathbb{R} - (-2, 2)$
	(V) $\{\sec^{-1} 1, \sec^{-1} 2\}$

The correct match is

[TS EAMCET 09-09-20\_Shift-1]

- |    |   |    |     |    |    |     |    |    |    |
|----|---|----|-----|----|----|-----|----|----|----|
| 1. | A | B  | C   | D  | 2. | A   | B  | C  | D  |
|    | V | IV | I   | II |    | III | IV | II | I  |
| 3. | A | B  | C   | D  | 4. | A   | B  | C  | D  |
|    | V | II | III | IV |    | III | II | I  | IV |

29. The domain of the function

$$f(x) = \sec^{-1}(3x-4) + \tanh^{-1}\left(\frac{x+3}{5}\right) \text{ is}$$

[TS EAMCET 09-09-20\_Shift-1]

1.  $(-8, 1) \cup \left(\frac{5}{3}, 2\right)$   
2.  $\left(1, \frac{5}{3}\right)$   
3.  $[-8, 1] \cup \left[\frac{5}{3}, 2\right]$   
4.  $(-8, 1] \cup \left[\frac{5}{3}, 2\right)$

30. The function  $f: \mathbb{R} \rightarrow \mathbb{R}$  defined by

$$f(x) = \frac{x}{\sqrt{1+x^2}} \text{ is}$$

[TS EAMCET 09-09-20\_Shift-2]

1. surjective but not injective  
2. bijective  
3. injective but not surjective  
4. neither injective nor surjective

31. The set of values of  $\alpha$  such that  $f: \mathbb{R} \rightarrow \left[0, \frac{\pi}{2}\right)$

defined by  $f(x) = \tan^{-1}(x^2 + x + \alpha^2)$  is onto is

[TS EAMCET 09-09-20\_Shift-2]

1.  $\left(\frac{-1}{2}, \frac{1}{2}\right)$   
2.  $\left(\frac{-1}{4}, \frac{1}{4}\right)$   
3.  $\left(-\infty, \frac{-1}{2}\right) \cup \left(\frac{1}{2}, a\right)$   
4.  $\left(-\infty, \frac{-1}{4}\right) \cup \left(\frac{1}{4}, \infty\right)$

32. Let  $f: [0, 10] \rightarrow [1, 20]$  be a function defined

$$\text{as } f(x) = \begin{cases} \frac{60-5x}{3}, & 0 \leq x \leq 6 \\ 10, & 6 \leq x \leq 7 \\ 31-3x, & 7 \leq x \leq 10 \end{cases} \text{ then } f \text{ is}$$

[TS EAMCET 10-09-20\_Shift-1]

1. bijective function  
2. one-one but not onto function  
3. onto but not one-one function  
4. neither one-one nor onto function

33. The domain of the function

$$f(x) = \sqrt{\log_{10}\left(\frac{5x-x^2}{4}\right)}$$

is

[TS EAMCET 10-09-20\_Shift-1]

1.  $[0,1]$                       2.  $[1,4]$   
3.  $[4,5]$                       4.  $(-\infty, \infty)$

34. If  $f(x) = x - \frac{1}{x}, x \neq 0$ , then  $3f(x) =$

[TS EAMCET 10-09-20\_Shift-2]

1.  $3[f(x)]^2 - f(x^2)$       2.  $[f(x)]^2 - f(x^3)$   
3.  $f(x^3) - [f(x)]^3$       4.  $f(x^3) - f(x^2)$

35. Let  $[.]$  denote greatest integer function. If

$$f(x) = [x] \text{ and } g(x) = 3\left[\frac{x}{3}\right], \text{ then the set of}$$

all real  $x$  such that  $f(x) = g(x)$  is

[TS EAMCET 10-09-20\_Shift-2]

1.  $R$   
2.  $\{x \in R / x = 3k, k \in Z\}$   
3.  $\{x \in R / 3k - 1 < x \leq 3k, k \in Z\}$   
4.  $\{x \in R / 3k \leq x < 3k + 1, k \in Z\}$

36. If  $f: [-3, 2] \rightarrow [0, \sqrt[3]{x}]$  is an onto function

$$\text{defined by } f(n) = \begin{cases} 2 + \sqrt[3]{n}, & -3 \leq n \leq -1 \\ \frac{2}{n^3}, & -1 \leq n \leq 2 \end{cases} \text{ then}$$

$x =$  [TS EAMCET 11-09-20\_Shift-1]

1. 1                      2. 2  
3. 4                      4. 6

37. Let  $[x]$  denote the greatest integer not more than  $x$ . If  $A$  and  $B$  are the domains of the

$$\text{functions } f(x) = \frac{x - [x]}{\sqrt{|x|} - x} \text{ and } g(x) = \frac{x - [x]}{\sqrt{|x|} + x}$$

respectively, then

[TS EAMCET 11-09-20\_Shift-1]

1.  $A \cup B = R$               2.  $A \cap B = \phi$   
3.  $A - B = (-\infty, 0)$       4.  $B - A = (0, \infty)$

38. The number of bijective functions  $f: Z \rightarrow Z$  such that  $f(x+y) = f(x) + f(y) \forall x, y \in Z$ , is

[TS EAMCET 11-09-20\_Shift-2]

1. 2                      2. 4  
3. 0                      4. Infinitely many

39. For each  $n \in N$ , let  $A_n = \{(n+1)k / k \in N\}$  and

$$X = \bigcup_{n \in N} A_n. \text{ A mapping } f: X \rightarrow N \text{ defined by}$$

$$f(x) = x, \forall x \in X \text{ is}$$

[TS EAMCET 11-09-20\_Shift-2]

1. one-one and onto  
2. one-one but not onto  
3. onto but not one-one  
4. neither one-one nor onto

40. Given that for any  $n \in N$  there exist an odd integer  $q$  and a non-negative integer  $r$  such that,  $n$  can be written uniquely as  $n = q \times 2^r$ .

Let  $f: N \rightarrow N \times N$  be function defined by

$$f(n) = \left(r+1, \frac{q+1}{2}\right). \text{ Then,}$$

[TS EAMCET 14-09-20\_Shift-1]

1.  $f$  is one-one but not onto  
2.  $f$  is onto but not one-one  
3.  $f$  is bijection  
4. only  $f^{-1}(1,1)$  does not exist because  $f$  is not a bijection.

41. If  $f: R \rightarrow R$  be defined by

$$f(x) = x + 2|x+1| + 2|x-1|, \text{ then the element in the codomain, which has unique pre image in the domain is}$$

[TS EAMCET 14-09-20\_Shift-1]

1. 3                      2. 1  
3. 2                      4. 5

42. For  $n \in N$ , if  $f(n) = (\cos nx)(\sec x)^n$

And  $g(n) = (\sin nx)(\sec x)^n$ , then

$$f(2020) - f(2019) + (\tan x)g(2019) =$$

[TS EAMCET 14-09-20\_Shift-1]

1.  $\sin x$                       2.  $\cos x$   
3. 0                      4. 1

43. If  $f: Z \rightarrow N$  is defined by

$$f(n) = \begin{cases} 2n, & \text{if } n > 0 \\ 1, & \text{if } n = 0 \\ -2n-1 & \text{if } n < 0 \end{cases}$$

[TS EAMCET 14-09-20\_Shift-2]

1. one-one but not onto
2. onto but not one-one
3. both one-one and onto
4. neither one-one nor onto

44. Domain of  $\cos^{-1}[\log_5(x^2 + 7x + 15)]$  is

[TS EAMCET 14-09-20\_Shift-2]

1. The set of all real numbers
2.  $(-\infty, -5] \cup [-2, \infty)$
3.  $R - \{-5, -2\}$ , where  $R$  is the set of real numbers
4.  $[-5, -2]$

45. Let  $f(n) = A(-2)^n + B(-3)^n \forall A, B \in R$  and  $n \in N - \{1, 2\}$ . If  $f(n) + af(n-1) + bf(n-2) = 0$ , then  $(a+b)(b-a) =$

[TS EAMCET 14-09-20\_Shift-2]

1. 0
2. 5
3. 7
4. 11

46. Let  $f: R \rightarrow R$  and  $g: R \rightarrow R$  be defined by

$$f(x) = 2x + 1 \text{ \& } g(x) = x^2 - 2$$

$$(g \circ f)(x) =$$

[AP EAMCET 19-08-2021\_Shift-1]

1.  $2x^2 - 3$
2.  $4x^2 + 4x - 1$
3.  $4x^2 + 4x + 1$
4.  $2x^2 - 4$

47. Given the function  $f(x) = \frac{a^x + a^{-x}}{2}, (a > 2)$

then  $f(x+y) + f(x-y)$  is equal to

[AP EAMCET 19-08-2021\_Shift-1]

1.  $f(x) - f(y)$
2.  $f(y)$
3.  $2f(x)f(y)$
4.  $f(x)f(y)$

48. If  $f$  is a function define on  $(0, 1)$  by

$$f(x) = \min\{x - [x], -x - [x]\}$$

then  $(f \circ f \circ f \circ f)(x) = \underline{\hspace{1cm}}$  ( $[.]$  greatest integer

function) [AP EAMCET 19-08-2021\_Shift-1]

1.  $x$
2.  $-x$
3.  $4x$
4.  $2x$

49. The real valued function  $f(x) = \frac{x}{e^x - 1} + \frac{x}{2} + 1$

defined on  $R \setminus \{0\}$  is

[AP EAMCET 19-08-2021\_Shift-2]

1. An odd function
2. An even function
3. Both even & odd function
4. Neither even nor odd function

50. The domain of the function  $f(x) = \frac{1}{[x]-1}$ ,

where  $[x]$  is the greatest integer function of  $x$  is [AP EAMCET 19-08-2021\_Shift-2]

1.  $R - (1, 2)$
2.  $R - \{1\}$
3.  $R - \{0, 1\}$
4.  $R - [1, 2)$

51. Let  $f: R \rightarrow R$  be a function defined by

$$f(x) = \frac{4^x}{4^x + 2}, \text{ what is the value of}$$

$$f\left(\frac{1}{4}\right) + 2f\left(\frac{1}{2}\right) + f\left(\frac{3}{4}\right) = ?$$

[AP EAMCET 19-08-2021\_Shift-2]

1. 4
2. 3
3. 2
4. 1

52.  $f(x) = \sin x + \cos x, g(x) = x^2 - 1$  then

$g(f(x))$  is invertible if

[AP EAMCET 20-08-2021\_Shift-1]

1.  $-\frac{\pi}{4} \leq x \leq \frac{\pi}{4}$
2.  $-\frac{\pi}{2} \leq x \leq 0$
3.  $-\frac{\pi}{2} \leq x \leq \pi$
4.  $0 \leq x \leq \frac{\pi}{2}$



63. The inverse of the function  $y = \frac{10^x - 10^{-x}}{10^x + 10^{-x}}$  is

[AP EAMCET 25-08-2021\_Shift-1]

1.  $\frac{1}{2} \log_{10} \left( \frac{1+x}{1-x} \right)$       2.  $\frac{1}{2} \log_{10} \left( \frac{2+x}{2-x} \right)$   
 3.  $\frac{1}{2} \log_{10} \left( \frac{1-x}{1+x} \right)$       4.  $\frac{1}{2} \log_{10} \left( \frac{2-x}{2+x} \right)$

64. If  $f: R \rightarrow R$  is defined as

$$f(x+y) = f(x) + f(y) \quad \forall x, y \in R \text{ and}$$

$$f(1) = 5, \text{ then find the value of the following;}$$

$$\sum_{r=1}^n f(r) = ?$$

[AP EAMCET 25-08-2021\_Shift-2]

1.  $\frac{5n(n+1)}{2}$       2.  $\frac{7n(n-1)}{2}$   
 3.  $\frac{5n(n-1)}{2}$       4.  $\frac{7n(n+1)}{2}$

65. What is the range of the function

$$h(x) = \frac{x-2}{x+3} ?$$

[AP EAMCET 25-08-2021\_Shift-2]

1.  $(-\infty, 2) \cup (2, \infty)$   
 2.  $(-\infty, 1) \cup (1, \infty)$   
 3.  $(-\infty, -3) \cup (-3, \infty)$   
 4.  $(-\infty, -1) \cup (-1, \infty)$

66. Define  $f: R \rightarrow R$  by

$$f(x) = \cos \left( \tan^{-1} \left( \sin \left( \tan^{-1} x \right) \right) \right), \text{ then}$$

$$\lim_{x \rightarrow \infty} (f \circ f) x =$$

[AP EAMCET 25-08-2021\_Shift-2]

1.  $\frac{3}{2\sqrt{3}}$       2.  $\frac{\sqrt{2}}{3}$   
 3.  $\sqrt{\frac{2}{3}}$       4.  $\frac{2}{3\sqrt{3}}$

67. Let  $f: A \rightarrow B, g: B \rightarrow A$  defined as

$$f(x) = x^2, \forall x \in A \text{ and } g(x) = x^{1/2}, \forall x \in B.$$

$f(x)$  and  $g(x)$  are inverse functions to each other when

[TS EAMCET 04-08-2021\_Shift-2]

1.  $A = B = R$       2.  $A = R \setminus R^+; B = R \setminus R^+$   
 3.  $A = R; B = R \setminus R^+$       4.  $A = B = R \setminus R^+$

68. The function of  $f(x) = \log(x + \sqrt{x^2 + 1})$  is

[TS EAMCET 04-08-2021\_Shift-2]

1. An even function  
 2. An odd function  
 3. A periodic function  
 4. Neither an even function nor an odd function

69. If  $f: R \rightarrow R$  is defined as

$$f(x+y) = f(x) + f(y), \forall x, y \in R \text{ and}$$

$$f(1) = 10, \text{ then } \sum_{r=1}^n (f(r))^2 =$$

[TS EAMCET 04-08-2021\_Shift-1]

1.  $\frac{7}{2}n(n+1)$       2.  $5n(n+1)$   
 3.  $\frac{50}{3}n(n+1)(2n+1)$       4.  $\frac{100}{4}n^2(n+1)^2$

70. If  $f: R \rightarrow R$  is defined as

$$f(x) = \frac{3^x + 3^{-x}}{2}, \forall x \in R \text{ and it satisfies}$$

$$f(x+y) + f(x-y) = af(x)f(y), \text{ then } a =$$

[TS EAMCET 04-08-2021\_Shift-1]

1. 2      2. 1  
 3. 4      4. 8

71. A and B are subsets of R. Every element x of A is mapped to an element of B by the rule

$$y(x) = \begin{cases} \frac{5x}{(x-3)(x+3)} & \text{if } x \neq -1 \\ -1 & \text{if } x = -1 \end{cases},$$

Then A = [TS EAMCET 05-08-2021\_Shift-1]

1.  $R / \{-3, +3, -0\}$       2.  $R / \{-3, +3\}$   
 3.  $R / \{-3, +3, 0, -1\}$       4.  $R$

72. The domain and range of  $y(x) = \cos x - 3$  are respectively

[TS EAMCET 05-08-2021\_Shift-1]

1.  $R$  and  $[-1, 1]$
2.  $R$  and  $[-4, -2]$
3.  $R / \{0\}$  and  $[0, 1]$
4.  $R / \left\{ (2n+1)\frac{\pi}{2} \right\}$  and  $[-1, 1]$

73. Let  $f(x) = \frac{1}{2} - \tan\left(\frac{\pi x}{2}\right)$ ,  $-1 < x < 1$  and

$g(x) = \sqrt{3 + 4x - 4x^2}$  then the domain  $(f + g)$

is [TS EAMCET 05-08-2021\_Shift-2]

1.  $\left[\frac{1}{2}, 1\right)$
2.  $\left[\frac{-1}{2}, \frac{1}{2}\right)$
3.  $\left[-\frac{1}{2}, 1\right)$
4.  $\left[-\frac{1}{2}, -1\right]$

74. Let  $a > 1$  be a constant. If  $f : A \rightarrow A$  and

$(x, y) \in f$  satisfy  $a^x + a^y = a$ , then  $A =$

[TS EAMCET 05-08-2021\_Shift-2]

1.  $(0, a]$
2.  $[0, a]$
3.  $(-\infty, 1)$
4.  $(-\infty, a+1)$

75. If  $f : R \setminus \{0\} \rightarrow R$  is such that

$$2f(x) + f\left(\frac{1}{x}\right) = 4x, \text{ and}$$

$S = \{x \in R : f(x) = -f(x)\}$ , then the number of elements in  $S$  is

[TS EAMCET 06-08-2021\_Shift-2]

1. 0
2. 1
3. 2
4. At least three

76. If a function  $f : (-1, 1) \rightarrow B (\subseteq R)$  is defined as

$f(x) = x + x^2 + x^3 + \dots + \infty$ , then in order to have the inverse function of  $f$ ,  $B =$

[TS EAMCET 06-08-2021\_Shift-2]

1.  $\left(-\infty, \frac{1}{2}\right)$
2.  $\left(\frac{-1}{2}, \infty\right)$
3.  $(-1, 1)$
4.  $R$

77.  $f : [-2, 2] \rightarrow [-2, 2]$ ,  $g : [-2, 2] \rightarrow [0, 4]$  are two functions defined as

$$f(x) = \begin{cases} -2, & -2 \leq x \leq 0 \\ x^2 - 2, & 0 \leq x \leq 2 \end{cases} \text{ and}$$

$$g(x) = |f(x)| + f(|x|), \text{ then}$$

[TS EAMCET 06-08-2021\_Shift-1]

1.  $f$  and  $g$  are injective mappings
2.  $f$  and  $g$  are surjective mappings
3.  $f$  is bijective mapping and  $g$  is injective mapping
4.  $f$  is not bijective mapping and  $g$  is surjective mapping

78. The domain of the function  $f(x) = \frac{1}{\sqrt{|x| - x}}$  is

[TS EAMCET 06-08-2021\_Shift-1]

1.  $R$
2.  $(-\infty, 0)$
3.  $(0, \infty)$
4.  $(-\infty, 1)$

79. The range of the real valued function

$$f(x) = \sqrt{\frac{x^2 + 2x + 8}{x^2 + 2x + 4}}$$
 is

[AP EAMCET 04-07-2022\_Shift-1]

1.  $\left[\sqrt{\frac{7}{3}}, \infty\right)$
2.  $(0, \infty)$
3.  $(1, \infty)$
4.  $\left(1, \sqrt{\frac{7}{3}}\right]$

80. If  $f(x) = \sqrt{4 - x^2}$  and  $g(x) = \log(1 - x)$  are two real valued functions then the domain of the function  $(f + g)(x)$  is

[AP EAMCET 04-07-2022\_Shift-1]

1.  $[-2, 2]$
2.  $[-2, 1)$
3.  $(-\infty, 1)$
4.  $(1, 2]$

81.  $\left\{ x \in R / \frac{\sqrt{|x|^2 - 2|x| - 8}}{\log(2 - x - x^2)} \text{ is a real number} \right\} =$

[AP EAMCET 04-07-2022\_Shift-2]

1.  $(-\infty, -4] \cup [4, \infty)$
2.  $\emptyset$
3.  $(-1, 2)$
4.  $(-\infty, -4] \cup (-1, 2) \cup [4, \infty)$

82. The domain of the real valued function

$$f(x) = \sin \left( \log \left( \frac{\sqrt{4-x^2}}{1-x} \right) \right) \text{ is}$$

[AP EAMCET 04-07-2022\_Shift-2]

1. (1,4)                      2. (-1,1)  
3. (-2,1)                    4. (-2,4)

83.  $f(x) = \log \left( \left( \frac{2x^2-3}{x} \right) + \sqrt{\frac{4x^4-11x^2+9}{|x|}} \right)$  is

[AP EAMCET 05-07-2022\_Shift-1]

1. An odd function    2. An even function  
3. A polynomial function    4. Not a function

84. Let  $f: R - \left\{ \frac{-1}{2} \right\} \rightarrow R$  be defined by

$$f(x) = \frac{x-2}{2x+1}. \text{ If } \alpha, \beta \text{ satisfy the equation}$$

$$f(f(x)) = -x, \text{ then } 4(\alpha^2 + \beta^2) =$$

[AP EAMCET 05-07-2022\_Shift-1]

1. 17                      2. 12  
3. 24                      4. 34

85. The range of the real valued function

$$f(x) = \frac{x^2+x+1}{x} \text{ is}$$

[AP EAMCET 05-07-2022\_Shift-2]

1.  $(-\infty, 1) \cup (1, \infty)$     2.  $(-\infty, -1] \cup [1, \infty)$   
3.  $(-\infty, -2] \cup [3, \infty)$     4.  $(-\infty, -1] \cup [3, \infty)$

86. If a function  $f: R - \{l\} \rightarrow R - \{m\}$  defined by

$$f(x) = \frac{x+3}{x-2} \text{ is a bijection. Then } 3l+2m =$$

[AP EAMCET 05-07-2022\_Shift-2]

1. 10                      2. 12  
3. 8                        4. 14

87. Let  $f: R \rightarrow R$  defined by  $f(x) = 5x^4 + 2$ .

Then [AP EAMCET 06-07-2022\_Shift-1]

1.  $f$  is one-one but not onto  
2.  $f$  is onto but not one-one  
3.  $f$  is both one-one and onto  
4.  $f$  is neither one-one nor onto

88. Let  $f$  be a function defined by  $f(xy) = \frac{f(x)}{y}$  for all positive real number  $x$  and  $y$ . If  $f(30) = 20$ , then  $f(40) =$

[AP EAMCET 06-07-2022\_Shift-1]

1. 10                      2. 15  
3. 25                      4. 17

89. The domain of the real valued function  $f(x) = \frac{\log_2(x+3)}{\sqrt{x^2+3x+2}}$  is

[AP EAMCET 06-07-2022\_Shift-2]

1.  $(-3, \infty)$   
2.  $(-3, -1) \cup (-1, \infty)$   
3.  $(-3, -2) \cup (-2, -1) \cup (-1, \infty)$   
4.  $(-3, -2) \cup (-1, \infty)$

90. The domain of the real valued function  $f(x) = \frac{\sqrt{2-x} + \sqrt{1+x}}{\sqrt{x+3}}$  is

[AP EAMCET 06-07-2022\_Shift-2]

1.  $[-1, 2]$                       2.  $(-1, 2)$   
3.  $[-1, \infty)$                       4.  $[2, \infty)$

91. If a function  $f$  satisfies

$$f(x+1) + f(x-1) = \sqrt{2}f(x), \text{ then } f(x+2) + f(x-2) =$$

[AP EAMCET 07-07-2022\_Shift-1]

1.  $2.f(x)$                       2.  $f(x+1) - f(x-1)$   
3.  $4.f(x)$                       4. 0

92. The domain of the real valued function

$$f(x) = \frac{\sqrt{\log_{0.5}(x-3)}}{\sqrt{x-1}}$$

[AP EAMCET 07-07-2022\_Shift-1]

1.  $(3, 4]$                       2.  $[4, \infty)$   
3.  $(1, \infty)$                       4.  $(1, 3)$

93. Let  $f: R \rightarrow R$  be defined by  $f(x) = 2x+3$ , If  $\alpha, \beta$  are the roots of the equation

$$f(x^2) - 2f\left(\frac{x}{2}\right) - 1 = 0 \text{ then } \alpha^2 + \beta^2 =$$

[AP EAMCET 07-07-2022\_Shift-2]

1. 13                      2. 25  
3. 5                        4. 18

94. If  $f$  is a relation from set of positive real numbers to the set of positive real numbers defined by  $f(x) = 3x^2 - 2$  then  $f$  is

[AP EAMCET 07-07-2022\_Shift-2]

1. one-one but not onto
2. onto but not one-one
3. a bijection
4. not a function

95. If  $f: R \rightarrow R$  is defined as  $f(x) = x^2 - 2x - 3$

then  $f$  is [AP EAMCET 08-07-2022\_Shift-1]

1. One-one but not onto
2. Onto but not one-one
3. Neither one-one nor onto
4. A bijection

96. Let  $f(x) = \sqrt{\frac{x+1}{x+3}}$  and  $g(x) = \sqrt{\frac{2-x}{x+3}}$  be two

real valued functions. Then the domain of  $f/g$

is [AP EAMCET 08-07-2022\_Shift-1]

1.  $(-\infty, -3) \cup [-1, \infty)$
2.  $[-1, 2]$
3.  $(-3, 2)$
4.  $(-\infty, -3) \cup [2, \infty)$

97. If the function  $f: R \rightarrow R$  is defined by

$$f(x) = x|x|, \text{ then}$$

[AP EAMCET 08-07-2022\_Shift-2]

1.  $f$  is one-one but not onto
2.  $f$  is onto but not one-one
3.  $f$  is both one-one and onto
4.  $f$  is neither one-one nor onto

98. The domain of the real valued function

$$f(x) = \sqrt{\frac{2-|x|}{3-|x|}} \text{ is}$$

[AP EAMCET 08-07-2022\_Shift-2]

1.  $(-\infty, \infty)$
2.  $(-\infty, -3) \cup (2, \infty)$
3.  $(-\infty, -3) \cup (-2, 2) \cup [3, \infty)$
4.  $(-\infty, -3) \cup [-2, 2] \cup (3, \infty)$

99. The domain of the real valued function

$$f(x) = \frac{\sqrt{6x^2 + 5x - 6}}{\sqrt{4-x} - \sqrt{x+4}} \text{ is}$$

[TS EAMCET 18-07-2022\_Shift-1]

1.  $\left[-4, -\frac{3}{2}\right] \cup \left[\frac{2}{3}, 4\right]$
2.  $\left(-\infty, -\frac{3}{2}\right] \cup \left[\frac{2}{3}, \infty\right)$
3.  $[-4, 4]$
4.  $\left[-\frac{3}{2}, \frac{2}{3}\right]$

100. If  $[x]$  represents the greatest integer  $\leq x$ , then the range of the real valued function

$$f(x) = \frac{1}{\sqrt{[x]^2 + [x] - 2}} \text{ is}$$

[TS EAMCET 18-07-2022\_Shift-1]

1.  $(-\infty, 0] \cup \left(\frac{1}{2}, \infty\right)$
2.  $\left(0, \frac{1}{2}\right]$
3.  $(-\infty, 0) \cup [2, \infty)$
4.  $(0, 2]$

101. Let  $R$  be the set of all real numbers.

Statement I: The function  $f: \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \rightarrow R$

defined by  $f(x) = \sec x + \tan x$  is a one - one function.

Statement II: The function  $f: [0, \infty) \rightarrow R$

defined by  $f(x) = x^2$  is a one-one function

Which of the above statements is(are) true?

[TS EAMCET 18-07-2022\_Shift-2]

1. Statement I is true, but Statement II is false
2. Statement II is true, but Statement I is false
3. Both Statement I and Statement II are true
4. Both Statement I and Statement II are false

102. Let  $R$  be the set of all real numbers. Let

$f: R \rightarrow R$  be a function defined by

$$f(x) = \begin{cases} 2x-5, & \text{if } x < -3 \\ x+2, & \text{if } -3 \leq x < 5 \\ 3x+1, & \text{if } x \geq 5 \end{cases}$$

Match the following

	List - I		List - II
A)	$f(-5) + f(0) + f(-1) =$	I.	16
B)	$f(f(5) + 10f(-3)) =$	II.	40
C)	$f( f(-4) ) =$	III.	-32
D)	$f(f(f(1))) =$	IV.	-12
		V.	19

The connect match is

[TS EAMCET 18-07-2022\_Shift-2]

1. A - III; B - II; C - V; D - I
2. A - V; B - IV; C - I; D - III
3. A - IV; B - V; C - II; D - I
4. A - IV; B - V; C - III; D - I

103. The set of all real values of  $x$  for which

$$f(x) = \log_2(2^x - 2) + \sqrt{1-x} \text{ is also real is}$$

[TS EAMCET 19-07-2022\_Shift-I]

1.  $\mathbb{R}$
2.  $(1, \infty)$
3.  $(-\infty, 1]$
4.  $\phi$

104. Let  $f(x) = 1 - x$ ,  $g(x) = \frac{1}{1-x}$ ,  $h(x) = \frac{1}{x}$  be

three functions, for  $x \neq 0, 1$ . If a function  $F(x)$  satisfies  $f(F(h(x))) = g(x)$ , then

[TS EAMCET 19-07-2022\_Shift-I]

1.  $F(2022) = f(2022)$
2.  $F(2022) = g(2022)$
3.  $F(2022) = h(2022)$
4.  $F(2022) = \frac{1}{2022} f(2022)$

105. Let  $f: A \rightarrow B$  be defined as

$$f(x) = \frac{1}{2} - \tan\left(\frac{\pi x}{2}\right) \text{ and } g: B \rightarrow C \text{ be defined}$$

as  $g(x) = \sqrt{3+4x-4x^2}$ . If  $A, B, C$  are subsets of  $\mathbb{R}$  and  $f$  is an onto function then the range of the function  $f(x)$  is

[TS EAMCET 19-07-2022\_Shift-2]

1.  $(-\infty, \infty)$
2.  $[0, \infty)$
3.  $\left[-\frac{1}{2}, \frac{3}{2}\right]$
4.  $[-1, 1]$

106. If  $D$  is the domain and  $G$  is the range of the

$$\text{real valued function } f(x) = \sqrt{\frac{1-x^2}{1+x^2}}, \text{ then}$$

$$D \cap G =$$

[TS EAMCET 19-07-2022\_Shift-2]

1.  $[0, \infty)$
2.  $[0, 1]$
3.  $\left[0, \frac{1}{2}\right]$
4.  $[-1, 1]$

107. The domain of the real valued function

$$f(x) = \sqrt{\frac{2x^2 - 7x + 5}{3x^2 - 5x - 2}} \text{ is}$$

[TS EAMCET 20-07-2022\_Shift-1]

1.  $\left(-\infty, -\frac{1}{3}\right) \cup [1, 2) \cup \left[\frac{5}{2}, \infty\right)$
2.  $(-\infty, 1) \cup (2, \infty)$
3.  $\left[-\frac{1}{3}, \frac{5}{2}\right]$
4.  $\left(-\infty, -\frac{1}{3}\right] \cup \left[\frac{5}{2}, \infty\right)$

108. The range of the real valued function

$$f(x) = |x-2| + |x-3| \text{ is}$$

[TS EAMCET 20-07-2022\_Shift-1]

1.  $[3, \infty)$
2.  $[1, \infty)$
3.  $[2, \infty)$
4.  $(0, 2] \cup [3, \infty)$

109. If  $[x]$  represents the greatest integer function, then the set of all real values of  $x$  for which

$$f(x) = \sqrt{\frac{[x]-x}{x-[x]}} \text{ is real is}$$

[TS EAMCET 20-07-2022\_Shift-2]

1.  $\phi$
2.  $\mathbb{R}$
3.  $\mathbb{Z}$
4.  $\mathbb{R} - \mathbb{Z}$

110. If  $[x]$  denotes the greatest integer  $\leq x$ , then the range of the real valued function

$$f(x) = \frac{1}{\sqrt{x-[x]}} \text{ is}$$

[TS EAMCET 20-07-2022\_Shift-2]

1.  $[0, 1)$
2.  $(0, 1)$
3.  $(1, \infty)$
4.  $[1, \infty)$

111. If  $f(x) = x^3 - x$  and  $g(x) = \sin 2x$ ,

$$\text{then } f\left(g\left(\frac{\pi}{12}\right)\right) = \quad \text{[15th May 2023 Shift 1]}$$

1. 0
2. 1
3.  $-\frac{3}{8}$
4. 2

112. For  $x \in \mathbb{R}$  if  $f(x) = \sqrt{\log_{10}\left(\frac{3-x}{x}\right)}$ , then the domain of  $f$  is [15th May 2023 Shift 1]

1.  $\left[0, \frac{3}{2}\right]$                       2.  $\left(0, \frac{3}{2}\right]$   
 3.  $[0, 1]$                               4.  $(0, 1]$

113. If  $f(a) = \log\left|\frac{1-a}{1+a}\right|$  for  $a \neq \{-1, 1\}$ , then the set

of values of all 'a', for which  $f\left(\frac{2a}{1+a^2}\right) > 0$  is [15th May 2023 Shift 2]

1.  $(0, \infty) - \{1\}$                       2.  $(-\infty, 0) - \{-1\}$   
 3.  $(-\infty, \infty) - \{-1, 1\}$               4.  $(-1, 1)$

114. If a real valued function is defined by

$$f(x) = \frac{ax + \sqrt{a^2 - x^2}}{bx}, \text{ then } f \text{ is}$$

[15th May 2023 Shift 2]

1. Only one- one  
 2. Only onto  
 3. Both one - one and onto  
 4. Neither one- one nor onto

115. Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be a function defined by

$$f(x) = \frac{2x+1}{3}, \text{ if } \alpha \text{ is an element in the}$$

domain of  $f$  whose image is  $\frac{1}{\alpha}$ , then the sum of all possible values of such  $\alpha$  is

[16th May 2023 Shift 1]

1.  $\frac{-1}{2}$                                       2.  $\frac{1}{2}$   
 3.  $\frac{5}{2}$                                         4. 0

116. Let  $f(x) = |x|$  and  $g(x) = |x| + a, (a > 0)$ :

For  $0 \leq x \leq b, \{(x, y) / g(x) \leq y \leq f(x)\}$

Represents all the points in the interior of

[16th May 2023 Shift 1]

1. A parallelogram                      2. A triangle  
 3. A Square                                4. A circle

117. Let  $A \subseteq \mathbb{R}, B \subseteq \mathbb{R}$  and  $f: A \rightarrow B$  be define  $d$  by  $x^2 - 3x + 2$ . If  $f$  is a bijection then

[16th May 2023 Shift 2]

1.  $A = (-\infty, 0], B = \left(-\infty, \frac{-1}{4}\right]$     2.  $A = \left(-\infty, \frac{3}{2}\right], B = \left[\frac{-1}{4}, \infty\right)$   
 3.  $A = \left[\frac{3}{2}, \infty\right), B = \left(-\infty, \frac{-1}{4}\right]$     4.  $A = (-\infty, \infty), B = \left[\frac{-1}{4}, \infty\right)$

118. If  $f: \mathbb{R} \rightarrow \mathbb{R}$  is defined by

$$f(x+y) = f(x) + f(y) \forall x, y \in \mathbb{R} \text{ and}$$

$$f(1) = 7 \text{ then } \sum_{r=1}^n f(r) =$$

[16th May 2023 Shift 2]

1.  $\frac{3n(n+2)}{4}$                                       2.  $\frac{n(n-1)}{2}$   
 3.  $\frac{7n(n+1)}{2}$                                       4.  $\frac{(n+1)(n+2)}{4}$

119. If a set  $A$  has  $n$  elements, then the number of functions defined from  $A$  to  $A$  that are not one - one is [17th May 2023 Shift 1]

1.  $(n)^{n^2}$   
 2.  $n! - ({}^n C_0 + {}^n C_1 + {}^n C_2 + \dots + {}^n C_n)$   
 3.  $n^n - n!$   
 4.  $n^n$

120. If  $f(x)$  is the signum function, then in terms of  $f(x)$ , the constant function  $g(x) = 1, \forall x \in \mathbb{R}$  is

[17th May 2023 Shift 1]

$$1. g(x) = \begin{cases} 2 - f(x), & x < 0 \\ f(x), & x \geq 0 \end{cases}$$

$$2. g(x) = \begin{cases} f(x) + f(-x), & x < 0 \\ f(x)f(-x), & x \geq 0 \end{cases}$$

$$3. g(x) = \begin{cases} 1 + f(x), & x > 0 \\ 1 - f(x), & x \leq 0 \end{cases}$$

$$4. g(x) = \begin{cases} f(x) + 2, & x < 0 \\ 1 + f(x), & x = 0 \\ f(x), & x > 0 \end{cases}$$

121. If  $f(0)=0$ ,  $f(1)=1$ ,  $f(2)=2$  and  $f(x)=f(x-2)+f(x-3)$  for  $x=3,4,5,\dots$ , then  $f(10)=$   
**[17th May 2023 Shift 2]**
1. 13
  2. 9
  3. 11
  4. 10

122. If  $f(x) = \frac{\cos^2 x + \sin^4 x}{\sin^2 x + \cos^4 x} \forall x \in \mathbb{R}$ , then  $f(2023)=$   
**[17th May 2023 Shift 2]**
1. 1
  2. 0
  3. 2
  4.  $\pi$

123. If A is the domain and B is the range of the function  $f(x) = \begin{cases} 3x-1, & x > 1 \\ x^2+1, & x \leq 1 \end{cases}$ , then  $A-B=$   
**[18th May 2023 shift -1]**
1.  $(1, \infty)$
  2.  $(-\infty, 1)$
  3.  $\mathbb{R} - (-1, 1)$
  4.  $(-1, 1)$

124. If  $f: \mathbb{R} \rightarrow \mathbb{R}$  and  $g: \mathbb{R} \rightarrow \mathbb{R}$  are defined by  $f(x) = x^3 - x$  and  $g(x) = \sin 2x$ , then the value of  $x \in (0, 2\pi)$  that satisfy  $f(g(x)) > 0$ , lie in the interval **[18th May 2023 shift -1]**
1.  $\left(\frac{\pi}{2}, \pi\right)$
  2.  $\left(0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right)$
  3.  $\left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\frac{3\pi}{4}, \pi\right)$
  4.  $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

125. The range of the real valued function  $f(x) = \sqrt{9-x^2}$  is **[18th May 2023 Shift 2]**
1.  $[-3, 3]$
  2.  $[-3, 0]$
  3.  $[0, 3]$
  4.  $[-2, 2]$

126. If  $f: \mathbb{R} \setminus \{0\} \rightarrow \mathbb{R}$  is defined by  $f(x) = x + \frac{1}{x}$ , then the value of  $(f(x))^2 =$   
**[18th May 2023 Shift 2]**
1.  $f(x)+f(0)$
  2.  $f(x^2)+f(2)$
  3.  $f(x^3)+f(0)$
  4.  $f(x^2)+f(1)$

127. The range of the function

$$f(x) = \begin{cases} 4x-1, & x > 3 \\ x^2-2, & -2 \leq x \leq 3 \\ 3x+4, & x < -2 \end{cases}$$

- [19th May 2023 Shift 1]**
1.  $(-\infty, \infty)$
  2.  $\mathbb{R} - (-3, 3)$
  3.  $\mathbb{R} - (7, 11]$
  4.  $(7, 11]$

128. The domain of the function  $y = f(x)$ , where x and y are related by  $2^x + 2^y = 2$  is **[19th May 2023 Shift 1]**
1.  $(-\infty, \infty)$
  2.  $(-\infty, 1)$
  3.  $(0, \infty)$
  4.  $(1, \infty)$

129. The range of the function  $f(x) = \log_{0.5}(x^4 - 2x^2 + 3)$  is **[12th MAY 2023 SHIFT-1]**
1.  $(-\infty, \infty)$
  2.  $(-\infty, -1]$
  3.  $[-1, \infty)$
  4.  $[-1, 1]$

130. If  $f(x) = -|x|$ , then  $(f \circ f \circ f)(x) + (f \circ f \circ f)(-x) =$   
**[12th MAY 2023 SHIFT-1]**
1.  $-2f(x)$
  2.  $|f(x)|$
  3.  $2f(x)$
  4.  $-|f(x)|$

131. If  $f: [2, \infty) \rightarrow \mathbb{R}$  is defined by  $f(x) = x^2 - 4x + 5$ , then the range of f is **[12th MAY 2023 SHIFT-1]**
1.  $\mathbb{R}$
  2.  $[1, \infty)$
  3.  $[4, \infty)$
  4.  $[5, \infty)$

132. The domain of the function  $f(x) = \text{Sin}^{-1}\left(\log_2\left(\frac{x^2}{2}\right)\right)$  is **[12th MAY 2023 SHIFT -2]**
1.  $[-2, 0) \cup (0, 1)$
  2.  $[1, \infty) \cap [-2, 2]$
  3.  $[-2, -1] \cup [1, 2]$
  4.  $(-\infty, 1] \cap [-2, 2]$

133. The range of the function

$$f(x) = -\sqrt{-x^2 - 6x - 5} \text{ is}$$

[12<sup>TH</sup> MAY 2023 SHIFT -2]

1.  $[0, 2]$
2.  $[-2, 0]$
3.  $[-2, 2]$
4.  $(-\infty, 2]$

134. If  $f: \mathbb{R} \rightarrow \mathbb{R}$  is defined by  $f(x) = 2x + \sin x$ ,  $x \in \mathbb{R}$ , then  $f$  is

[12<sup>TH</sup> MAY 2023 SHIFT -2]

1. One-one and onto
2. One-one but not onto
3. Onto but not one-one
4. Neither one-one nor onto

135. If the function  $f: \mathbb{R} \rightarrow \mathbb{R}$  is defined by

$$f(x) = \begin{cases} 2x-3, & \text{if } x < -2 \\ x^2-1, & \text{if } -2 \leq x \leq 2 \\ 3x+2, & \text{if } x > 2 \end{cases} \text{ then } f \text{ is}$$

[13<sup>TH</sup> MAY 2023 SHIFT-1]

1. An injection but not a surjection
2. A surjection but not an injection
3. A bijection
4. Neither injection nor surjection

136. The domain of the real valued function

$$f(x) = \frac{\sqrt{\log_{10}\left(\frac{x}{x-2}\right)}}{\sqrt{[x]^2 - 5[x] + 6}} \text{ is (Here } [x] \text{ denotes}$$

the greatest integer function)

[13<sup>TH</sup> MAY 2023 SHIFT-1]

1.  $(-\infty, 0) \cup (2, \infty)$
2.  $(2, \infty)$
3.  $(-\infty, 2) \cup (4, \infty)$
4.  $[4, \infty)$

137. The range of the real valued function

$$f(x) = \frac{1}{x-|x|} \text{ is}$$

[13<sup>TH</sup> MAY 2023 SHIFT-1]

1.  $(0, \infty)$
2.  $(-\infty, 0)$
3.  $(-\infty, 0) \cup (0, \infty)$
4.  $(-\infty, \infty)$

138. Which one of the following functions is a

bijection? [EAPCET 13-05-23 SHIFT-2]

1.

$$f: \mathbb{R} - \mathbb{Z} \rightarrow [0, 1] \text{ defined by}$$

$$f(x) = \sqrt{x - [x]}. \text{ (Here } [x] \text{ represents the greatest integer function).}$$

2.  $f: \mathbb{R} \rightarrow (-\infty, 2)$  defined by

$$f(x) = 4x - x^2 - 3.$$

3.  $f: (5, \infty) \rightarrow \mathbb{R} - \{0\}$  defined by

$$f(x) = \frac{1}{\sqrt{x-5}}$$

4.  $f: [0, 4] \rightarrow [0, 4]$

$$\text{defined by } f(x) = \sqrt{16 - x^2}$$

139. The domain of the real valued function

$$f(x) = \frac{\sqrt{|x| - x}}{\sqrt{x - [x]}} \text{ is}$$

[EAPCET 13-05-23 SHIFT-2]

1.  $\mathbb{Z}$
2.  $\phi$
3.  $\mathbb{R} - \mathbb{Z}$
4.  $\mathbb{R}$

140. The range of the function defined by

$$f(x) = \begin{cases} 2x-3, & \text{if } x < -1 \\ 1-x^2, & \text{if } -1 \leq x \leq 1 \\ 3x^2+2, & \text{if } x > 1 \end{cases} \text{ is}$$

[EAPCET 13-05-23 SHIFT-2]

1.  $\mathbb{R}$
2.  $(-\infty, -5) \cup [0, 1] \cup (5, \infty)$
3.  $(-\infty, -1) \cup (1, \infty)$
4.  $(-\infty, -3) \cup (0, 1) \cup (3, \infty)$

### KEY

- |     |   |     |   |     |   |     |   |     |   |
|-----|---|-----|---|-----|---|-----|---|-----|---|
| 1)  | 2 | 2)  | 4 | 3)  | 2 | 4)  | 3 | 5)  | 3 |
| 6)  | 4 | 7)  | 2 | 8)  | 2 | 9)  | 3 | 10) | 1 |
| 11) | 2 | 12) | 4 | 13) | 3 | 14) | 4 | 15) | 1 |
| 16) | 4 | 17) | 4 | 18) | 2 | 19) | 1 | 20) | 3 |
| 21) | 4 | 22) | 3 | 23) | 3 | 24) | 3 | 25) | 2 |
| 26) | 4 | 27) | 3 | 28) | 1 | 29) | 4 | 30) | 3 |

31) 3	32) 3	33) 2	34) 3	35) 4
36) 3	37) 2	38) 1	39) 2	40) 3
41) 1	42) 3	43) 2	44) 4	45) 4
46) 2	47) 3	48) 1	49) 2	50) 4
51) 3	52) 1	53) 2	54) 4	55) 3
56) 2	57) 3	58) 2	59) 2	60) 3
61) 2	62) 3	63) 1	64) 1	65) 2
66) 1	67) 4	68) 2	69) 3	70) 1
71) 2	72) 2	73) 3	74) 3	75) 3
76) 2	77) 2	78) 2	79) 4	80) 2
81) 2	82) 3	83) 1	84) 1	85) 4
86) 3	87) 4	88) 2	89) 4	90) 1
91) 4	92) 1	93) 3	94) 4	95) 3
96) 2	97) 3	98) 4	99) 1	100) 2
101) 3	102) 3	103) 4	104) 2	105) 3
106) 2	107) 1	108) 2	109) 1	110) 3
111) 3	112) 2	113) 2	114) 2	115) 2
116) 1	117) 2	118) 3	119) 3	120) 4
121) 1	122) 1	123) 2	124) 1	125) 3
126) 4	127) 4	128) 2	129) 2	130) 3
131) 2	132) 3	133) 2	134) 1	135) 4
136) 4	137) 2	138) 4	139) 3	140) 2

### SOLUTIONS

1.  $([x]^2 - [x] - 2) > 0$

$$([x] + 1)([x] - 2) > 0$$

$$\Rightarrow [x] < -1, [x] > 2$$

$$\Rightarrow x \in (-\infty, -1) \cup [3, \infty)$$

$$\Rightarrow R - [-1, 3)$$

2. Here  $f(x) = kx, k \in R$

$\Rightarrow$  infinitely many bijections exist

3.  $-1 \leq \frac{x-3}{2} \leq 1$  and  $4-x > 0$

$$1 \leq x \leq 5 \quad x < 4$$

$$\Rightarrow x \in [1, 4)$$

4. Given  $f: R \rightarrow R$

let

$$y = \frac{x}{\sqrt{1+x^2}}$$

$$y^2 = \frac{x^2}{1+x^2}$$

$$y^2(1+x^2) = x^2$$

$$x^2 = \frac{y^2}{1-y^2} \Rightarrow x = \frac{y}{\sqrt{1-y^2}} \text{ is defined}$$

$$1-y^2 > 0; y^2 \neq 1$$

$$y^2 < 1; y \neq -1, 1$$

$$y \in (-1, 1)$$

$$y \in (-1, 1) - \{0\}$$

range  $\neq$  co-domain

$\therefore f$  is not surjective

$$f(x) = \frac{x}{\sqrt{1+x^2}}$$

Different values of  $x$  has different images

$\therefore f$  is 1, -1

Injective but not surjective

5. Given

$$f(x) = \frac{\sin \pi[x]}{1+[x]} + \frac{x}{2+3x}$$

$$\text{let } g(x) = \frac{\sin \pi[x]}{1+[x]}, h(x) = \frac{x}{2+3x}$$

domains:

$$1+[x] \neq 0, 2+3x \neq 0$$

$$[x] \neq -1, x \neq \frac{-2}{3}$$

$$x \notin [-1, 0)$$

$\therefore$  domain of  $f(x) = R - [-1, 0)$

Range of  $f(x)$ :

$$\frac{x}{2+3x} = y$$

$$\Rightarrow x = \frac{2y}{1-3y} = h^{-1}(y)$$

$$\therefore h^{-1}(x) = \frac{2x}{1-3x}$$

$$\Rightarrow 1-3x \neq 0 \Rightarrow x \neq \frac{1}{3}$$

$$D(h^{-1}(x)) = R - \left\{ \frac{1}{3} \right\}$$

$$\text{range of } h(x) = D(h^{-1}(x)) = R - \left\{ \frac{1}{3} \right\}$$

$$\therefore \text{Range of } f(x) = R - \left\{ \frac{1}{3} \right\}$$

6. Let

$$f(x) = a^n$$

$$f(x+y) = f(x) + f(y) \text{ is true } \forall x, y \in \mathbb{Z}$$

$\therefore$  Infinite functions exists.

7.  $f(x+y) = f(x) + f(y), x, y \in \mathbb{R}$

$$\text{Let } x=1 \text{ and } b=1$$

$$f(2) = 2f(1)$$

$$f(3) = 3f(1)$$

$$\sum_{t=1}^{39} f(t) = f(1) + f(2) + \dots + f(39)$$

$$= f(1) + 2f(1) + \dots + 39f(1)$$

$$= f(1)(1+2+3+\dots+39)$$

$$= \frac{7 \times 39 \times 40}{2} = 5460$$

8.  $(g \circ f)(t-2) = t$

$$\text{Let } t-2 = z$$

$$(g \circ f)(z) = z+2$$

$$g(f(z)) = z+2$$

$$g(3z-2) = z+2$$

$$\text{let } 3z-2 = x$$

$$g(x) = \frac{x+8}{3}$$

9.  $f(x) = \frac{x^6}{x^6+2020}$

$$= 1 - \frac{2020}{x^6+2020}$$

<1

as  $x=0$ , we have

$$f(x) = 0$$

$\therefore$  Range is  $[0,1)$

10.  $f(x)=0$  is only function

11.  $f(x) = \frac{2020^x}{2020^x + \sqrt{2020}}$

$$f(1-x) = \frac{2020^{1-x}}{2020^{1-x} + \sqrt{2020}}$$

$$f(1-x) = \frac{\sqrt{2020}}{2020^x + \sqrt{2020}}$$

$$\therefore f(x) + f(1-x) = 1$$

Now,

$$= 2 \left[ f\left(\frac{1}{4040}\right) + f\left(\frac{2}{4040}\right) + \dots + f\left(\frac{4038}{4040}\right) + f\left(\frac{4039}{4040}\right) \right]$$

$$= 2 \cdot \frac{4039}{2} = 4039$$

12.  $(f \circ g)(x) = e^{\log x} = x$

Which is an identity function

Every identity function must be a both one-one and onto

13. Given  $|x| - x \geq 0$

$$\Rightarrow |x| \geq x, \forall x \in \mathbb{R}$$

Domain is  $(-\infty, \infty)$

14. Data insufficient (concept)

15. Take  $f(x) = x + \frac{1}{x}$

$$\text{Now } (f(x))^2 = \left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2 \dots (1)$$

$$\text{And } f(x^2) + f(1) = x^2 + \frac{1}{x^2} + 2 \dots (2)$$

From (1) & (2), we get

$$(f(x))^2 = f(x^2) + f(1)$$

16. Conceptual

17. Conceptual

18. Conceptual

19. Conceptual

20.  $f(x) = \sin x - \cos x$

$f(-x) = -\sin x - \cos x$

$f(x) \neq f(-x)$

$f(x) \neq -f(x)$

$\therefore f(x)$  is neither even nor odd.

21. Conceptual

22. Conceptual

23.  $x - [x]$  period is 1

24.  $(f \circ f)(x) = f(f(x))$

$= f\left[2020 - x^{2019}\right]^{\frac{1}{2019}}$

$= \left[2020 - \left(2020 - x^{2019}\right)^{2019}\right]^{\frac{1}{2019}} = x$

$(f \circ f \circ f \circ f)(x) = (f \circ f)(x) = x$

25. Conceptual

26.  $f(x) = \frac{e^{|x|} - e^{-x}}{e^x + e^{-x}} + \cos^3\left(\frac{x}{2}\right)$

$$f(x) = \begin{cases} \frac{e^x - e^{-x}}{e^x + e^{-x}} + \cos^3\left(\frac{x}{2}\right) & , x > 0 \\ \cos^3\frac{x}{2} & , x \leq 0 \end{cases}$$

$\therefore f(x) \neq 0, \forall x \in R$

$\therefore f(x)$  is not bijective.

27.  $A = \{1, 2, 3, \dots, n\}$  and  $B = \{a, b\}$

No. of onto functions = 62

$2^n - 2 = 62 \Rightarrow n = 6$

No. of subsets of A containing exactly three elements is  ${}^n C_3 = {}^6 C_3 = 20$

28. A)  $[1 + \cos^2 x] = 1$  or  $2$

Range of  $\sec^{-1}[1 + \cos^2 x]$  is

$\{\sec^{-1} 1, \sec^{-1} 2\}$

B)  $f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2$

$f(y) = y^2 - 2$

$y = x + \frac{1}{x} \geq 2$  or  $\leq -2$

Domain is  $\mathbb{R} - (-2, 2)$

C)  $f(x) = kx$

$f(1) = 5 \Rightarrow k = 5$

$f(x) = 5x$  is an odd function

D)  $\sin^{-1} x - \cos^{-1} x + \sin^{-1}(1-x) = 0$

It satisfies for  $x = 0, \frac{1}{2}$

29.  $f(x) = \sec^{-1}(3x-4) + \tanh^{-1}\left(\frac{x+3}{5}\right)$

$3x-4 \leq -1$  or  $3x-4 \geq 1$  and  $-1 \leq \frac{x+3}{5} \leq 1$

$\Rightarrow x \in (-8, 1] \cup [5/3, 2)$

30. Different elements have different images

$f(x) = 1$

$\frac{x}{\sqrt{1+x^2}} = 1 \Rightarrow x^2 = 1+x^2 \exists \text{ no } x \in \mathbb{R}$

Not surjective

31.  $0 \leq \tan^{-1}(x^2 + x + \alpha^2) < \frac{\pi}{2}$

$0 \leq (x^2 + x + \alpha^2) < \infty$

$x^2 + x + \alpha^2 \geq 0$

$\Delta \leq 0$

$1 - 4\alpha^2 \leq 0$

$4\alpha^2 - 1 \geq 0$

$\left(-\alpha, \frac{-1}{2}\right) \cup \left[\frac{1}{2}, \alpha\right)$

32. Clearly  $f(6) = 10$  and  $f(7) = 10$

Different elements have same f-image

$\therefore f(x)$  is not 1-1

$\therefore$  Range = co-domain

$\therefore f$  is onto

33.  $\log_{10}\left(\frac{5x-x^2}{4}\right) \geq 0$  and  $\frac{5x-x^2}{4} > 0$

$\frac{5x-x^2}{4} \geq 1$  and  $x^2 - 5x < 0$

$x \in [1, 4]$  and  $x \in (0, 5)$

Domain of  $f(x) = [1, 4]$

34.  $f(x) = x - \frac{1}{x}$

$3f(x) = 3x - 3$

By option verification

$f(x^3) - (f(x))^3 = x^3 - \frac{1}{x^3} - \left(x - \frac{1}{x}\right)^3$

$$f(x^3) - (f(x))^3 = 3f(x)$$

35.  $f(x) = g(x)$

$$[x] = 3 \left[ \frac{x}{3} \right]$$

$$\frac{[x]}{3} = \left[ \frac{x}{3} \right]$$

$$x \in [3n, 3n+1), n \in \mathbb{Z}$$

36. Given function  $f: [-3, 2] \rightarrow [0, \sqrt[3]{x}]$ ,

When  $-3 \leq n \leq -1$ ,  $f(n) = 2 + \sqrt[3]{n}$

And  $-1 \leq n \leq 2$ ,  $f(n) = n^{\frac{2}{3}}$

Now,  $\left. \begin{aligned} f(-3) &= 2 - \sqrt[3]{3} \\ f(-2) &= 2 - \sqrt[3]{2} \\ f(-1) &= 1 \end{aligned} \right\} \forall n \in [-3, -1]$

And  $\left. \begin{aligned} f(0) &= 0 \\ f(1) &= 1 \\ f(2) &= \sqrt[3]{4} \end{aligned} \right\} \forall n \in [-1, 2]$

w.k.t  $f(2) = \sqrt[3]{x}$

by comparing  $x=4$  [ $\therefore$  The function is onto, range=codomain]

37.  $f(x) = \frac{x - [x]}{\sqrt{|x| - x}}$   $g(x) = \frac{x - [x]}{\sqrt{|x| + x}}$

To define  $f(x)$

To define  $g(x)$

$$|x| - x > 0 \quad |x| + x > 0$$

$$|x| > x \quad |x| > -x$$

$$\forall x \in (-\infty, 0) \quad \forall x \in (0, \infty)$$

i.e.  $A \in (-\infty, 0)$  i.e.  $B \in (0, \infty)$

Clearly  $A \cap B = \emptyset$

38.  $f(x+y) = f(x) + f(y), \forall x, y \in \mathbb{Z}$

$$f(x) = Kx$$

39.  $A_1 = \left\{ \frac{2K}{K \in \mathbb{N}} \right\}, A_2 = \left\{ \frac{3K}{K \in \mathbb{N}} \right\}, \dots, A_n = \left\{ \frac{nK}{K \in \mathbb{N}} \right\}$

$$X = A_1 \cup A_2 \cup \dots \cup A_n$$

$$f: X \rightarrow \mathbb{N}, f(x) = x$$

Has no pre image so it is not onto but it is one-

one

40.  $f(n_1) = f(n_2)$

$$\left( r_1 + 1, \frac{q_1 + 1}{2} \right) = \left( r_2 + 1, \frac{q_2 + 1}{2} \right)$$

$$\Rightarrow r_1 = r_2 \text{ and } q_1 = q_2$$

$\therefore f$  is one-one

Let  $\left( r + 1, \frac{q + 1}{2} \right) \in \mathbb{N} \times \mathbb{N}$

Let  $r + 1 = t \in \mathbb{N}$

$$r = t - 1 \in \mathbb{N}$$

$$\frac{q + 1}{2} = m \in \mathbb{N}$$

$$q = 2m - 1 \in \mathbb{N}$$

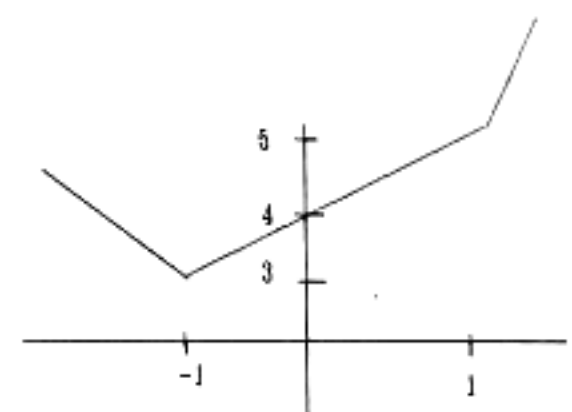
$$\therefore q \in \mathbb{N}$$

$\exists q \in \mathbb{N}$  such that  $f(n) = \left( r + 1, \frac{q + 1}{2} \right)$

$\therefore f$  is onto

$\therefore f$  is bijection.

41.  $f(x) = \begin{cases} -3x & , x < -1 \\ x + 4 & , -1 < x < 1 \\ 5x & , x > 1 \end{cases}$



By graph, element in codomain 3 has unique pre-image in domain.

42.  $f(n) = \cos nx (\sec x)^n$

$$f(2020) - f(2019) + \tan x \cdot g(2019) = \cos(2020x)(\sec x)^{2020}$$

$$- \cos(2019x)(\sec x)^{2019} + \tan x \sin(2019x)(\sec x)^{2019}$$

$$= (\sec x)^{2019} \left[ \cos 2020x \cdot \sec x - \cos 2019x + \frac{\sin x}{\cos x} \cdot \sin 2019x \right]$$

$$= (\sec x)^{2020} [\cos 2020x - \cos 2020x] = 0$$

43.  $f$  is not one-one because  $f(0) = f(-1) = 1$   
 And  $f(Z) = N \Rightarrow f$  is onto

44.  $-1 \leq \log_5 [x^2 + 7x + 15] \leq 1$

$$\Rightarrow \frac{1}{5} \leq [x^2 + 7x + 15] \leq 5$$

$$\Rightarrow \frac{1}{5} \leq x^2 + 7x + 15 \text{ and } x^2 + 7x + 15 \leq 5$$

$$\Rightarrow 5x^2 + 35x + 74 \geq 0 \quad \forall x \in R$$

$$\text{and } x^2 + 7x + 10 \leq 0$$

$$\Rightarrow x \in R \text{ and } -5 \leq x \leq -2$$

$$\text{domain} = [-5, -2] \cap R = [-5, -2]$$

45.  $f(n) = A(-2)^n + B(-3)^n \quad \forall A, B \in R \text{ and } n \in N - \{1, 2\}$

put  $n = 0, 1, 2, 3$

$$\text{we get } f(0) = A + B, f(1) = -2A - 3B,$$

$$f(2) = 4A + 9B, f(3) = -8A - 27B$$

Now

$$f(n) + af(n-1) + bf(n-2) = 0$$

put  $n = 2, 3$

$$f(2) + af(1) + bf(0) = 0 \text{ and}$$

$$f(3) + af(2) + bf(1) = 0$$

And substitute above in it, we get  $a = 5, b = 6$

46.  $(g \circ f)(x) = g[f(x)]$

$$= g[2x+1]$$

$$= (2x+1)^2 - 2$$

$$= 4x^2 + 4x - 1$$

47.  $f(x+y) = \frac{a^x a^y + a^{-x} a^{-y}}{2}$

$$f(x-y) = \frac{a^x a^{-y} + a^{-x} a^y}{2}$$

$$f(x+y) + f(x-y) = \frac{a^x(a^y + a^{-y}) + a^{-x}(a^{-y} + a^y)}{2}$$

$$= \frac{(a^x + a^{-x})(a^y + a^{-y})}{2}$$

$$= 2 \times \left( \frac{a^x + a^{-x}}{2} \right) \left( \frac{a^y + a^{-y}}{2} \right)$$

$$= 2f(x)f(y)$$

48.  $x \in (0, 1) \Rightarrow [x] = 0$

$$f(x) = \min\{x, -x\}$$

$$(f \circ f \circ f \circ f)(x) = f[f[f[f(-x)]]]$$

$$= f[f(x)]$$

$$= f(-x) = x$$

49.  $f(-x) = \frac{-x}{e^{-x}-1} - \frac{x}{2} + 1$

$$= \frac{-xe^x}{1-e^x} - \frac{x}{2} + 1$$

$$= \frac{xe^x}{e^x-1} - \frac{x}{2} + 1$$

$$= \frac{x[(e^x-1)+1]}{e^x-1} - \frac{x}{2} + 1$$

$$= x + \frac{x}{e^x-1} - \frac{x}{2} + 1$$

$$= \frac{x}{e^x-1} + \frac{x}{2} + 1 = f(x)$$

$\therefore f(x)$  is an even function

50.  $[x] - 1 \neq 0$

$$[x] \neq 0$$

$$x \neq [1, 2)$$

Domain  $R - [1, 2)$

51.  $f(x) = \frac{4^x}{4^x + 2}$

$$f(1-x) = \frac{4^{1-x}}{4^{1-x} + 2} = \frac{2}{4^x + 2}$$

$$f(x) + f(1-x) = \frac{4^x + 2}{4^x + 2} = 1$$

$$= f\left(\frac{1}{4}\right) + f\left(1 - \frac{1}{4}\right) + f\left(\frac{1}{2}\right) + f\left(1 - \frac{1}{2}\right)$$

$$= 1 + 1 = 2$$

52.  $g(f(x)) = \sin 2x$  is invertible  $-\frac{\pi}{2} \leq 2x \leq \frac{\pi}{2}$

$$-\frac{\pi}{2} \leq x \leq \frac{\pi}{4}$$

$$\begin{aligned}
 53. \quad f(x) &= x^9 - 11x^8 - 2x^7 + 22x^6 \\
 &+ x^4 - 12x^3 + 11x^2 + 11x - 3 \\
 &= 11^9 - 11 \times 11^8 - 2 \times 11^7 + 22 \times 11^6 \\
 &+ 11^4 - 12 \times 11^3 + 11^2 + 11 - 3 \\
 &= 8
 \end{aligned}$$

$$\begin{aligned}
 54. \quad (f \circ g)(x) &= (g \circ f)(x) \\
 f[g(x)] &= g(f(x)) \\
 (3^x)^3 &= 3^{x^3} \\
 x(x^2 - 3) &= 0
 \end{aligned}$$

Given  $x \neq 0$  so  $x^2 - 3 = 0$

$$\begin{aligned}
 55. \quad f(x) &= (x+2)^2 - 2 = y \\
 x &= \sqrt{y+2} - 2 \quad (\because x \geq -2) \\
 f^{-1}(y) &= \sqrt{y+2} - 2 \\
 f^{-1}(x) &= \sqrt{x+2} - 2
 \end{aligned}$$

$$\begin{aligned}
 56. \quad (g \circ f)\left(\frac{-5}{3}\right) &= g\left(f\left(\frac{-5}{3}\right)\right) \\
 &= g(-2) = 2
 \end{aligned}$$

$$\begin{aligned}
 57. \quad (f \circ g)(x) &= f[g(x)] \\
 &= f[ax^3 + d] \\
 &= acx^3 + ad + b = y \\
 x &= \left(\frac{y - ad - b}{ac}\right)^{\frac{1}{3}}
 \end{aligned}$$

$$f^{-1}(x) = \left(\frac{x - ad - b}{ac}\right)^{\frac{1}{3}}$$

58. Insufficient data

$$\begin{aligned}
 59. \quad |x| - x &> 0 \\
 |x| &> x \\
 x &\in (-\infty, 0)
 \end{aligned}$$

$$60. \quad g(x) = 1 + \{x\}$$

$$f(g(x)) = \begin{cases} -3 & \text{if } 1 + \{x\} < 0 \rightarrow (1) \\ 0 & \text{if } 1 + \{x\} = 0 \rightarrow (2) \\ 5 & \text{if } 1 + \{x\} > 0 \end{cases}$$

(1) and (2) always not possible

$$f(g(x)) = 5$$

$$61. \quad A = \{1, 2, 3, 4, 5, 6\}$$

No of ways selecting  $m + n = 7$  is  ${}^6C_1 = 6$

These are (1,6)(6,1)(2,5)(5,2)(3,4)(4,3)

(1,6)(6,1) maps to f images so that

$$f(1) + f(6) = 7 \text{ (or)}$$

$$f(6) + f(1) = 7 \text{ in 6 ways}$$

Similarly (2,5)(5,2) maps to 6 ways

(3,4)(4,3) maps to 6 ways

$$62. \quad \text{let } A = \left\{ \begin{bmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{bmatrix} / k \in R \right\}$$

No two scalar matrices have same determinant

$f$  is one-one

$f$  is onto

$f$  is bijective

$$63. \quad y = \frac{(10^x)^2 - 1}{(10^x)^2 + 1}$$

$$10^{2x} = \frac{1+y}{1-y}$$

$$x = \frac{1}{2} \log_{10} \left( \frac{1+y}{1-y} \right)$$

$$f^{-1}(x) = \frac{1}{2} \log_{10} \left( \frac{1+x}{1-x} \right)$$

$$64. f(1) = 5$$

$$f(2) = 2 \times 5$$

$$f(3) = 3 \times 5$$

$$f(n) = n \times 5$$

$$\sum_{r=1}^n f(r) = 5[1+2+\dots+n]$$

$$= \frac{5n(n+1)}{2}$$

$$65. y = \frac{x-2}{x+3}$$

$$\Rightarrow x = \frac{3y+2}{1-y}$$

$$1-y \neq 0$$

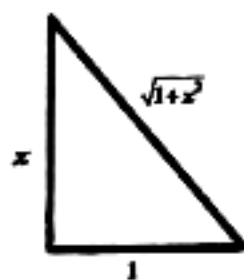
$$y \neq 1$$

$$\therefore \text{Range} = (-\infty, 1) \cup (1, \infty)$$

$$66. \tan^{-1} x = \alpha$$

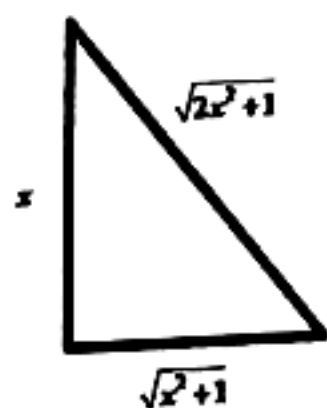
$$\tan \alpha = \frac{x}{1}$$

$$\sin \alpha = \frac{x}{\sqrt{1+x^2}}$$



$$f(x) = \cos \left[ \tan^{-1} \left( \frac{x}{\sqrt{1+x^2}} \right) \right]$$

$$\tan \beta = \frac{x}{\sqrt{x^2+1}}$$



$$f(x) = \sqrt{\frac{x^2+1}{2x^2+1}}$$

$$(f \circ f)(x) = f(f(x))$$

$$= \sqrt{\frac{3x^2+2}{4x^2+3}}$$

$$\lim_{x \rightarrow \infty} (f \circ f)(x) = \sqrt{\frac{3}{4}}$$

$$= \frac{3}{2\sqrt{3}}$$

67. Conceptual

$$68. f(x) = \log(x + \sqrt{x^2+1})$$

$$f(-x) = \log(-x + \sqrt{x^2+1})$$

$$= \log \left( \frac{x^2+1-x^2}{\sqrt{x^2+1}+x} \right)$$

$$= -\log(x + \sqrt{x^2+1})$$

$$= -f(x)$$

$f(x)$  is an odd function.

$$69. f(x) = kx$$

$$f(1) = k \Rightarrow k = 10$$

$$\sum_{r=1}^n (f(r))^2 = \sum_{r=1}^n (10r)^2$$

$$= 100 \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{50n(n+1)(2n+1)}{3}$$

$$70. f(x) = \frac{3^x + 3^{-x}}{2}$$

$$f(x+y) + f(x-y)$$

$$= \frac{3^{x+y} + 3^{-(x+y)}}{2} + \frac{3^{x-y} + 3^{-(x-y)}}{2}$$

$$= \frac{3^x(3^y + 3^{-y}) + 3^{-x}(3^y + 3^{-y})}{2}$$

$$= \frac{(3^x + 3^{-x})(3^y + 3^{-y})}{2} \times \frac{2}{2}$$

$$= 2f(x)f(y)$$

71. Given

$$y(x) = \begin{cases} \frac{5x}{(x-3)(x+3)} & \text{if } x \neq -1 \\ -1 & \text{if } x = -1 \end{cases}$$

Case (i):

$$y = \frac{5x}{x^2 - 9}, x \neq -1$$

$$x^2 - 9 \neq 0$$

$$x \neq \{-3, 3\}$$

Case (ii):

$$y = -1, x = -1$$

Clearly  $x \in \mathbb{R} - \{-3, 3\}$

i.e.,  $\forall x \in \mathbb{R} - \{-3, 3\} \exists y \in \mathbb{R} \ni f(x) = y$

72. Let  $f(x) = \cos x - 3$

W.k.t Domain of  $\cos x$  is  $\mathbb{R}$ ,

$\therefore$  Domain of  $f(x)$  is  $\mathbb{R}$ .

and Range of  $f(x)$  is,

$$-1 \leq \cos x \leq 1$$

$$-4 \leq \cos x - 3 \leq -2$$

$$-4 \leq f(x) \leq -2$$

$$\therefore f(x) \in [-4, -2]$$

73. Domain of  $f(x)$  is  $\frac{\pi x}{2} \neq (2n+1)\frac{\pi}{2}$

$$x \neq (2n+1), n \in \mathbb{Z}$$

Domain of  $g(x)$  is  $3 + 4x - 4x^2 \geq 0$

$$4x^2 - 4x - 3 \leq 0$$

$$\Rightarrow (2x-3)(2x+1) \leq 0$$

$$\Rightarrow x \in \left[ \frac{-1}{2}, \frac{3}{2} \right]$$

Domain of  $f + g$  is  $\left[ \frac{-1}{2}, 1 \right)$

74. Given  $a^x + a^y = a$

$$\Rightarrow a^x = a - a^y$$

$$a^x < a \quad (\because a^y > 0)$$

$$x < \log_a a$$

$$x < 1$$

$$x \in (-\infty, 1)$$

75.  $2f(x) + f\left(\frac{1}{x}\right) = 4x \dots (1)$

Replace 'x' by ' $\frac{1}{x}$ ',

$$2f\left(\frac{1}{x}\right) + f(x) = \frac{4}{x} \dots (2)$$

$$(2) \times (1) - (2) \Rightarrow 3f(x) = 8x - \frac{4}{x}$$

$$f(x) = \frac{8x^2 - 4}{3x}$$

Now take  $f(x) = f(-x) \Rightarrow x = \pm \frac{1}{\sqrt{2}}$

No. of elements is  $S = 2$ .

76.  $f(x) = \frac{x}{1-x}$

$$\text{Let } f(x) = y \Rightarrow \frac{x}{1-x} = y \Rightarrow x = \frac{y}{1+y}$$

We have  $-1 < x < 1$

$$-1 < \frac{y}{1+y} < 1$$

$$\frac{y}{1+y} > -1 \quad \text{and} \quad \frac{y}{1+y} < 1$$

$$\frac{y}{1+y} > 0 \quad \text{and} \quad \frac{y}{1+y} - 1 < 0$$

$$\frac{y}{y+1} + 1 > 0 \quad \text{and} \quad \frac{-1}{1+y} < 0$$

$$\frac{2y+1}{y+1} > 0 \quad \text{and}$$

$$y \in (-\infty, -1) \cup \left( \frac{-1}{2}, \infty \right) \quad \text{and} \quad y > -1$$

$$y \in (-1, \infty)$$

$$\therefore B = \left( \frac{-1}{2}, \infty \right)$$

77. conceptual

78.  $f(x) = \frac{1}{\sqrt{|x|} - x}$

$$|x| - x > 0 \Rightarrow |x| > x$$

By verifying  $x \in (-\infty, 0)$

$$79. f(x) = \sqrt{\frac{x^2 + 2x + 8}{x^2 + 2x + 4}} = \sqrt{1 + \frac{4}{(x+1)^2 + 3}}$$

At  $x = 1$  is has maximum value is

At  $x = \infty$  is has minimum value

$$\text{Range} = [1, \sqrt{7/3}]$$

$$80. f(x) = \sqrt{4 - x^2} \quad g(x) = \log(1 - x)$$

$$4 - x^2 \geq 0 \quad 1 - x > 0$$

$$x^2 - 4 \leq 0 \quad 1 > x$$

$$x \in [-2, 2] \quad x < 1$$

$$81. |x^2| - 2|x| - 8 \geq 0 \text{ and } 2 - x^2 - x > 0$$

$$x > 0, x^2 - 2x - 8 \geq 0$$

$$(x - 4)(x + 2) \geq 0$$

$$x \in (-\infty, -2] \cup [4, \infty)$$

$$x < 0, x^2 + 2x - 8 \geq 0$$

$$(x + 4)(x - 2) \geq 0$$

$$x \in (-\infty, -4] \cup [2, \infty)$$

$$2 - x - x^2 > 0$$

$$x^2 + x - 2 < 0$$

$$(x + 2)(x - 1) < 0$$

$$x \in (-2, 1)$$

domain is  $\phi$

$$82. \frac{\sqrt{4 - x^2}}{1 - x} > 0$$

$$4 - x^2 > 0 \quad \text{and} \quad 1 - x > 0$$

$$x^2 - 4 < 0 \quad 1 - x$$

$$(x + 2)(x - 2) < 0 \quad x < 1$$

$$x \in (-2, 2) \quad x \in (-2, 1)$$

$$\text{domain} = (-2, 1)$$

83. Data in correct

$$84. f(x) = \frac{x - 2}{2x + 1}$$

$$f[f(x)] = -x$$

$$f\left[\frac{x - 2}{2x + 1}\right] = -x$$

$$2x^2 - 3x - 2 = 0$$

$$2x^2 - 4x + x - 2 = 0$$

$$2x(x - 2) + 1(x - 2) = 0$$

$$(x - 2)(2x + 1) = 0$$

$$(x - 2) \text{ or } x = -1/2$$

$$4(\alpha^2 + \beta^2) = 4\left(4 + \frac{1}{4}\right) = 4 \times \frac{17}{4} = 17$$

$$85. f(x) = \frac{x^2 + 1 + x}{x} = x + \frac{1}{x} + 1$$

$$x + \frac{1}{x} \geq 2 \text{ or } x + \frac{1}{x} \leq -2$$

$$\text{Range} = (-\infty, -1] \cup [3, \infty)$$

$$86. f(x) = \frac{x + 3}{x - 2} \quad y = \frac{x + 3}{x - 2}$$

$$x - 2 \neq 0 \quad x = \frac{3 + 2y}{y - 1}$$

$$x \neq 2 \quad y \neq 1$$

$$R - \{2\} \quad R - \{1\}$$

$$1 = 2 \quad m = 1 \quad 31 + 2m = 8$$

87.  $a \in R$   $a, -a$  have same image

$\therefore f$  is not one-one

-ve real numbers have no pre image

$\therefore f$  is not an onto

$$88. f(xy) = \frac{f(x)}{y}$$

$$f(30) = f(10 \times 3) = \frac{f(10)}{3}$$

$$20 = \frac{f(10)}{3}$$

$$f(10) = 60$$

$$f(40) = f(10 \times 4)$$

$$= \frac{f(10)}{4}$$

$$= \frac{60}{4}$$

$$= 15$$

$$89. x + 3 > 0 \text{ and } x^2 + 3x + 2 > 0$$

$$x > -3 \text{ and } (x + 2)(x + 1) > 0$$

$$x \in (-3, \infty) \text{ and } x \in (-\infty, -2) \cup (-1, \infty)$$

$$\text{Domain} = (-3, -2) \cup (-1, \infty)$$

$$90. 2 - x \geq 0 \text{ and } 1 + x \geq 0 \text{ and } x + 3 > 0$$

$$x \leq 2 \text{ and } x \geq -1 \text{ and } x > -3$$

$$\text{Domain} = [-1, 2]$$

91. Data in correct

$$92. \log_{0.5}(x - 3) \geq 0 \text{ and } x - 3 > 0 \text{ and } x - 1 > 0$$

$$0 < x - 3 \leq 1$$

$$x > 3$$

$$x > 1$$

$$x \leq 4$$

$$\text{domain} = (3, 4]$$

93.  $f(x^2) - 2f(x/2) - 1 = 0$

$$2x^2 - 2x - 4 = 0$$

$$x^2 - x - 2 = 0$$

$$(x-2)(x+1) = 0$$

$$x = 2 \text{ or } -1$$

$$\alpha = 2 \quad \beta = -2$$

$$\alpha^2 + \beta^2 = 4 + 1 = 5$$

94. Conceptual

95.  $f(x) = x^2 - 2x - 3$

$$f'(x) = 2x - 2$$

$$f'(x) > 0 \text{ for } x > 1$$

$$f'(x) < 0 \text{ for } x < 1$$

$\therefore f(x)$  is not one-one

$$f(x) = (x-1)^2 - 4$$

Minimum value of  $f(x)$  is  $-4$

$\therefore f$  is not onto

96.  $(x+1)(x+3) \geq 0$  and  $(x-2)(x+3) \leq 0$

and  $x+3 \neq 0$

$$x \in (-\infty, -3] \cup [-1, \alpha) \text{ and } x \in [-3, -2]$$

and  $x \neq -3$

$$\text{Domain} = [-1, 2]$$

97.  $f(x) = x|x| = \begin{cases} x^2 & x \geq 0 \\ -x^2 & x < 0 \end{cases}$

$\therefore f$  is one-one and onto

98.  $(2-|x|)(3-|x|) \geq 0$  and  $3-|x| \neq 0$

$$(|x|-2)(|x|-3) \geq 0 \text{ and } |x| \neq 3$$

$$x \neq \pm 3$$

$$\therefore x \in (-\infty, -3) \cup [-2, 2] \cup (3, \infty)$$

99.  $6x^2 + 5x - 6 \geq 0$

$$(2x+3)(3x-2) \geq 0$$

$$x \in \left(-\infty, -\frac{3}{2}\right) \cup \left[\frac{2}{3}, \infty\right)$$

$$4-x \geq 0 \text{ and } 4+x \geq 0 \text{ and } \sqrt{4-x} \neq \sqrt{x+4}$$

$$x \leq 4 \text{ and } x \geq -4 \text{ and } 4-x \neq x+4$$

$$x \neq 0$$

$$\text{Domain} = [-4, -3/2] \cup \left[\frac{2}{3}, 4\right)$$

100.  $[x]^2 + [x] - 2 > 0$

$$([x]+2)([x]-1) > 0$$

$$[x] \in (-\infty, -3] \cup [2, \infty)$$

$$\text{Range} = \left(0, \frac{1}{2}\right]$$

101.  $f(x) = \frac{1+\sin x}{\cos x}$

$$f'(x) = \frac{1+\sin x}{\cos^2 x} > 0$$

$f(x)$  is one-one

Clearly  $f(x)$  is onto

102. By verification

103.  $2^x - 2 > 0$  and  $1-x \geq 0$

$$2^x > 2 \text{ and } 1 \geq x$$

$$x > 1$$

$$\text{domain} = \phi$$

104.  $f\left(F\left(\frac{1}{x}\right)\right) = \frac{1}{1-x}$

$$1 - F\left(\frac{1}{x}\right) = \frac{1}{1-x}$$

$$F\left(\frac{1}{x}\right) = \frac{x}{x-1}$$

$$F(2022) = \frac{-1}{2022}$$

Verify options

105.  $3+4x-4x^2 \geq 0$

$$4x^2 - 4x + 3 \leq 0$$

$$(2x+1)(2x-3) \leq 0$$

$$x \in \left[-\frac{1}{2}, \frac{3}{2}\right]$$

Domain of  $g = \text{range of } f$

106.  $\frac{1-x^2}{1+x^2} \geq 0$

$$x^2 - 1 \leq 0$$

$$x \in [-1, 1]$$

$$D = [-1, 1]$$

$$f(x) = \sqrt{1 - \frac{2x^2}{1+x^2}}$$

$$f(x) = \sqrt{1 - \frac{2}{x + \frac{1}{x}}}$$

$$\text{range} = [0, 1] = G$$

$$D \cap G = [0, 1]$$

107. conceptual

108. Put  $x=2$  or  $3$  we get minimum value

$$\text{range} = [1, \infty)$$

109.  $([x] - x)(x - [x]) \geq 0$  &  $[x] \neq x$

The above is not possible

$$\text{Domain} = \phi$$

110.  $x - [x] > 0$  for  $x \in R - Z$

If  $x \in R - Z$  then

$$0 < x - [x] < 1$$

$$\text{range} = (1, \infty)$$

$$111. g\left(\frac{\pi}{2}\right) = \sin 2 \times \frac{\pi}{12}$$

$$= \frac{1}{2}$$

$$f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^3 - \frac{1}{2} = \frac{-3}{8}$$

$$112. \log_{10}\left(\frac{3-x}{x}\right) \geq 0, \quad \frac{3-x}{x} > 0$$

$$\frac{3-x}{x} \geq 1 \quad x(3-x) > 0$$

$$x \leq \frac{3}{2} \quad x(x-3) < 0$$

$$x \in (0, 3)$$

$$\text{C.S} \rightarrow \left(0, \frac{3}{2}\right]$$

$$113. f\left(\frac{2a}{1+a^2}\right) > 0$$

$$\log\left(\frac{1 - \frac{2a}{1+a^2}}{1 + \frac{2a}{1+a^2}}\right) > 0$$

$$\log\left(\frac{1-a}{1+a}\right)^2 > 0$$

$$\left(\frac{1-a}{1+a}\right)^2 > 1$$

$$\frac{(1-a)^2}{(1+a)^2} - 1 > 0$$

$$\frac{-2a}{(1+a)^2} > 0$$

$$\boxed{a < 0}, a \neq \{-1, 1\}$$

$$a \in (-\infty, 0)_{-1}$$

114. CONCEPTUAL

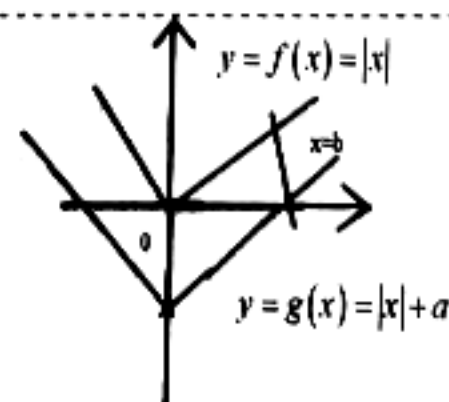
$$115. \text{Given function } f(x) = \frac{2x+1}{3}$$

$$\frac{2\alpha+1}{3} = \frac{1}{\alpha}$$

$$2\alpha^2 + \alpha - 3 = 0$$

Sum of all possible values  $\alpha$  is  $\frac{1}{2}$

116.



$$y = f(x) = |x|$$

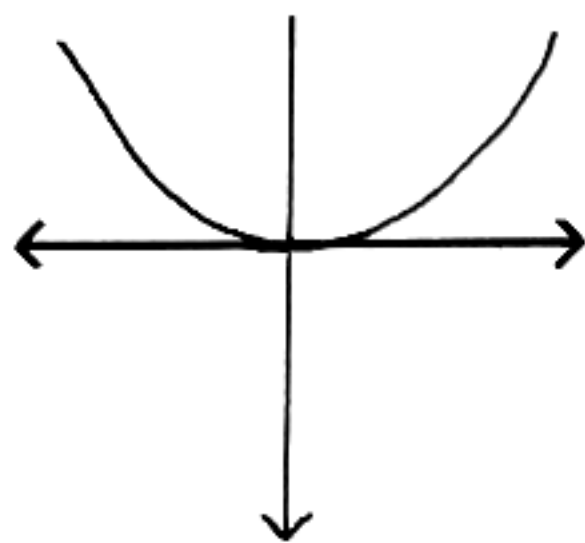
$$y = g(x) = |x| + a$$

From diagram form a "parallelogram"

117.  $f(x) = x^2 - 3x + 2$

$$a = 1 > 0$$

$$\text{Range} = \left[ \frac{4ac^2 - b^2}{4a}, \infty \right) \quad \mathbf{B} = \left[ \frac{-1}{4}, \infty \right)$$



$$x = -b/2a = 3/2$$

$f(x)$  is bijection if

$$x \in (-\infty, 3/2] = A$$

(or)

$$x \in [3/2, \infty) = A$$

118.  $f(x+y) = f(x) + f(y)$

$$\Rightarrow f(x) = kx$$

Given  $f(1) = 7$

$$\Rightarrow k = 7$$

$$\therefore f(x) = 7x$$

$$\begin{aligned} \sum_{r=1}^n f(r) &= 7 \sum_{r=1}^n r \\ &= 7 \frac{n(n+1)}{2} \end{aligned}$$

119.  $n(A) = n$

No. of functions that are

$$\text{not one-one} = n^n - {}^n P_n$$

$$= n^n - n!$$

120.  $f(x) = \frac{|x|}{x} = \begin{cases} 1 & ; x > 0 \\ 0 & ; x = 0 \\ -1 & ; x < 0 \end{cases}$

$$g(x) = 1, \forall x \in R$$

By verification

$$g(x) = \begin{cases} f(x) + 2 & ; x < 0 \\ 1 + f(x) & ; x = 0 \\ f(x) & ; x > 0 \end{cases}$$

121.  $f(3) = 1$

$$f(4) = 2 + 1 = 3$$

$$f(5) = 1 + 2 = 3$$

$$f(6) = 3 + 1 = 4$$

$$f(7) = 3 + 3 = 6$$

$$f(8) = 4 + 3 = 7$$

$$f(9) = 6 + 4 = 10$$

$$f(10) = 7 + 6 = 13$$

122.

$$\begin{aligned} f(x) &= \frac{1 - \sin^2 x + \sin^4 x}{1 - \cos^2 x + \cos^4 x} \\ &= \frac{1 - \sin^2 x (1 - \sin^2 x)}{1 - \cos^2 x (1 - \cos^2 x)} \\ &= \frac{1 - \sin^2 x \cos^2 x}{1 - \cos^2 x \sin^2 x} = 1 \\ f(2023) &= 1 \end{aligned}$$

123.  $A = \text{Domain} = R$

$$\begin{array}{l|l} x > 1 & x \leq 1 \\ 3x > 3 & x^2 \geq 0 \\ 3x - 1 > 2 & x^2 + 1 \geq 1 \end{array}$$

$$\text{Range} = [1, \infty) = B$$

$$\therefore A - B = (-\infty, 1)$$

$$124. f(\sin 2x) > 0$$

$$(\sin 2x)^3 - \sin 2x > 0$$

$$(\sin 2x)(\sin^2 2x - 1) > 0$$

$$\sin 2x \cos^2 2x < 0$$

$$\sin 2x < 0 \quad (\because \cos^2 2x > 0)$$

$$\pi < 2x < 2\pi$$

$$\frac{\pi}{2} < x < \pi$$

$$x \in \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\frac{3\pi}{4}, \pi\right)$$

$$125. f(x) = \sqrt{9-x^2}$$

$$x^2 \geq 0$$

$$-x^2 \leq 0$$

$$9-x^2 \leq 9$$

$$\sqrt{9-x^2} \leq 3$$

$$\text{Range} = [0, 3]$$

$$126. f(x) = x + \frac{1}{x} \quad (f(x))^2 = ?$$

$$(f(x))^2 = \left(x + \frac{1}{x}\right)^2$$

$$= x^2 + \frac{1}{x^2} + 2 \cdot x \cdot \frac{1}{x}$$

$$(f(x))^2 = x^2 + \frac{1}{x^2} + 2$$

Option verification

$$127. y = 4x - 1$$

$$x = \frac{y+1}{4} > 3$$

$$\Rightarrow \boxed{y > 11}$$

$$y = x^2 - 2 \Rightarrow x = \sqrt{y+2}$$

$$0 \leq y+2 \leq 9$$

$$\Rightarrow \boxed{y \leq 7}$$

$$y = 3x + 4$$

$$\Rightarrow x = \frac{y-4}{3} < -2$$

$$\Rightarrow \boxed{y < -2}$$

$$\therefore \text{Range of } f(x) = (-\infty, 7] \cup (11, \infty)$$

$$128. 2^y = 2 - 2^x < 2$$

$$\Rightarrow 2^y < 2$$

$$\Rightarrow y < \log_2 2$$

$$\Rightarrow \boxed{y < 1}$$

$$129. (x^2 - 1)^2 + 2 \geq 2$$

$$\log_2 \left( (x^2 - 1)^2 + 2 \right) \geq 1$$

$$-\log(x^2 - 2x^2 + 3) \leq -1$$

$$y = \log_{0.5}(x^4 - 2x^2 + 3) \leq -1$$

$$y \in (-\infty, -1]$$

$$130 \text{ Given } f(x) = -|x|$$

$$\text{Now } (f \circ f \circ f)(x) + (f \circ f \circ f)(-x)$$

$$= -|x| + (-|-x|) = -2|x| = 2 \cdot f(x)$$

$$131. f(x) = (x-2)^2 + 1 \geq 1$$

$$\text{Range} = [1, \infty)$$

$$132. f(x) = \sin^{-1}(\log_2(x^2/2))$$

$$-1 \leq \log_2 \frac{x^2}{2} \leq 1$$

$$\frac{1}{2} \leq \frac{x^2}{2} \leq 2$$

$$1 \leq x^2 \leq 4$$

$|x|$  lies b/w 1 and 2

$$1 \leq |x| \leq 2$$

$$\therefore \text{domain} = [-2, -1] \cup [1, 2]$$

$$133. f(x) = -\sqrt{-x^2 - 6x - 5}$$

$$-x^2 - 6x - 5 \geq 0 \quad \text{max of } f(x) = 0$$

$$x^2 + 6x + 5 \leq 0 \quad \text{min of } f(x) = -2$$

$$(x+1)(x+5) \leq 0$$

$$\text{domain } x \in [-1, -5]$$

$$\therefore \text{Range of } f(x) = [-2, 0]$$

134.  $f: R \rightarrow R \quad f(x) = 2x + \sin x, x \in R$

$f'(x) = 2 + \cos x$

$f'(x) > 0 \quad \forall x \in R$

$f(x)$  is strictly increasing

$\therefore f(x)$  is 1-1 and onto

135.  $f(x) = 2x - 3, \text{ if } x < -2 \quad f(x) = x^2 - 1 \text{ if } -2 \leq x \leq 2$

$f(-3) = -9$

$f(-2) = 3$

$f(-4) = -11$

$f(2) = 3$

$f(x)$  is not a one-one

$f(x) \rightarrow \text{Range} \neq \text{codomain}$

$\therefore f(x)$  is neither injection nor surjection

136. Denominator:

Numerator :

$[x]^2 - 5[x] + 6 > 0$

$([x] - 2)([x] - 3) > 0$

$(-\infty, 2) \cup [4, \infty)$

$\log_{10} \frac{x}{x-2} \geq 0$

$\frac{x}{x-2} \geq 1$

$x \geq x - 2$  (does not exist)

$\frac{x}{x-2} > 0$

$x > 0$

$\therefore \frac{Nr}{Dr} \Rightarrow [4, \infty)$

137.  $f(x) = \frac{1}{x - |x|}$

$x - |x| \neq 0$

It is satisfied only when  $x < 0$

Range =  $(-\infty, 0)$

138.  $f(x) = \sqrt{16 - x^2}; f: [0, 4] \rightarrow [0, 4]$  onto

Let  $f(x) = y \Rightarrow x = \sqrt{16 - y^2}$

By definition,

$y \in \text{Codomain} \exists \sqrt{16 - y^2} \in \text{Domain}$

$f(\sqrt{16 - y^2}) = \sqrt{16 - (\sqrt{16 - y^2})^2}$   
 $= y$

$\therefore f(x)$  is onto

139.  $f(x) = \frac{\sqrt{|x|} - x}{\sqrt{x - [x]}}$

It is defined when

$|x| - x \geq 0$  and  $x - [x] > 0$

$|x| \geq x$

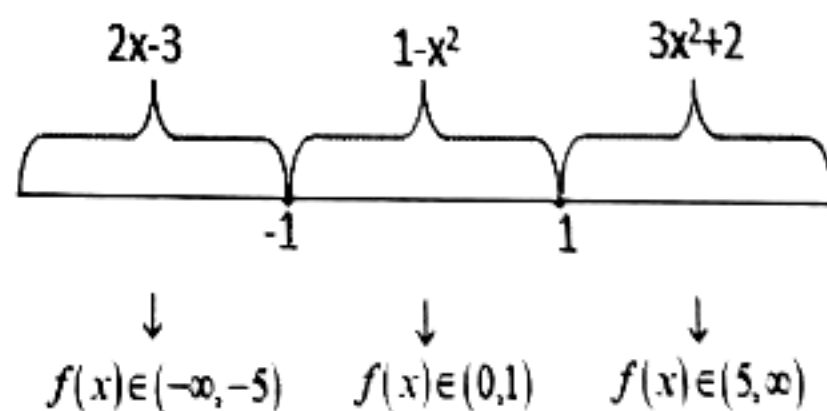
$x > [x]$

$x \in R$

$x \in R - Z$

$\therefore \text{Domain} \Rightarrow x \in R - Z$

140.  $f(x) = \begin{cases} 2x - 3, & x < -1 \\ 1 - x^2, & -1 \leq x \leq 1 \\ 3x^2 + 2, & x > 1 \end{cases}$



$\therefore \text{Range} = (-\infty, -5) \cup [0, 1] \cup (5, \infty)$